Project 1: Computational Physics - FYS3150

Fredrik Hoftun & Mikkel Metzsch Jensen

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1 Introduction

In this project we will investegate different approaches to solving the one-dimensional

2 Method

3 Implementation?

4 Results

5 Concluding remarks

6 Part a

The solution can be shown be doing the following rewritting of the Poisson equation:

$$-u''(x_i) = f(x_i)$$

$$-\frac{v_{i+1} + 2v_i - v_{i+1}}{h^2} = f(x_i)$$

$$-v_{i-1} + 2v_i - v_{i+1} = h^2 f(x_i)$$

As we try to setup the equation for f($\mathbf{v}$) for each individual component the matrix A starts to appear.

$$\begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} v_0 \\ \vdots \\ v_{n+1} \end{bmatrix} = h^2 f(x_0)$$

$$\begin{bmatrix} 2 & -1 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & \cdots \end{bmatrix} \begin{bmatrix} v_0 \\ \vdots \\ v_{n+1} \end{bmatrix} = h^2 \begin{bmatrix} f(x_0) \\ f(x_1) \end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix} 2 & -1 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & \cdots \\ 0 & -1 & 2 & -1 & 0 & \cdots \\ & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & & -1 & 2 & -1 \\ 0 & \cdots & & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_0 \\ \vdots \\ v_{n+1} \end{bmatrix} = h^2 \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f_{n+1} \end{bmatrix}$$

By using the definition from the assignment description we arrive at the the expression

$$\mathbf{A}\mathbf{v} = \tilde{\mathbf{b}}$$

We assume that $f(x) = 100e^{-10x}$. The solution is given to be $u(x) = 1 - (1 - e^{-10})x - e^{-10x}$. We can ensure that this is true by inserting it into the Poisson equation. We first find the double derivative of u(x):

$$u'(x) = -(1 - e^{-10}) + 10e^{-10x}, \quad u''(x) = -100e^{-10x}$$

We now see that the solution satisfy the Poisson equation:

$$-u''(x) = 100e^{-10x} = f(x)$$