Assignment

1. Which types of inversion are exemplified in the paper[2]? List at least 4 examples, their index sets I and O and their inversion type.

2. Define subtraction of unary numbers as a CCS, and check that your system rewrites $sub(s(s(0)), s(0))$ into $\langle s(0) \rangle$. An example of subtraction of unary numbers defined in mosML:
   ```
   fun sub z oo = z    (* datatype unum = oo | s of unum *)
   | sub (s z) (s x) = let val y = sub z x in y end;
   ```

3. Use the inversion tool http://topps.di.ku.dk/pirc/inversion-tool/ to experiment with the four rule inverters. Choose an interesting semi-inversion, i.e., a pair of I and O, of *fall*, and use each of the four rule inverters to semi-invert *fall*. Include the outputs in your report. Summarize whether the inverters produce different results. For each of the inverters and its result, provide a brief reason why/how its ruleinv-function causes that result, e.g., one or two sentences.
   $$fall(v, h, 0) \rightarrow \langle v, h \rangle$$
   $$fall(v_0, h_0, s(t)) \rightarrow \langle v, h \rangle \Leftarrow add(v_0, s^{10}(0)) \rightarrow \langle v_n \rangle \land height(h_0, v_n) \rightarrow \langle h_n \rangle \land fall(v_n, h_n, t) \rightarrow \langle v, h \rangle$$
   $$height(h_0, v_n) \rightarrow \langle h_n \rangle \Leftarrow add(h_0, s^5(0)) \rightarrow \langle h_{temp} \rangle \land sub(h_{temp}, v_n) \rightarrow \langle h_n \rangle$$

4. Provide a useful program (less than 20 lines), and its useful semi-inversion, partial inversion, or full inversion -you can use the tool to create the inversions: (1) Briefly describe their useful purposes, e.g., one or two sentences. (2) The programs should be specified as CCSs. You can choose to also provide the program in their original language, e.g., see slide 8.

[2] Kirkeby, M. H. & Glück, R. *Semi-inversion of conditional constructor term rewriting systems*. in *Logic-based Program Synthesis and Transformation. Proceedings* (ed Gabbrielli, M.) (Springer, 2020), 243–259.