CPS Capital

15-424: Final Project

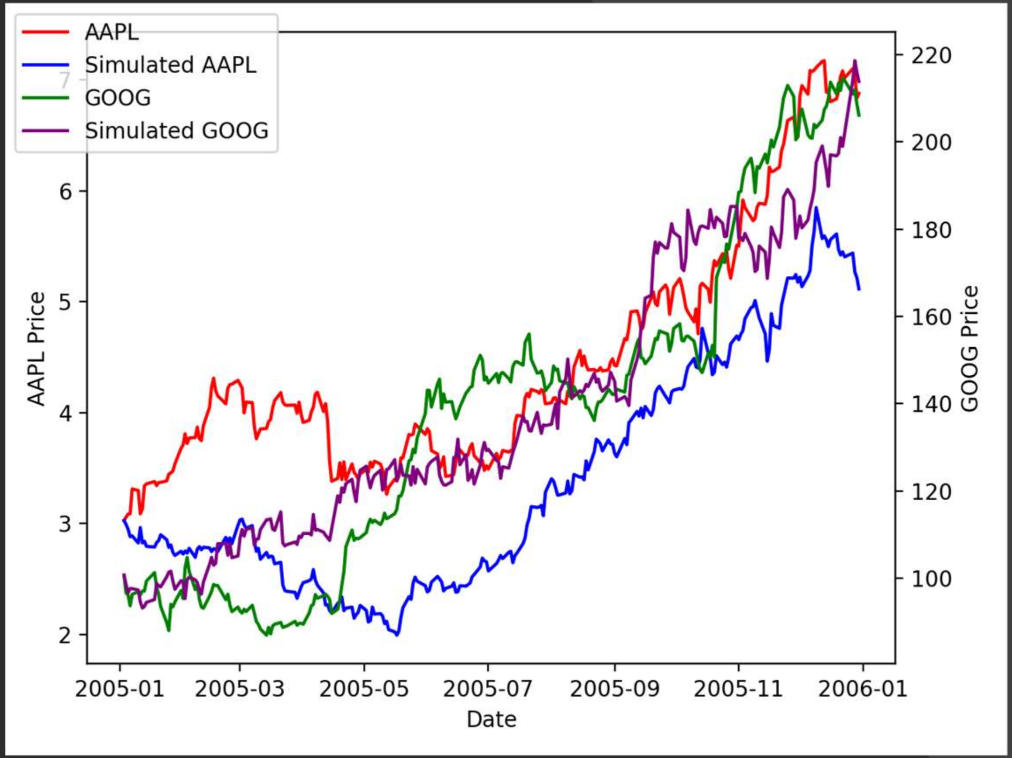
Abhishek Bhargava
Michael You

Markets are becoming more complex

- Big data and more financial assets
- Difficult to accurately model
- Global models don’t perform well
    - Need local models based on world state
- 2008 Recession
- 2010 Flash Crash

Brownian Motion is Essential to Market Simulation

- Random evolution
  - Suspended particles in fluid
- $dS_t = S_t(\mu\, dt + \sigma\, dB_t)$
- Stock prices follow correlated GBMs



SMC can make safety guarantees about trading strategies

- Markov Decision Process
- Bound the probability that we lose money
- Henriques et al. 2012: resolve nondeterminism

Definitions

Trader and Scheduler

- Trader
  - Chooses a strategy
    - How to allocate wealth in each time period
  - Goal: Make as much money as possible
- Scheduler
  - Chooses “world state” - how the market behaves
  - Tries to make trader lose money

Assets and Portfolio

- Controlled by the trader
- 8 assets
  - 7 stocks
  - 1 risk-free
- Allocation vector

$$\begin{bmatrix} \text{Bank} \\ \text{AAPL} \\ \text{MSFT} \\ \text{GOOG} \\ \text{F} \\ \text{JNJ} \\ \text{JPM} \\ \text{XOM} \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \\ 0.1 \\ 0 \\ -1 \\ 1 \\ 0.5 \end{bmatrix}$$

World States

- Influences how stock prices behave
  - μ: drift
  - Σ: volatility
- Based on S&P500
  - Ex: S&P500 going up in price
- Transition to other world states

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Transition matrix for 4 world states

Model

New probabilistic syntax to SdL

$$[\![[\alpha]_\theta P]\!] = \left\{ \begin{matrix} \omega \mid \nu \in [\![P]\!] \text{ for at most } \theta \text{ proportion} \\ \text{of all } \nu \text{ such that } (\omega, \nu) \in [\![\alpha]\!] \end{matrix} \right\}$$

- Defined for a finite state space

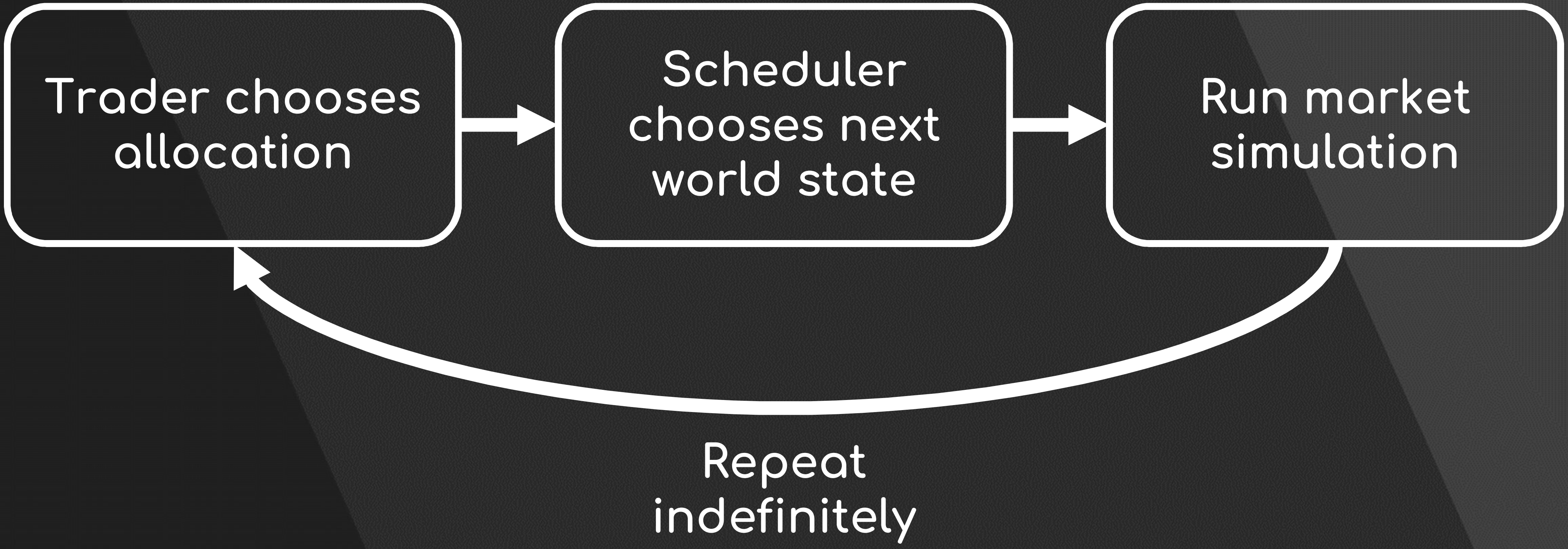
Model

- Preconditions
- Hybrid Program
- Postcondition

Preconditions

- $T_0 > 0$: duration of each iteration
- $X > 0$: Starting portfolio value
- $J \in \{J_1, J_2, \ldots, J_{N_J}\}$: starting world state
- $S_i > 0$: all stock prices are positive

Hybrid Program



Postcondition and Formula

- $\varphi \coloneqq X < X_0$: trader loses money
- Let θ be the probability we want to prove

$$\mathrm{PRE} \rightarrow [\![\mathrm{HP}]\!]_{\theta}\varphi$$

Simulation & Training

Market Simulation

- Given a set of world transitions, for each transition:
    - Let trader choose portfolio allocation
    - Stock prices evolve according to correlated GBMs
    - Stock returns → portfolio returns ($\alpha^T \cdot R$)
- Use historical data to compute parameters for each world state
- Calculate Sharpe ratio

Making an Evil Scheduler

- Find optimal adversarial scheduler
- Algorithm based on Henriques et al. 2012
- Reinforcement learning
    - Evaluate
    - Improve
    - Optimize

Training example: Evaluate

$$\begin{bmatrix} 0.08 & 0.92 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.04 & 0.15 & 0.81 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.05 & 0.15 & 0.80 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.18 & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.55 & 0.27 & 0.18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.98 & 0.02 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.85 & 0.15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow p = 0.95$$

Training example: Improve

$$q = \begin{bmatrix} 1 & 0.98 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.88 & 099 & 0.99 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.99 & 0.99 & 0.99 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.99 & 0.99 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

New Scheduler

Training example: Evaluate

$$\begin{bmatrix} 0.04 & 0.96 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.03 & 0.56 & 0.42 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.02 & 0.66 & 0.32 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.28 & 0.52 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.96 & 0.04 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.55 & 0.45 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow p = 0.95$$

Training example: Converge

$$\begin{bmatrix} 0.02 & 0.98 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.09 & 0.90 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.57 & 0.16 & 0.27 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.66 & 0.32 & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.69 & 0.06 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.60 & 0.14 & 0.26 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.64 & 0.29 & 0.07 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.92 & 0.08 \end{bmatrix} \Rightarrow p = 1$$

Results

Sampling and Metric

- Monte Carlo sampling
- Trading strategy metric m

$$\Pr(\text{Sharpe} < -m) = 0.4$$

Results: Optimal Scheduler

$$\begin{bmatrix} 0.02 & 0.98 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.09 & 0.90 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.57 & 0.16 & 0.27 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.66 & 0.32 & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.69 & 0.06 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.60 & 0.14 & 0.26 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.64 & 0.29 & 0.07 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.92 & 0.08 \end{bmatrix}$$

$$m = -1.95$$

Results: Optimal Scheduler

$$\begin{bmatrix} 0.02 & 0.98 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.09 & 0.90 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.57 & 0.16 & 0.27 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.66 & 0.32 & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.69 & 0.06 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.60 & 0.14 & 0.26 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.64 & 0.29 & 0.07 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.92 & 0.08 \end{bmatrix}$$

$$m = -1.95$$

Results: Optimal Scheduler Moves

4, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3,
2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2,
2, 2, 2, 3, 3, 3, 2, 2, 2, 2, 2, ...

Results: Optimal Scheduler Moves

4, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3,
2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2,
2, 2, 2, 3, 3, 3, 2, 2, 2, 2, 2, ...

Tweaks to Strategy

- Trader was losing money when S&P500 performed poorly
    - Take on less risk during bad market conditions
- Scheduler illuminated certain orderings of world states that made trader lose money
    - Modify trader to account for these orderings

Results: Optimal Scheduler Improved

$$\begin{bmatrix} 0.62 & 0.38 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0.27 & 0.72 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.02 & 0.75 & 0.23 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.36 & 0.63 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.75 & 0.20 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.91 & 0.09 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$m = -1.00$$

Results: Metric Improvement

	Long Short	Improved Long Short
m	-1.95	-1.00

Discussion

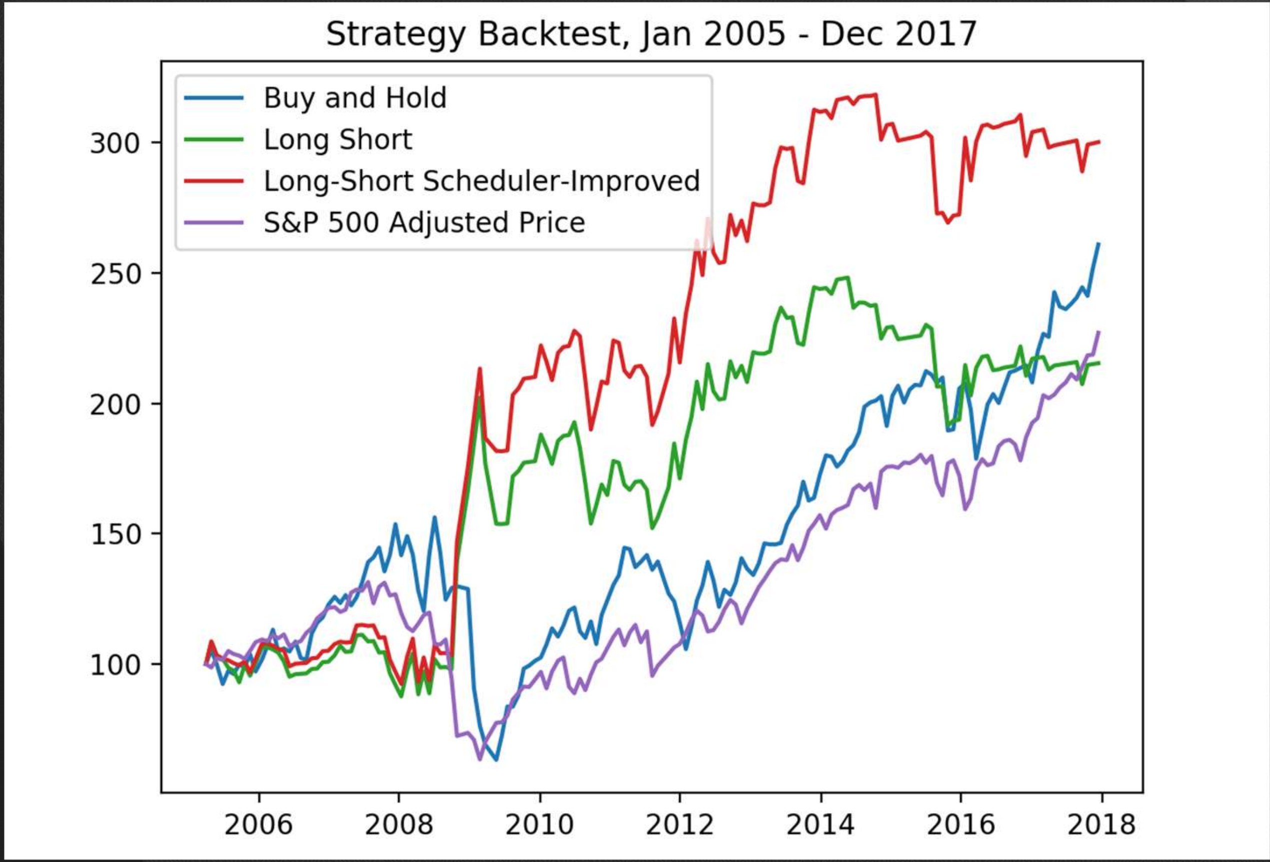
Discussion

- Buy and hold does reasonably well in real life
    - Beats most active mutual funds
- Reinforcement learning made strong schedulers
- Schedulers gave insight into constructing better trading strategies
- Real life is not that evil (usually)

Backtesting on real data

- Buy and hold does better than most strategies
- Our system helped us come up with a decent long-short strategy
- Our modified long-short strategy was even better
    - Constructed based on what the scheduler told us about original strategy

Backtesting on real data



Conclusion

- Statistical model checking can be used in portfolio optimization
- Optimal schedulers give good real-world insight into when a trading strategy loses money
    - Extremely important for hedge funds and investment banks

- Can be extended to virtually anything that can be expressed as an MDP or state transition diagram
    - Mortgage pricing, options pricing, lattice-based term-structure modeling

Future Work

- Simulate Brownian Motion more accurately
    - Brownian bridge rather than sequential simulation

- Experiment with different world states and trading strategies

- Optimize trading strategies with trader and scheduler locked in a two-player zero-sum game
    - Generative Adversarial Networks (zero-sum game between neural networks)

References

[1] Ermogenous Angeliki. Brownian motion and its applications in the stock market. http://ecommons.udayton.edu/mth_epumd/15, 2006.

[2] Fischer Black and Myron Scholes. The pricing of options and corporate liabilities. Journal of Political Economy, 81(3):637–654, 1973.

[3] Tim Bollerslev. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3):307 – 327, 1986.

[4] Gavin Cassar and Joseph J. Gerakos. Do risk management practices work? evidence from hedge funds. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1722250, Dec 2010.

[5] Joe Chang. Brownian motion conditional distributions. http://disi.unal.edu.co/~gjhernandezp/mathcomm/slides/bm.pdf. Accessed: 2018-11-18.

[6] D. Henriques, J. G. Martins, P. Zuliani, A. Platzer, and E. M. Clarke. Statistical model checking for markov decision processes. In 2012 Ninth International Conference on Quantitative Evaluation of Systems, pages 84–93, Sept 2012.

[7] D. Henriques, J. G. Martins, P. Zuliani, A. Platzer, and E. M. Clarke. Statistical model checking for markov decision processes. In 2012 Ninth International Conference on Quantitative Evaluation of Systems, pages 84–93, Sept 2012.

[8] Marta Kwiatkowska, Gethin Norman, and David Parker. Prism: Probabilistic symbolic model checker. In Tony Field, Peter G. Harrison, Jeremy Bradley, and Uli Harder, editors, Computer Performance Evaluation: Modelling Techniques and Tools, pages 200–204, Berlin, Heidelberg, 2002. Springer Berlin Heidelberg.

[9] Andrew W. Lo. Risk management for hedge funds: Introduction and overview. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=283308, Sep 2001.

[10] Grant Olney Passmore and Denis Ignatovich. Formal verification of financial algorithms. In Leonardo de Moura, editor, Automated Deduction – CADE 26, pages 26–41, Cham, 2017. Springer International Publishing.

[11] Simon Peyton Jones, Jean-Marc Eber, and Julian Seward. Composing contracts: An adventure in financial engineering (functional pearl). SIGPLAN Not., 35(9):280–292, September 2000.

[12] André Platzer. Stochastic differential dynamic logic for stochastic hybrid programs. In Nikolaj Bjørner and Viorica Sofronie-Stokkermans, editors, Automated Deduction – CADE-23, pages 446–460, Berlin, Heidelberg, 2011. Springer Berlin Heidelberg.17

[13] Dr Reddy and V Clinton. Simulating stock prices using geometric brownian motion: Evidence from australian companies. 10:23–47, 01 2016.

[14] Karl Sigman. Simulating normal (gaussian) rvs with applications to simulating brownian motion and geometric brownian motion in one and two dimensions. http://www.columbia.edu/~ks20/4703-Sigman/4703-07-Notes-BM-GBM-I.pdf. Accessed: 2018-11-18.

Acknowledgments

- Professor André Platzer
- TAs Yong Kiam, Irene, Brandon, CPS Lab
- Sponsors