Mellin transform

Definition 0.1 (Mellin transform): Let $f(t)$ be defined on $0 < t < \infty$. $\mathcal{M}$ is the Mellin transformation mapping $f \to F$ defined on the complex plane by:

$$\mathcal{M}[f; s] \equiv F(s) = \int_0^\infty f(t) t^{s-1} \, \mathrm{d}t$$

with $F(s)$ being the transform of f.

In general this integral exists for complex values $s = a + ib$ where $a_1 < a < a_2$ and a_1, a_2 depend on $f(t) \longrightarrow$ strip of definition $S(a_1, a_2)$, this would be the whole plane if $a_1 = -\infty$ and $a_2 = +\infty$.

Example 0.1: Given

$$f(t) = H(t - t_0) t^z$$

then

$$\mathcal{M}[f; s] = -\frac{t_0^{z+s}}{z + s}$$

Example 0.2: Given

$$f(t) = e^{-pt} \quad p > 0$$

then

$$\mathcal{M}[f; s] = p^{-s} \Gamma(s)$$

by definition of the gamma function.