Complex Networks 2022

Node Vector Distances: Methods and Applications

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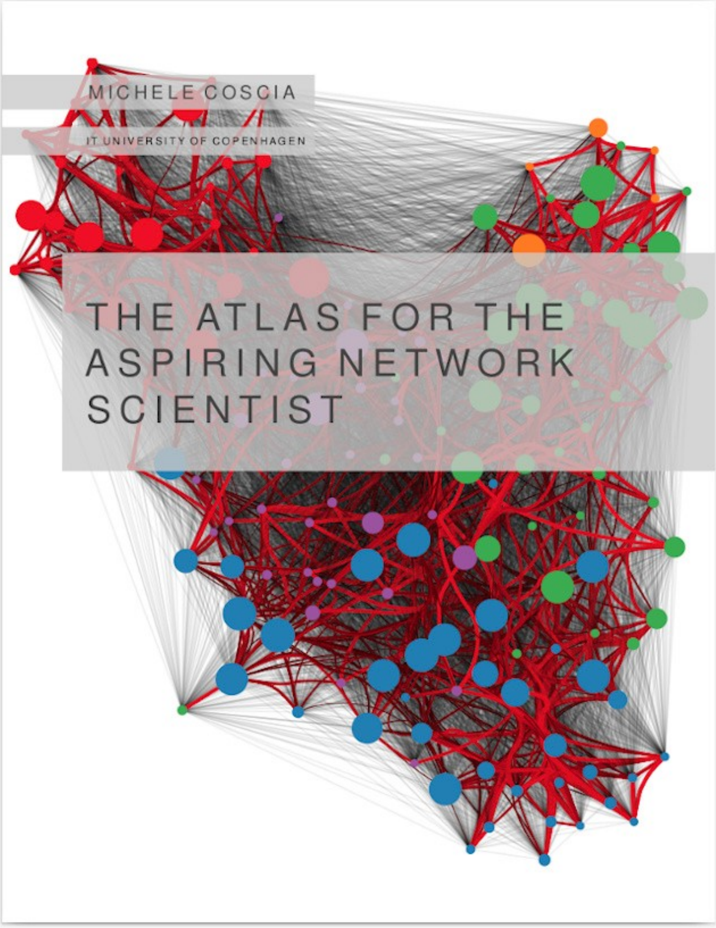
Download the Material

https://github.com/mikk-c/complexnetworks22-tutorial

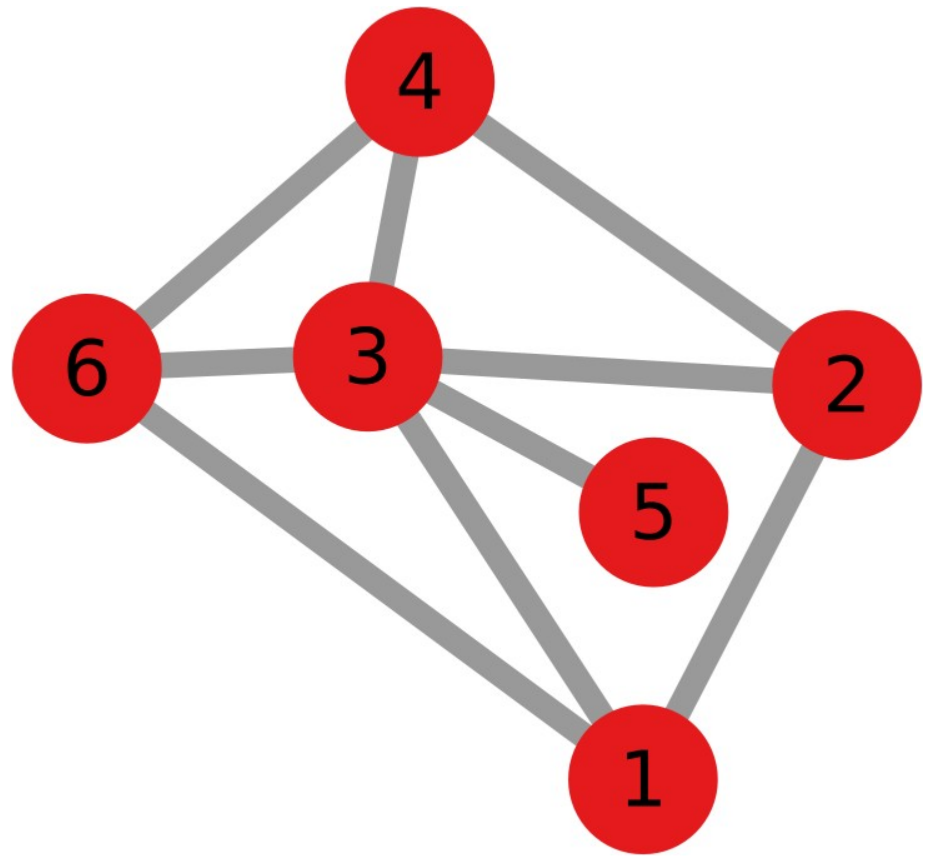
Readings

- The Atlas for the Aspiring Network Scientist
  - Chapter 40
  - https://www.networkatlas.eu/
- The node vector distance problem in complex networks, ACM Computing Surveys (CSUR) 53 (6), 1-27



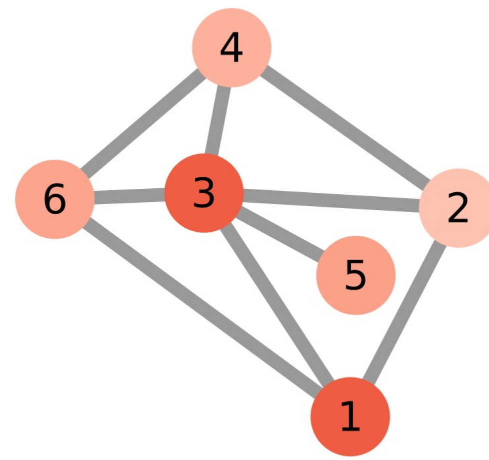
NVD: What?

- A graph G



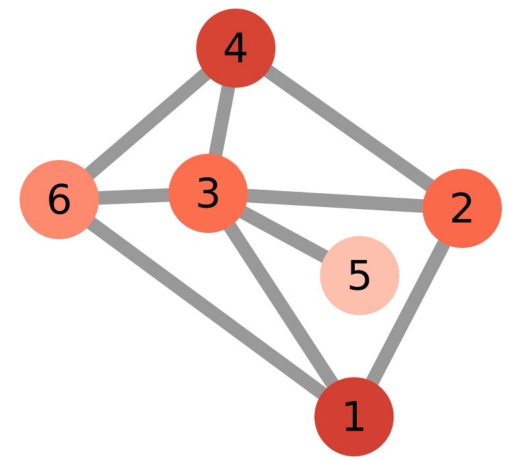
NVD: What?

- A graph G
- Two vectors v_1 and v_2
  - One value per node
- How far is v_1 from v_2?



0.57
0.13
0.57
0.20
0.27
0.25

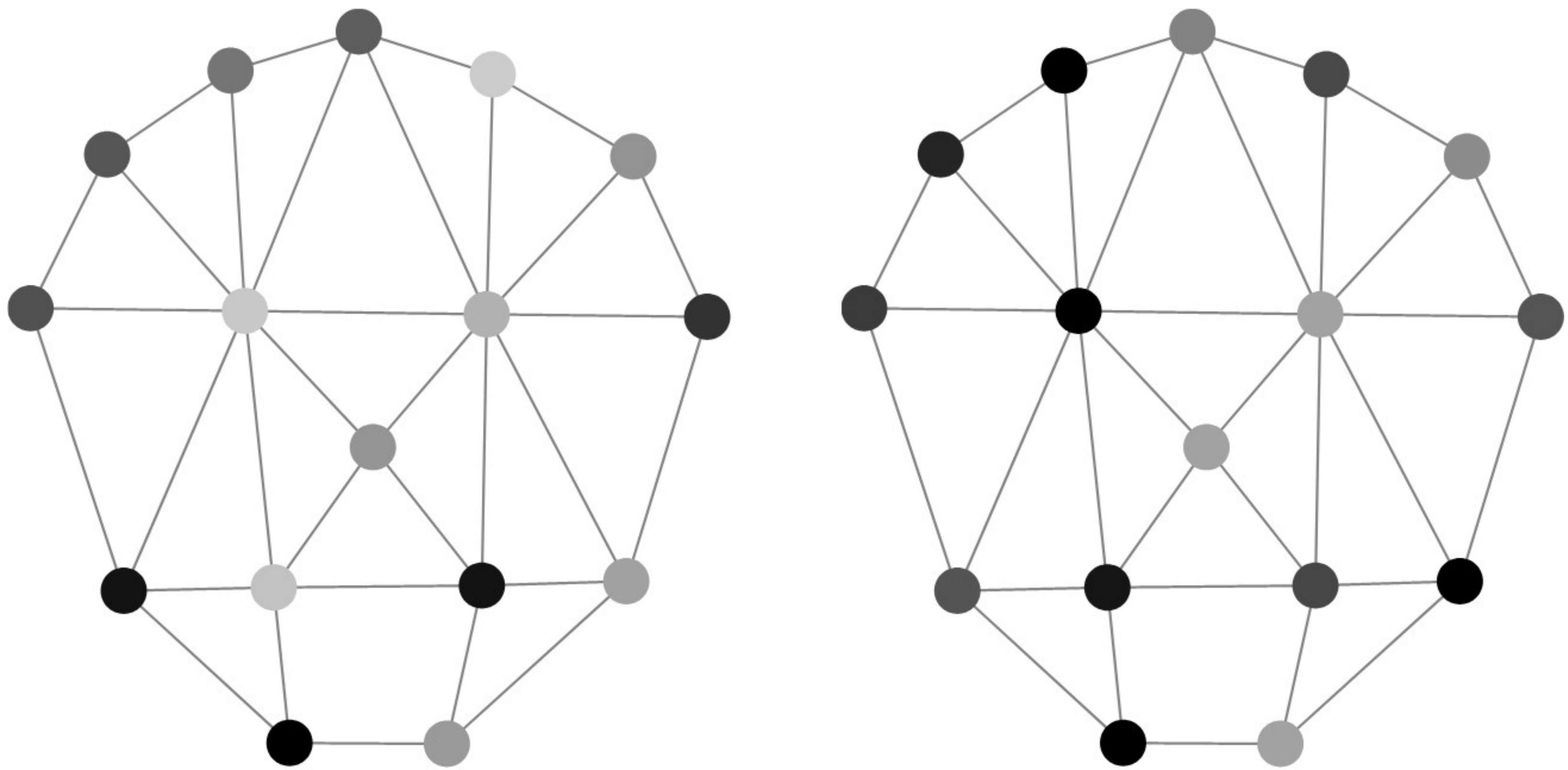
v_1



0.73
0.51
0.48
0.71
0.13
0.37

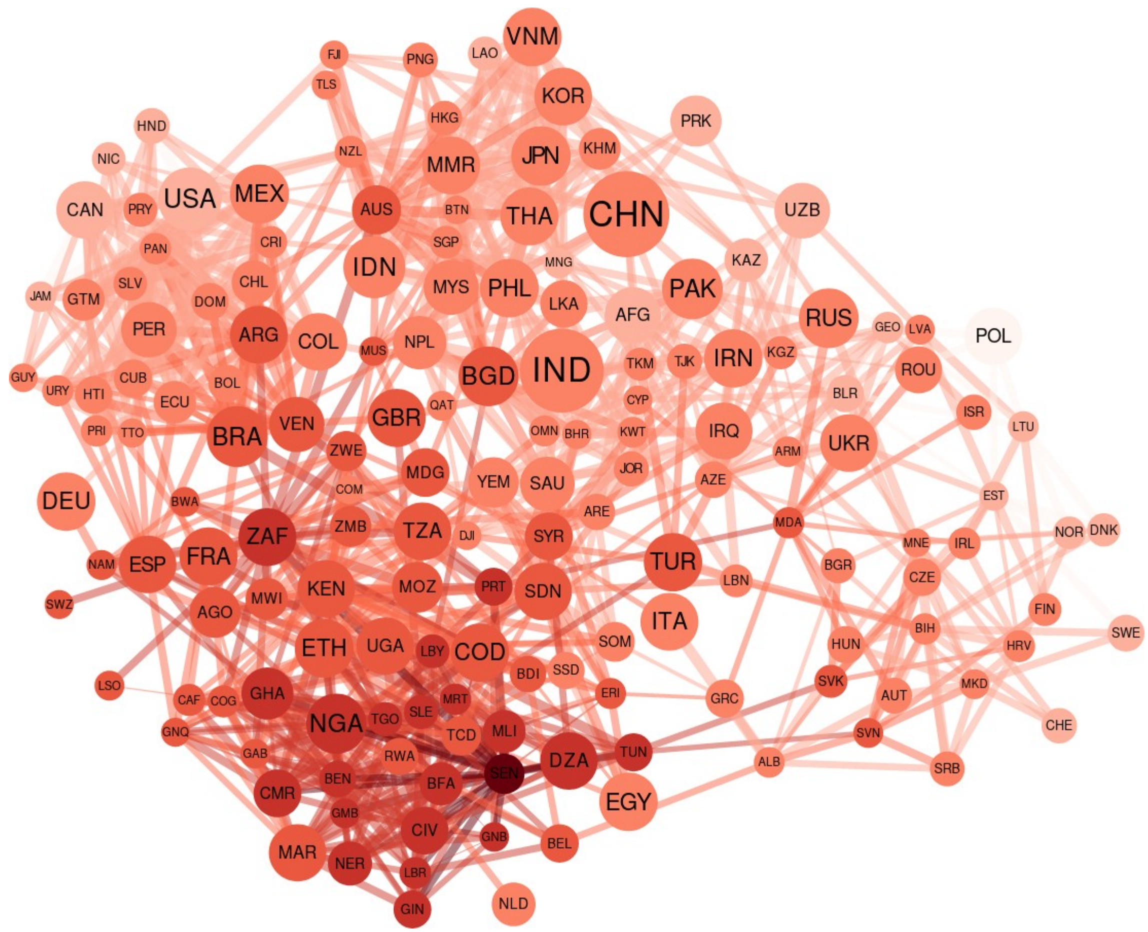
v_2

NVD: Why?



(a) Face #1 (b) Face #2

NVD: Why?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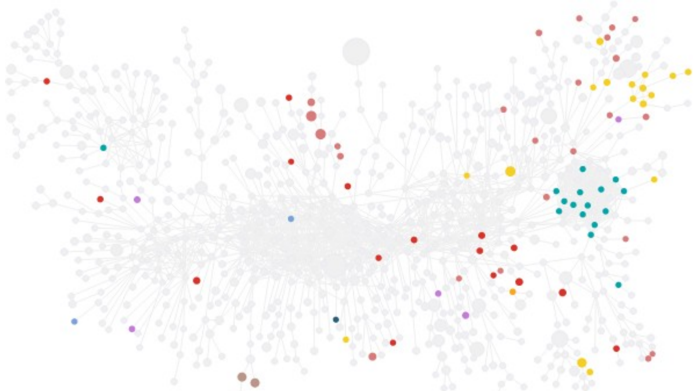
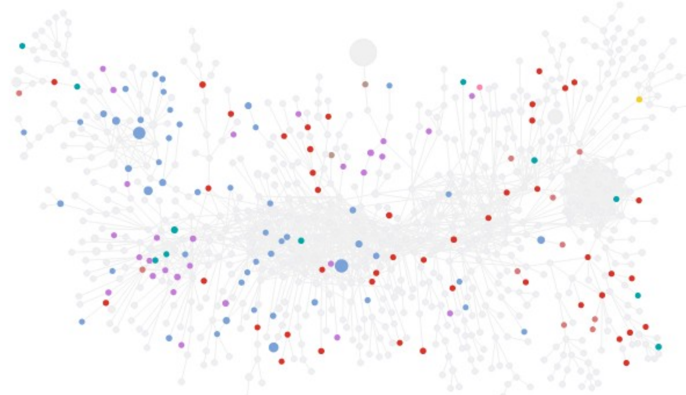
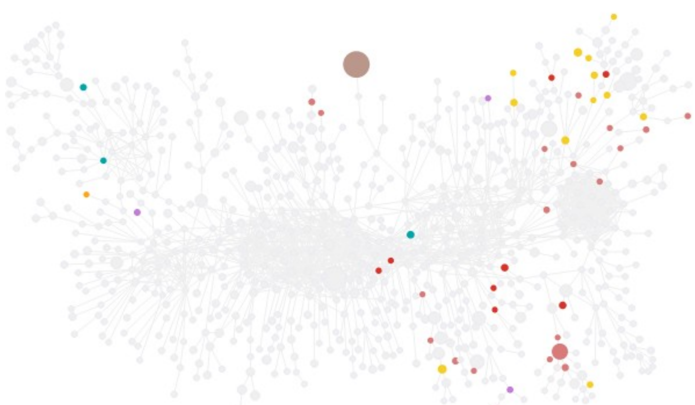
NVD: Why?



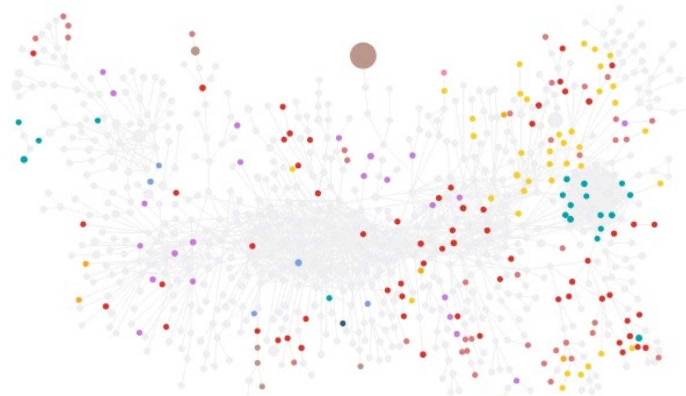
(a) Korea 1962



(b) Korea 2013

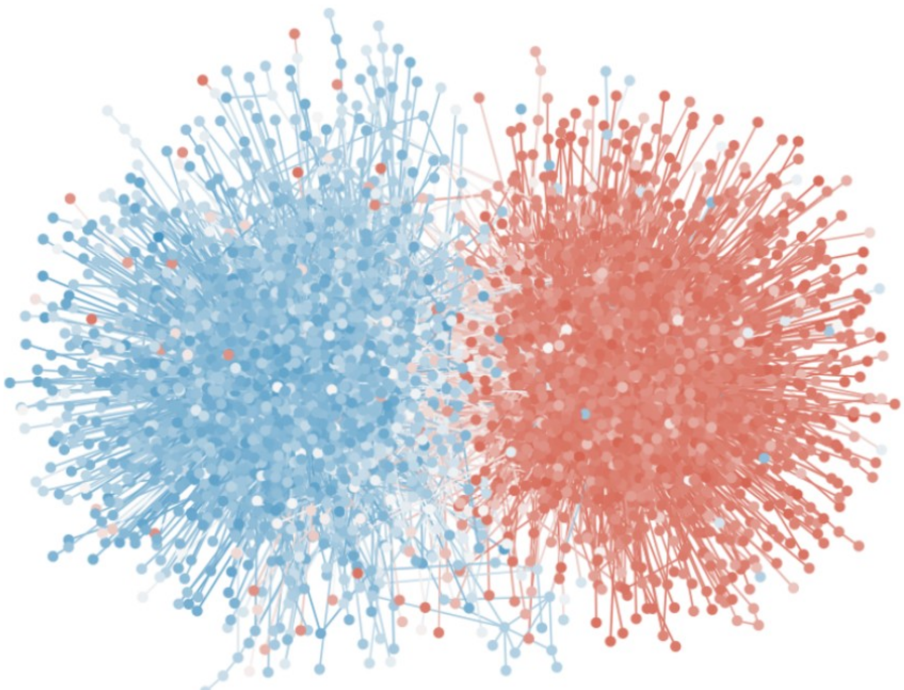


(c) Egypt 1962



(d) Egypt 2013

NVD: Why?



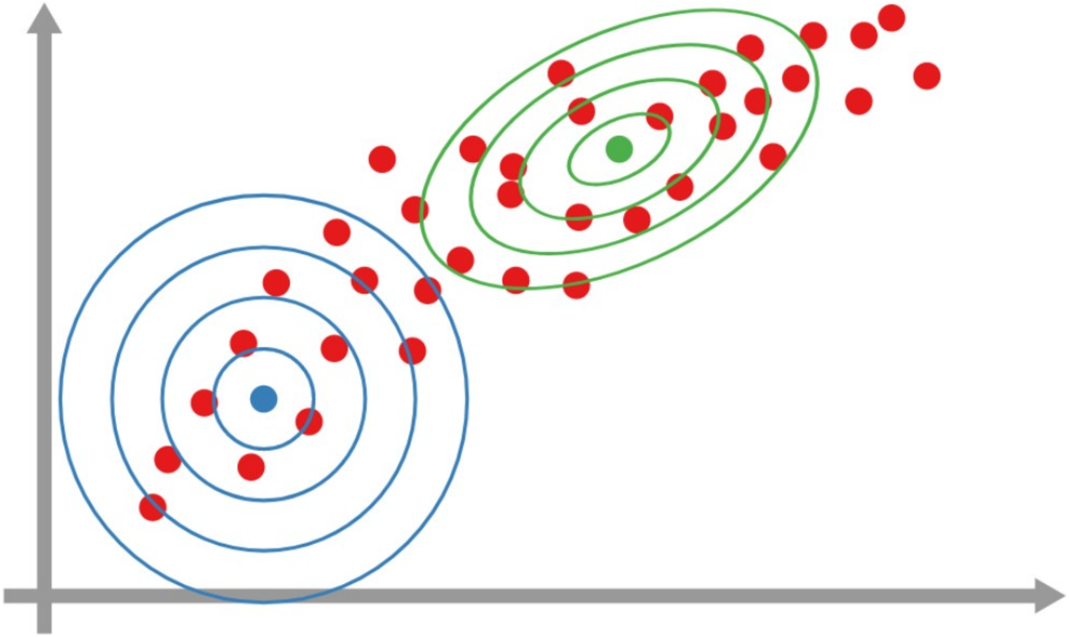
Re-Cap on Distances

- Euclidean “straight line”

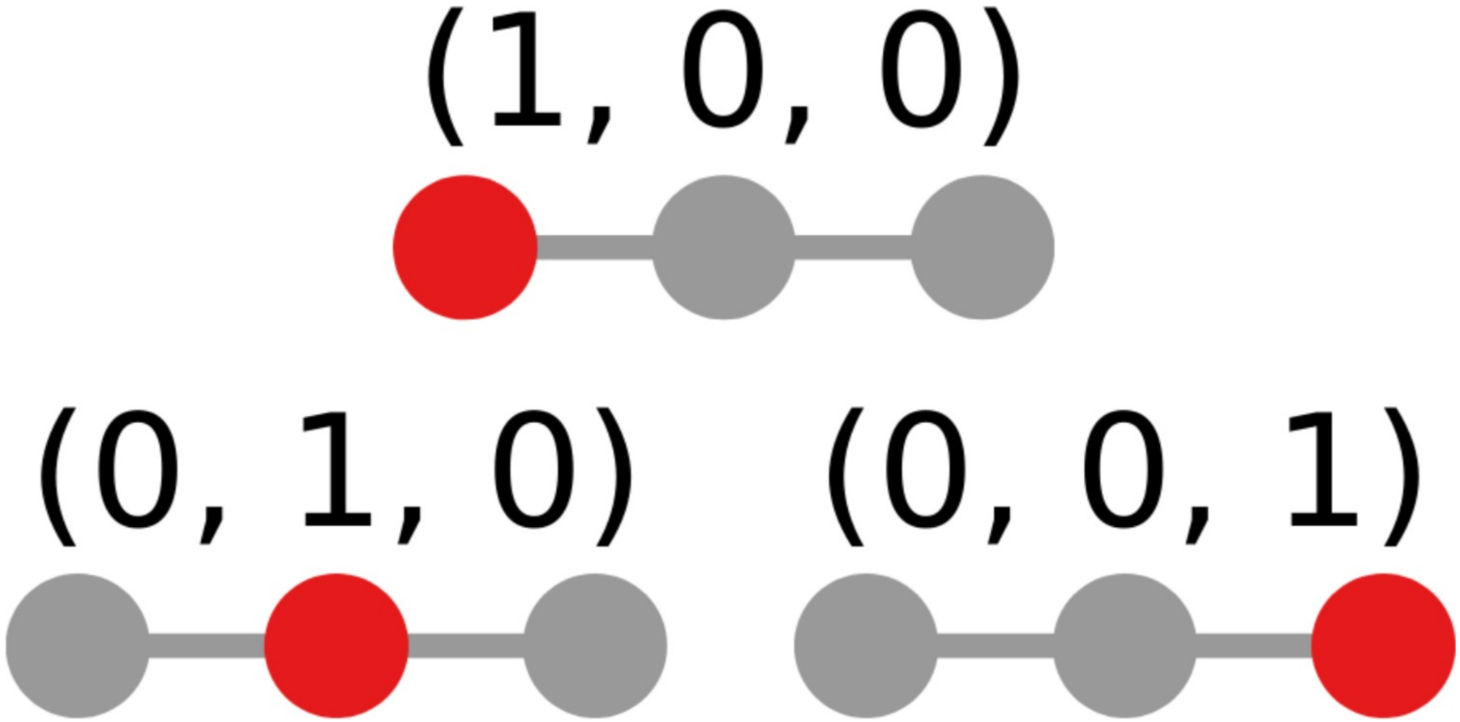
$$\sqrt{(p-q)^T I (p-q)}$$

- Mahalanobis “bendy” space

$$\sqrt{(p-q)^T \operatorname{cov}(p,q)^{-1} (p-q)}$$

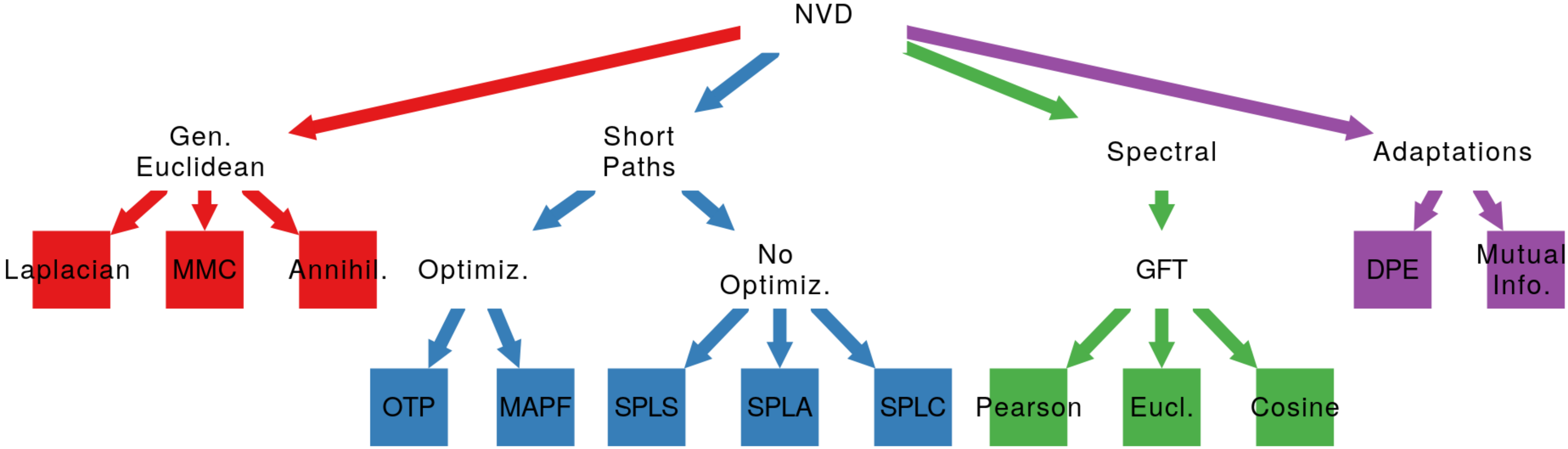


Euclidean vs Network



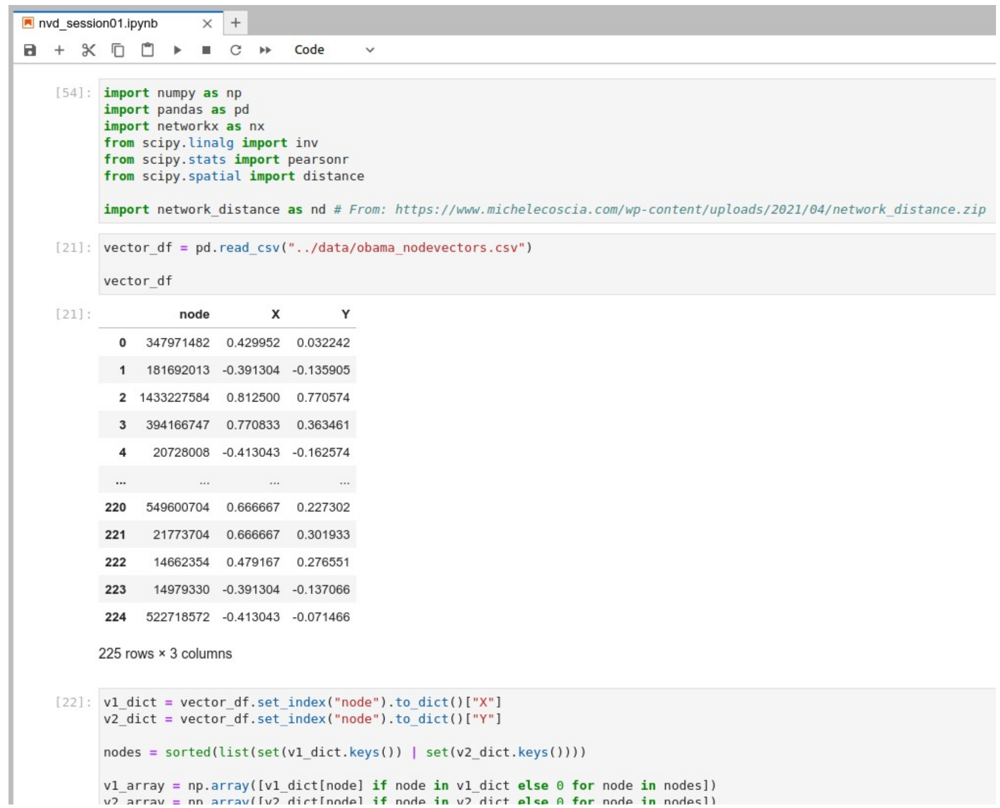
Same Euclidean distances, different network distances
The network “bends” the space, like in Mahalanobis

Current Methods



Tutorial Part #1

- Objectives:
  - Refresher on vector distances
  - Understand I/O of library
  - Set up the data




```python
import numpy as np
import pandas as pd
import networkx as nx
from scipy.linalg import inv
from scipy.stats import pearsonr
from scipy.spatial import distance

import network_distance as nd # From: https://www.michelecoscia.com/wp-content/uploads/2021/04/network_distance.zip
```



```python
vector_df = pd.read_csv("../data/obama_nodevectors.csv")

vector_df
```


	node	X	Y
0	347971482	0.429952	0.032242
1	181692013	-0.391304	-0.135905
2	1433227584	0.812500	0.770574
3	394166747	0.770833	0.363461
4	20728008	-0.413043	-0.162574
...	...	...	...
220	549600704	0.666667	0.227302
221	21773704	0.666667	0.301933
222	14662354	0.479167	0.276551
223	14979330	-0.391304	-0.137066
224	522718572	-0.413043	-0.071466

225 rows × 3 columns

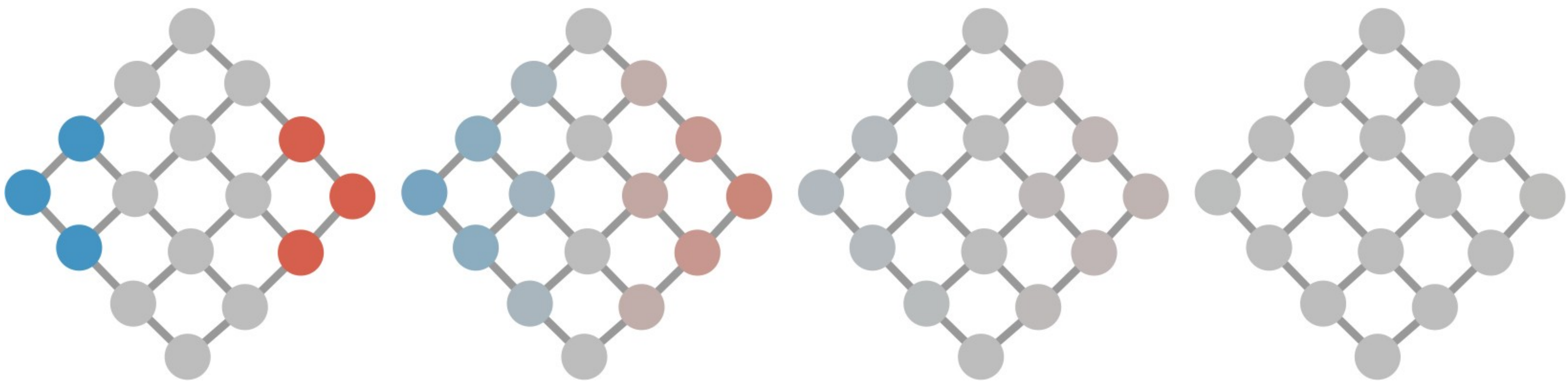

```python
v1_dict = vector_df.set_index("node").to_dict()["X"]
v2_dict = vector_df.set_index("node").to_dict()["Y"]

nodes = sorted(list(set(v1_dict.keys()) | set(v2_dict.keys())))

v1_array = np.array([v1_dict[node] if node in v1_dict else 0 for node in nodes])
v2_array = np.array([v2_dict[node] if node in v2_dict else 0 for node in nodes])
```

Generalized Euclideans

Heat Diffusion



- Imagine each node as a thermometer
- Connected to other thermometers to pass heat
- If we know the heat h at time t
- What would be the heat at t + 1?

$$\frac{\partial h}{\partial t} = -Lh$$

Laplacian

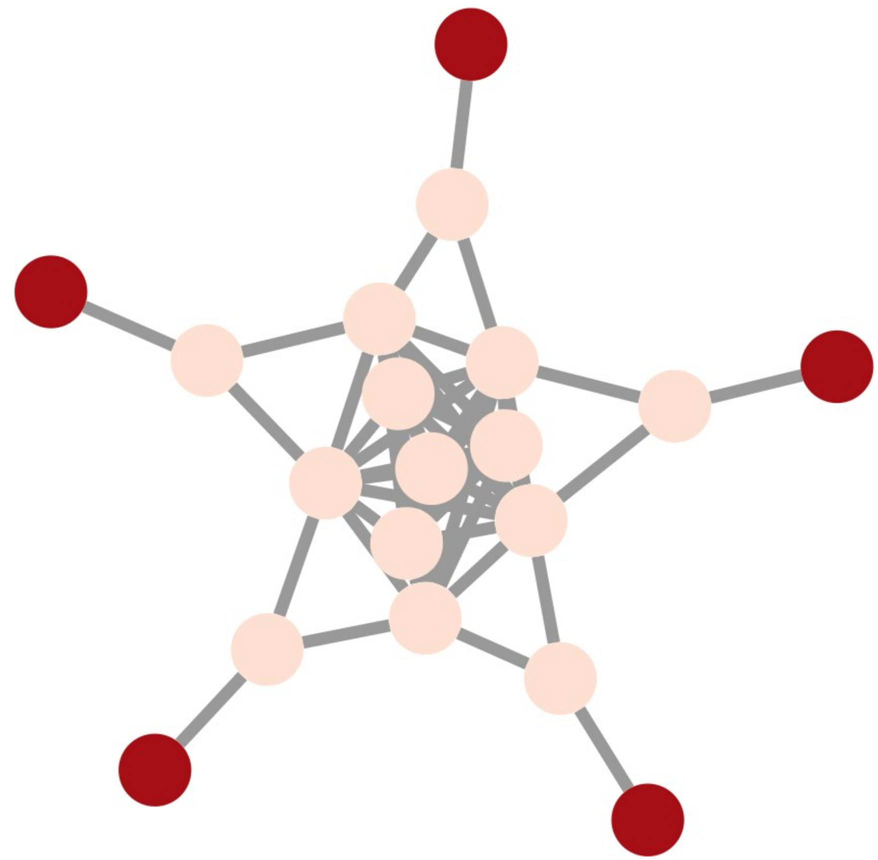
Generalized Euclidean: Laplacian

- L tells us how easy it is to pass heat between nodes
- Which is via random walks
- Thus its inverse gives a sense of distance

$$\sqrt{(p-q)^T L^{-1} (p-q)}$$

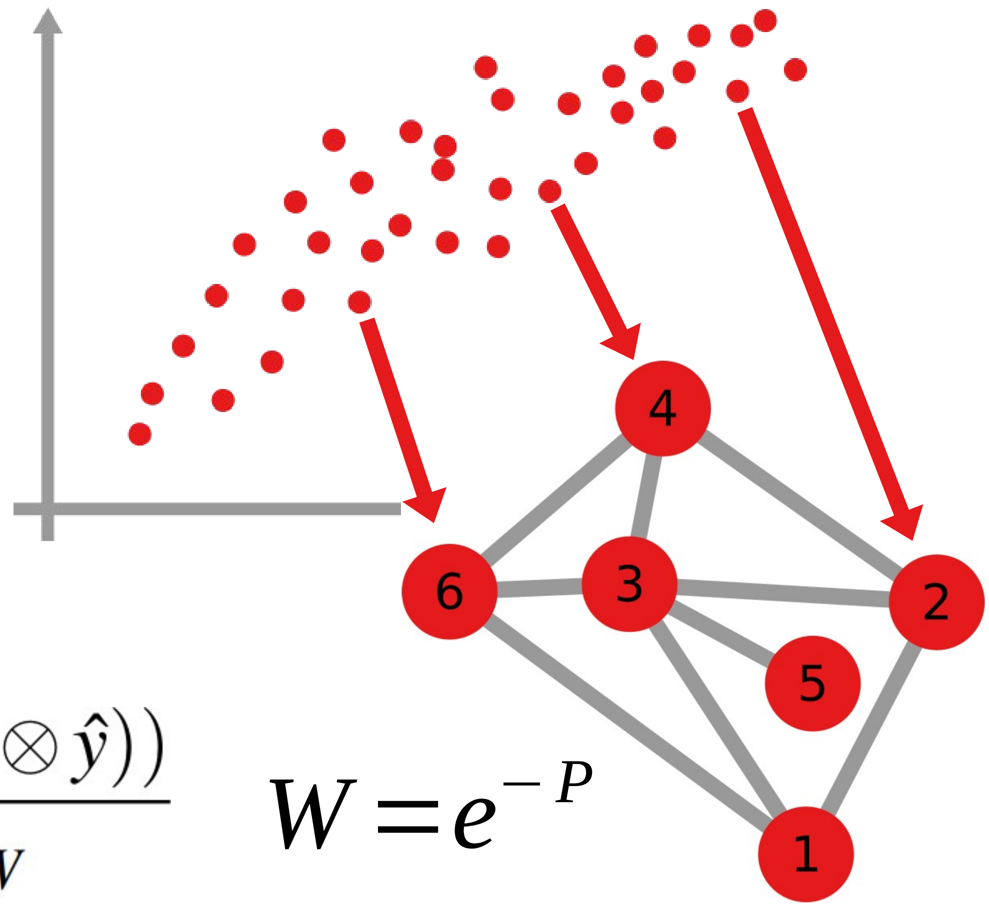
Network Variance

- Not only about distances!
- Vector v: how spread out is on the network?
- Same as variance, now network variance



Network Correlation

- How are two variables related?
- Pearson works in a flat space
- What if we have a network?
- E.g. correlation between age and sharing activity in a social network



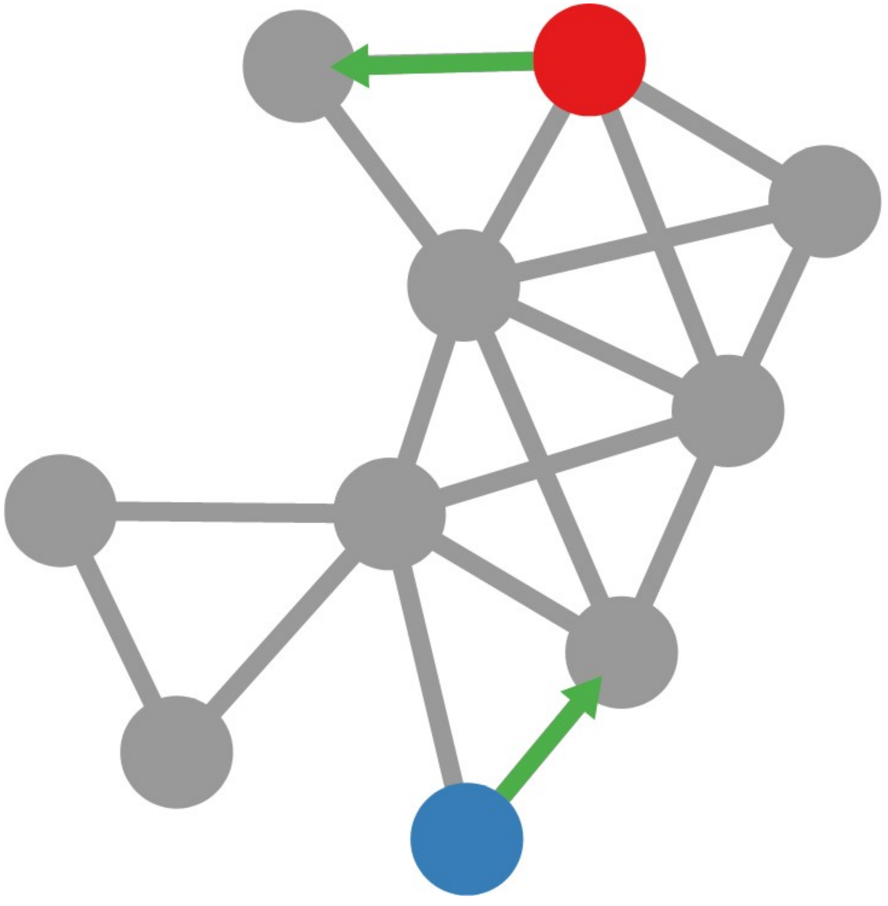
$$\rho_{x,y,G} = \frac{\mathrm{sum}(W \times (\hat{x} \otimes \hat{y}))}{\sigma_{x,W}\sigma_{y,W}}$$

$$W = e^{-P}$$

Random Walks

- If we move randomly, we expect bump into each other
- The later we do so from what we expected, the farther we were
- Z-scores!

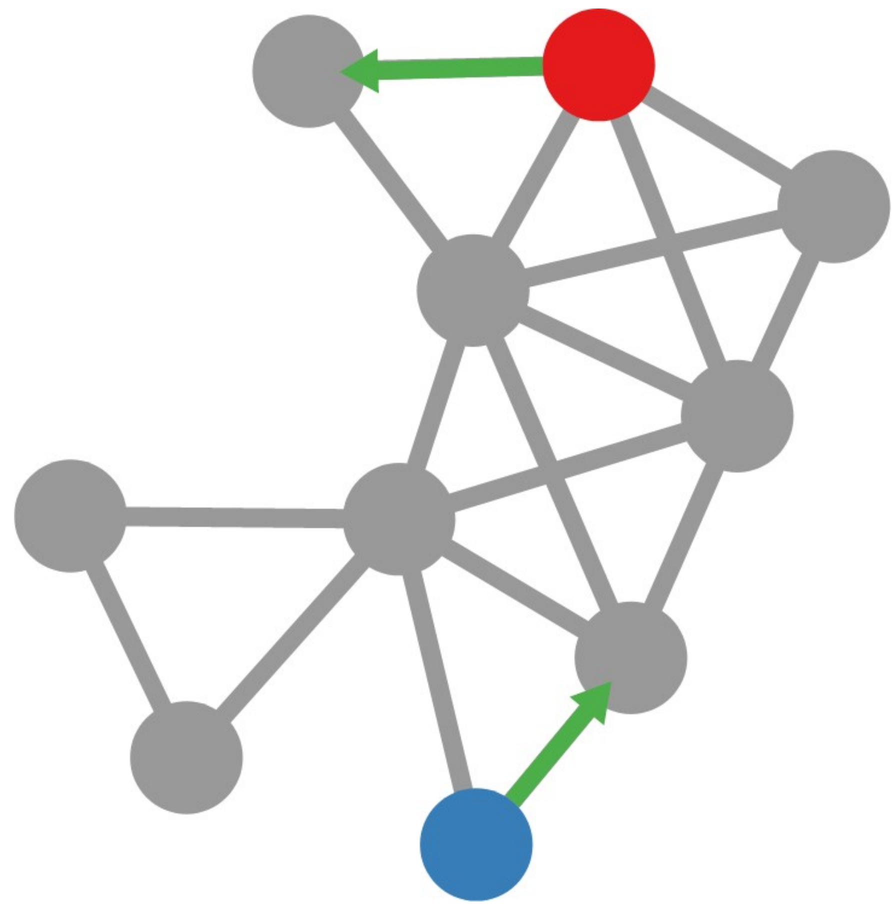
$$\sqrt{(p-q)^T Z^{-1} (p-q)}$$



Random Walks #2

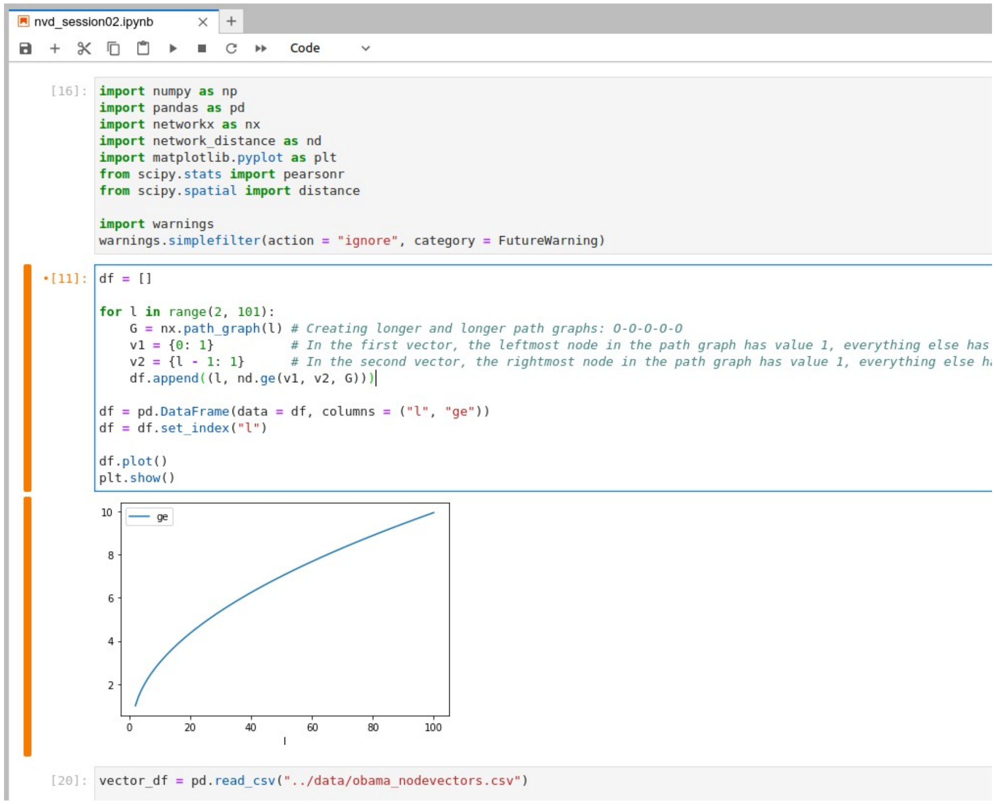
- A^k: p of going from v_1 to v_2 in k steps (RW)
- For which k would our vectors completely overlap each other?

$$\sqrt{(p-q)^T \sum_{k=0}^{\infty} A^k (p-q)}$$



Tutorial Part #2

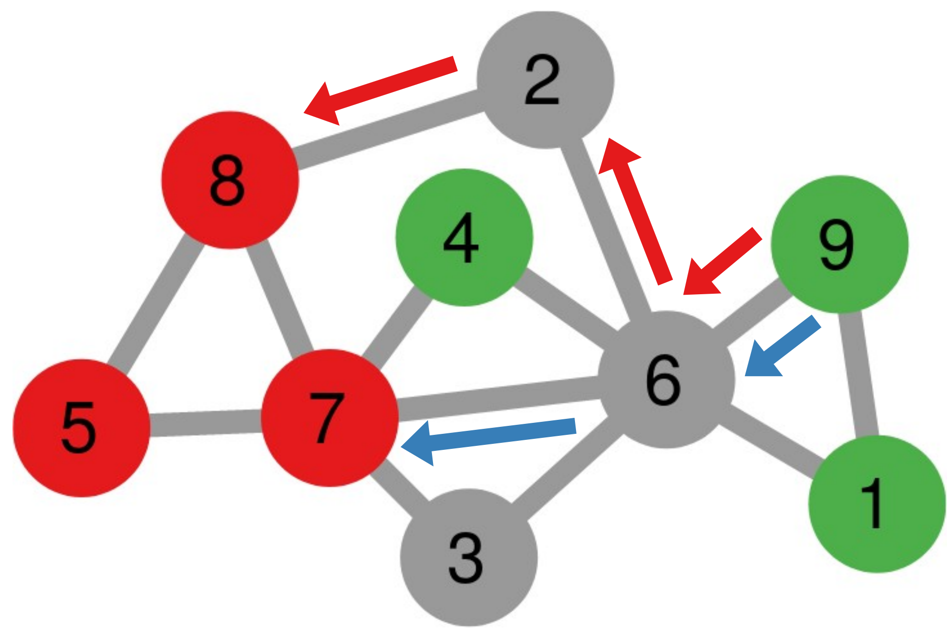
- Objectives:
  - Build an intuition of the measure with a simple toy experiment
  - Calculate Euclidean, variance, and correlation on a network
  - Calculate alternative Euclideans



Shortest Path Distances

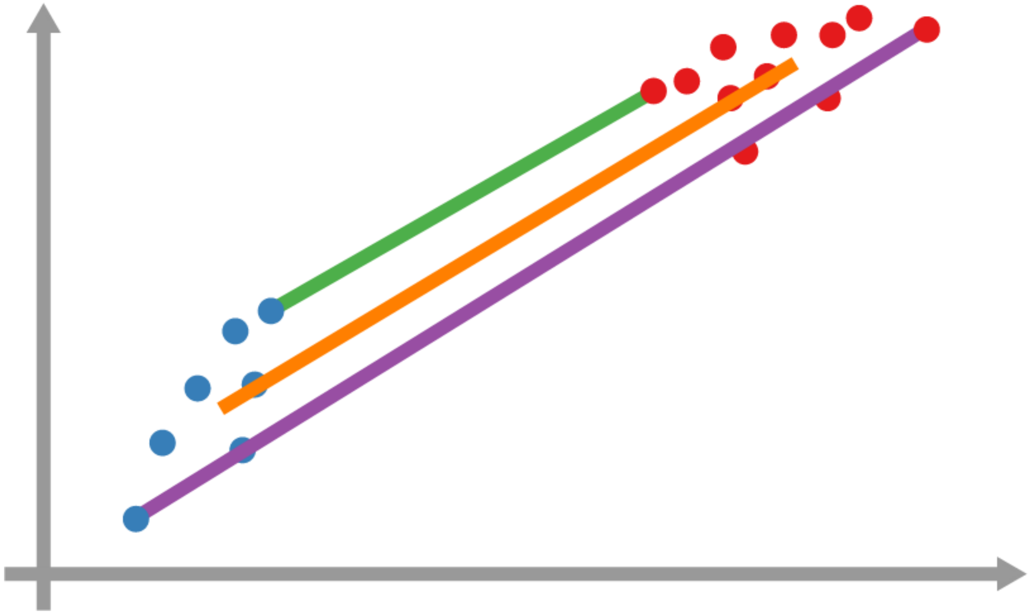
NVD Seems Easy

- Just calculate shortest paths!
- But then: which ones do we choose?



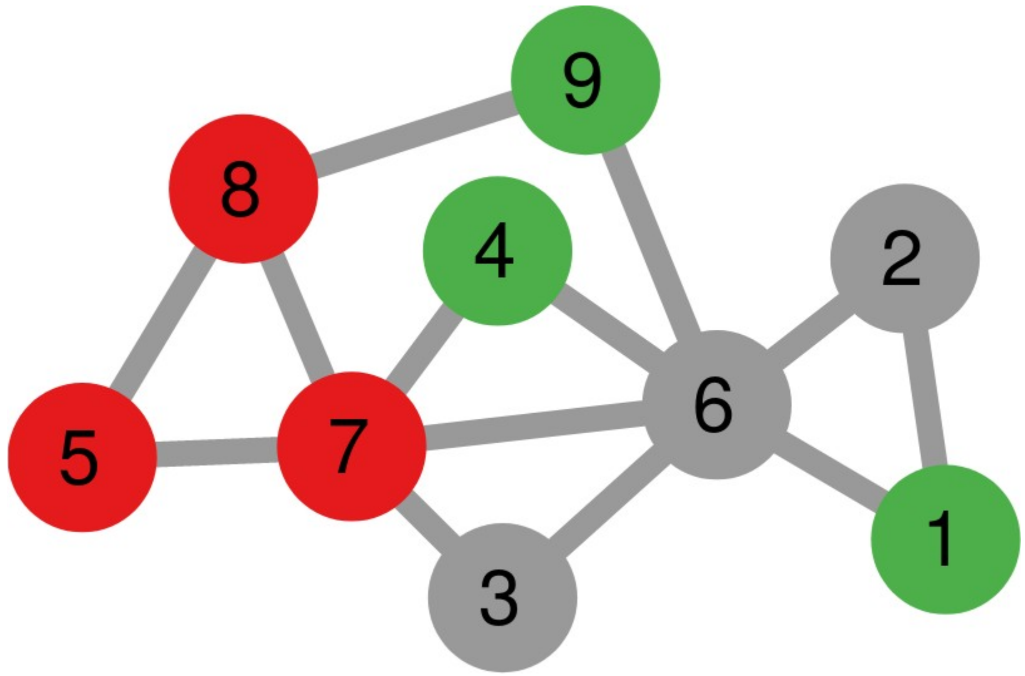
Non-Optimized

- **Single** linkage
  - Always pick the shortest available
- **Complete** Linkage
  - Always pick the longest available
- **Average** Linkage
  - Pick them all and weigh the alternatives



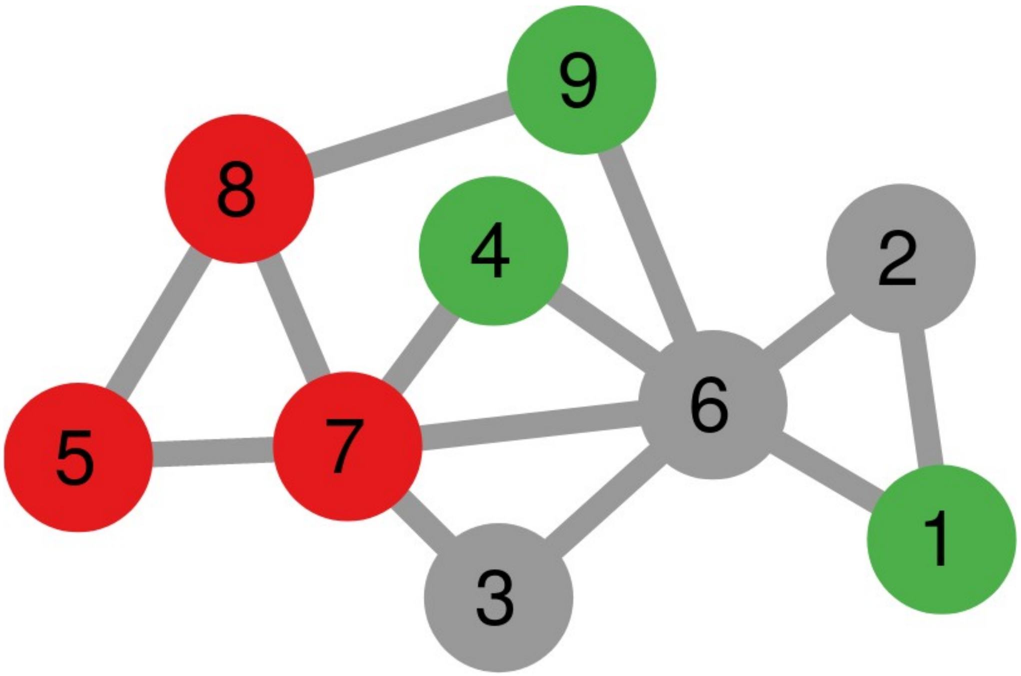
Non-Optimized

- Single Linkage
  - 1 + 1 + 3
- Complete Linkage
  - 3 + 2 + 2
- Average Linkage
  - 18 / 3



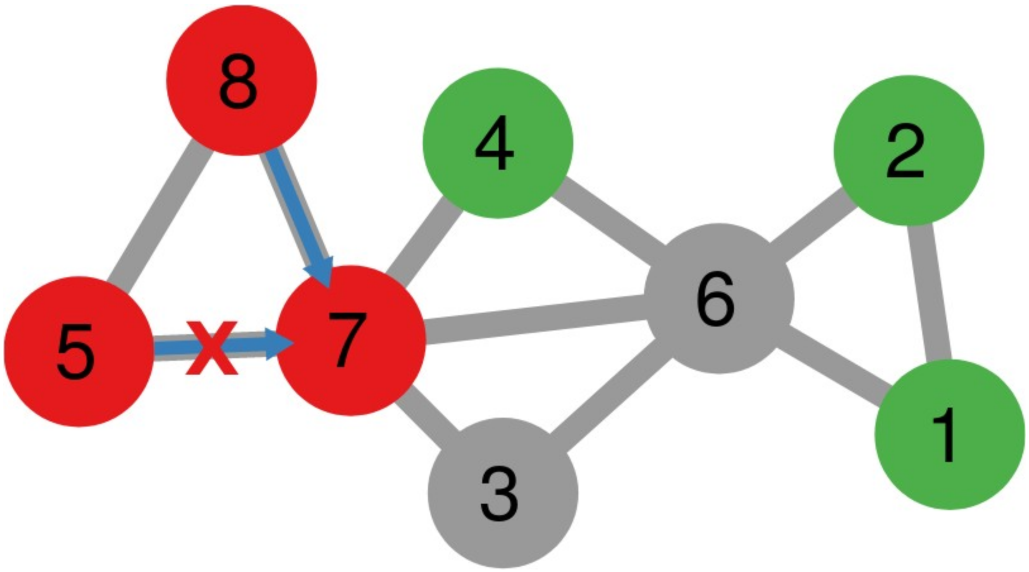
Earth Mover Distance

- Find the best set of paths
- Minimize the cost
- Similar to single linkage



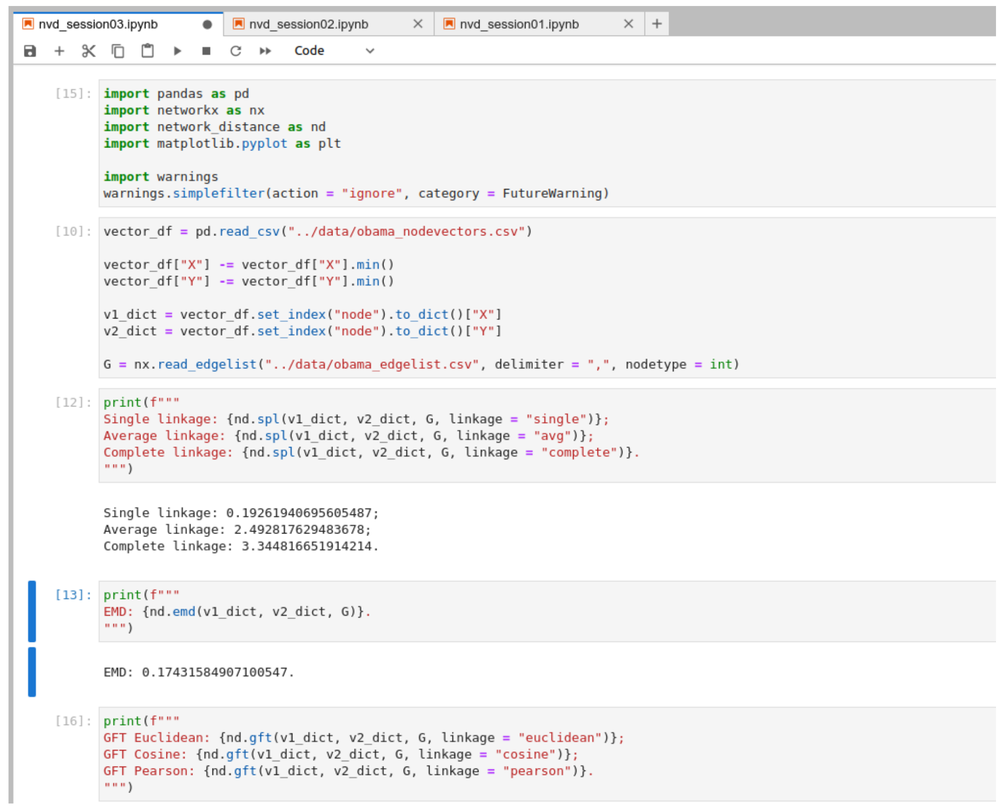
Multi Agent Path Finding

- Similar to EMD
- With constraints
- Nodes & edges have capacity limits
- E.g. cannot use the same link at the same time



Tutorial Part #3

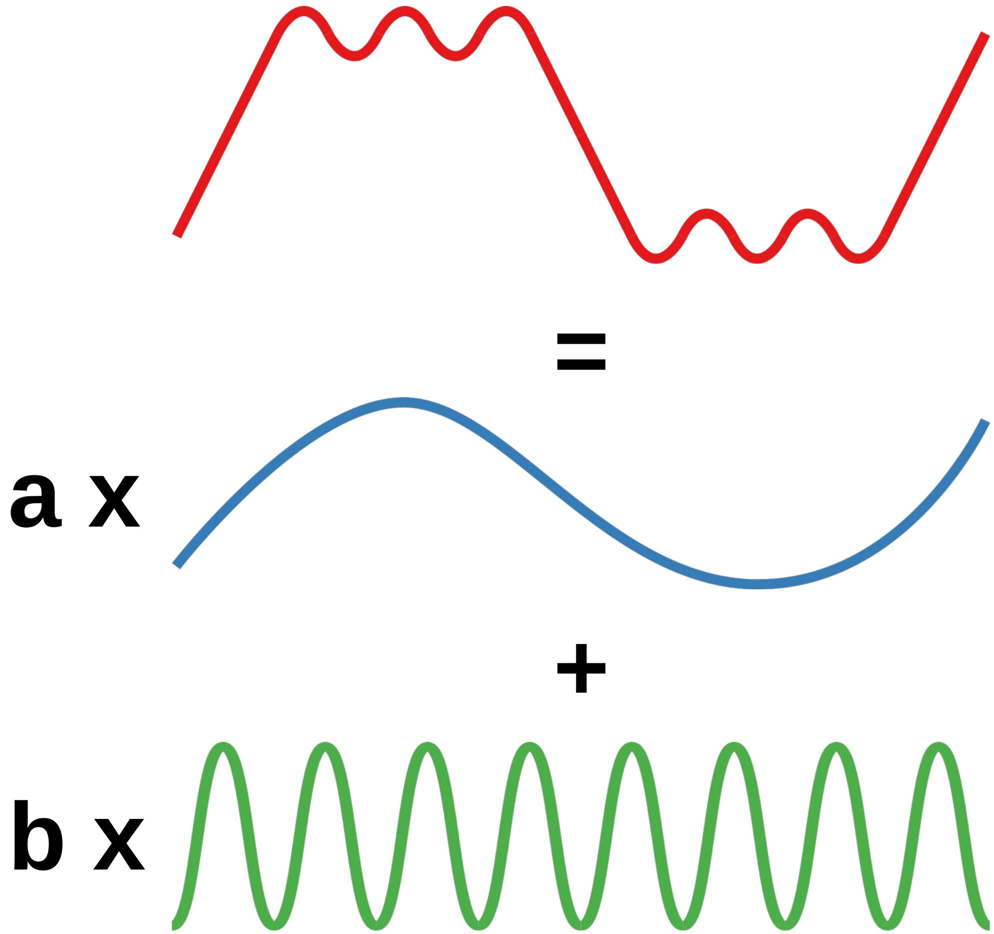
- Objectives:
  - Experiment with different linkage criteria
  - Use optimal EMD solution
  - (Skipping ahead) Test spectral distance



Spectral Methods

Graph Fourier Transform

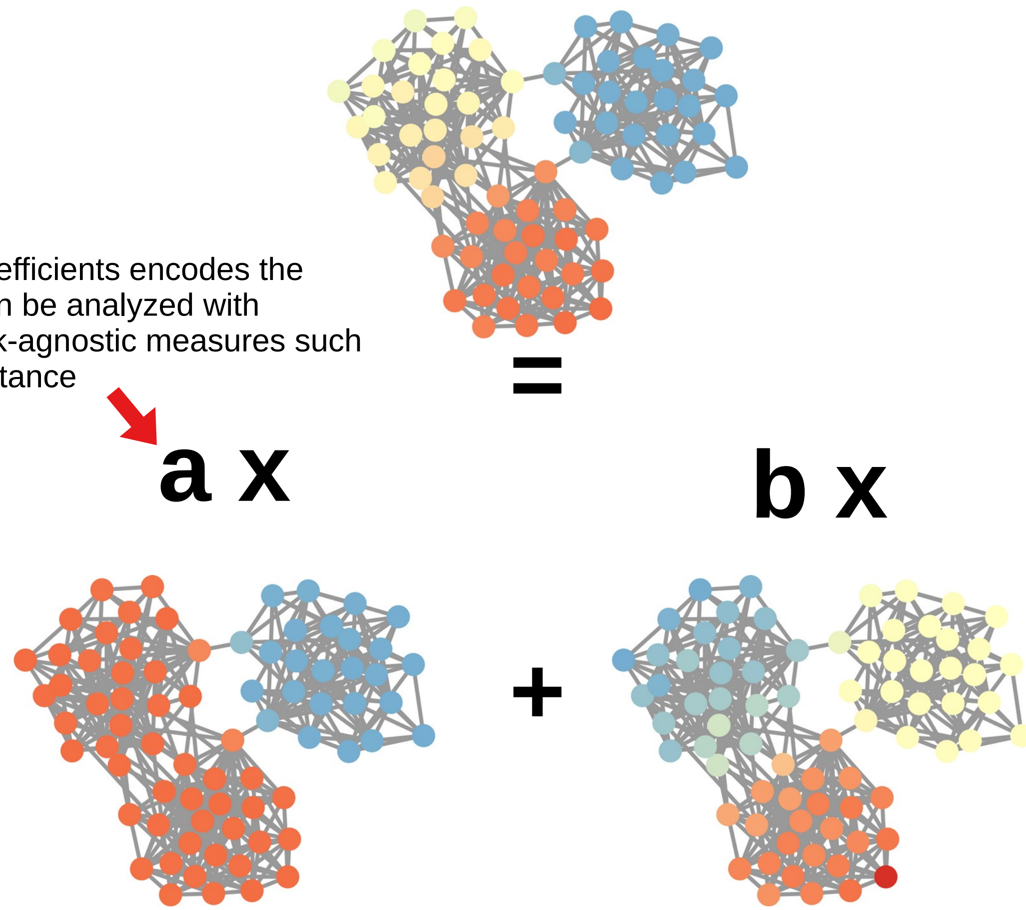
- FT: decompose any signal into its component frequencies



Graph Fourier Transform

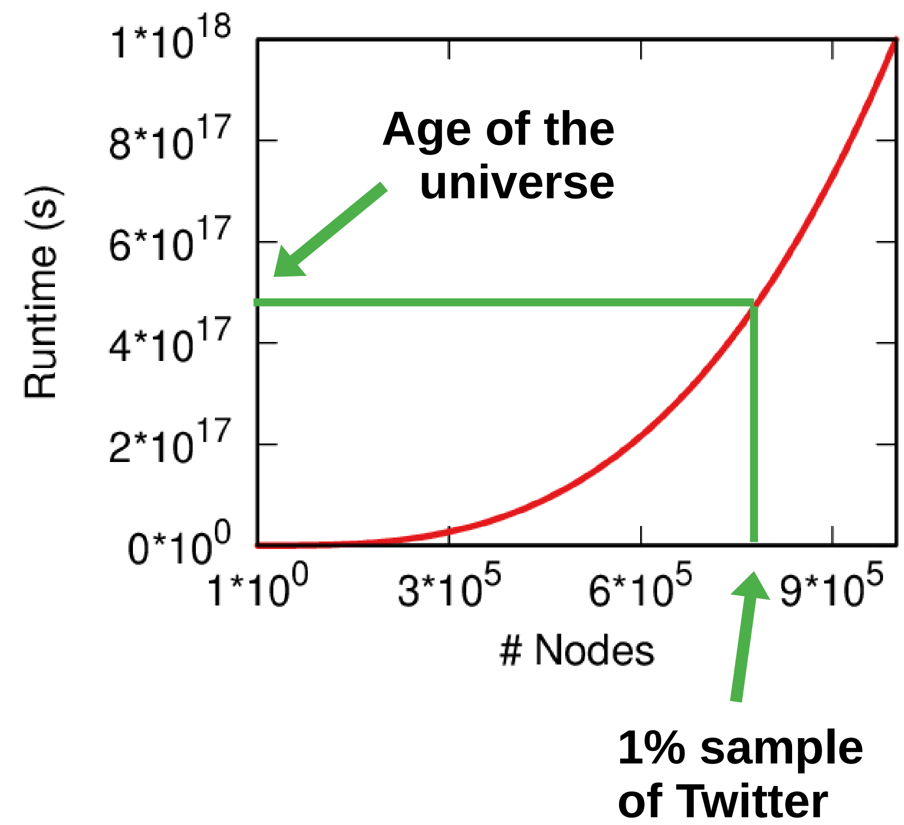
- FT: decompose any signal into its component frequencies
- GFT: same, but the components are the eigenvectors of the graph
- Remember: eigenvectors encode structure

The vector of coefficients encodes the structure and can be analyzed with classical network-agnostic measures such as Euclidean distance



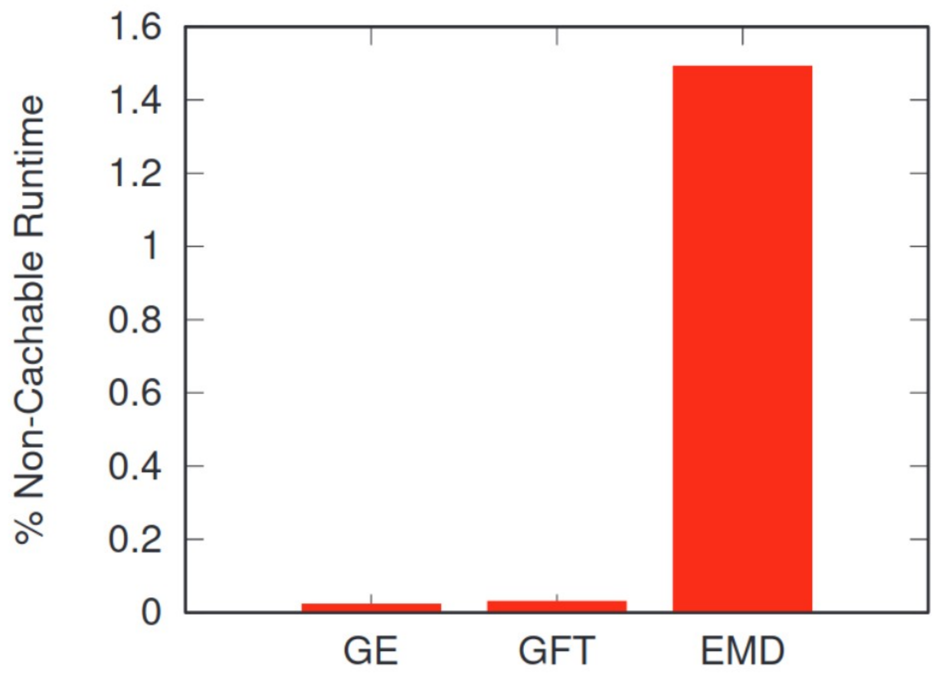
Computational Efficiency

- Expensive steps can be $O(|V|^3)$…
  - Pseudoinverse for GE
  - Shortest paths
  - Eigenvectors of Laplacian



Some Tricks

- Expensive part needs to be done once per network
- You can re-use it for all node vector pairs you have
- E.g. disease spreading on an unchanging social network



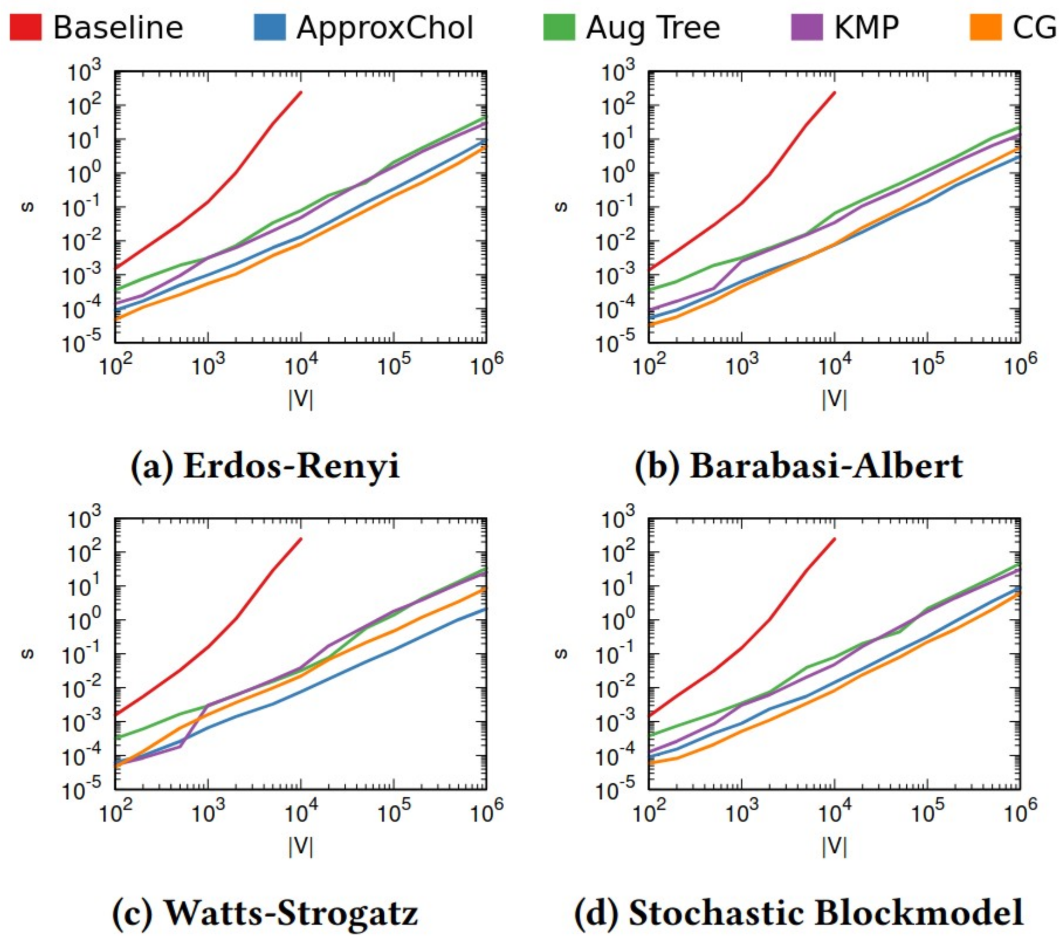
Laplacian Solvers

$$\sqrt{(p-q)^T \boxed{L^{-1}(p-q)}}$$

This part can be calculated efficiently

Without pseudo-inverting the Laplacian!

With Laplacian solvers (in Julia)



Tutorial Parts #4 & #5

- Objectives:
  - Testing runtimes
  - Experimenting with Laplacian solvers
  - Trying out some application scenarios