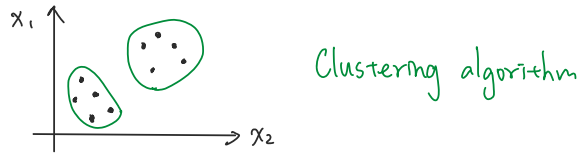


- Unsupervised learning introduction

- Unsupervised learning

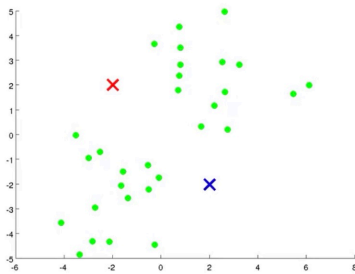


- Applications of clustering

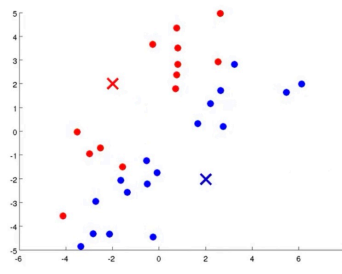
eg. Market segmentation

Social network analysis

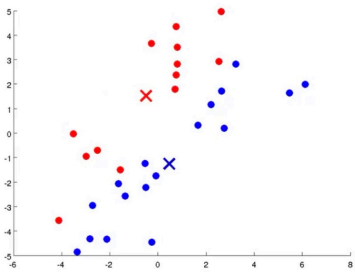
- K-means algorithm



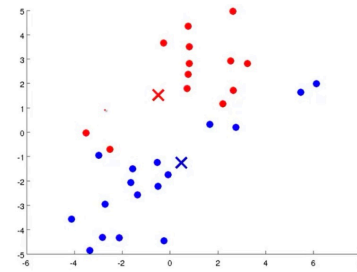
隨機取兩點(cluster centroids)



距離紅點近染為紅色, 反之

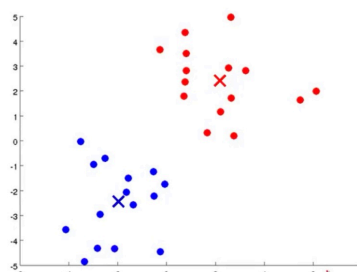
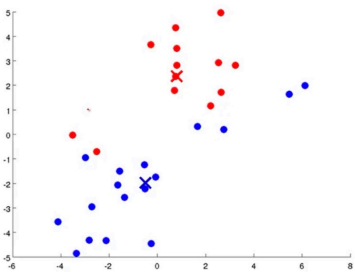


紅色圓點計算 cluster centroids



距離紅點近染為紅色, 反之

新的位置



紅色圓點計算 cluster centroids

距離紅點近染為紅色, 反之 收斂

新的位置

- K-means algorithm

input: K (number of clusters)

Training Set  $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$

$x^{(i)} \in \mathbb{R}^n$  (drop  $x_0 = 1$  convention)

Randomly initialize K cluster centroids  $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

Repeat {

for  $i = 1$  to  $m$

$c^{(i)} = \text{index (for } 1 \text{ to } K) \text{ of centroid closest to } x^{(i)}$

cluster assignment step  
minimize  $J(\dots)$  cost  $c^{(1)}, \dots, c^{(m)}$   
(holding  $\mu_1, \dots, \mu_K$  fixed)

for  $k = 1$  to  $K$

$$\min_k \|x^{(i)} - \mu_k\|^2$$

move centroid

$\mu_k = \text{average (mean) of points assigned to cluster } k$

$$x^{(1)}, x^{(5)}, x^{(6)}, x^{(10)} \rightarrow c^{(1)} = 2, c^{(5)} = 2, c^{(6)} = 2, c^{(10)} = 2$$

}

$$\mu_2 = \frac{1}{4} [x^{(1)} + x^{(5)} + x^{(6)} + x^{(10)}] \in \mathbb{R}^n \rightarrow \text{location of cluster centroid}$$

• Optimization objective

- K-means optimization objective

$\mu_c^{(i)} = \text{cluster centroid of cluster to which example } x^{(i)} \text{ has been assigned}$

$$x^{(i)} \rightarrow 5 \quad c^{(i)} = 5 \quad \mu_c^{(i)} = \mu_5$$

Optimization objective:

$$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K) = \frac{1}{m} \sum_{i=1}^m \|x^{(i)} - \mu_{c^{(i)}}\|^2$$

$$\min_{\substack{c^{(1)}, \dots, c^{(m)} \\ \mu_1, \dots, \mu_K}} J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$$

• Random initialization 如何避開局部最優 (local optima)

解決方法: 多次隨機初始化

For  $i = 1$  to 100 {  
     $\hookrightarrow 50-1000$

Randomly initialize K-means.

Run K-means. Get  $c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K$

\* Pick k distinct random integers

$i_1, \dots, i_k$  from  $\{1, \dots, m\}$

Set  $\mu_1 = x^{(i_1)}, \dots, \mu_k = x^{(i_k)}$

Compute cost function (distortion)

$$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_k)$$

}

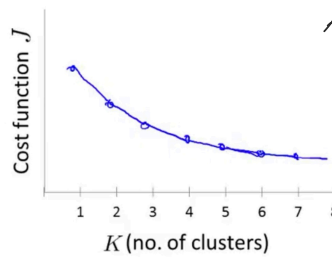
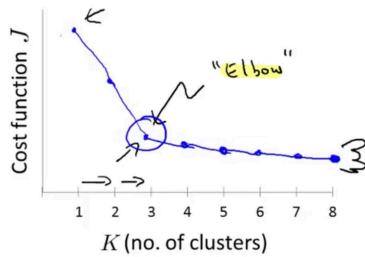
Pick clustering that gave lowest cost  $J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_k)$

$k = 2-10$

• Choosing the number of clusters

- Elbow method

↙ 大部份的情況很難用 Elbow method  
做出判斷



- Sometimes, you're running K-means to get clusters to use for some later / downstream purpose.

Evaluate K-means based on a metric for how well it performs for that later purpose.

→ 由人工決定