Stožnice

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
$D = B^2 - 4AC$
$D < 0$: elipsa (če $A = C$ in $B = 0$, krožnica)
$D = 0$: parabola
$D > 0$: hiperbola

Limite

$\lim_{t\to 0} t^\alpha \cdot \log t^{2k} = 0$ (za $\alpha > 0, k \in \mathbb{Z}$)
$\lim_{t\to 0^+} t^\alpha \cdot \log t^{\beta} = 0$ (za $\alpha > 0$)
$\lim_{t\to 0} t^\alpha \cdot \sin t^{-\beta} = 0$ (za $\alpha, \beta \in \mathbb{Z}^+$)

$$\lim_{t\to 0} t^{-\alpha} \cdot \sin t^{\beta} = \begin{cases} 0 & \alpha < \beta \\ 1 & \alpha = \beta \\ \pm\infty & \alpha > \beta \end{cases}$$

Kotne funkcije

$\sin(\frac{\pi}{2} - x) = \cos x$ $\qquad$ $\tan(\frac{\pi}{2} - x) = \cot x$
$\cos(\frac{\pi}{2} - x) = \sin x$ $\qquad$ $\cot(\frac{\pi}{2} - x) = \tan x$

Adicijski izreki

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$
$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$
$\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$
$\sin(x \pm y) = \sin x\cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x\cos y \mp \sin x \sin y$
$\tan(x \pm y) = (\tan x \pm \tan y)/(1 \mp \tan x \tan y)$
$\sin 2x = 2\sin x\cos x = 2\tan x/(1+\tan^2 x)$
$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$
$= 1 - 2\sin^2 x = (1-\tan^2 x)/(1+\tan^2 x)$
$\tan 2x = 2\tan x/(1-\tan^2 x)$

Vrednosti

	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$	$5\pi/12$	$\pi/2$
sin	0	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	1
cos	1	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	0
tan	0	$2-\sqrt{3}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$	$\pm\infty$

Inverzne kotne funkcije

$\arcsin : [-1,1] \to [-\pi/2, \pi/2]$
ODV: $1/\sqrt{1-x^2}$, INT: $x\arcsin x + \sqrt{1-x^2}$
$\arccos : [-1,1] \to [0, \pi]$
ODV: $-1/\sqrt{1-x^2}$, INT: $x\arccos x - \sqrt{1-x^2}$
$\arctan : (-\infty, \infty) \to (-\pi/2, \pi/2)$
ODV: $1/(x^2+1)$, INT: $x\arctan x - 1/2 \cdot \ln(x^2+1)$

Integrali

$\int u\,dv = uv - \int v\,du$
$\int a^x dx = a^x/\ln a$
$\int e^{kx} dx = e^{kx}/k$
$\int \sin^2 x\,dx = -1/4(\sin(2x) - 2x), [0, 2\pi] \to \pi$
$\int \cos^2 x\,dx = 1/2(\cos x \sin x + x), [0, 2\pi] \to \pi$
$\int dx/\sin^2 x = -\cot x$
$\int dx/\cos^2 x = -\tan x$
$\int f'(x)/f(x)dx = \ln|f(x)|$
$\int x e^2 dx = e^x(x-1)$
$\int \log_a x\,dx = x\log_a x - x/\ln a$
$\int dx/(x^2+a^2) = 1/a \cdot \arctan(x/a)$
$\int dx/(x^2-a^2) = 1/2a \cdot \ln|(x-a)/(x+a)|$
$\int dx/(a^2-x^2) = 1/2a \cdot \ln|(a+x)/(a-x)|$
$\int dx/\sqrt{x^2 \pm a} = \ln|x + \sqrt{x^2 \pm a}|$
$\int dx/\sqrt{a-x^2} = \arctan(x/\sqrt{a-x^2}) = \arcsin(x/\sqrt{a})$
$\int x \cdot dx/\sqrt{x^2-a} = \sqrt{x^2-a}$
$\int dx/\sqrt{ax^2+bx+c} =$

$$\begin{cases} 1/\sqrt{a} \cdot \ln|2ax + b + 2\sqrt{a}\sqrt{ax^2+bx+c}| & ; a > 0 \\ -1/\sqrt{-a}\arcsin((2ax+b)/\sqrt{D}) & ; a < 0 \end{cases}$$

$\int e^{ax}\sin(bx)dx = e^{ax}/(a^2+b^2) \cdot (a\sin(bx) - b\cos(bx))$
$\int e^{ax}\cos(bx)dx = e^{ax}/(a^2+b^2) \cdot (a\cos(bx) + b\sin(bx))$

Substitucije

$u = \tan(x/2), \sin x = 2u/(1+u^2), \cos x = (1-u^2)/(1+u^2), dx = 2du/(1+u^2)$
$t = \tan x, \sin^2 x = t^2/(1+t^2), \cos^2 x = 1/(1+t^2), dx = dt/(1+t^2)$

Funkcije, preslikave

Definicija: Naj bo $D \subset \mathbb{R}^n$. Funkcija $f\colon D \to \mathbb{R}$ je v notranji točki $a \in D$ *diferenciabilna*, če obstaja tak vektor $A \in \mathbb{R}^n$, da je $f(a+h) = f(a) + A \cdot h + o(h)$, kjer je $\lim_{h\to 0} o(h)/\|h\|| = 0$.

Izrek: *Če je funkcija diferenciabilna v točki $a \in D$, potem je tam zvezna in parcialno odvedljiva na vse spremenljivke. Tedaj velja*

$$A = \left| \frac{\partial f}{\partial x_1}(a), \frac{\partial f}{\partial x_2}(a), \ldots, \frac{\partial f}{\partial x_n}(a) \right|.$$

Izrek: *Če je funkcija zvezno parcialno odvedljiva na vse spremenljivke, je diferenciabilna.*

Izrek: *Naj bo $g\colon \mathbb{R}^m \to \mathbb{R}^n$ diferenciabilna v a in $h\colon \mathbb{R}^n \to \mathbb{R}^p$ diferenciabilna v $g(a)$. Potem je $h \circ g$ diferenciabilna v a in velja $D(h \circ g)(a) = Dh(g(a)) \circ Dg(a)$.*

Izrek (Vpeljava novih spremenljivk): *Naj bo $f(x) = f(x_1, \ldots, x_n)$ diferenciabilna funkcija in $g(y) = g(y_1, \ldots, y_n) = f(x(y))$. Potem velja*

$$\frac{\partial}{\partial x_k} = \sum_{i=1}^{n} \frac{\partial y_i}{\partial x_k} \cdot \frac{\partial}{\partial y_i}.$$

(x_k so spremenljivke prvotne funkcije, $y_i(\ldots, x_k, \ldots)$ so funkcije (spremenljivke), ki jih vpeljemo)

Definicija (Laplace): Za dvakrat zvezno odvedljivo funkcijo $u(x,y)$ definiramo $\triangle u = u_{xx} + u_{yy}$. V polarnih koordinatah: $\triangle u = u_{rr} + 1/r^2 u_{\varphi\varphi} + 1/r u_r$. Dvakrat zvezno odvedljiva funkcija u je *harmonična*, če velja $\triangle u = 0$.