Munkres Topology Solutions

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Contents

Forward

These solutions are for the 2nd edition Topology textbook by Munkres.

Chapter 1

Set Theory and Logic

1.1 Fundamental Concepts

Exercise 1.1.1
We will check $\cup, \cap$ in DeMorgan's laws.

Let's use

- $A = \{1, 2, 3, 4\}$
- $B = \{-1, 2, 3, 5\}$
- $C = \{3, 9, 11\}$

Check

$$\begin{aligned} A - (B \cup C) &= \{1, 2, 3, 4\} - \{-1, 2, 3, 5, 9, 11\} \\ &= \{1, 4\} \\ &= (A - B) \cap (A - C) \\ &= \{1, 4\} \cap \{1, 2, 4\} = \{1, 4\} \end{aligned}$$

$$\begin{aligned} A - (B \cap C) &= \{1, 2, 3, 4\} - \{3\} \\ &= \{1, 2, 4\} \\ &= (A - B) \cup (A - C) \\ &= \{1, 4\} \cup \{1, 2, 4\} = \{1, 2, 4\} \end{aligned}$$

Exercise 1.1.2

(a) $\implies$ is true. $\impliedby$ is not true, consider $A = \{1, 2, 3\}, B = \{1, 3\}, C = \{2\}$.

(b) $\implies$ is true. $\impliedby$ is not true, consider $A = \{1, 2, 3\}, B = \{1, 3\}, C = \{2\}$.

(c) True.

(d) $\implies$ is not true. Consider $A = \{1\} \subset B = \{1, 2\}, C = \emptyset$. $\impliedby$ is true.

(e) Not true. Consider $A = \{1\}, B = \{2\}$. I think $\subset$ works.

(f) Not true. Consider $A = \{1, 2\}, B = \{2, 3\}$. LHS is equivalent to A, so this should be $\supset$.

(g) True.

(h) $\supset$

(i) True.

(j) True.

(k) Not true, if $A = \emptyset$ for example, we have $(A \times B) \subset (C \times D) = \emptyset \subset (C \times D)$, but we can set B to whatever and this statement is still true, so we can make B have an element that is not in D, and therefore $B \not\subset D$.

(l) True.

(m) $\subset$

(n) $\subset$

(o) True.

(p) I think this is true at first glance...at least $\subset$ looks good.

(q) $\supset$

Exercise 1.1.3

(a)
- **Original**: If $x < 0$ then $x^2 - x > 0$. True.
- **Contrapositive**: If $x^2 - x \leq 0$ then $x \geq 0$. True.
- **Converse**: If $x^2 - x > 0$ then $x < 0$. False.

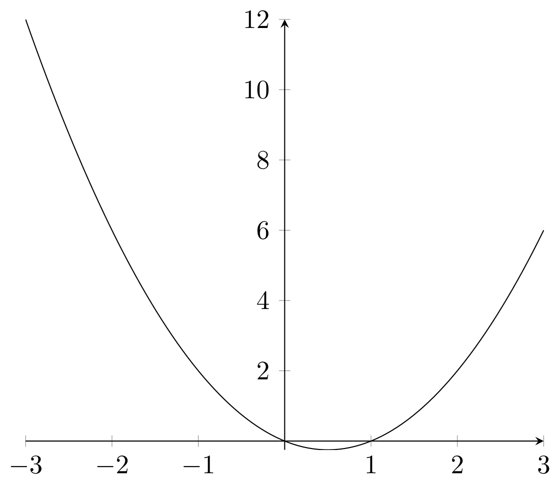

Figure 1.1: Showing how to visualize where $x^2 - x > 0$

(b)
- **Original**: If $x > 0$ then $x^2 - x > 0$. False.
- **Contrapositive**: If $x^2 - x \leq 0$ then $x \leq 0$. False.
- **Converse**: If $x^2 - x > 0$ then $x > 0$. False.

Exercise 1.1.4

(a) $\exists a \in A$ such that $a^2 \notin B$

(b) $\forall a \in A, a^2 \notin B$

(c) $\exists a \in A$ such that $a^2 \in B$.

(d) $\exists a \notin A$ such that $a^2 \notin B$.

Exercise 1.1.5

(a) True. True.

(b) False. True.

(c) True. False.

(d) True. True.

Exercise 1.1.6
TODO too lazy

Exercise 1.1.7

$$
\begin{aligned}
D &= A \cap (B \cup C) \\
E &= (A \cap B) \cup C \\
F &= A
\end{aligned}
$$

For F, I was thinking $x \in B \implies x \in C$ means that either $x \in B$ and $x \in C$, or $x \notin B$ and x can be anything. This sounds like x can be anything in the second case, so we have $A \cap \mathcal{U} = A$.

Exercise 1.1.8
$A = \{0, 1\}$. $\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$.
If A has one element, $|\mathcal{P}(A)| = 2$. It is called the power set because it contains all the subsets of A, and that $|\mathcal{P}(A)| = 2^{|A|}$.

Exercise 1.1.9
TODO: You can honestly find this everywhere online. Standard proof.

Exercise 1.1.10
(a) $\mathbb{Z} \times \mathbb{R}$

(b) $\mathbb{R} \times (0, 1]$

(c) No. You can do a contradiction proof with cases that the first and second set are disjoint, and then that they are not disjoint.

(d) Yes, $(\mathbb{R} - \mathbb{Z}) \times \mathbb{Z}$

(e) No. The cartesian product will produce a box, while this set is a circle.