Lucky numbers of Euler

Euler's “lucky” numbers are positive integers n such that for all integers k with $1 \leq k < n$, the polynomial $k^2 - k + n$ produces a prime number.

When k is equal to n, the value cannot be prime anymore since $n^2 - n + n = n^2$ is divisible by n. Since the polynomial can be rewritten as $k(k-1) + n$, using the integers k with $-(n-1) < k \leq 0$ produces the same set of numbers as $1 \leq k < n$.

Leonhard Euler published the polynomial $k^2 - k + 41$ which produces prime numbers for all integer values of k from 1 to 40. Only 7 lucky numbers of Euler exist, namely 1, 2, 3, 5, 11, 17 and 41 (sequence A014556 in the OEIS).

The primes of the form $k^2 - k + 41$ are

> 41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, ... (sequence A005846 in the OEIS).

The terminology is ambiguous: "Euler's lucky numbers" are neither the same as, neither related to the "lucky numbers" defined by a sieve algorithm. In fact, the only number which is both lucky and Euler-lucky is 3, since all other Euler-lucky numbers are congruent to 2 mod 3, but no lucky numbers are congruent to 2 mod 3.

See also

- Heegner number
- List of topics named after Leonhard Euler
- Formula for primes
- Ulam spiral

References

- Le Lionnais, F. *Les Nombres Remarquables*. Paris: Hermann, pp. 88 and 144, 1983.

External links

- Weisstein, Eric W. "Lucky Number of Euler" (http://mathworld.wolfram.com/LuckyNumberofEuler.html). *MathWorld*.

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