CS113 Lab 6 - SML

```
SML
PolyML.print_depth 100;
infix --;
fun (i -- n) = if (i > n)
    then [ ]
    else i::(i + 1 -- n);

fun elementOf(a, [ ]) = false |
    elementOf(a, x::xs) = if (a = x)
    then true
    else elementOf(a, xs);

fun insert(x, [ ]) = [x] |
    insert(x, ys) = x::ys;
```

SML functions

- (i – n) creates a list of number
  - $A = \{i,\ i+1,\ i+2, ...,\ n\}$
- elementOf checks whether a is a member of x
  - $x \in A$
- insert inserts an element x into ys

SML

```
fun remove(x, [ ]) = [ ] |
    remove(x, y::ys) = if (x = y)
    then remove(x, ys)
    else y::remove(x, ys);

fun subset([ ], _) = true |
    subset(x::xs, ys) = if not (elementOf(x, ys))
    then false
    else subset(xs, ys);

fun subsetEqual(A, B) = subset(A, B) andalso subset(B, A);

fun properSubset(A, B) =
    subset(A, B) andalso not(subset(B, A));
```

SML functions

- remove removes an element x from ys
- subset checks whether x is subset of y
  - $A \subseteq B$
- subsetEqual checks whether two sets are equal
  - $A = B$ *that* $A \subseteq B$ *and* $B \subseteq A$
- properSubset checks whether A is a proper subset of B
  - $A \subset B$ *that* $A \subseteq B$ *and* $A \neq B$

SML

```
fun relativeComplement(xs, [ ]) = [ ] |
    relativeComplement([ ], ys) = ys |
    relativeComplement(x::xs, ys) = if (elementOf(x, ys))
    then relativeComplement(xs, remove(x, ys))
    else relativeComplement(xs, ys);

fun union(xs, [ ]) = xs |
    union([ ], ys) = ys |
    union(x::xs, ys) = if not (elementOf(x, ys))
    then x::union(xs, ys)
    else union(xs, ys);
```

SML functions

- relativeComplement returns the relative complement of A with respect to B
  - $B \setminus A$
- union returns the union of A and B
  - $A \cup B$

SML

```
fun intersection([ ], _) = [ ] |
    intersection(_, [ ]) = [ ] |
    intersection(x::xs, ys) = if (elementOf(x, ys))
    then x::intersection(xs, ys)
    else intersection(xs, ys);

fun disjointSets(A, B) = if (intersection(A, B) = [ ])
    then true
    else false;

fun symmetricDifference(xs, [ ]) = xs |
    symmetricDifference([ ], ys) = ys |
    symmetricDifference(x::xs, ys) = if (elementOf(x, ys))
    then symmetricDifference(xs, remove(x, ys))
    else x::symmetricDifference(xs, ys);
```

SML functions

- intersection returns the intersection of A and B
  - $A \cap B$
- disjointSets checks whether A and B are disjoint sets
  - $A \cap B = \varnothing$
- symmetricDifference returns the symmetric difference of A and B
  - $A \triangle B$

SML

```
fun cartesianProduct([ ], _) = [ ] |
    cartesianProduct(x::xs, ys) =
    cartesianProduct(xs, ys) @ (map (fn v => (x, v)) ys);

fun powerSet([ ]) = [ [ ] ] |
    powerSet(x::xs) =
    powerSet(xs) @ (map (fn S => x::S) (powerSet(xs)));
```

SML functions

- cartesianProduct returns the Cartesian product of given sets
  - $\mathbf{A_1} \times \mathbf{A_2} \times \cdots \times \mathbf{A_n} = \{(\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_n}) | \mathbf{a_1} \in \mathbf{A_2}, \mathbf{a_1} \in \mathbf{A_2}, \cdots, \mathbf{a_n} \in \mathbf{A_n}\}$
- powerSet returns the power set of A
  - $\mathcal{P}(A) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

CS113 Lab 6 - Example 12.4

- Suppose that $A = \{2, 4, 6\},\ B = \{2, 6\},\ and\ C = \{4, 6\}$.
  Determine which of these sets are subsets of which other(s) of these sets.

SML

```
val A = [2, 4, 6];
val B = [2, 6];
val C = [4, 6];
subset(B, A);
subset(C, A);
```

$$B \subseteq A \text{ and } C \subseteq A \blacksquare$$

```
> val A = [2, 4, 6]: int list
> val B = [2, 6]: int list
> val C = [4, 6]: int list
> val it = true: bool
> val it = true: bool
```

CS113 Lab 6 - Example 12.6

- Determine whether each of the following pairs of sets are equal.
  (a) $\{1, 3, 5\} and \{5, 3, 1\}$

SML

```
val A = [1, 3, 5];
val B = [5, 3, 1];
subsetEqual(A, B);
```

$$A = B \blacksquare$$

```
> val A = [1, 3, 5]: int list
> val B = [5, 3, 1]: int list
> val it = true: bool
```

CS113 Lab 6 - Example 12.10

- Let $A = \{a, b, c\}, B = \{b, c, d\},$ *and* $C = \{b, c, e\}$.
  (a) Find $A \cup (B \cap C), (A \cup B) \cap C,$ *and* $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?
  (b) Find $A \cap (B \cup C), (A \cap B) \cup C,$ *and* $(A \cap B) \cup (A \cap C)$. Which of these sets are equal?
  (c) Find $A \setminus (B \setminus C)$ *and* $(A \setminus B) \setminus C$. Are these sets equal?

SML

```
val A = [#"a", #"b", #"c"]; val B = [#"b", #"c", #"d"]; val C = [#"b", #"c", #"e"];
val a1 = union(A, intersection(B, C));
val a2 = intersection(union(A, B), C);
val a3 = intersection(union(A, B), union(A, C));
val b1 = intersection(A, union(B, C));
val b2 = union(intersection(A, B), C);
val b3 = union(intersection(A, B), intersection(A, C));
val c1 = relativeComplement(A, relativeComplement(B, C));
val c2 = relativeComplement(relativeComplement(A, B), C);
```

CS113 Lab 6 - Example 12.10

SML

```
subsetEqual(a1, a3);
subsetEqual(b1, b3);
subsetEqual(c1, c2);
```

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

c) $A \setminus (B \setminus C) \neq (A \setminus B) \setminus C$ ■

```
> val A = [#"a", #"b", #"c"]: char list
> val B = [#"b", #"c", #"d"]: char list
> val C = [#"b", #"c", #"e"]: char list
> val a1 = [#"a", #"b", #"c"]: char list
> val a2 = [#"b", #"c"]: char list
> val a3 = [#"a", #"b", #"c"]: char list
> val b1 = [#"b", #"c"]: char list
> val b2 = [#"b", #"c", #"e"]: char list
> val b3 = [#"b", #"c"]: char list
> val c1 = [#"e"]: char list
> val c2 = [#"b", #"c", #"e"]: char list
> val it = true: bool
> val it = true: bool
> val it = false: bool
```

- The **symmetric difference** of A and B, denoted by $A \triangle B$, is the set containing those elements in either A or B but not both.
- Find $A \triangle B$ *if* $A = \{1, 3, 5\}$ *and* $B = \{1, 2, 3\}$.

SML

```
val A = [1, 3, 5];
val B = [1, 2, 3];
symmetricDifference(A, B);
```

$$A \triangle B = \{2, 5\} \blacksquare$$

```
> val A = [1, 3, 5]: int list
> val B = [1, 2, 3]: int list
> val it = [5, 2]: int list
```

CS113 Lab 6 - Example 12.13

- Let $A = \{x, y\}, B = \{1, 2, 3\},$ *and* $C = \{a, b\}$.
  (b) Find $(A \times B) \times C$.

$$(A \times B) \times C =$$
$$\{((x,1),a),((x,2),a),((x,3),a),((y,1),a),$$
$$((y,2),a),((y,3),a),((x,1),b),((x,2),b),$$
$$((x,3),b),((y,1),b)((y,2),b),((y,3),b)\}$$

SML

```
val A = [#"x", #"y"];
val B = [#"1", #"2", #"3"];
val C = [#"a", #"b"];
cartesianProduct(cartesianProduct(A, B), C);
```

```
> val A = [#"x", #"y"]: char list
> val B = [#"1", #"2", #"3"]: char list
> val C = [#"a", #"b"]: char list
> val it =
   [((#"x", #"3"), #"a"), ((#"x", #"3"), #"b"), ((#"x", #"2"), #"a"),
    ((#"x", #"2"), #"b"), ((#"x", #"1"), #"a"), ((#"x", #"1"), #"b"),
    ((#"y", #"3"), #"a"), ((#"y", #"3"), #"b"), ((#"y", #"2"), #"a"),
    ((#"y", #"2"), #"b"), ((#"y", #"1"), #"a"), ((#"y", #"1"), #"b")]:
  ((char * char) * char) list
```

CS113 Lab 6 - Problem 12.3

- Which of the following sets are equal?
  (a) $\{a, b, c, d\}$
  (b) $\{d, e, a, c\}$
  (c) $\{d, b, a, c\}$
  (d) $\{a, a, d, e, c, e\}$

SML

```
val A = [#"a", #"b", #"c", #"d"];
val B = [#"d", #"e", #"a", #"c"];
val C = [#"d", #"b", #"a", #"c"];
val D = [#"a", #"a", #"d", #"e", #"c", #"e"];
subsetEqual(A, C);
subsetEqual(B, D);
```

$$A = C,$$

$$B = D \blacksquare$$

```
> val A = [#"a", #"b", #"c", #"d"]: char list
> val B = [#"d", #"e", #"a", #"c"]: char list
> val C = [#"d", #"b", #"a", #"c"]: char list
> val D = [#"a", #"a", #"d", #"e", #"c", #"e"]: char list
> val it = true: bool
> val it = true: bool
```

CS113 Lab 6 - Problem 12.4

- Let $A = \{c, d, f, g\}, B = \{f, i\},$ *and* $C = \{d, g\}$.
  Answer each of the following questions.
  Give resons for your answers.
  (a) Is $B \subseteq A$?
  (b) Is $C \subseteq A$?
  (c) Is $C \subseteq C$?
  (d) Is $C \subset A$?

a) $B \nsubseteq A$
b) $C \subseteq A$
c) $C \subseteq C$
d) $C \subset A$ ■

SML

```
val A = [#"c", #"d", #"f", #"g"];
val B = [#"f", #"i"];
val C = [#"d", #"g"];
subset(B, A);
subset(C, A);
subset(C, C);
properSubset(C, A);
```

```
> val A = [#"c", #"d", #"f", #"g"]: char list
> val B = [#"f", #"i"]: char list
> val C = [#"d", #"g"]: char list
> val it = false: bool
> val it = true: bool
> val it = true: bool
> val it = true: bool
```

CS113 Lab 6 - Problem 12.5

(a) Is $3 \in \{1, 2, 3\}$?

(e) Is $1 \in \{1\}$?

(g) Is $\{1\} \subseteq \{1, 2\}$?

(j) Is $\{1\} \subseteq \{1\}$?

SML

```
elementOf(1, 1 -- 3);
elementOf(1, 1 -- 1);
subset(1 -- 1, 1 -- 2);
subset(1 -- 1, 1 -- 1);
```

a) $3 \in \{1, 2, 3\}$
e) $1 \in \{1\}$
g) $\{1\} \subseteq \{1, 2\}$
j) $\{1\} \subseteq \{1\}$ ■

```
> val it = true: bool
> val it = true: bool
> val it = true: bool
> val it = true: bool
```

CS113 Lab 6 - Problem 12.6

Let $A = \{b, c, d, f, g\}$ *and* $B = \{a, b, c\}$.
Find each of the following:
(a) $A \cup B$
(b) $A \cap B$
(c) $A \setminus B$
(d) $B \setminus A$

SML

```
val A = [#"b", #"c", #"d", #"f", #"g"];
val B = [#"a", #"b", #"c"];
union(A, B);
intersection(A, B);
relativeComplement(A, B);
relativeComplement(B, A);
```

a) $A \cup B = \{a, b, c, d, f, g\}$
b) $A \cap B = \{b, c\}$
c) $A \setminus B = \{a\}$
d) $B \setminus A = \{d, f, g\}$ ■

```
> val A = [#"b", #"c", #"d", #"f", #"g"]: char list
> val B = [#"a", #"b", #"c"]: char list
> val it = [#"d", #"f", #"g", #"a", #"b", #"c"]: char list
> val it = [#"b", #"c"]: char list
> val it = [#"a"]: char list
> val it = [#"d", #"f", #"g"]: char list
```

CS113 Lab 6 - Problem 12.8

Let $A = \{x, y, z, w\}$ *and* $B = \{a, b\}$.
List the elements of each of the following:
(a) $A \times B$
(b) $B \times A$
(c) $A \times A$
(d) $B \times B$

SML

```
val A = [#"x", #"y", #"z", #"w"];
val B = [#"a", #"b"];
cartesianProduct(A, B);
cartesianProduct(B, A);
cartesianProduct(A, A);
cartesianProduct(B, B);
```

```
> val A = [#"x", #"y", #"z", #"w"]: char list
> val B = [#"a", #"b"]: char list
> val it =
   [(#"w", #"a"), (#"w", #"b"), (#"z", #"a"), (#"z", #"b"), (#"y", #"a"),
    (#"y", #"b"), (#"x", #"a"), (#"x", #"b")]: (char * char) list
> val it =
   [(#"b", #"x"), (#"b", #"y"), (#"b", #"z"), (#"b", #"w"), (#"a", #"x"),
    (#"a", #"y"), (#"a", #"z"), (#"a", #"w")]: (char * char) list
> val it =
   [(#"w", #"x"), (#"w", #"y"), (#"w", #"z"), (#"w", #"w"), (#"z", #"x"),
    (#"z", #"y"), (#"z", #"z"), (#"z", #"w"), (#"y", #"x"), (#"y", #"y"),
    (#"y", #"z"), (#"y", #"w"), (#"x", #"x"), (#"x", #"y"), (#"x", #"z"),
    (#"x", #"w")]: (char * char) list
> val it = [(#"b", #"a"), (#"b", #"b"), (#"a", #"a"), (#"a", #"b")]:
   (char * char) list
```

CS113 Lab 6 - Problem 12.9

(a) Find all possible subsets of the set $A = \{a, b, c\}$.
(b) How many proper subsets are there?

SML

```
val A = [#"a", #"b", #"c"];
powerSet(A);
```

a) 8 possible subsets
b) 8 - 1(the euqal subset), therefore 7

```
> val A = [#"a", #"b", #"c"]: char list
> val it =
    [[], [#"c"], [#"b"], [#"b", #"c"], [#"a"], [#"a", #"c"], [#"a", #"b"],
     [#"a", #"b", #"c"]]: char list list
```

CS113 Lab 6 - Problem 12.12

- Let A be the set of the firsrt five composite numbers and B be the set of positive integers less than or equal to 8. Find $A \setminus B$ and $B \setminus A$.

SML

```
val A = [4, 6, 8, 9, 10];
val B = (1 -- 8);
relativeComplement(A, B);
relativeComplement(B, A);
powerSet(A);
```

```
> val A = [4, 6, 8, 9, 10]: int list
> val B = [1, 2, 3, 4, 5, 6, 7, 8]: int list
> val it = [1, 2, 3, 5, 7]: int list
> val it = [9, 10]: int list
```

a) $A \setminus B = \{1, 2, 3, 5, 7\}$
b) $B \setminus A = \{9, 10\}$ ■

CS113 Lab 6 - Problem 12.14

- Let A be the set of natural numbers less than 0 and $B = \{1, 3, 7\}$. Find $A \cup B$ and $A \cap B$.

SML

```
val A = [ ];
val B = [1, 3, 7];
union(A, B);
intersection(A, B);
```

Notice that A = $\varnothing$, therefore
$A \cup B = \{1, 3, 7\}$
$A \cap B = \varnothing$ ■

```
> val A = []: 'a list
> val B = [1, 3, 7]: int list
> val it = [1, 3, 7]: int list
> val it = []: int list
```

CS113 Lab 6 - Problem 12.15

- Let
  $A = \{x \in \mathbb{N} | 4 \geq x \geq 8\}$
  $B = \{x \in \mathbb{N} | x$ even and $x \geq 10$.
  Find $A \cup B$ and $A \cap B$.

SML

```
val A = (4 -- 8);
val B = [2, 4, 6, 8, 10];
union(A, B);
intersection(A, B);
```

```
> val A = [4, 5, 6, 7, 8]: int list
> val B = [2, 4, 6, 8, 10]: int list
> val it = [5, 7, 2, 4, 6, 8, 10]: int list
> val it = [4, 6, 8]: int list
```

$A \cup B = \{2, 4, 6, 8, 10\}$
$A \cap B = \{4, 6, 8\}$ ■