Munkres Topology Solutions

Michael You

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Forward

These solutions are for the 2$^{\text{nd}}$ edition Topology textbook by Munkres.

I am slowly going to evolve this guide as I work through the problems, but I'm going to start summarizing each chapter before I do the problems, not only to help me learn each chapter section, but also as a good study guide and reference while doing the problems.

Chapter 1

Set Theory and Logic

1.1 Fundamental Concepts

Exercise 1.1.1
We will check $\cup, \cap$ in DeMorgan's laws.
Let's use

- $A = \{1, 2, 3, 4\}$
- $B = \{-1, 2, 3, 5\}$
- $C = \{3, 9, 11\}$

Check

$$\begin{aligned} A - (B \cup C) &= \{1, 2, 3, 4\} - \{-1, 2, 3, 5, 9, 11\} \\ &= \{1, 4\} \\ &= (A - B) \cap (A - C) \\ &= \{1, 4\} \cap \{1, 2, 4\} = \{1, 4\} \end{aligned}$$

$$\begin{aligned} A - (B \cap C) &= \{1, 2, 3, 4\} - \{3\} \\ &= \{1, 2, 4\} \\ &= (A - B) \cup (A - C) \\ &= \{1, 4\} \cup \{1, 2, 4\} = \{1, 2, 4\} \end{aligned}$$

Exercise 1.1.2
(a) $\implies$ is true. $\impliedby$ is not true, consider $A = \{1, 2, 3\}, B = \{1, 3\}, C = \{2\}$.

(b) $\implies$ is true. $\impliedby$ is not true, consider $A = \{1, 2, 3\}, B = \{1, 3\}, C = \{2\}$.

(c) True.

(d) $\implies$ is not true. Consider $A = \{1\} \subset B = \{1, 2\}, C = \emptyset$. $\impliedby$ is true.

(e) Not true. Consider $A = \{1\}, B = \{2\}$. I think $\subset$ works.

(f) Not true. Consider $A = \{1, 2\}, B = \{2, 3\}$. LHS is equivalent to A, so this should be $\supset$.

(g) True.

(h) $\supset$

(i) True.

(j) True.

(k) Not true, if $A = \emptyset$ for example, we have $(A \times B) \subset (C \times D) = \emptyset \subset (C \times D)$, but we can set B to whatever and this statement is still true, so we can make B have an element that is not in D, and therefore $B \not\subset D$.

(l) True.

(m) $\subset$

(n) $\subset$

(o) True.

(p) I think this is true at first glance...at least $\subset$ looks good.

(q) $\supset$

Exercise 1.1.3

(a)
- **Original**: If $x < 0$ then $x^2 - x > 0$. True.
- **Contrapositive**: If $x^2 - x \leq 0$ then $x \geq 0$. True.
- **Converse**: If $x^2 - x > 0$ then $x < 0$. False.

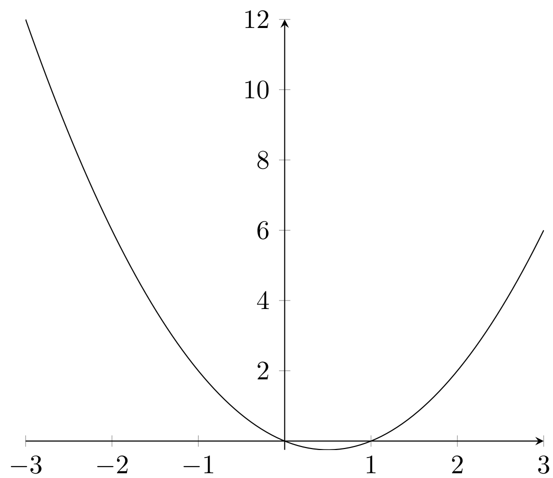

Figure 1.1: Showing how to visualize where $x^2 - x > 0$

(b)
- **Original**: If $x > 0$ then $x^2 - x > 0$. False.
- **Contrapositive**: If $x^2 - x \leq 0$ then $x \leq 0$. False.
- **Converse**: If $x^2 - x > 0$ then $x > 0$. False.

Exercise 1.1.4

(a) $\exists a \in A$ such that $a^2 \notin B$

(b) $\forall a \in A, a^2 \notin B$

(c) $\exists a \in A$ such that $a^2 \in B$.

(d) $\exists a \notin A$ such that $a^2 \notin B$.

Exercise 1.1.5

(a) True. True.

(b) False. True.

(c) True. False.

(d) True. True.

Exercise 1.1.6
TODO too lazy

Exercise 1.1.7

$$
\begin{aligned}
D &= A \cap (B \cup C) \\
E &= (A \cap B) \cup C \\
F &= A
\end{aligned}
$$

For F, I was thinking $x \in B \implies x \in C$ means that either $x \in B$ and $x \in C$, or $x \notin B$ and x can be anything. This sounds like x can be anything in the second case, so we have $A \cap \mathcal{U} = A$.

Exercise 1.1.8
$A = \{0, 1\}$. $\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$.

If A has one element, $|\mathcal{P}(A)| = 2$. It is called the power set because it contains all the subsets of A, and that $|\mathcal{P}(A)| = 2^{|A|}$.

Exercise 1.1.9
TODO: You can honestly find this everywhere online. Standard proof.

Exercise 1.1.10

(a) $\mathbb{Z} \times \mathbb{R}$

(b) $\mathbb{R} \times (0, 1]$

(c) No. You can do a contradiction proof with cases that the first and second set are disjoint, and then that they are not disjoint.

(d) Yes, $(\mathbb{R} - \mathbb{Z}) \times \mathbb{Z}$

(e) No. The cartesian product will produce a box, while this set is a circle.

1.2 Functions

Definition 1.2.1
We define the **composite** $g \circ f$ of $f : A \to B$ and $g : B \to C$ as the function $g \circ f : A \to C$ defined by the equation

$$(g \circ f)(a) = g(f(a)) \tag{1.1}$$

and

$$\{(a, c) \mid \text{for some } b \in B, f(a) = b \text{ and } g(b) = c\} \tag{1.2}$$

Definition 1.2.2
A function f is **injective** if

$$[f(a) = f(a')] \implies [a = a'] \tag{1.3}$$

Definition 1.2.3
A function f is **surjective** if

$$[b \in B] \implies [\exists a \in A, b = f(a)] \tag{1.4}$$

Lemma 1 *Let $f : A \to B$. If there are functions $g : B \to A$ and $h : B \to A$ such that $g(f(a) = a, \forall a \in A$ and $f(h(b)) = b, \forall b \in B$, then f is bijective and $g = h = f^{-1}$.*

Definition 1.2.4
$f(A_0)$ is the **image** of A_0, and

$$f(A_0) = \{b \mid \exists a \in A_0, b = f(a)\} \tag{1.5}$$

Definition 1.2.5
$f^{-1}(B_0)$ is the preimage of B_0 under f, formally

$$f^{-1}(B_0) = \{a \mid f(a) \in B_0\} \tag{1.6}$$

Exercises

Exercise 1.2.1

(a) Suppose $x \in A_0$. Then consider

$$f^{-1}(f(A_0)) = \{x' \mid f(x') \in f(A_0)\}.$$

Since $x \in A_0$, we know $f(x) \in f(A_0)$, and so we can conclude $x \in f^{-1}(f(A_0))$.

If f is injective, then we know $f(x) = f(x'), x' \in A_0$ implies that $x = x' \implies x \in A_0$, proving equality.

(b) Suppose $y \in f(f^{-1}(B_0)) = \left\{y' \mid \exists x \in f^{-1}(B_0), y' = f(x)\right\}$.

We have $x \in \{x' \mid f(x') \in B_0\}$, which means $y' = f(x) \in B_0$, so therefore $y \in B_0$.

If f is surjective, then we know $y \in B_0 \implies \exists x \in A$ such that $f(x) = y$.

In particular, $f(x) = y \in B_0$, this set of $x \in f^{-1}(B_0)$, so we can conclude that $y \in f(f^{-1}(B_0))$.

Exercise 1.2.2

(a) We can write some definitions first

- $f^{-1}(B_0) = \{x \mid f(x) \in B_0\}$
- $f^{-1}(B_1) = \{x \mid f(x) \in B_1\}$

If we know $B_0 \subset B_1$, then for some $x \in B_0$, we know $x \in B_1$.

This means for some $f(x) \in B_0, f(x) \in B_1$ as well, so therefore $f^{-1}(B_0) \subset f^{-1}(B_1)$.

(b)

$$\begin{aligned}
f^{-1}(B_0 \cup B_1) &= \{x \mid f(x) \in B_0 \cup B_1\} \\
&= \{x \mid f(x) \in B_0 \text{ or } f(x) \in B_1\} \\
&= \{x \mid f(x) \in B_0\} \cup \{x \mid f(x) \in B_1\} \\
&= f^{-1}(B_0) \cup f^{-1}(B_1)
\end{aligned}$$

(c) Basically the same proof as (b).

(d) Basically the same proof as (b).

(e) Suppose $x \in A_0$ means $x \in A_1$ as well. Consider $y \in f(A_0) = \{y' \mid y' = f(x) \text{ for some } x \in A_0\}$. Because of our assumptions, it is also the case that $y \in \{y' \mid y' \text{ for some } x \in A_1\} = f(A_1)$.

Notice that $f(A_0) \subset f(A_1)$ does not imply that $A_0 \subset A_1$. E.g. think parabola.

(f)

$$\begin{aligned}
f(A_0 \cup A_1) &= \{y \mid y = f(x) \text{ for some } x \in A_0 \cup A_1\} \\
&= \{y \mid y = f(x) \text{ for some } x \in A_0 \text{ or } x \in A_1\} \\
&= \{y \mid y = f(x) \text{ for some } x \in A_0\} \cup \{y \mid y \text{ for some } x \in A_1\} \\
&= f(A_0) \cup f(A_1)
\end{aligned}$$

(g)

$$f(A_0 \cap A_1) = \{y \mid y = f(x) \text{ for some } x \in A_0 \cap A_1\}$$

which implies that $f(A_0 \cap A_1) \in f(A_0)$, since $x \in A_0$ and $f(A_0 \cap A_1) \in f(A_1)$, since $x \in A_1$.

This means that $f(A_0 \cap A_1) \subset f(A_0) \cap f(A_1)$.

Now, if f is injective, then if we start with $y \in f(A_0) \cap f(A_1)$, we know $y \in \{y' \mid y' = f(x), x \in A_0\}$ and $y \in \{y' \mid y' = f(x), x \in A_1\}$. Since f is injective, the common y' values in $f(A_0)$ and $f(A_1)$ will map to the same x values in A_0 and A_1, which means $y \in \{y' \mid y' = f(x), x \in A_0 \cap A_1\} = f(A_0 \cap A_1)$.

Exercise 1.2.3
too lazy

Exercise 1.2.4

(a) Consider

$$\begin{aligned}
(g \circ f)^{-1}(C_0) &= \{a \mid (g \circ f)(x) \in C_0\} && \text{(by definition)} \\
&= \{a \mid \text{for some } b \in B, f(a) = b, g(b) = c \in C_0\} \\
&= \big\{a \mid \text{for some } b \in B, f(a) = b, b \in g^{-1}(C_0)\big\} && \text{(we know } b \in g^{-1}(C_0) \text{ since } C_0 \subset C) \\
&= \big\{a \mid f(a) \in g^{-1}(C_0)\big\} \\
&= f^{-1}\big(g^{-1}(C_0)\big)
\end{aligned}$$

Just a note for this problem, it's easy to get caught up with definitions and forget why we need assumptions. It might seem easy to do this problem without the fact that $C_0 \subset C$, but if you look at the step where we use that property, if $C_0 \not\subset C$, we cannot assume that $b \in g^{-1}(C_0)$, e.g. if C_0 contains elements that are not in C.

(b) Suppose we have

$$\begin{aligned}
(g \circ f)(a) &= (g \circ f)(a') \\
g(f(a)) &= g(f(a')) && \text{(by def.)} \\
f(a) &= f(a') && \text{(because } g \text{ is injective)} \\
a &= a' && \text{(because } f \text{ is injective)}
\end{aligned}$$

therefore we conclude that $g \circ f$ is also injective.

(c) If we know that $g \circ f$ is injective,

- AFSOC f is not injective. Then $\exists a_1, a_2$ such that $f(a_1) = f(a_2)$ but $a_1 \neq a_2$. If this is the case, then $(g \circ f)(a_1) = g(f(a_1)) = g(f(a_2)) = (g \circ f)(a_2)$ which shows that $g \circ f$ is not injective. Which is a contradiction. Therefore, f must be injective.
- It is possible for g to not be injective. We can have some $b \in B$ that $\nexists a \in A$ such that $f(a) = b$. In this case, we will not be able to find some input $a \neq a' \in$ where we break injectivity for $g \circ f$.

(d) Suppose f and g are surjective. Now consider some $c \in C$. Since g is surjective, we know $\exists b \in B$ such that $g(b) = c$. For this $b \in B$, since f is surjective, we know that $\exists a \in A$ such that $f(a) = b$. This means for any $c \in C$, we know $\exists a$ such that $g(f(a)) = (g \circ f)(a) = c$, which means $g \circ f$ is surjective.

(e) If we know that $g \circ f$ is surjective,

- It is possible for f to not be surjective. Intuitively, a counterexample would show that there is some $b \in B$ such that $\nexists a \in A$ such that $f(a) = b$. But all we have to make sure in our example is that whatever $g(b) = c$ maps to, $\exists a' \neq a \in A$ such that $(g \circ f)(a') = c$.
- AFSOC g is not surjective. Then $\exists c \in C$ such that $\nexists b \in B$ such that $g(b) = c$. If this is the case, then $\nexists a \in A$ such that $(g \circ f)(a) = c$, which means $g \circ f$ is not surjective. This is a contradiction, therefore g must be surjective.

(f) Summary should be pretty clear :) from the results above.

Exercise 1.2.5

Alright let's consider the two cases in this problem

- f has a left inverse, i.e. $\exists g, g \circ f = i_A$.

  AFSOC f is not injective. This means that $f(a) = f(a') = b \in B$ but $a \neq a' \in A$.

  If this is the case, then $g(b) = a$ or $g(b) = a'$, but cannot be both, by the definition of a function, which means that $g \circ f$ is not i_A by counterexample of either a or a'.

- f has a right inverse, i.e. $\exists h, f \circ h = i_B$.

  AFSOC f is not surjective. Then $\exists b \in B$ such that $\nexists a \in A$ such that $f(a) = b$. If this is the case, then for this b, $(f \circ h)(b) \neq b$, in which case $f \circ h \neq i_B$.

The proofs above in 1.2.5 are by contradiction, and illustrate how to construct such a counterexample.

The proofs above in 1.2.5 are by contradiction, and illustrate how to construct such a counterexample.

No, these left/right inverses are unique. Not rigorous, but if you have an identity mapping and you change any of the mappings, it will no longer be an identity mapping.

So we have f that has both a left and right inverse, g, h respectively.In that case, we know that f is injective and surjective, by the results of 1.2.5, so we can conclude that f is bijective. We have $g = h = f^{-1}$ by Lemma 2.1 in the text.

(We could've just used Lemma 2.1 directly, but I think it's important to remind ourselves that showing a function is injective and surjective is a problem solving technique for showing a function is bijective.)

Exercise 1.2.6
Let us draw out our function first,

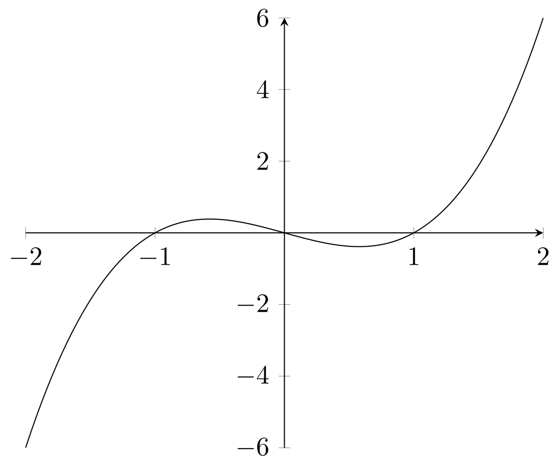

Figure 1.2: Plotting $f(x) = x^3 - x$

A less formal way to imagine injective functions is to use the horizontal line test. If you sweep a horizontal line and it intersects the plot in more than 1 spot, then you know the function is not injective. We see our function $f(x)$ here has issues between $[-1, 1]$.

For surjectivity, we just need to map all of $\mathbb{R}$ on the y-axis. Our function does this nicely already, so we just have to make our function injective, and then make sure to keep the surjectivity.

We have two choices here, we can either restrict our domain to be

- $D = (-\infty, -1) \cup [-1, \infty)$
- $D = (-\infty, -1] \cup (-1, \infty)$

If we use the first choice, we will get the following plot as g:

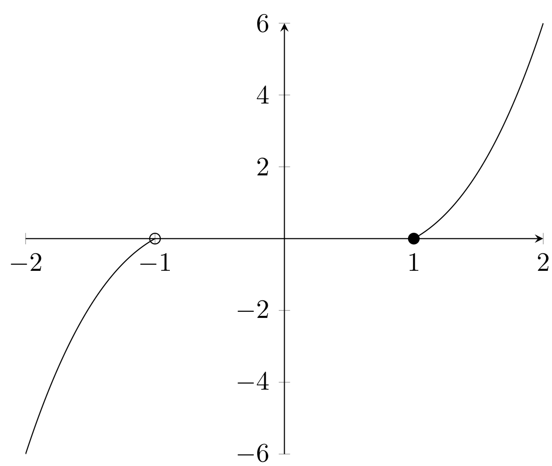

Figure 1.3: Plotting $g(x)$, a bijective function. Open circles are exclusive, closed circles are inclusive.

To find g^{-1}, a classic algebra way to do this is to solve the function in terms of x. Because of the horizontal line issues from before, we will encounter, some issues, but because of our domain restriction, things should be ok.

Instead of solving for x, since we already have a plot, we can just do a reflection across $y = x$, and we will get the inverse function g^{-1}. The intuitive way to think about this is that we are essentially swapping all (x, y) coordinates to become (y, x).

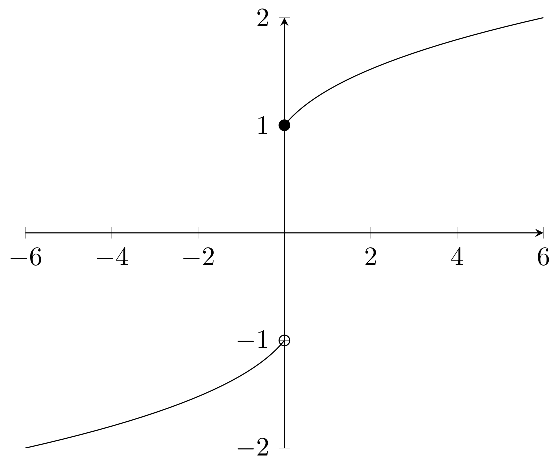

Figure 1.4: Plotting $g(x)$, a bijective function. Open circles are exclusive, closed circles are inclusive.

1.3 Relations

Definition 1.3.1
A **relation** on a set A is a subset C of the cartesian product $A \times A$.

For a relation C on A, we use the notation xCy to mean $(x, y) \in C$, or "x is in the relation C to y."

Definition 1.3.2
An **equivalence relation** on a set A is a relation C on A having the following 3 properties: (we use $\sim$ to denote the equivalence relation)

1. Reflexivity: $xCx \quad \forall x \in A$ $(x \sim x)$

2. Symmetry: If xCy then yCx $(x \sim y \implies y \sim x)$

3. Transitivity: If xCy and yCz then xCz $(x \sim y \wedge y \sim z \implies x \sim z)$

Definition 1.3.3
We call a subset of E of A the **equivalence class** determined by x as the equation

$$E = \{y \mid y \sim x\} \tag{1.7}$$

Definition 1.3.4
A **partition** of a set A is a collection of disjoint nonempty subsets of A whose union is all of A.

Definition 1.3.5
A relation C on a set A is called an **order relation** if it has the following properties:

1. Comparability: For every $x, y \in A$ for which $x \neq y$, either xCy or yCx

2. Nonreflexitivity: For no $x \in A$ does xCx hold

3. Transtivity: If xCy and yCz then xCz

Definition 1.3.6
if X is a set and $<$ is an order relation on X, and if $a < b$, we use the notation (a, b) to denote the set

$$\{x \mid a < x < b\}; \tag{1.8}$$

it is called an **open interval** in X. If this set if empty, we call

- a the **immediate predecessor** of b

- b the **immediate successor** of a

Definition 1.3.7
Suppose A, B are two sets with order relations $<_A$ and $<_B$ respectively. We say that A, B have the same **order type** if there is a bijective correspondence between them that preserves order; that is, if there exists a bijective function $f : A \to B$ such that

$$a_1 <_A a_2 \implies f(a_1) <_B f(a_2) \tag{1.9}$$

Definition 1.3.8
An ordered set A has the **least upper bound property** if every nonempty subset A_0 of A that is bounded above has a least upper bound. Analogously, the set A is said to have the **greatest lower bound property** if every nonempty subset A_0 of A that is bounded below has a greatest lower bound.

Exercises

Exercise 1.3.1
Check equivalence relation:

- Reflexivity is obvious
- Equality is symmetric, so the relation is too
- Equality is transitive, so the relation is too

This looks like a bunch of parabolas of the form $y = x^2 + C$ on the plane, see figure 1.5

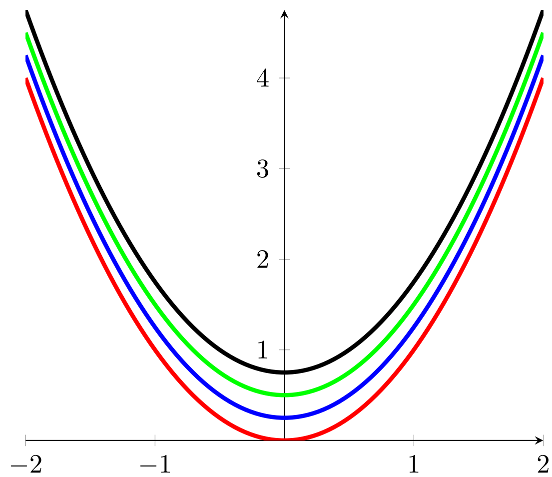

Figure 1.5: Plotting the partition defined by the equivalence relation

Exercise 1.3.2
Reflexivity will still hold in A_0, since $A_0 \in A$, and C applies to any element $x \in A$. Symmetry still holds, since $x, y \in A$, and transitivity also holds since $x, y, z \in A$. The idea is that A_0's elements are contained in A, so all the equivalence relation properies still hold.

Exercise 1.3.3
We are assuming $\exists b$ such that aCb. If there is no such b, then we do not have aCa.

Exercise 1.3.4

(a) Let us check the properties

- Reflexive: $f(a) = f(a)$ is trivial
- Symmetric: If we have $f(a) = f(b)$, then $f(b) = f(a)$
- Transitive: Equality is transitive, so this also holds

(b) A^* is a partition of B, so a bijective correspondence exists.

Exercise 1.3.5
(a) S' is an equivalence relation because we can imagine partitions of $y - x = z$ for $z \in \mathbb{Z}$. In S, we notice that $y - x = 1$, and $1 \in \mathbb{Z}$, so we know every relation in S is also in S', therefore $S \subset S'$. See 1.6 for how to visualize these equivalence classes.

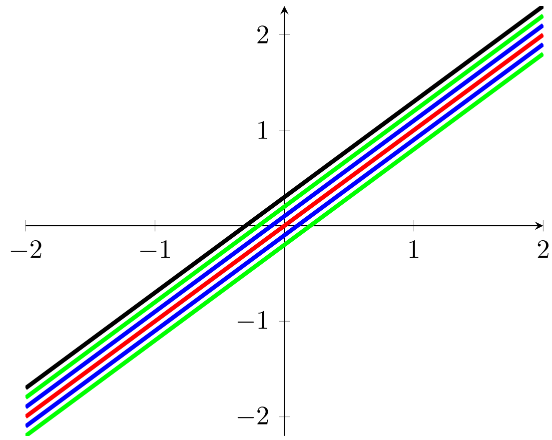

Figure 1.6: Plotting the partition defined by the equivalence relation

(b) If their intersection is empty, then this is trivially true.

Otherwise, if we have some nonempty intersection, since we know any elements in this intersection are also part of some equivalence relation, all the equivalence relation properties apply, so this intersection is also an equivalence relation on A.

(c) **TODO**: I'm pretty confused about this question. Wouldn't the intersection of all equivalence relations that contain S just end up with S?

Exercise 1.3.6
Showing it is an order relation

- Comparability: Per the rule, we will always have xCy or yCx.
- Nonreflexitivity: We have a tiebreaker rule that prevents xCx.
- Transitivity: Intuitively, there is an ordering at the highest level with $y - x^2$ value. If there are ties there, we check with x values for ordering.

Yeah sorry I didn't feel like doing the order relation formally, but hopefully the geometric solution can help explain more. There's just a lot of casework and mundane proof so I didn't feel like writing it out.

Geometrically, this is like the partition in 1.5, except parabolas that are higher up are ordered "higher". Within a parabola, the values on the left are less than the values on the right.

Exercise 1.3.7
A restriction is a subset of the larger set, so all the order relation properties will still hold.

Exercise 1.3.8
I assume the author wants us to check the $x^2 < y^2$, if $x^2 = y^2$, then $x < y$.

1. Comparability: If $x^2 \neq y^2$, either $x^2 < y^2$ or $x^2 > y^2$. If $x^2 = y^2$, then it must be the case that $x, y \neq 0$, and one of x, y is negative and the other positive, so we either have $x > y$ or $x < y$.
2. Nonreflexitivity: xCx means $x^2 = x^2$, so we would check $x <$ or $> x$, but this is not possible since $x = x$.
3. Transitivity: If xCy then $x^2 < y^2$ or $x < y$, and if we have yCz then $y^2 < z^2$ or $y < z$. In all cases, we can conclude $x^2 < z^2$ or $x < z$. I'm being lazy with casework here.

Exercise 1.3.9
We want to check that a dictionary order relation is an order relation.

1. Comparability: For any two $(a_1, b_1), (a_2, b_2)$, we have 2 cases:

(a) $a_1 <_A a_2$ or $a_2 <_A a_1$ since $<_A$ is an order relation on A. Then we know $(a_1, b_1) < (a_2, b_2)$ or $(a_1, b_1) > (a_2, b_2)$.

(b) $a_1 = a_2$. Then we use the same argument with b_1, b_2, that either $b_1 <_B b_2$ or $b_2 <_B b_1$, which then shows the corresponding $<$ and $>$ on the tuple.

2. Nonreflexitivity: if we have some (a, b), we know by $<_A$ that $a <_A a$ does not hold, so the overall order relation is not possible.

3. Transitivity: If we have $(a_1, b_1) < (a_2, b_2) < (a_3, b_3)$, then we have 2 cases for the first tuple, and 2 cases for the second tuple.

   (a) $a_1 <_A a_2 <_A a_3$: then we can use the transitive property of $<_A$

   (b) $a_1 <_A a_2 = a_3$: we can see that $a_1 <_A a_3$

   (c) $a_1 = a_2 <_A a_3$: we can see that $a_1 <_A a_3$

   (d) $a_1 = a_2 = a_3$: then we must have $b_1 <_B b_2 <_B b_3$, so we can use the transitive property of $<_B$

Exercise 1.3.10

- One way to see that this is an order preserving function is that the derivative is always positive between $(-1, 1)$,

$$f'(x) = \frac{x^2 + 1}{(1 - x^2)^2},$$

  which means the function is monotonically increasing, and thus will preserve the order, since monotonically increasing functions have the property that

$$a < b \implies f(a) < f(b).$$

- This is just an algebra exercise...pretty easy to verify

Exercise 1.3.11

AFSOC there is more than one immediate successor to some a, call them b and c. Then by order set properties, we know that either $b < c$ or $c < b$. In either case, we end up finding that b or c cannot be immediate successors, since for example, if $b < c$, we have that (a, c) is not empty.

The argument for immediate predecessor is symmetric to this argument.

To show there can only be one smallest element, we can AFSOC there is more than one. If we call these a, a', we know from ordering properties that WLOG $a < a'$, then a' is not the smallest element, so this is a contradiction.

The argument for the largest element is symmetric.

Exercise 1.3.12

(i*) Every element has an immediate predecessor. For some (x, y), the immediate predecessor is $(x, y+1)$. There is no smallest element, since you can always find a smaller element, i.e. for any (x, y), $(x-1, y) < (x, y)$.

(ii*) The immediate predecessor for some (x, y) is $(x+1, y+1)$. There is no smallest element, since you can always find $(x-1, y) < (x, y)$ for any (x, y).

(iii*) The immediate predecessor for some (x, y) is $(x-1, y+1)$. There is no smallest element, since for any (x, y), you have $(x-1, y) < (x, y)$.

Not rigorous, but geometrically, the first ordering is like a zigzag on the plane, the second one is like $y = x + C$, and the third is $y = -x + C$, so these orderings are all different.

Exercise 1.3.13

Exercise 1.3.14

(a) If C is symmetric, then $(a, b) \in C \implies (b, a) \in C$, which means $D \subset C$.

If $C = D$, then $(a, b) \in C \implies (b, a) \in D = C$, so therefore C is symmetric.

(b) We will check the order relation properties for D

   (a) Comparability: For any $(b, a) \in D$, we know $(a, b) \in C$, so we know either $b < a$ or $a < b$.
   (b) Nonreflexitivity: $(b, b) \in D$ would imply $(b, b) \in C$, but C is an order relation so this is not possible.
   (c) Transitivity: $(c, b) \in D, (b, a) \in D$. We know that $(a, b), (b, c) \in C$, so we know $(a, c) \in C$, so therefore $(c, a) \in D$, which proves transitivity.

(c)

Exercise 1.3.15