Project 1: Computational Physics - FYS3150

Fredrik Hoftun & Mikkel Metzsch Jensen

September 09, 2020

1 Introduction

In this project we will investegate different approaches to solve the one-dimensional Poisson equation with Dirichlet boundary conditions given as follows:

$$u''(x) = f(x), \quad x \in (0,1), \quad u(0) = u(1) = 0$$

We will rewrite this as a set of linear equations, and solve it by a number of different computational approaches on either gaussian elimination or LU decomposition. We will solve the equation above with the function:

$$f(x) = 100e^{-10x}$$

Where the analytical solution then is given as:

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x}$$

We will use the analytical solution to evaluate the precision of the numerical solutions for different steplength between the discretized gridpoints x_i.

2 Method

2.1 Rewritting the equation as a set of linear equations

2.2 General solution using Gausian elimination

Forward / Backward sub FLOPS

2.3 Simplified problem specific solution

FLOPS

2.4 LU decomposition

FLOPS

2.5 Comparing precision and error

3 Implementation?

4 Results

5 Concluding remarks

6 Part a

The solution can be shown be doing the following rewritting of the Poisson equation:

$$-u''(x_i) = f(x_i)$$

$$-\frac{v_{i+1} + 2v_i - v_{i+1}}{h^2} = f(x_i)$$

$$-v_{i-1} + 2v_i - v_{i+1} = h^2 f(x_i)$$

As we try to setup the equation for f($\mathbf{v}$) for each individual component the matrix A starts to appear.

$$\begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} v_0 \\ \vdots \\ v_{n+1} \end{bmatrix} = h^2 f(x_0)$$

$$\begin{bmatrix} 2 & -1 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & \cdots \end{bmatrix} \begin{bmatrix} v_0 \\ \vdots \\ v_{n+1} \end{bmatrix} = h^2 \begin{bmatrix} f(x_0) \\ f(x_1) \end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix} 2 & -1 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & \cdots \\ 0 & -1 & 2 & -1 & 0 & \cdots \\ & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & & -1 & 2 & -1 \\ 0 & \cdots & & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_0 \\ \vdots \\ v_{n+1} \end{bmatrix} = h^2 \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f_{n+1} \end{bmatrix}$$

By using the definition from the assignment description we arrive at the the expression

$$\mathbf{Av} = \tilde{\mathbf{b}}$$

We assume that $f(x) = 100e^{-10x}$. The solution is given to be $u(x) = 1 - (1 - e^{-10})x - e^{-10x}$. We can ensure that this is true by inserting it into the Poisson equation. We first find the double derivative of u(x):

$$u'(x) = -(1 - e^{-10}) + 10e^{-10x}, \quad u''(x) = -100e^{-10x}$$

We now see that the solution satisfy the Poisson equation:

$$-u''(x) = 100e^{-10x} = f(x)$$