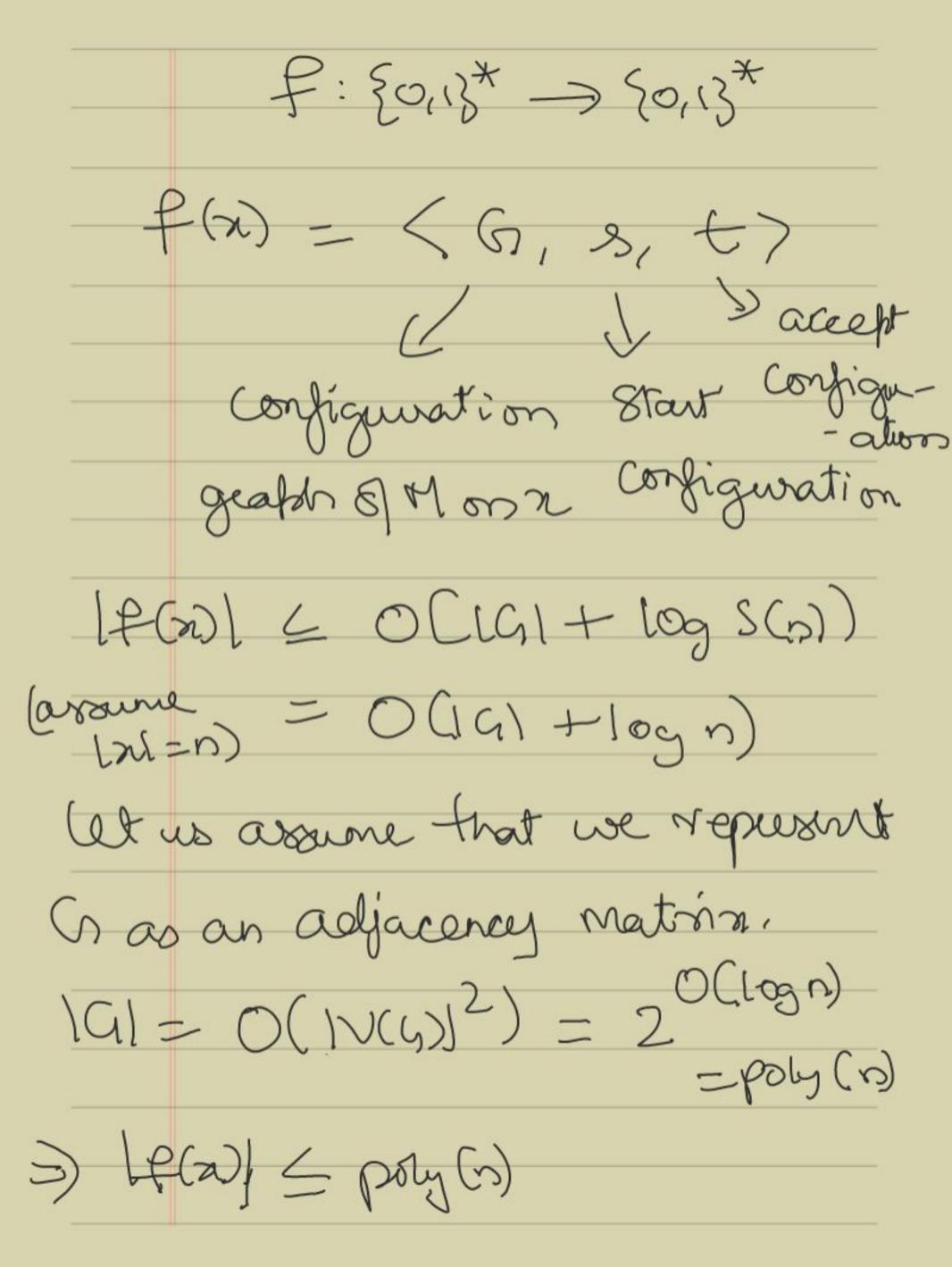
Cast Week: Space Complexity LCNLCPCNPSPACE We know LG PSPACE Cinfactallo
NLG PSPACE) but suspect most (all?) inclusions (i.e L G NL G P G NP G PSPACE) Antagonist in toclays lecture: NL NL = ELSSO, 13\* I I decided by an Space 0 (10g n) }

Recall: Reductions is NI are defined via implicitly compu -table Coj-space functions le f: {0,1}\* -> {0,1}\* such that (1) LPGO) ( poly ( lai) (2) {(xi) | P(x) = 1) EL 3 {(x,i) | i 4 1 f co) } EL A GB: At reduces to B via an (inplicitly computable) Coyspace reduction plogspace Probuties: 1) ACEB LBGL=)AGL (2) ALQBEBLLC=) ALQC

Thin 1: Path = E(G, S, t) | Japoth Jeon stot Jeon stot in 93 is NL-complete. Cira log-space reduction) Need to show 2 things 1) Path ENL 2) Any Conjugge AGNL veduces to Path. For (2), fix A ENL & let M & den NTM woing space SGn= OGogns) decidens A. Define the reduction as follows:

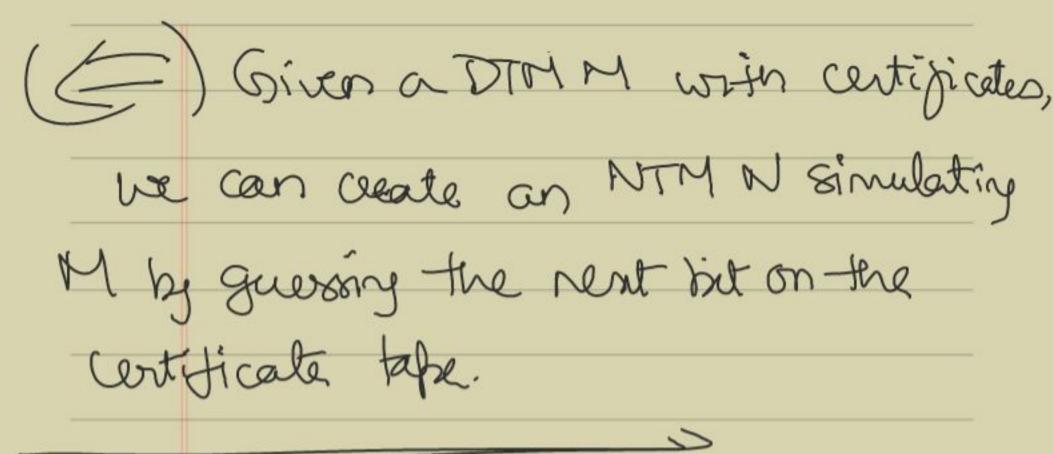


The Cength of & (a) is easily confaitable in space O (bogs) Leaucisi Finally, to show of is implicitly Loy space computable, we need to able to compute any but I) G, sont in space Ollogs). -) 8 let all easy (they are only O agn) hits long) -> For G, we need to recall that for 2 vutices upe & G, we can check if (4,14) (FECG) on not in Space O(logn). Finishes Bert

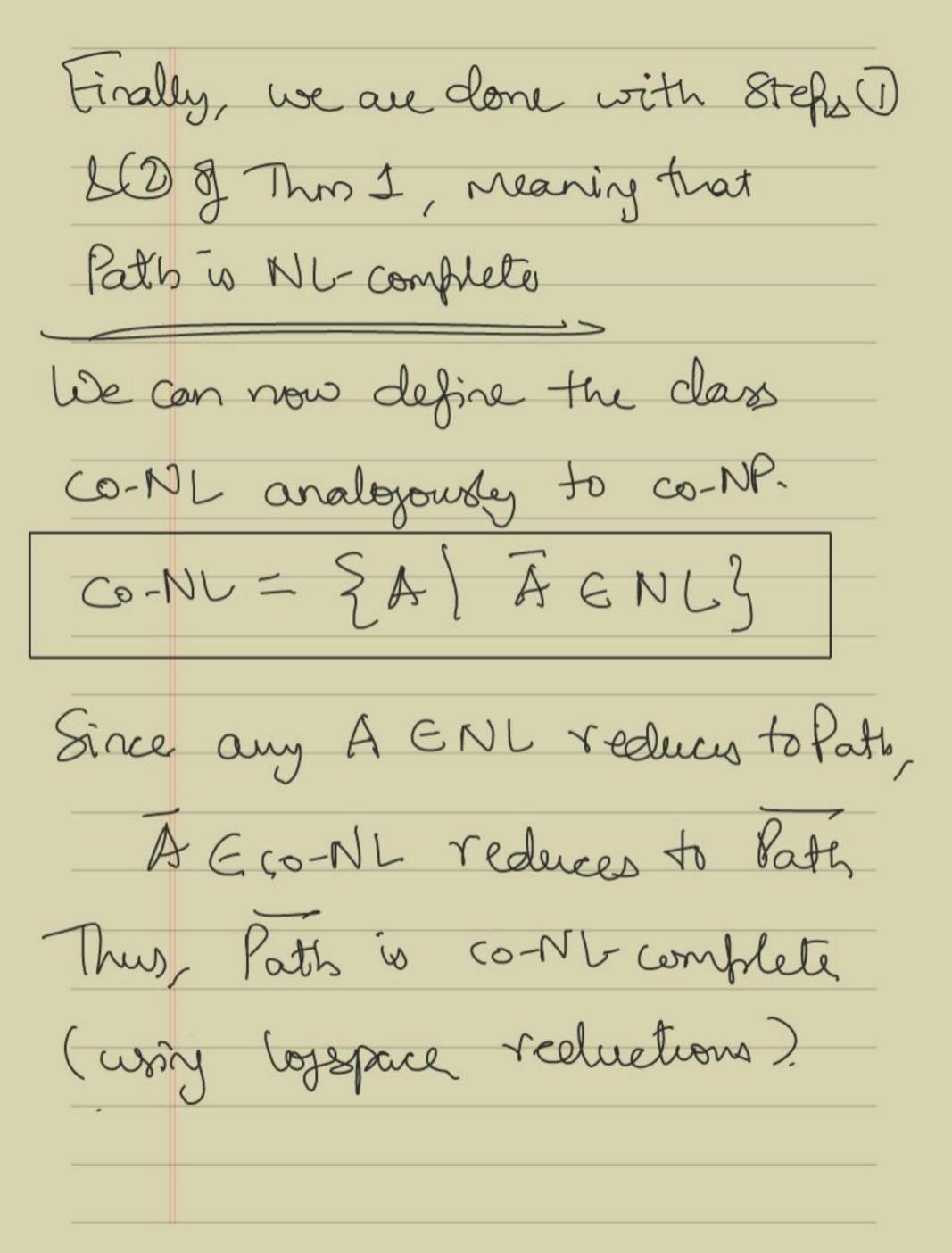
In to	Oin Thus.
-	
	any to do via NTMs.
-> le	l's see another way, via
	Certificates
Rec	all: A ENP if & only if
	there is a DTM M Overifice?
	running in polynomial time such
71 6	A (=>) Jy 1yl & polylixi) &
	M(24y) =1.
Hou	can we modify this to give
	similar characterization of
	Nr;

Two changes: (i) Make Ma DTM running is logspace. (i.e Space (M, 2) = O (logn)
ushue n=1x1) (1) Make the certificate read-once 1-e the TM M receives the certifical on a new take (" Certificate take") where the heard can only move signit. Without the "voed-once" condition on the certificate take. This definition actually captures all of NP. J

Claim: AENL if Lonly if there is a DTM M with a read-once certificate running is logspace such that n(x)) gr (y) L pay(x)) M(G1,y)=1 input ( Lo on certificate take The proof is quite large sketched below: (=>) IF A ENL, then it has an NTM N in Cosspace. Use it to awate a DIM M that simulates of using The certificate topa to simulate the non-deterministic choices

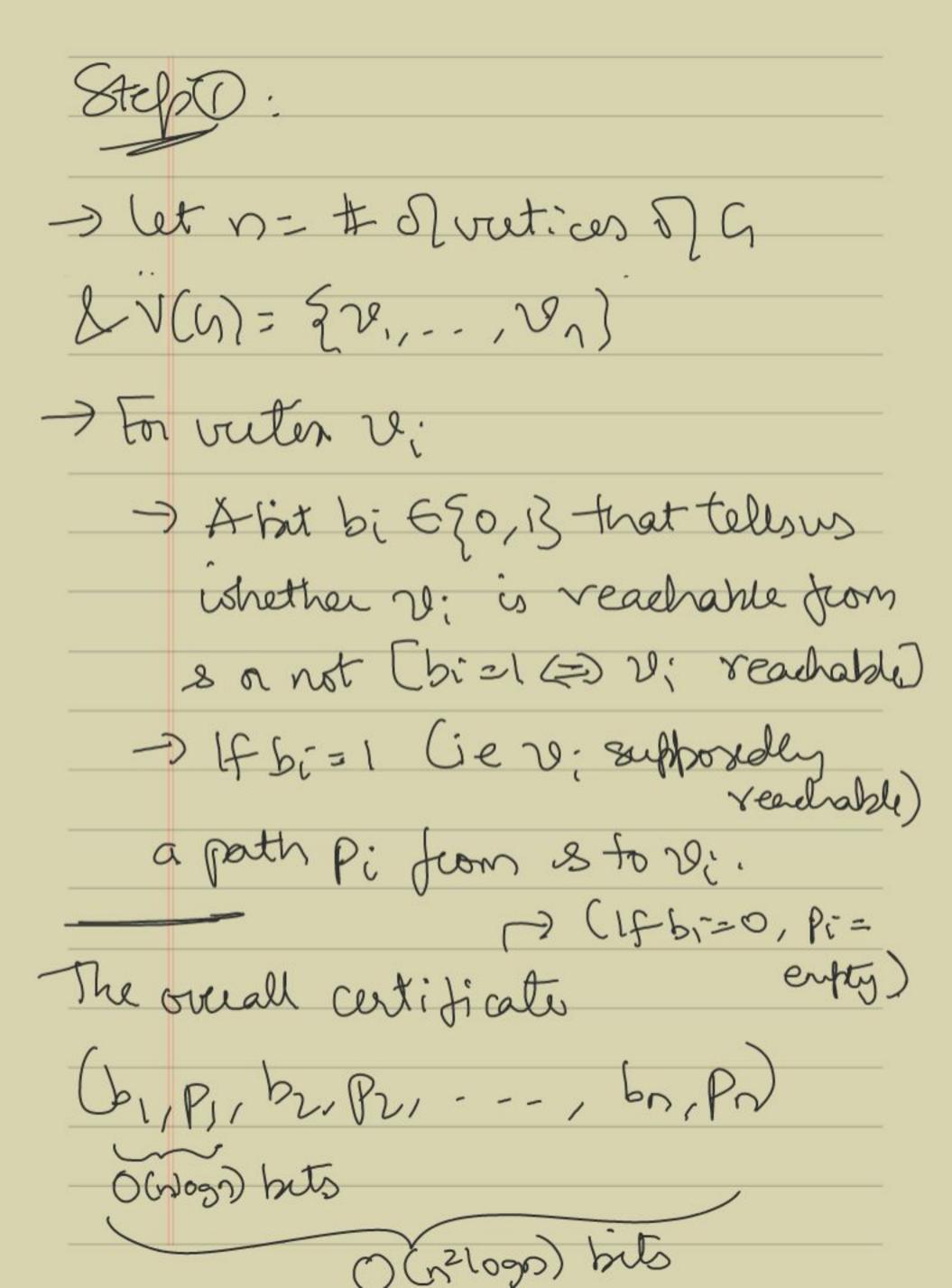


Showing Path ENL via certificates To show (q,s,t) in Path, the Certificate is just a path (or none Specifically a walk ) from s to t is Grusts at most or vertices where n= N(cs). The machine M just needs to check (a) the first verten is & & the last is t (b) If the path is 23=8, 21, ---, 72 = t then (Di, Vin) (= E(G)) for all i E {0,-, K-1}-



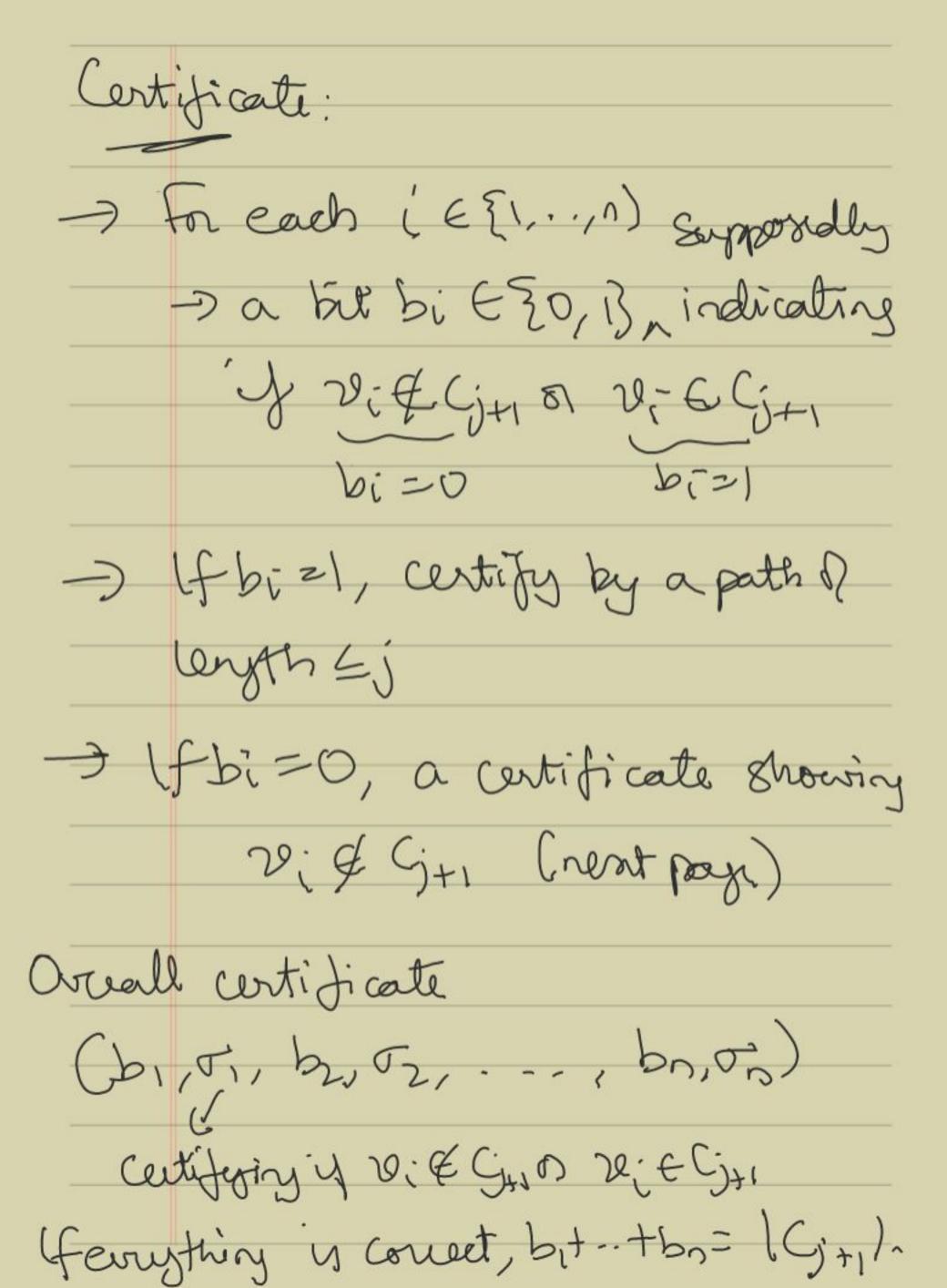
Like	NP vs co-NP, we can also
aver	"It NL=co-NL. Here, we
	ea surprisny (?) annu.
Then	2 (Immerman - Szelepcsényi heoreas)
	NL = CO - NL
To 8	now this, we only need to
8/10	on that co.NL ENL Conercial
200	Path is (o-Nh complete) is enough to show that
-1	is enough to show that
	s: (also the Is theorem)
	Path ENL.

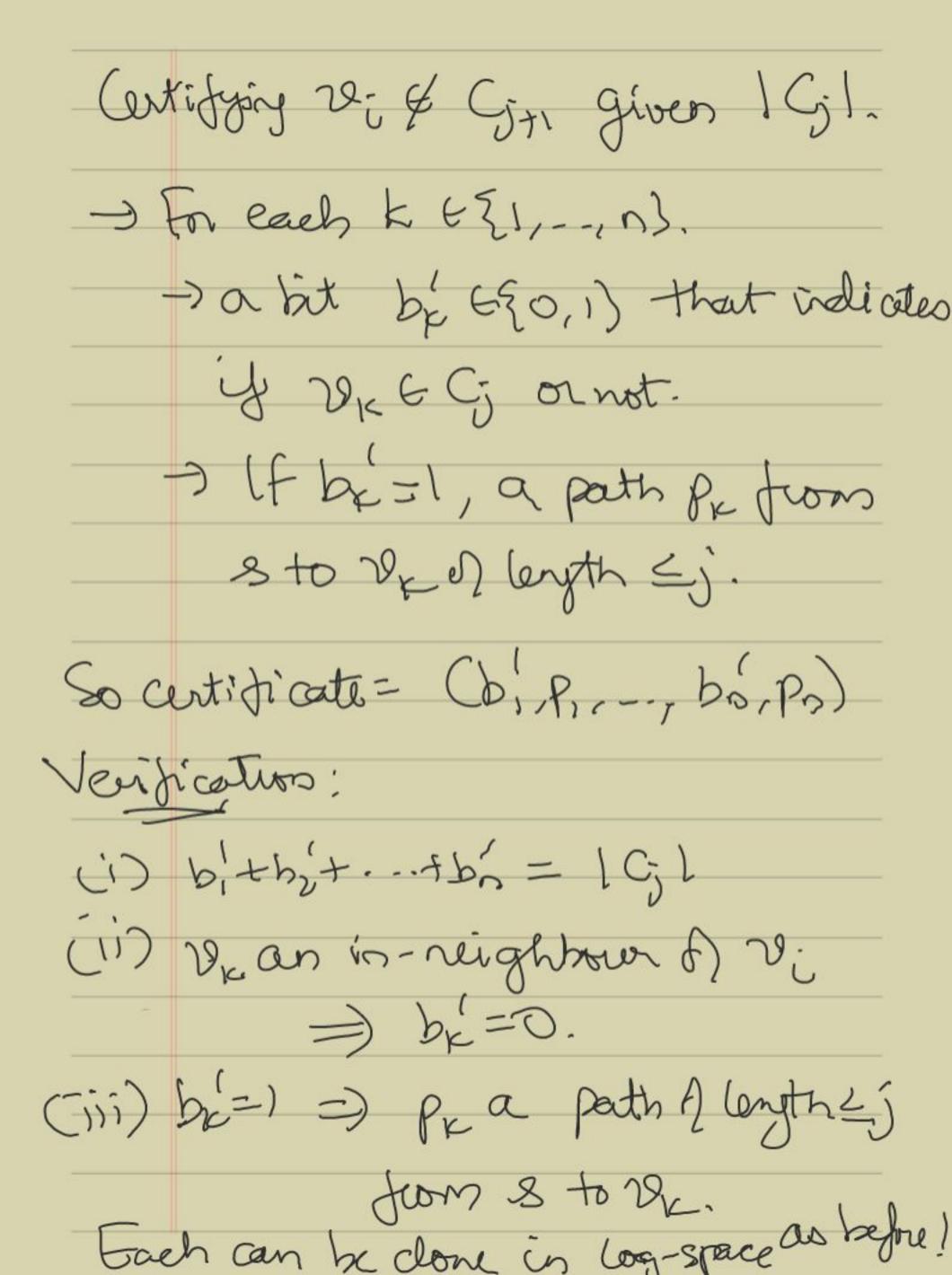
Proof: Need to give a certificate for (4,818) & Path (j. e. the certificale Shows that there is no path from & so ti) Moceove, the certificate is chechable by a DTM M that was by space & reads the certificate on a readonce take. Two steps: D Arnens we know C= # of vertices reachable from & & give a certificate for (9,8,t) = Path 3 Give a certificate for a



Verifying this certificate: The vuification process needs to check 3 things: (i) bit...tbn= C (ii) If t=22; , bé= 0 (iii) Each p: gives a valid path from s to ve. Each can be checked in parallel beg a by-space machino is read-Once forshion! To all the thee checks in parallel. Takes space O (logn) x3 = O (logn).

Stell	
et Cj	= {v;   v; reachable from -s by a path of length 4 is 3
	a path () length 40 5
Obs:	Co = { s} C, C, C, C, C, C,
81	$C_{n} = c$
Carry 1	rutes reachable from a is
rea	chable by path of (eight &n)
So 10	Col-1 is known & we want to
Certi	Jy 1 Cn1 = c.
For ea	achjes, we will certify
	lGtil given [Gl.





Verification of overall certificate for 1 Gnl given 161.

For each i & 21,-, n) -> If bi=1, check yr; is a paths from 3 to 20; of Cenyth & 7) If bi=0, chech Ti as on the previous page. -> Compute | Cj+1 = b,+.. +bn+1. Note: Space re-used between iterations. The only additional space we need is to store i, c; the current sum O (logn) buts.

tira	l certificate for Path
( ) 1	
( 4,	1921 19n, y)
-	
4.	- Certifies l'Gil given
7)	
	(G-1)
~ ·	
4	Contifies (9, s, t) & Path
	gwin   Cn  = c.
	0
178.	1-0(31-)
150	$1 = O(n^3 \log n)$
	7. N. 1. W. J. (~ 5/1)
	Durvall length= O(n bogn)
	= bops (2).
-	