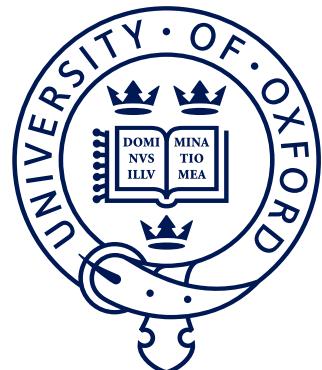


¹ **CP violation in $B^\pm \rightarrow Dh^\pm$ decays**
² **where $D \rightarrow K_S^0 h'^+ h'^-$**

³ *A measurement of the CKM angle γ at LHCb and understanding
⁴ the impact of neutral kaon CP violation*



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⁹ A thesis submitted for the degree of
¹⁰ *Doctor of Philosophy*
¹¹ Trinity 2020

¹²

Acknowledgements

¹³ suitable thank you's

Abstract

14

¹⁵ World's best measurement of γ . Details to be added.

Contents

16

17	Preface	vii
18	1 Introduction	1
19	1.1 Structure of the thesis	1
20	2 Theoretical background	2
21	2.1 The C, P and T symmetries and their violation	2
22	2.2 CP violation in the Standard Model	4
23	2.2.1 The CKM matrix and the Unitarity Triangle	5
24	2.2.2 Measuring γ in tree level decays	8
25	2.3 Measuring γ using multi-body D final states	12
26	2.3.1 Dalitz plots and the phase space of multi-body decays	12
27	2.3.2 The GGSZ method to measure γ	14
28	2.3.3 A model-independent approach	16
29	2.3.4 Measuring strong-phase inputs at charm factories	18
30	2.3.5 Global CP asymmetry and the relation to GLW and ADS measurements	22
31	2.4 Strategy for the LHCb measurement	26
33	3 The LHCb experiment	29
34	3.1 The LHCb subdetectors	31
35	3.1.1 The VELO	32
36	3.1.2 Magnet and tracking stations	34
37	3.1.3 The RICH detectors	36
38	3.1.4 Calorimeters	38
39	3.1.5 Muon detectors	40
40	3.2 Reconstruction	41
41	3.2.1 Track reconstruction	41
42	3.2.2 Particle identification	43
43	3.3 The LHCb trigger system	44
44	3.3.1 The level-0 hardware trigger	46
45	3.3.2 High-level triggers	46
46	3.3.3 Offline data filtering: the LHCb stripping	48
47	3.4 Simulation	48

48	4 Neutral kaon CP violation and material	50
49	interaction in BPGGSZ measurements	
50	4.1 CP violation and material interaction of neutral kaons	50
51	4.1.1 A first look at the impact on γ measurements	54
52	4.2 Impact on BPGGSZ measurements of γ :	
53	principles	56
54	4.2.1 Modified symmetry relations	56
55	4.2.2 Relationship between the K_S^0 and K_L^0 amplitudes	57
56	4.2.3 Modification of the BPGGSZ yield equations	58
57	4.3 Impact on BPGGSZ measurements of γ :	
58	LHCb and Belle II measurements	61
59	4.3.1 Detector geometries	62
60	4.3.2 Kaon momentum distributions	63
61	4.3.3 Experimental time acceptance	64
62	4.3.4 Detector material budget	65
63	4.3.5 Calculation procedure	67
64	4.3.6 Results	68
65	4.3.7 Coupled $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ measurements	70
66	4.4 Concluding remarks	71
67	5 A BPGGSZ measurement of γ with $B^\pm \rightarrow Dh^\pm$ decays	73
68	5.1 Candidate reconstruction and selection	73
69	5.1.1 Initial requirements	74
70	5.1.2 Boosted decision tree	76
71	5.1.3 Particle-identification requirements	81
72	5.1.4 Final requirements	82
73	5.1.5 Selected candidates	82
74	5.2 Signal selection efficiencies	85
75	5.2.1 Efficiency of the PID requirements	86
76	5.2.2 Efficiency profile over the Dalitz plot	87
77	5.3 Background studies	89
78	5.3.1 Charmless decays	91
79	5.3.2 Background from four-body D decays	93
80	5.3.3 Semi-leptonic backgrounds	95
81	5.3.4 Cross-feed from other $D \rightarrow K_S^0 h^+ h^-$ decays	101
82	5.3.5 Swapped-track backgrounds	101
83	5.4 Signal and background mass shapes	103
84	5.4.1 Signal decays	104
85	5.4.2 Cross-feed between $B^\pm \rightarrow Dh^\pm$ channels	105

86	5.4.3	Partially reconstructed backgrounds	108
87	5.4.4	Combinatorial background	113
88	5.4.5	Fit results	113
89	5.5	Measurement of the CP-violation observables	119
90	5.5.1	Fit setup	119
91	5.5.2	Main results	126
92	5.5.3	Cross checks	128
93	5.6	Systematic uncertainties	138
94	5.6.1	Strong phase uncertainties	138
95	5.6.2	Efficiency-profile-related systematic uncertainties	140
96	5.6.3	Mass shapes	143
97	5.6.4	CP violation and material interaction of the K_S^0	150
98	5.6.5	Impact of D mixing	150
99	5.6.6	PID efficiencies	150
100	5.6.7	Dalitz-coordinate resolution	151
101	5.6.8	The fixed yield fractions	152
102	5.6.9	Systematic uncertainty due to backgrounds that are not modelled in fit	152
103	5.6.10	Bias correction	154
104	5.6.11	Charmless backgrounds	155
105	5.6.12	Summary of systematic uncertainties	155
106	5.7	Obtained constraints on γ	157
107	5.7.1	Statistical approach	157
108	5.7.2	Interpretation results	160
109	5.7.3	Compatibility with other measurements	162
110	6	Summary and outlook	167
111	Appendices		
112	A	Projections of the main fit to data	169
113	Bibliography		175

Preface

¹¹⁶ The work presented in this thesis has been resulted in two papers, either under
¹¹⁷ review or published in the Journal of High Energy Physics. These are

¹¹⁸ [1] *Measurement of the CKM angle γ using $B^\pm \rightarrow [K_S^0 h^+ h^-]_D h^\pm$ decays*,
¹¹⁹ submitted to JHEP.

¹²⁰ This paper describes a measurement of the CKM angle γ using pp collision
¹²¹ data taken with the LHCb experiment during the Run 1 of the LHC, in 2011
¹²² and 2012, and during the full Run 2, in 2015–2018. The measurement uses the
¹²³ decay channels $B^\pm \rightarrow D h^\pm$ where $D \rightarrow K_S^0 h'^+ h'^-$, in which h and h' denotes
¹²⁴ pions or kaons. It obtains a value of $\gamma = (69 \pm 5)^\circ$, which constitutes the
¹²⁵ world’s best single-measurement determination of γ . The work is the main
¹²⁶ focus of this thesis and described in detail in Chapter 5.

¹²⁷ [2] *CP violation and material interaction of neutral kaons in measurements*
¹²⁸ *of the CKM angle γ using $B^\pm \rightarrow DK^\pm$ decays where $D \rightarrow K_S^0 \pi^+ \pi^-$* , JHEP
¹²⁹ 19 (2020) 106.

¹³⁰ This paper describes a phenomenological study of the impact of neutral-
¹³¹ kaon CP violation and material interaction on measurements of γ . With the
¹³² increased measurement precision to come in the near future, an understanding
¹³³ of these effects is crucial, especially in the context of $B \rightarrow D\pi$ decays; however
¹³⁴ no detailed study had been published at the start of this thesis. The study is
¹³⁵ the subject of Chapter 4. Some text excerpts and figures from the paper have
¹³⁶ been reproduced in the thesis.

¹³⁷ All of the work described in the thesis is my own, except where clearly referenced
¹³⁸ to others. Furthermore, I contributed significantly to an analysis of $B^\pm \rightarrow DK^\pm$
¹³⁹ decays with LHCb data taken in 2015 and 2016, now published in

¹⁴⁰ [3] *Measurement of the CKM angle γ using $B^\pm \rightarrow DK^\pm$ with $D \rightarrow K_S^0 \pi^+ \pi^-$,*
¹⁴¹ *$K_S^0 K^+ K^-$ decays*, JHEP 08 (2018) 176.

¹⁴² I was responsible for the selection and analysis of the signal channel, studies of
¹⁴³ systematic uncertainties, and the interpretation of the measured observables in
¹⁴⁴ terms of underlying physics parameters. The measurement is superseded by that
¹⁴⁵ of Ref. [1] and is not described in detail in the thesis.

¹⁴⁶ Within the LHCb collaboration, I took part in the preparation for Run 3 by
¹⁴⁷ working as a *migration coordinator* for the *B-decay-to-open-charm* (B2OC) physics
¹⁴⁸ working group, responsible¹ for the migration of the working group's centralised,
¹⁴⁹ offline selections (so called *stripping lines*) to the software-trigger framework being
¹⁵⁰ developed for the LHCb Upgrade.² With more than 800 lines in the current B2OC
¹⁵¹ stripping module this is a major task; I took a leading role in the initial design and
¹⁵² testing of the upgraded B2OC module, and in helping the first analysts implement
¹⁵³ their selections within it. I have also undertaken shift work as RICH piquet and
¹⁵⁴ Data Manager, and acted as the liaison between the B2OC physics working group
¹⁵⁵ and the *particle-identification* performance working group.

¹Along with Alessandro Bertolin and Shunan Shang.

²The current software trigger and *stripping* framework is presented in Section 3.3.

Le roi est mort, vive le roi!

— Traditional French proclamation at the death
of one monarch and the ascension of a new

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157

Introduction

158 All the big picture stuff: constraints on New Physics from high precision measurements,
159 a small nod to matter-antimatter asymmetry questions etc.

160 This thesis places itself at the forefront of the latter effort, presenting the
161 World's most precise measurement of the CP -violating phase γ ; a fundamental
162 parameter in the Standard Model, in which it describes the sole source of matter-
163 antimatter asymmetries.

164 Naturally, the work presented here is far from the last word on the value of
165 γ . In the very near future, several important results will be updated by the
166 LHCb collaboration, based on the full data set collected during Run 2 of the LHC.
167 In the slightly longer term, both the LHCb and Belle II collaborations expect
168 to collect data samples of B decays that are orders of magnitudes larger than
169 the current ones during the coming decade, pushing the obtainable precision on
170 γ towards, even below, one degree. An outlook towards this exciting future of
171 CP -violation measurements is given in Chapter 6, along with a summary of the
172 contributions made in the present thesis.

173 1.1 Structure of the thesis

2

174

175

Theoretical background

176 This chapter lays out the theoretical framework of the thesis. Section 2.1 introduces
177 charge and parity symmetry violation in general, while Section 2.2 covers the
178 description in the Standard Model and the general theory behind charge-parity
179 symmetry violation measurements in charged B decays. Section 2.3 focuses on
180 the theory of measurements using $B^\pm \rightarrow Dh^\pm$ decays with multi-body D final
181 states, after which the specific analysis strategy for the measurement described
182 in the thesis is outlined out in Section 2.4.

183 2.1 The C, P and T symmetries and their vio- 184 lation

185 The concept of symmetry play a fundamental role in modern physics. By Noether's
186 theorem [4], the simple assumption of invariance of our physical laws under universal
187 temporal and spatial translations leads to the very non-trivial prediction of conserved
188 energy and momentum; within the field of particle physics, the interactions and
189 dynamics of the Standard Model (SM) follow completely simply from requiring
190 the fundamental particle fields to satisfy a local $U(1) \times SU(2) \times SU(3)$ gauge
191 symmetry [5]; and one of the short-comings of the SM, is that it fails to explain
192 the apparent *lack* of symmetry in our matter-dominated universe [6]. Indeed, it is
193 important to experimentally establish the symmetries of our world at a fundamental
194 level, and the degree to which they are broken.

195 Three discrete symmetries of importance are the symmetries under

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duction.

- 196 1. The charge operator C , which conjugates all internal quantum numbers of a
 197 quantum state and thus converts particles into their anti-particle counter parts.
 198 For example, C transforms the electric charge of a particle state $Q \rightarrow -Q$.
- 199 2. The parity operator P , which inverts the spatial dimensions of space time:
 200 $\vec{x} \rightarrow -\vec{x}$. As such, it transforms left-handed particle fields into right-handed
 201 particle fields and vice versa.
- 202 3. The time-inversion operator T , which inverts the temporal dimension of space
 203 time: $t \rightarrow -t$.

204 These are fundamentally related by the *CPT* theorem [7], which states that any
 205 Lorentz-invariant Quantum Field Theory (QFT) must be symmetric under the
 206 simultaneous application of *all* three operators. However, any one of the symmetries
 207 can be broken individually, and experiments have shown the physical laws of our
 208 world to violate each of the C , P , and T symmetries.

209 Such a symmetry-breaking effect was established for the first time in 1956, when
 210 Chien-Shiung Wu observed parity violation in weak decays of Co-60 nuclei [8], after
 211 carrying out an experiment that was proposed by Yang Chen-Ning and Tsung-Dao
 212 Lee [9]. While this experiment established the breaking of P symmetry, it left open
 213 the possibility that the physical laws are invariant under a combination of a charge-
 214 and parity inversion; that they are *CP* symmetric. However, this was disproved in
 215 1964 when Kronin and Fitch observed that long-lived kaons, which predominantly
 216 decay to the *CP*-odd 3π state, could also decay to the *CP*-even $\pi\pi$ states [10].

217 Since then *CP* violation has been found in the B^0 system by the BaBar and Belle
 218 collaborations [11, 12] during the early 2000's; the B factories, along with CDF, also
 219 saw evidence for *CP* violation in B^\pm decays [13–18] later confirmed by LHCb [19],
 220 and *CP* violation was measured for the B_s^0 meson by LHCb in 2013 [20]; within the
 221 last year and a half, the first observation of *CP*-violation in D^0 decays has also been
 222 made by the LHCb collaboration [21], and most recently evidence for *CP*-violation in
 223 the neutrino sector has been reported by the T2K collaboration [22]. The observed
 224 effects can be divided into distinct classes. The conceptually simplest case is

- 225 1. *CP-violation in decay*, where $|A/\bar{A}| \neq 1$ for some decay amplitude A , and the
 226 amplitude \bar{A} of the *CP*-conjugate decay. The result is different decay rates in
 227 two *CP*-conjugate decays

$$\Gamma(M \rightarrow f) \neq \Gamma(\bar{M} \rightarrow \bar{f}). \quad (2.1)$$

This type of CP violation was not seen until the late 1980ies [23, 24], more than 20 years after the first observation of CP violation, and only finally established around the year 2000 [25, 26]. Also this discovery was made in $K \rightarrow \pi\pi$ decays.

CP -violation in decay is the only type possible for charged initial states, and it is thus the main focus of the thesis. Two additional CP -violating effect are possible for neutral initial states (a situation that will be the main focus of Chapter 4). These effects are

2. CP -violation in mixing, which denotes the case where the mixing rates between the M^0 and \bar{M}^0 states differ

$$\Gamma(M^0 \rightarrow \bar{M}^0) \neq \Gamma(\bar{M}^0 \rightarrow M^0). \quad (2.2)$$

The CP violation first observed by Kronin and Fitch in the neutral kaon sector [10] is (dominantly) of this type.

3. CP -violation in interference between mixing and decay, which can be present for a neutral initial states M^0 decaying into a final state f common to both M^0 and \bar{M}^0 . The decay rate includes an interference term between two amplitudes: the amplitude for a direct $M^0 \rightarrow f$ decay and the amplitude for a decay after mixing: $M^0 \rightarrow \bar{M}^0 \rightarrow f$. Even in the absence of the two aforementioned effects, the rates $\Gamma(M^0 \rightarrow f)$ and $\Gamma(\bar{M}^0 \rightarrow \bar{f})$ can differ due to the interference term. Such CP asymmetries have been measured in eg. $B^0 \rightarrow J/\psi K$ by LHCb and the B factories, and in $B_s^0 \rightarrow J/\psi \phi$ decays by the LHC and Tevatron experiments [27].

CP violation measurements thus have a long, rich, and still-developing history.

2.2 CP violation in the Standard Model

All existing measurements of CP violation in the quark sector are naturally explained in the SM; indeed, the need to explain the observation CP violation in neutral kaons was a driving force in the development of the model in the first place, when it lead Kobayashi and Maskawa to predict the existence of then-unknown particles in 1973 [28] (now known to be the third generation quarks).

2.2.1 The CKM matrix and the Unitarity Triangle

The SM contains three generations of quarks, each consisting of an up-type quark (u , c , and t) and a down-type quark (d , s , and b). The charged weak interaction of the W^\pm boson couples up and down-type quarks. The quark states that couple to the W are not (a priori) identical to the mass eigenstates, and can be denoted $(u', c', \text{ and } t')$ and $(d', s', \text{ and } b')$. A basis for the quark states can be chosen such that the weakly coupling up-quark states are identical to the propagating quark states, $u = u'$, but then the down-type quark state are different: $d' \neq d$. The two bases of the down-type quarks are related via the Cabibbo-Kobayashi-Maskawa (CKM) matrix [28, 29]¹

$$\begin{pmatrix} d' \\ s' \\ t' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ t \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ t \end{pmatrix}. \quad (2.3)$$

Thus the Lagrangian terms representing the coupling of a W^\pm boson with a u - and a d -type quark is

$$\mathcal{L}_{W^+} = -\frac{g}{\sqrt{2}} V_{ud} (\bar{u} \gamma^\mu W_\mu^+ d) \quad \mathcal{L}_{W^-} = -\frac{g}{\sqrt{2}} V_{ud}^* (\bar{d} \gamma^\mu W_\mu^- u) \quad (2.4)$$

where g is the weak coupling constant, γ_u are the Dirac matrices, and u and d represent the left-handed components of the physical quark states.

The CKM matrix is a unitary complex 3×3 matrix, and hence has $3^2 = 9$ independent, real parameters. However, 5 of these can be absorbed into non-physical phases of the quark states (both mass and weak eigenstates) and hence the matrix has 4 real, physical parameters: 3 mixing angles and a single phase. Chau and Keung [30] proposed the parameterisation

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 1 \\ 0 & 0 & 1 \end{pmatrix} \\ = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \quad (2.5)$$

which is the preferred standard by the PDG [31]. Here, $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$ denote the sine and cosine of three rotation angles in quark space; $\theta_{12} = \theta_C$ being the usual Cabibbo angle [29].

¹ A basis for the quarks can of course be chosen, such that neither the up-quarks or the down-quarks are expressed in their mass eigenstates. In that case the CKM matrix is recovered as $V = U_u^* U_d$, where $U_{u/d}$ is the unitary transformation matrices that brings the u/d quarks into their mass eigenstates.

The presence of the complex phase δ_{CP} in the Lagrangian term of the W coupling causes CP violation because, as evident from Eq. (2.4), if δ_{CP} enters the amplitude for some decay mediated by a W boson, $A = |A|e^{i(\delta_0 + \delta_{CP})}$, then it will enter the CP conjugate decay amplitude with the opposite sign: $\bar{A} = |A|e^{i(\delta_0 - \delta_{CP})}$. In these expressions, δ_0 denotes a CP conserving phase that is not caused by complex terms in the Lagrangian, but arises due to potential intermediate states in the decay amplitude.² Usually the underlying mechanism is due to QCD effects, and these CP conserving phases are therefore generally dubbed *strong* phases, as opposed to the CP violating *weak* phase of the W coupling [31]. This terminology will be applied throughout the thesis.

Experimentally, it has been observed that the CKM matrix elements of Eq. (2.5) satisfy $s_{13} \ll s_{23} \ll s_{12}$. This motivates an often used, alternative parameterisation of the matrix, where the elements are expressed as power series in a parameter λ that naturally incorporates this hierarchy: the Wolfenstein parameterisation [32].
The definitions

$$\begin{aligned} s_{12} &\equiv \lambda \\ s_{23} &\equiv \lambda^2 A \\ s_{13} &\equiv \lambda^3 (\rho - i\eta) \end{aligned} \tag{2.6}$$

are made, after which the unitarity conditions (or Eq. 2.5) determine the remaining elements to any order in λ .³ To $\mathcal{O}(\lambda^5)$ the Wolfenstein parameterisation of the CKM matrix is [34, 35]

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{\lambda^5}{2}A^2(1 - 2(\rho + i\eta)) & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - (\rho + i\eta)(1 - \frac{\lambda^2}{2})) & -A\lambda^2(1 - \frac{\lambda^2}{2}(1 - 2(\rho + i\eta))) & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \tag{2.7}$$

The unitarity condition $V^\dagger V = \mathbb{1}$ of the CKM matrix defines 9 relations between the CKM elements of the form

$$\sum_j V_{jq}^* V_{jq} = 1 \quad , \quad q \in \{d, s, b\} \quad \text{along the diagonal} \tag{2.8a}$$

$$\sum_j V_{jq}^* V_{jq'} = 0 \quad , \quad q, q' \in \{d, s, b\}, q \neq q' \quad \text{off-diagonal.} \tag{2.8b}$$

²It is generally true that all phases of a single term in a given amplitude will be convention dependent, but that the phase differences between terms are not.

³Other variants of the Wolfenstein parameterisation do exist [33]. They all agree at the lowest orders of λ .

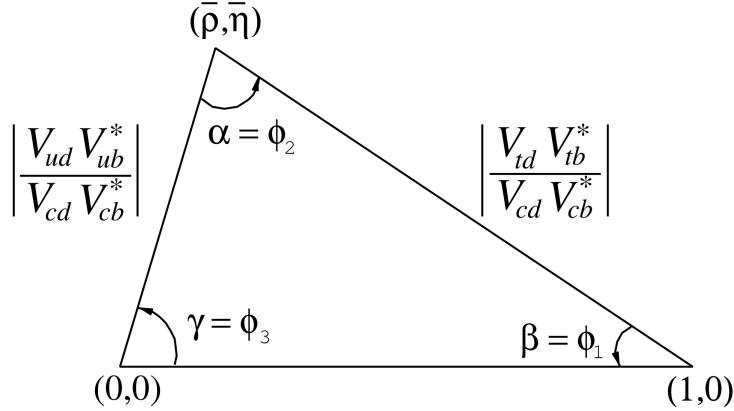


Figure 2.1: Definition of the lengths and sides of the Unitarity Triangle. Figure is taken from the *CKM Quark-Mixing Matrix* review of the PDG [31].

298 The off-diagonal conditions constrain three complex numbers to sum to zero, and
 299 can thus be visualised as triangles in the complex plane, the so-called unitarity
 300 triangles. Of these, the triangle corresponding to the (d, b) elements plays a
 301 special role, because all three sides are of the same order of magnitude, $\mathcal{O}(\lambda^3)$.
 302 When expressed in the form

$$\frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} + \frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}} + 1 = 0, \quad (2.9)$$

303 it is often referred to as the singular Unitarity Triangle, illustrated in Fig. 2.1 where
 304 the usual names for the three angles are also given.

305 Over-constraining the unitarity triangle by making separate measurements of all
 306 sides and angles, in as many different decay channels as possible, is an important,
 307 and non-trivial test of the SM. The current experimental constraints are in agreement
 308 with the SM predictions, as visualised in Fig. 2.2. The CKM angle

$$\gamma \equiv \arg(-V_{ud} V_{ub}^* / V_{cd} V_{cb}^*) = \arg(-V_{cb} V_{cd}^* / V_{ub} V_{ud}^*) \quad (2.10)$$

309 is unique among the CKM parameters, in that it can be measured in tree-level pro-
 310 cesses without significant theoretical uncertainty from lattice QCD calculations [36].
 311 Because tree-level processes are less likely to be affected by Beyond-Standard-Model
 312 (BSM) effects, direct measurements of γ can be considered a SM benchmark, which
 313 can be compared to estimates based on measurements of other CKM elements that
 314 are measured in loop-level processes, and thus are more likely to be affected by
 315 BSM effects [37]. The current, worldwide combination of direct measurements,
 316 published by the CKMFitter group, is $\gamma = (72.1^{+5.4}_{-5.7})^\circ$, to be compared with the
 317 estimate from loop-level observables of $\gamma = (65.66^{+0.90}_{-2.65})^\circ$ [38]. Other world averages

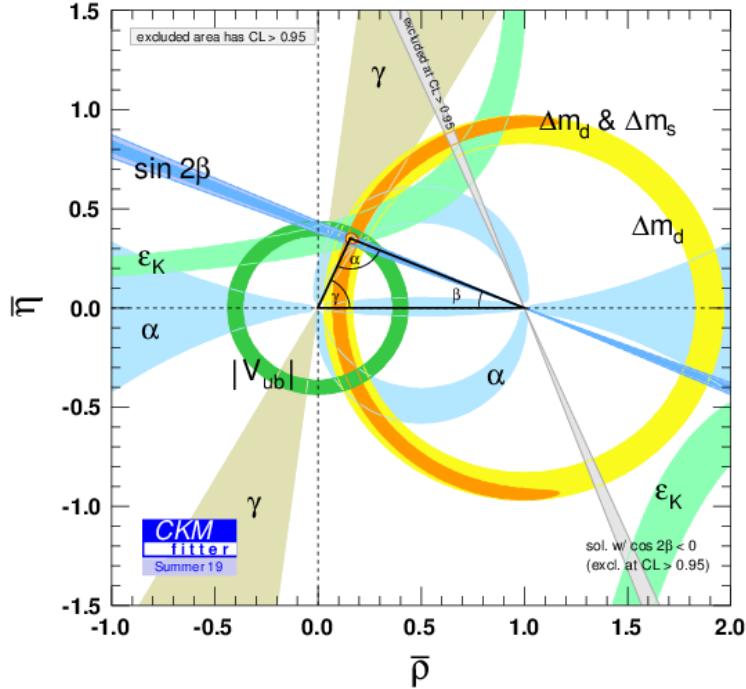


Figure 2.2: Current constraints on the Unitarity Triangle parameters as determined by the CKMFitter group for the EPS 2019 conference [38].

exist [27, 39], but the overall picture is the same: the ability to constrain BSM physics is currently limited by the uncertainty of the direct measurements. Hence further precision measurements of γ are highly motivated. Presently, the precision is driven by time-integrated measurements of direct CP -violation in $B^\pm \rightarrow DK^\pm$ decays; such a measurement is the topic of this thesis and the theory behind is treated in detail in the following section. It is also possible to measure γ in time-dependent mixing analyses of $B_s^0 \rightarrow D_s^\mp K^\pm$, $B^0 \rightarrow D^\mp \pi^\pm$ and related decays, by measuring CP violation in interference between mixing and decay. These modes are expected to provide competitive measurements in the future [40, 41].

2.2.2 Measuring γ in tree level decays

The phase γ can be measured in tree-level processes with interference between $b \rightarrow c\bar{s}u$ and $b \rightarrow \bar{c}s u$ transitions. The canonical example, also the subject of this thesis, is based on measurements sensitive to interference between the $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$ decay amplitudes. As illustrated in Fig. 2.3 for the case of B^- decays, the electro-weak phase difference between the two decays

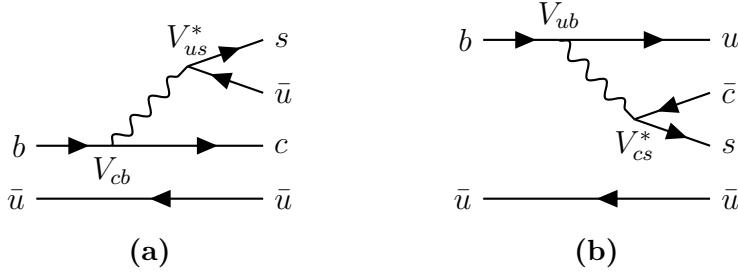


Figure 2.3: Tree level Feynman diagrams describing (a) $B^- \rightarrow D^0 K^-$ and (b) $B^- \rightarrow \bar{D}^0 K^-$ decays. The electro-weak phase difference between the two decays is $\Delta\phi = \arg(V_{cb}V_{us}^*/V_{ub}V_{cs}^*) \simeq \gamma$.

is $\Delta\phi = \arg(V_{cb}V_{us}^*/V_{ub}V_{cs}^*)$. While $\Delta\phi$ is not identical to the definition of γ in Eq. (2.10), the ratio of the involved CKM matrix elements is [42]

$$\begin{aligned} -\frac{V_{cd}^*/V_{ud}^*}{V_{us}^*/V_{cs}^*} &= -\frac{-\lambda[1 - \frac{\lambda^4}{2}A^2(1 - 2(\rho - i\eta))](1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}(1 + 4A^2))}{\lambda(1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{4})} \\ &= 1 - \lambda^4 A^2(1 - 2(\rho - i\eta)) + \mathcal{O}(\lambda^5). \end{aligned} \quad (2.11)$$

The ratio equals unity to $\mathcal{O}(\lambda^4) \simeq 2.6 \times 10^{-3}$, and thus $\Delta\phi \simeq \gamma$ is a good approximation within current experimental uncertainties. For the remainder of this thesis the approximation will be used without further comment. The diagrams in Fig. 2.3 describe the leading order contributions to the two amplitudes

$$\begin{aligned} A[B^- \rightarrow D^0 K^-] &\equiv A_B \\ A[B^- \rightarrow \bar{D}^0 K^-] &\equiv \bar{A}_B \equiv r_B A_B e^{i(\delta_B - \gamma)}, \end{aligned} \quad (2.12a)$$

where the last equality introduces two new parameters: the amplitude magnitude ratio $r_B \equiv |\bar{A}_B|/|A_B|$, and δ_B , the strong-phase difference between the decay amplitudes. Since all CP -violation is attributed to the electro-weak phase in the SM, the CP -conjugate decay amplitudes are [43]

$$\begin{aligned} A[B^+ \rightarrow \bar{D}^0 K^+] &= A_B \\ A[B^+ \rightarrow D^0 K^+] &= \bar{A}_B = r_B A_B e^{i(\delta_B + \gamma)}. \end{aligned} \quad (2.12b)$$

In an experimental setting, the D^0 and \bar{D}^0 mesons are reconstructed in some final state, f or its CP -conjugate \bar{f} . In analogy with the B^\pm decays, the D decay amplitude can be related⁴

$$\begin{aligned} A[D^0 \rightarrow f] &= A[\bar{D}^0 \rightarrow \bar{f}] = A_D \\ A[\bar{D}^0 \rightarrow f] &= A[D^0 \rightarrow \bar{f}] = r_D A_D e^{i\delta_D}. \end{aligned} \quad (2.13)$$

⁴In this notation δ_D is thus phase of the suppressed D -decay amplitude minus the phase of the favoured D -decay amplitude. This is the opposite convention to that used in the LHCb measurements with the ADS technique, but aligns with the notation used in the literature on γ measurements in $D \rightarrow K_S^0 \pi^+ \pi^-$ decays.

where the assumption has been made that CP violation in the D decays is negligible, and δ_D denotes a CP -conserving strong-phase difference. While CP -violation in D decays has recently been measured [21], the size of the effect is small and it is considered negligible in this thesis. Based on Eqs. 2.12 and (2.13), the decay rates of B^+ and B^- mesons into the possible final states can be seen to satisfy

$$\Gamma(B^- \rightarrow D(\rightarrow f)K^-) \propto 1 + r_D^2 r_B^2 + 2r_B r_D \cos [\delta_B + \delta_D - \gamma], \quad (2.14a)$$

$$\Gamma(B^+ \rightarrow D(\rightarrow \bar{f})K^+) \propto 1 + r_D^2 r_B^2 + 2r_B r_D \cos [\delta_B + \delta_D + \gamma], \quad (2.14b)$$

$$\Gamma(B^- \rightarrow D(\rightarrow \bar{f})K^-) \propto r_D^2 + r_B^2 + 2r_B r_D \cos [\delta_B - \delta_D - \gamma], \quad (2.14c)$$

$$\Gamma(B^+ \rightarrow D(\rightarrow f)K^+) \propto r_D^2 + r_B^2 + 2r_B r_D \cos [\delta_B - \delta_D + \gamma]. \quad (2.14d)$$

The processes in Eqs. (2.14a) and (2.14b) are CP -conjugate and it is clear how, in the general case where $\delta_B + \delta_D \neq 0$, a non-zero value of γ leads to CP violation in the form of differing decay rates. The same is true for the processes in Eqs. (2.14c) and (2.14d). Depending on the choice of D final state, these expressions can be used to relate γ to various observables that are experimentally accessible. This thesis concerns the choice $f = K_S^0 \pi^+ \pi^-$ or $f = K_S^0 K^+ K^-$, where the terms related to the D decay all have a non-trivial variation over the phase space of the decay. However, it is useful to first analyse the simpler case where f is a two-body state.

The simplest case is when f is chosen to be a CP eigenstate, so that $f = \pm \bar{f}$ and the rate equations of (2.14a)–(2.14d) simplify, because $r_D = 1$ and $\delta_D \in \{0, \pi\}$. Measurements of γ in such decay modes are denoted GLW measurements, after Gronau, London, and Wyler who described the approach in the early 90ies [43, 44]. Experimentally it is preferable to measure yield ratios rather than absolute rates, and the observables of interest are thus the CP asymmetry

$$\begin{aligned} A_{CP=\pm 1} &= \frac{\Gamma[B^- \rightarrow D_{CP} K^-] - \Gamma[B^+ \rightarrow D_{CP} K^+]}{\Gamma[B^- \rightarrow D_{CP} K^-] + \Gamma[B^+ \rightarrow D_{CP} K^+]} \\ &= \frac{\pm r_B \sin \delta_B \sin \gamma}{1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma}, \end{aligned} \quad (2.15a)$$

as well as the ratio

$$\begin{aligned} R_{CP=\pm 1} &= 2 \frac{\Gamma[B^- \rightarrow D_{CP} K^-] + \Gamma[B^+ \rightarrow D_{CP} K^+]}{\Gamma[B^- \rightarrow D^0 K^-] + \Gamma[B^+ \rightarrow \bar{D}^0 K^+]} \\ &= 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma. \end{aligned} \quad (2.15b)$$

In practice, A_{CP} and R_{CP} are obtained from measured yield ratios that are corrected with appropriate branching fractions. A measurement of A_{CP} and R_{CP} alone is not sufficient to determine the underlying physics parameters (γ, r_B, δ_B) , and this is not solely due to the number of parameters exceeding the number of constraints:

the equations also allow for multiple, ambiguous solutions for (γ, δ_B) . One way to break the ambiguity, first noted in the original paper [43], is to make further measurements in additional B decays. These can be described with the formalism described above, but will not share the same ambiguous solutions because (r_B, δ_B) are unique to a given B decay. Another method is to analyse D decay final states that are not CP eigenstates.

A few years later, Atwood, Dunietz, and Sonis analysed an alternative choice of D final states: a simultaneous analysis of a Cabibbo-favoured (CF) decay $D^0 \rightarrow f$ and the doubly-Cabibbo-suppressed (DCS) decay $D^0 \rightarrow \bar{f}$ into the CP conjugate final state [45, 46]. Their suggested method is named the ADS method after the authors. The classical example is to take $f = K^-\pi^+$ and $\bar{f} = \pi^-K^+$. The relative suppression means that the r_D of Eq. (2.14) is small, typically of the same order of magnitude as r_B , and thus the CP asymmetry of the suppressed decay is $\mathcal{O}(1)$:

$$\begin{aligned} A_{ADS(\bar{f})} &= \frac{\Gamma[B^- \rightarrow D(\rightarrow \bar{f})K^-] - \Gamma[B^+ \rightarrow D(\rightarrow f)K^+]}{\Gamma[B^- \rightarrow D(\rightarrow \bar{f})K^-] + \Gamma[B^+ \rightarrow D(\rightarrow f)K^+]} \\ &= \frac{r_D r_B \sin(\delta_B - \delta_D) \sin \gamma}{r_D^2 + r_B^2 + 2r_D r_B \cos(\delta_B - \delta_D) \cos \gamma}. \end{aligned} \quad (2.16a)$$

The large CP asymmetry is a prime feature of the ADS method. However, also the suppressed-to-favoured yield ratio is sensitive to the physics parameters of interest:

$$\begin{aligned} R_{ADS(\bar{f})} &= \frac{\Gamma[B^- \rightarrow D(\rightarrow \bar{f})K^-] + \Gamma[B^+ \rightarrow D(\rightarrow f)K^+]}{\Gamma[B^- \rightarrow D(\rightarrow f)K^-] + \Gamma[B^+ \rightarrow D(\rightarrow \bar{f})K^+]} \\ &= \frac{r_B^2 + r_D^2 + 2r_D r_B \cos(\delta_B - \delta_D) \cos \gamma}{1 + r_D^2 r_B^2 + 2r_D r_B \cos(\delta_B + \delta_D) \cos \gamma}. \end{aligned} \quad (2.16b)$$

The interpretation of A_{ADS} and R_{ADS} in terms of (γ, r_B, δ_B) requires knowledge of the r_D and δ_D parameters, but these can be measured independently. In general, the constraints from a single set of ADS observables suffer the same ambiguities as in the GLW case. However, unlike the GLW case, each D decay mode provides an independent set of constraints, because the parameters related to the D decay vary.

The discussion of this section has centred on the classical case of $B^\pm \rightarrow DK^\pm$ decays with a two-body D final state. With minor modifications the techniques have been used to make measurements sensitive to γ in B^0 decays, with B decay final states including excited D mesons or kaons, and in $B^\pm \rightarrow D\pi^\pm$ decays (summaries of the measurements made by the B factories and LHCb can be found in Refs. [47–50]). The $B^\pm \rightarrow D\pi^\pm$ decay is also CP -violating, although the effect is much smaller than in the $B^\pm \rightarrow DK^\pm$ decay because $r_B^{D\pi} \simeq 0.005$ [51], whereas $r_B^{DK} \simeq 0.1$. Furthermore, it is possible to use multi-body D final states. However, in some cases, a better precision can then be obtained by exploiting phase-space dependent decay rates. This is the topic of the next section.

400 2.3 Measuring γ using multi-body D final states

401 In multi-body D decays, the r_D and δ_D parameters of the fundamental rate equations
 402 in Eq. (2.14) vary over the phase space of the D decay. This section describes a
 403 model-independent approach to measure γ in $B \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)h^\pm$ decays by
 404 exploiting this variation. The theory is identical for $D \rightarrow K_S^0K^+K^-$ decays, and
 405 similar ideas have been proposed for the $D \rightarrow K_S^0\pi^+\pi^-\pi^0$ [52] and $D \rightarrow 2\pi^+2\pi^-$
 406 modes [53]. First, however, the formalism for describing amplitudes of multi-
 407 body decays is briefly reviewed.

408 2.3.1 Dalitz plots and the phase space of multi-body decays

409 In general, the phase space of the n -body decay $P \rightarrow p_1 + p_2 + \dots + p_n$ consists of n
 410 four momenta, with a total of $4n$ components. The requirement that each of the final
 411 state particles is on-shell provides n constraints on these components, and energy-
 412 momentum conservation removes a further 4 degrees of freedom. If the original
 413 particle P is a scalar, the decay is isotropic, which removes an additional 3 degrees
 414 of freedom, leaving the total number of degrees of freedom at $3n - 7$. For the specific
 415 case of three-body decays, the available phase space can thus be parameterised with
 416 only two parameters. A practical and often used choice is the invariant masses

$$s_{12} = m^2(p_1 p_2) = (p_1^\mu + p_2^\mu)^2, \quad s_{13} = m^2(p_1 p_3) = (p_1^\mu + p_3^\mu)^2. \quad (2.17)$$

417 The choice of particle pairs is arbitrary, and the coordinates easily related

$$m_P^2 + m_{p_1}^2 + m_{p_2}^2 + m_{p_3}^2 = m^2(p_1 p_2) + m^2(p_1 p_3) + m^2(p_2 p_3). \quad (2.18)$$

418 A scatter plot of (s_{12}, s_{13}) values for a sample of particle decays is denoted a Dalitz
 419 plot [54]. It has the very useful feature that the presence of (narrow) resonances
 420 in the decay leads to visible bands in the scatter plot. Figure 2.4 illustrates how
 421 the limits of the Dalitz plot are defined by kinematic constraints, and shows an
 422 example of a Dalitz plot for $D \rightarrow K_S^0\pi^+\pi^-$ decays in which the $K^*(892)^\pm$ and $\rho(770)$
 423 resonances are clearly visible. The plot shows the sample of $B^+ \rightarrow D\pi^+$ decays
 424 used to make the measurement described in Chapter 5 and thus the D meson is in
 425 a superposition of D^0 and \bar{D}^0 states (as detailed in the following section).

426 In terms of the coordinates of Eq. (2.17) the differential decay rate is given by

$$d\Gamma = \frac{1}{32(2\pi)^3 m_P^3} |\mathcal{M}|^2 ds_{12} ds_{13}, \quad (2.19)$$

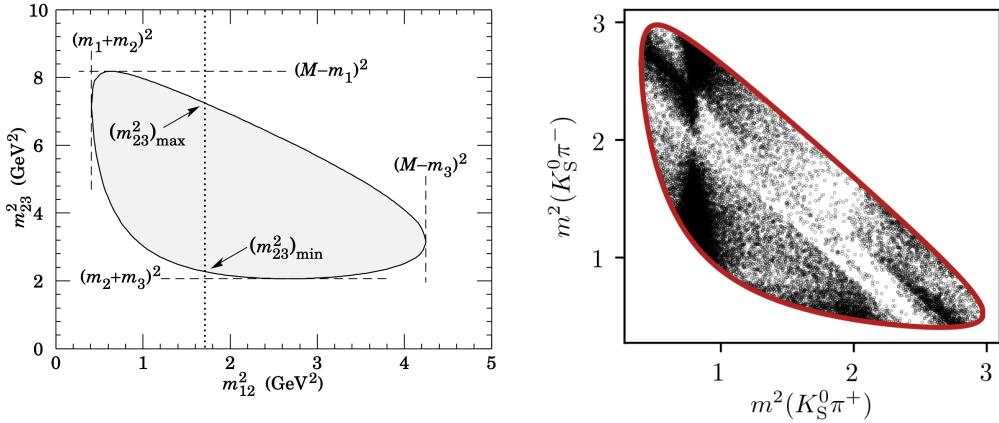


Figure 2.4: (Left) Schematic of a Dalitz plot and the limits of the kinematically allowed phase space limits. (Right) Example of a Dalitz plot for $D \rightarrow K_S^0 \pi^+ \pi^-$ decays where the D meson originates in a $B^+ \rightarrow D\pi^+$ decay; the decaying D meson is in a superposition of the D^0 and \bar{D}^0 states, but predominantly \bar{D}^0 -like.

where \mathcal{M} is the QFT matrix element, or total decay amplitude, corresponding to the decay. In general, it is not possible to calculate \mathcal{M} from first principles. Instead, a model is defined with an empirically well motivated form, in which a number of free parameters must be determined experimentally. The simplest case is that of an *isobar* model, where it is assumed that the full decay can be decomposed into consecutive two-body decays of the form $P \rightarrow R_{12}(\rightarrow p_1 + p_2)p_3$. Thus, \mathcal{M} is expressed as a non-resonant constant amplitude term, k_{NR} , plus a sum of resonance terms

$$\mathcal{M}(s_{12}, s_{13}) = k_{NR} + \sum_r k_r \mathcal{M}^r(s_{12}, s_{13}). \quad (2.20)$$

The exact form of the \mathcal{M}^r function depends on the resonance in question. An overview is given in the PDG review on resonances and references therein [31]. The isobar formalism breaks down when resonances in the decay are not well separated. In this case, models of the form in Eq. (4.27) can still be employed, if the contribution from overlapping resonances are collected in a single term. An example of such a model, is the amplitude model for $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays developed by the Belle collaboration for a measurement of the CKM angle β in 2018 [55]. In this model, individual terms are included for $D^0 \rightarrow K^*(\rightarrow K_S^0 \pi^\pm)\pi^\mp$ decays, whereas the $\pi\pi$ and $K\pi$ S -wave contributions are modelled with the so-called K -matrix- and LASS formalisms [56, 57]. The amplitude and phase of \mathcal{M} as predicted by this model are shown in Fig. 2.5.

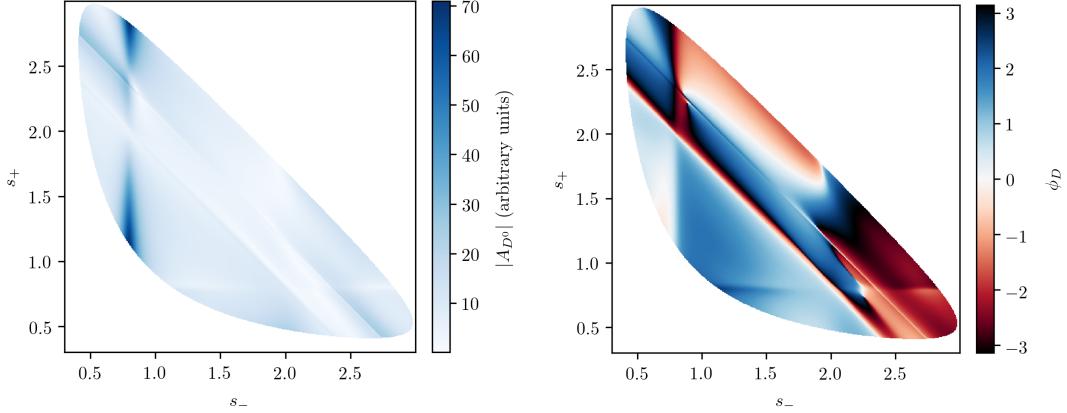


Figure 2.5: The (left) magnitude and (right) phase of the $D \rightarrow K_S^0 \pi^+ \pi^-$ amplitude in the Belle 2018 model [55].

2.3.2 The GGSZ method to measure γ

The non-trivial phase-space dependence of the $D \rightarrow K_S^0 \pi^+ \pi^-$ decay amplitude can be exploited to measure γ with $B^\pm \rightarrow DK^\pm$ or $B^\pm \rightarrow D\pi^\pm$ decays. This approach was proposed independently by Bondar [58], and by Giri, Grossman, Soffer, and Zupan [59] after whom it takes the commonly used acronym GGSZ. For this specific decay s_- and s_+ are used to describe the Dalitz coordinates $m^2(K_S^0 \pi^-)$ and $m^2(K_S^0 \pi^+)$, respectively, and the D decay amplitude is a function of these coordinates

$$A_S^{\bar{D}}(s_-, s_+) = A(\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-). \quad (2.21)$$

To a good approximation the K_S^0 meson is a CP eigenstate, meaning that the $K_S^0 \pi^+ \pi^-$ state is self-conjugate. Assuming this approximation to be exact, and that CP violation in the D decay is negligible, the D decay amplitude satisfies the symmetry relation

$$A_S^{\bar{D}}(s_-, s_+) = A_S^D(s_+, s_-). \quad (2.22)$$

The impact of the K_S^0 meson *not* being an exact CP eigenstate is treated in detail in Chapter 4. In order to simplify equations, the short-hand notation

$$(s_{-+}) = (s_-, s_+), \quad (s_{+-}) = (s_+, s_-), \quad (2.23)$$

will be employed for the remainder of the thesis, so that the relation in Eq. (2.22) can be expressed as $A_S^{\bar{D}}(s_{-+}) = A_S^D(s_{+-})$. Thus, the rate equations of Eq. (2.14)

460 for the $D \rightarrow K_S^0\pi^+\pi^-$ decay mode are

$$\begin{aligned} d\Gamma^-(s_{-+}) &\propto |\mathcal{A}_S^-|^2 = |A_B|^2 |A_{K_S^0}|^2 \\ &\times [|A_S^D(s_{-+})|^2 + r_B^2 |A_S^D(s_{+-})|^2 + 2r_B |A_S^D(s_{-+})| |A_S^D(s_{+-})| \\ &\times (\cos[\delta_D(s_{-+})] \cos[\delta_B - \gamma] + \sin[\delta_D(s_{-+})] \sin[\delta_B - \gamma])], \end{aligned} \quad (2.24a)$$

$$\begin{aligned} d\Gamma^+(s_{-+}) &\propto |\mathcal{A}_S^+|^2 = |A_B|^2 |A_{K_S^0}|^2 \\ &\times [|A_S^D(s_{+-})|^2 + r_B^2 |A_S^D(s_{-+})|^2 + 2r_B |A_S^D(s_{-+})| |A_S^D(s_{+-})| \\ &\times (\cos[\delta_D(s_{-+})] \cos[\delta_B + \gamma] - \sin[\delta_D(s_{-+})] \sin[\delta_B + \gamma])]. \end{aligned} \quad (2.24b)$$

461 Here, $\delta_D(s_{-+}) = \phi_D(s_{-+}) - \phi_D(s_{+-}) = -\delta_D(s_{+-})$, where $\phi_D(s_{-+})$ denotes the
 462 complex phase of the $A_S^D(s_{-+})$ amplitude, and a standard trigonometric relation
 463 have been employed to factorise the terms depending on the complex phases of the B
 464 and D decays. It can be seen that in the case where $\gamma = 0$ the B^+ and B^- decay rates
 465 are symmetric if the Dalitz coordinates are exchanged: $\Gamma^+(s_-, s_+) = \Gamma^-(s_+, s_-)$.
 466 The presence of CP violation in the B decay breaks the symmetry. Therefore it is
 467 possible to measure γ (and the nuisance parameters r_B and δ_B) from the phase-space
 468 distribution of $B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)K^\pm$ decays, given knowledge of $A_S^D(s_{-+})$.

469 A series of measurements of γ have been made that use amplitude models of the
 470 D decay [60–67]. However, a model-independent approach has been proposed by
 471 Bondar and Poluektov [68, 69] that relies on binning phase-space, in which case the
 472 necessary information on the D decay amplitude can be summarised in a small set
 473 of coefficients that can be measured in a separate experiment. That is the approach
 474 followed in this thesis, and has been used previously by the Belle [70] and LHCb
 475 collaborations [71]. It is described in detail in the following section.

476 Such a model-independent approach is favourable for two reasons. Firstly,
 477 uncertainty estimates related to model inputs and the choice of parameterisation in
 478 an amplitude model are non-trivial, yet would become the leading systematic with
 479 the very high precision expected for γ measurements in the near future. Secondly,
 480 amplitude models are notoriously hard to reproduce, and in a high-precision era it
 481 is favourable that any experiment is easy to reinterpret in various extensions of the
 482 SM. This is a lot easier for an experiment that measures a small set of well-defined
 483 observables, than for an experiment that fits a complicated amplitude model.

484 An alternative model-independent approach has recently been proposed by
 485 Poluektov [72] where the externally measured input on the D -decay phase are
 486 Fourier expansion coefficients, and which therefore avoids binning phase space; this
 487 approach may have the potential to improve the obtainable precision in the future.

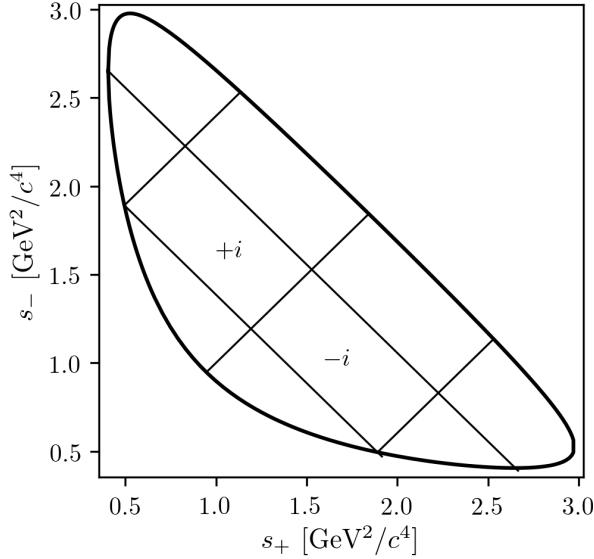


Figure 2.6: Illustration of the binning scheme used in GGSZ measurements: the bins are symmetric around the $m^2(K_S^0\pi^+) = m^2(K_S^0\pi^-)$ diagonal, and numbered so that opposite bins have the same number, but with opposite sign.

2.3.3 A model-independent approach

The phase-space distribution can be analysed in a model-independent way, if the D -decay phase space is split into regions, or bins, and the B decay yield in each bin determined experimentally. A measurement of γ using this approach is the main topic of the thesis. This section describes the fundamental principle, whereas the details pertaining to the exact experimental approach are delegated to Section 2.4.

The amplitude symmetry of Eq. (2.22) is exploited by defining $2N$ bins to be symmetric around the $s_- = s_+$ diagonal of the Dalitz plot, numbered $i = -N$ to N (omitting zero) such that if the point (s_-, s_+) is in bin i , then (s_+, s_-) is in bin $-i$, and by convention $i > 0$ for bins where $s_- > s_+$. The principle is illustrated in Fig. 2.6, but the binning schemes used in actual measurements are more complicated. The decay rates in Eq. (2.24) can be integrated over such bins, and give the bin yields

$$\begin{aligned} N_i^- &\propto h^- \left[K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}} (c_i x_- + s_i y_-) \right], \\ N_i^+ &\propto h^+ \left[K_{-i} + r_B^2 K_i + 2\sqrt{K_i K_{-i}} (c_i x_+ - s_i y_+) \right], \end{aligned} \quad (2.25)$$

where the parameters describing the B decay have been expressed in terms of the observables

$$x_\pm = r_B \cos(\delta_B \pm \gamma), \quad y_\pm = r_B \sin(\delta_B \pm \gamma), \quad (2.26)$$

and a number of phase-space integrated quantities related to the D -decay have been introduced. The K_i parameters denote fractional yield of a flavour-tagged D^0 decaying into bin i , defined as

$$K_i = \frac{1}{N_K} \int_i ds^2 |A_S^D(s_{-+})|^2, \quad N_K = \int ds^2 |A_S^D(s_{-+})|^2, \quad (2.27)$$

where $\int_i ds^2$ denotes integration over bin i of the Dalitz plot. The c_i and s_i denote the amplitude-weighted average of $\cos \delta_D(s_{-+})$ and $\sin \delta_D(s_{-+})$ over bin i

$$\begin{aligned} c_i &= \frac{\int_i ds^2 |A_S^D(s_{-+})| |A_S^D(s_{+-})| \cos[\delta_D(s_{-+})]}{\sqrt{\int_i ds^2 |A_S^D(s_{-+})|^2} \sqrt{\int_i ds^2 |A_S^D(s_{+-})|^2}}, \\ s_i &= \frac{\int_i ds^2 |A_S^D(s_{-+})| |A_S^D(s_{+-})| \sin[\delta_D(s_{-+})]}{\sqrt{\int_i ds^2 |A_S^D(s_{-+})|^2} \sqrt{\int_i ds^2 |A_S^D(s_{+-})|^2}}. \end{aligned} \quad (2.28)$$

By the symmetry properties of $\delta_D(s_{-+})$ these parameters satisfy $c_i = c_{-i}$ and $s_i = -s_{-i}$. The normalisation constants h^+ and h^- are identical in the ideal case, but it is convenient to define them separately for practical reasons: depending on the experimental setup, there may be overall production and detection asymmetries that affect the total signal yields. An experimental analysis can be made insensitive to these effects because they can be absorbed into the normalisation constants, as long as they are constant over the D -decay phase space. This comes at the cost that the information on x_\pm and y_\pm from the overall CP asymmetry is lost, but Section 2.3.5 will show the loss in precision to be minimal.

Thus, for a set of $2N$ bins, the bin yields of Eqs. (2.25) provide $4N$ constraints on a total of $4N + 6$ parameters: $(h^\pm, K_i, c_i, s_i, x_\pm, y_\pm)$. However, the K_i , c_i , and s_i parameters relate only to the D decay, and can thus, in principle, be measured in independent experiments. With such external inputs, a measurement of the $B^\pm \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-) K^\pm$ yields in a set of bins can be used to constrain x_\pm and y_\pm , and thereby (γ, r_B, δ_B) . The measurement presented in this thesis determines the K_i parameters directly, but uses externally measured values of c_i and s_i as input, as measured in quantum correlated D decays by the CLEO [73] and BESIII [74] collaborations. Because these measurements are the foundation of the approach, they are described in some detail in the following section. In the future, it is possible that the c_i and s_i parameters may be measured in quantum-correlated D decays in LHCb [75], and in charm-mixing measurements [76].

2.3.4 Measuring strong-phase inputs at charm factories

The strong-phase parameters c_i and s_i have been measured by the CLEO and BESIII collaborations, using quantum correlated $D^0\bar{D}^0$ pairs from decays of the $\psi(3770)$ resonance state, itself produced in e^+e^- collisions at the resonance energy. The $\psi(3770)$ has quantum-number $C = -1$, which is conserved in the strong decay into two D mesons, and thus the two D mesons are produced in an anti-symmetric wave function. By observing the decay of one D meson into a specific final state, say a CP eigenstate, the quantum state of the other D meson can be determined. The measurement is based on decays where both D decays are reconstructed, one in the $K_S^0\pi^+\pi^-$ final state, the other in one of several different tag categories. The main principles are outlined below, but most experimental considerations and implementation details are left out for the sake of brevity.

The simplest case is when one D meson decays into a final state that uniquely tags the flavour, such as $\bar{D}^0 \rightarrow K^+e^-\bar{\nu}_e$. In that case, the D meson decaying to $K_S^0\pi^+\pi^-$ is known to be in the D^0 state and the decay rate is simply determined by $A_S^D : \Gamma(s_{-+}) \propto |A_S^D(s_{-+})|^2$. This allows for a measurement of the K_i parameters.

If one D meson is reconstructed in a CP -even state, eg. K^+K^- , or a CP -odd state, eg. $K_S^0\pi^0$, the D meson decaying to $K_S^0\pi^+\pi^-$ is known to be in a state of opposite CP . Thus, for a tag-decay of $CP = \pm 1$ the decay rate has the form

$$\Gamma_{CP=\pm 1} \propto |A_S^D(s_{-+}) \mp A_S^D(s_{+-})|^2 \quad (2.29a)$$

and the bin yields will be given by

$$M_i^\pm \propto K_i + K_{-i} \mp 2\sqrt{K_i K_{-i}} c_i. \quad (2.29b)$$

Thus a simultaneous analysis of flavour and CP tagged decays allow for a determination of the K_i and c_i parameter sets.

Finally, the case where both D mesons, for now denoted D and D' , decay into the $K_S^0\pi\pi$ final state can be considered. The total amplitudes have contributions from the case where D is in the D^0 state and D' is in the \bar{D}^0 state, as well as the opposite flavour assignment. Thus the decay rate satisfies

$$\Gamma_{CP=\pm 1} \propto |A_S^D(s_{-+})A_S^{D'}(s'_{+-}) + A_S^D(s_{+-})A_S^{D'}(s'_{-+})|^2 \quad (2.30a)$$

where s_{-+} denotes the Dalitz-plot coordinates of the D meson, and s'_{-+} those of the D' meson. Defining M_{ij} to be the yield of decays where the D decay is in bin i and the D' in bin j , the bin yields satisfy

$$M_{ij} \propto K_i K_{-j} + K_j K_{-i} - 2\sqrt{K_i K_{-i} K_j K_{-j}} (c_i c_j + s_i s_j). \quad (2.30b)$$

558 Thus, analysing these decays in addition to the CP and flavour tagged decays provide
 559 information on all of K_i , c_i , and s_i . Note, however, that Eqs. (2.29) and (2.30) are
 560 invariant under the transformation $\delta_D \rightarrow -\delta_D$. In practice, the analysis is extended
 561 in a number of ways to enhance the statistics: using "flavour-tag" states that are not
 562 exact flavour tags, such as $K^-\pi^+$, using self-conjugate multi-body D -decay final
 563 states that are not exact CP eigenstates, such as $\pi^+\pi^-\pi^0$, and using the $K_L^0\pi^+\pi^-$
 564 final state as well. However, the main principles are the same as described above.

565 The measurements of c_i and s_i are made for a range of different binning schemes.
 566 It was noted already in Ref. [69] that a rectangular binning scheme, such as the
 567 example in Fig. 2.6, does not provide the optimal sensitivity to γ . A better sensitivity
 568 can be obtained if the bins are defined such that δ_D is approximately constant over
 569 a given bin, by defining bin i out of N via the condition

$$\text{bin}_i = \{(s_-, s_+) \mid 2\pi(i - 3/2)/N < \delta_D(s_-, s_+) < 2\pi \times (i - 1/2)/N\}. \quad (2.31)$$

570 In practice, the binning scheme is defined by splitting the D -decay phase-space
 571 into quadratic *micro bins* with a width of $0.0054 \text{ (GeV}/c^2)^2$ and assigning a bin
 572 number to each micro bin via the condition in (2.31) as evaluated in an amplitude
 573 model of choice. The obtained binning scheme when using an amplitude model
 574 developed by the BaBar collaboration in 2008 [61] is shown in Fig. 2.7a. In Ref [69]
 575 it was also shown that the binning can be even further optimised for sensitivity.
 576 The suggested figure of merit is

$$Q^2 = \frac{\sum_i \left(\frac{1}{\sqrt{N_i^B}} \frac{dN_i^B}{dx} \right)^2 + \left(\frac{1}{\sqrt{N_i^B}} \frac{dN_i^B}{dy} \right)^2}{\int ds^2 \left[\left(\frac{1}{|\Gamma^B(s_-)|} \frac{d|\Gamma^B(s_-)|^2}{dx} \right)^2 + \left(\frac{1}{|\Gamma^B(s_-)|} \frac{d|\Gamma^B(s_-)|^2}{dy} \right)^2 \right]} \quad (2.32)$$

577 which quantifies the statistical sensitivity for a given binning, relative to the one
 578 achievable in an unbinned analysis. The CLEO collaboration defined an *optimal*
 579 binning scheme by an iterative procedure where, starting from the equal binning
 580 scheme, a micro-bin is randomly reassigned new bin numbers in each step, and a
 581 step accepted if Q^2 increases. The optimisation is done for the case where $x = y = 0$
 582 and thus Q^2 simplifies to $Q_{x=y=0}^2 = \sum_i N_i^{x=y=0} (c_i^2 + s_i^2) / N_{total}^{x=y=0}$. The resulting
 583 binning scheme is shown in Fig. 2.7b. An additional binning scheme is defined,
 584 denoted the *modified optimal* scheme and shown in Fig. 2.7c, where the Q^2 figure
 585 of merit is modified to take into account the presence of backgrounds [73]. The
 586 modified optimal binning scheme has proven beneficial to use in measurements with
 587 small signal yields [77], but is not employed in the present thesis.

Table 2.1: The experimentally measured c_i and s_i values used in the thesis. The $D \rightarrow K_S^0\pi^+\pi^-$ values are the combined values from the BESIII and CLEO measurements published by BESIII [74]. The $D \rightarrow K_S^0K^+K^-$ values are measured by CLEO [73].

Optimal binning scheme: $D \rightarrow K_S^0\pi^+\pi^-$		
Bin i	c_i	s_i
1	-0.037 ± 0.049	0.829 ± 0.097
2	0.837 ± 0.067	0.286 ± 0.152
3	0.147 ± 0.066	0.786 ± 0.154
4	-0.905 ± 0.021	0.079 ± 0.059
5	-0.291 ± 0.041	-1.022 ± 0.062
6	0.272 ± 0.082	-0.977 ± 0.176
7	0.918 ± 0.017	-0.184 ± 0.065
8	0.773 ± 0.033	0.277 ± 0.118

2-bins binning scheme: $D \rightarrow K_S^0K^+K^-$		
Bin i	c_i	s_i
1	0.818 ± 0.107	-0.445 ± 0.215
2	-0.746 ± 0.083	-0.229 ± 0.220

Both the CLEO and BESIII collaborations have measured the values of c_i and s_i for the equal, optimal, and modified optimal binning schemes. The results are also shown in Fig. 2.7, where they are compared to the expectation from the latest amplitude model [55]. The measurements presented in this thesis are based on a combination of the BESIII and CLEO results for the optimal binning scheme, made by the BESIII collaboration [74] and tabulated in Table 2.1.

While the *definition* and *optimisation* of these binning schemes depend on knowledge of $A_S^D(s_-, s_+)$ via an amplitude model, it is important to note that no model information is needed when the binning schemes are used in the subsequent measurements of strong-phases⁵ or CP -observables. Therefore the measurements will not be biased by any modelling imperfections, although the obtained precision might be lower than expected.

The preceding discussion has been focusing on the $D \rightarrow K_S^0\pi^+\pi^-$ channel, however the $D \rightarrow K_S^0K^+K^-$ channel can be analysed completely analogously. The CLEO collaboration measure c_i and s_i values for this mode as well, in three binning schemes [73]. These are all equal-phase binning schemes, with 2, 3, and 4 bins,

⁵With the exception of minimal model-dependence introduced when the $K_L^0\pi^+\pi^-$ final state is employed to constrain the s_i parameters by the D -factories [73, 74], the impact of which is well under control.

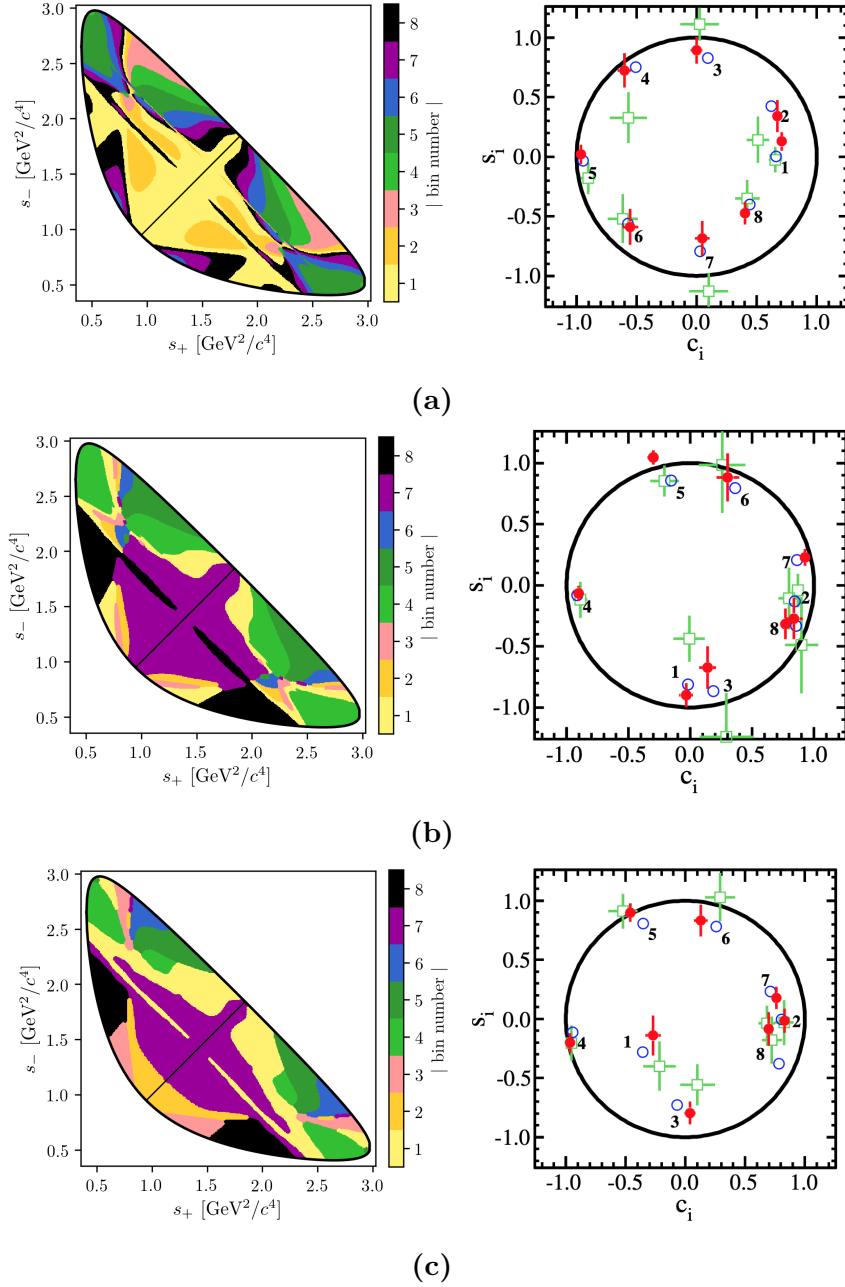


Figure 2.7: The (left) binning schemes and (right) measured values of (c_i, s_i) for (a) equal, (b) optimal, and (c) modified optimal binning schemes for $D \rightarrow K_S^0 \pi^+ \pi^-$ decays. The plots of the measured values are taken from Ref. [74] and show the results obtained by (red) BESIII, (green) CLEO, and (blue) the model expectation using the model from Ref. [55]. The measurement featured in this thesis used the optimal binning scheme.

respectively, shown in Fig. 2.8. The $D \rightarrow K_S^0 K^+ K^-$ decay amplitude is almost completely dominated by two $K^+ K^-$ resonances, the CP -odd $\phi(1020)$ and the CP -even $a_0(980)$, and this means that very little gain in sensitivity can be made by altering the equal-phase binning schemes. The measured c_i and s_i values are also shown in Fig. 2.8 and tabulated in Table 2.1 for the 2-bins scheme, which is used in this thesis. A BESIII measurement is in preparation, but has not been finished at the time of writing.

2.3.5 Global CP asymmetry and the relation to GLW and ADS measurements

The introduction of separate normalisation factors h^+ and h^- in Eq. (2.25) hides the fact that information on γ (in principle) can be obtained from the asymmetry in phase-space-integrated B^+ and B^- yields. In the ideal case where $h^- = h^+$ the total yield asymmetry is

$$\begin{aligned} A_{GGSZ} &= \frac{\sum_i N_- - N_i^+}{\sum_{i=-N}^N N_i - N_i^+} = \frac{\sum_{i=-N}^N \sqrt{K_i K_{-i}} c_i (x_- - x_+)}{1 + r_B^2 + 2 \sum_{i=-N}^N \sqrt{K_i K_{-i}} c_i (x_- + x_+)} \\ &= \frac{2 \sum_{i=1}^N \sqrt{K_i K_{-i}} c_i (x_- - x_+)}{1 + r_B^2 + 4 \sum_{i=1}^N \sqrt{K_i K_{-i}} c_i (x_- + x_+)}, \end{aligned} \quad (2.33)$$

where it has been exploited that $\sum_{i=-N}^N \sqrt{K_i K_{-i}} s_i = 0$ by definition. The size of the asymmetry is governed by the factor $\sum_{i=1}^N \sqrt{K_i K_{-i}} c_i$, which is small for $D \rightarrow K_S^0 \pi^+ \pi^-$ and $D \rightarrow K_S^0 K^+ K^-$ decays. The underlying reason is that $\delta_D(s_-, s_+)$ varies significantly across phase-space for these decays, as evident by the spread in the values of c_i in Table 2.1, which reduces the *average* of the asymmetry-generating $D^0 - \bar{D}^0$ interference term to being close to zero. The value of $\sum_{i=-N}^N \sqrt{K_i K_{-i}} c_i$ is closely related to the CP content of the final state in question: for a self-conjugate CP even (odd) final state

$$A_{D^0}(s_-, s_+) = {}^{(+)}_{(-)} A_{\bar{D}^0}(s_-, s_+) = {}^{(+)}_{(-)} A_{D^0}(s_+, s_-) \quad (2.34)$$

and thus $\sum_{i=1}^N \sqrt{K_i K_{-i}} c_i = {}^{(+)}_{(-)} 1$. This motivates the definition of the CP -even fraction of the decay

$$\mathcal{F}_+ \equiv \frac{1}{2} \left(1 + \sum_{i=1}^N \sqrt{K_i K_{-i}} c_i \right), \quad (2.35)$$

is equivalent to the definition in Ref. [79] for the case $N = 1$. With \mathcal{F}_+ in hand, the asymmetry in Eq. (2.33) can be rewritten

$$A_{GGSZ} = \frac{(2\mathcal{F}_+ - 1)r_B \sin \delta_B \sin \gamma}{1 + r_B^2(2\mathcal{F}_+ - 1)2r_B \cos \delta_B \cos \gamma}, \quad (2.36)$$

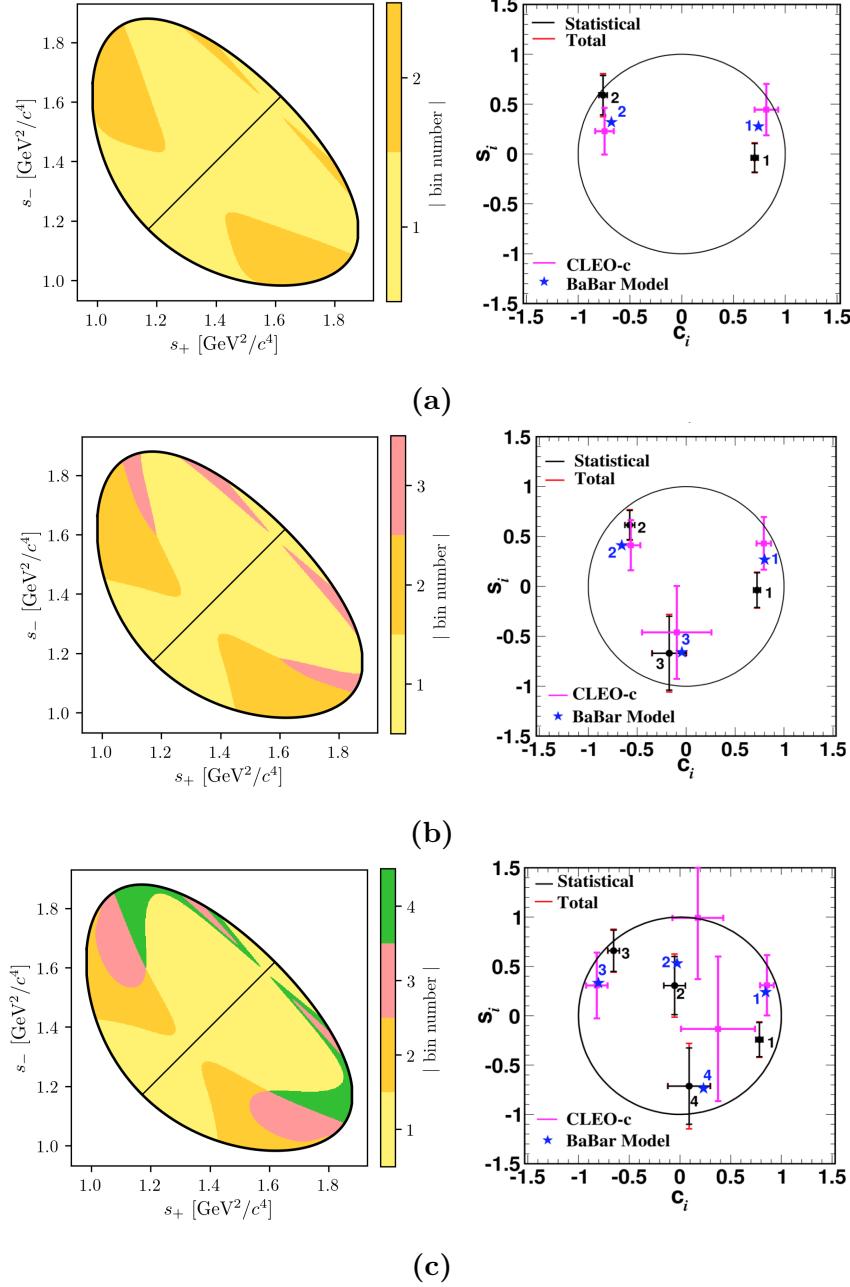


Figure 2.8: The (left) binning schemes and (right) measured values of (c_i, s_i) for the (a) 2-, (b) 3-, and (c) 4-bins binning schemes for $D \rightarrow K_0^0 K^+ K^-$ decays. The plots of the measured values are taken from Ref. [78] and show the (error bars) results obtained by (black) BESIII, (pink) CLEO, and (blue) the model expectation using the model from Ref. [62]. The measurement featured in this thesis uses the 2-bins scheme.

which is the usual form used in quasi-GLW measurements [79, 80]. The value of \mathcal{F}_+ is independent of the number and shape of bins in a given binning scheme, as long as the bin definitions follow the symmetry principles outlined in Section 2.3.3. For $D \rightarrow K_S^0\pi^+\pi^-$ and $D \rightarrow K_S^0K^+K^-$ decays the values of \mathcal{F}_+ are

$$\begin{aligned}\mathcal{F}_+(K_S^0\pi^+\pi^-) &= X? \\ \mathcal{F}_+(K_S^0K^+K^-) &= X?\end{aligned}\tag{2.37}$$

as evaluated with the Belle 2018 model for $D \rightarrow K_S^0\pi^+\pi^-$ decays and the BaBar 2010 model for $D \rightarrow K_S^0K^+K^-$ decays. Since $r_B^{DK^\pm} \sim 0.1$ the predicted global asymmetries are thus approximately 1–2 %, which is not resolvable with the current experimental yields. As shown in Chapter 4, CP violation in the K_S^0 sector leads to asymmetries of a similar size, further complicating the use of global asymmetries to constrain x_\pm and y_\pm . Thus these modes are ill-suited for quasi-GLW measurements, and ignoring global asymmetries leads to a negligible loss of information on γ in a GGSZ measurement. The reverse is true for a well-suited quasi-GLW mode, such as $D \rightarrow \pi^+\pi^-\pi^0$: if \mathcal{F}_+ is close to either zero or unity, it means that (c_i, s_i) will be close to $(\pm 1, 0)$ in all bins for *any* given binning scheme, and the set of bins will provide almost identical constraints on x_\pm and y_\pm . Thus, the binning of phase space leads to no significant gain in precision compared to a global analysis.

Indeed, a crucial quality of the GGSZ method, is that exactly because each bin-pair provides independent constraints on x_\pm and y_\pm , the method provides a single solution for (γ, r_B, δ_B) that does not suffer the ambiguities of the ADS and GLW approaches. In order to illustrate this further, it is useful to make one more comparison of the model-independent GGSZ formalism to the ADS and GLW formalisms. If there was no CP symmetry the B^+ yield in bin $+i$ would equal the B^- yield in bin $-i$. Therefore the relevant CP asymmetry for a given Dalitz bin is

$$\begin{aligned}A_{GGSZ}^i &\equiv \frac{N_i^- - N_{-i}^+}{N_i^- + N_{-i}^+} \\ &= \frac{\sqrt{K_i K_{-i}}(c_i(x_- - x_+) + s_i(y_- - y_+))}{K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(c_i(x_- + x_+) + s_i(y_- + y_+)})\end{aligned}\tag{2.38}$$

This expression is identical to the ADS asymmetry in Eq. (2.16a) if the effective D -decay parameters r_D^i and δ_D^i are defined via

$$\kappa_i \cos \delta_D^i \equiv c_i \quad , \quad \kappa_i \sin \delta_D^i \equiv s_i \quad , \quad r_D^i \equiv \sqrt{K_i / K_{-i}},\tag{2.39}$$

and a coherence factor, κ , is included in the interference terms of the ADS expression, as is standard for multi-body D decays [81]. These parameters allow us to classify

Table 2.2: Classification of the bins used in model-independent GGSZ measurements, in terms of whether the interplay between the D^0 and \bar{D}^0 amplitudes in the bin resemble typical GLW or ADS behaviour. The parameters are calculated using the 2018 Belle model [55] for $D \rightarrow K_S^0\pi^+\pi^-$ decays and the 2010 BaBar model [62] for $D \rightarrow K_S^0K^+K^-$ decays.

Optimal binning scheme: $D \rightarrow K_S^0\pi^+\pi^-$					
Bin i	\hat{r}_D	$\hat{\delta}_D$	\mathcal{F}_+	κ	Bin type
1	0.473	91.9°	48.97 %	0.81	Odd-even
2	0.164	11.1°	63.38 %	0.85	ADS-like
3	0.157	79.4°	52.50 %	0.89	ADS-like
4	0.768	175.3°	5.85 %	0.92	GLW-odd-like
5	0.759	-99.9°	42.84 %	0.87	Odd-even
6	0.223	-64.5°	57.92 %	0.87	ADS-like
7	0.651	-13.3°	89.44 %	0.89	GLW-even-like
8	1.745	21.0°	87.08 %	0.92	GLW-even-like

2-bins binning scheme: $D \rightarrow K_S^0K^+K^-$					
Bin i	\hat{r}_D	$\hat{\delta}_D$	\mathcal{F}_+	κ	Bin type
1	0.816	19.8°	86.14 %	0.78	GLW-even-like
2	0.775	154.5°	16.23 %	0.77	GLW-odd-like

656 a given pair of bins with number $\pm i$ as either *GLW-like*, if δ_D^i is close to 0 or π
 657 and r_D^i is close to unity, or *ADS-like* if $0 < r_D^i \ll 1$. The *CP*-even fraction of the
 658 D -decay can also be defined for a given bin-pair:

$$\mathcal{F}_+^i = \mathcal{F}_+^{-i} \equiv \frac{1}{2} \left(1 + 2c_i \frac{\sqrt{K_i K_{-i}}}{K_i + K_{-i}} \right) = \frac{1}{2} \left(1 + 2c_i \frac{r_D^i}{1 + r_D^i} \right). \quad (2.40)$$

659 A GLW-even-like bin pair will have $\mathcal{F}_+^i \simeq 1$ and a GLW-odd-like bin pair will
 660 have $\mathcal{F}_+^i \simeq 0$.

661 Table 2.2 summarises a classification of the bins for the optimal $D \rightarrow K_S^0\pi^+\pi^-$
 662 binning scheme and the 2-bins $D \rightarrow K_S^0K^+K^-$ binning scheme following these
 663 principles. Two bins are classified as *Odd-even*; in these bins, r_D^i is not particularly
 664 small but \mathcal{F}_+^i is close to 0.5. The name refers to the fact that for these
 665 bins A_{GGSZ}^i , as defined in Eq (2.38), will be positive and A_{GGSZ}^{-i} negative (or vice
 666 versa). The fact that multiple bin types appear for both the $D \rightarrow K_S^0\pi^+\pi^-$ and
 667 $D \rightarrow K_S^0K^+K^-$ modes underline that each mode benefits from being analysed in
 668 the GGSZ formalism, and that the bins provide independent constraints, allowing
 669 for a non-ambiguous solution for (γ, r_B, δ_B) .

2.4 Strategy for the LHCb measurement

The main topic of the thesis is a model-independent GGSZ measurement using $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays, and the two D final states $K_S^0\pi^+\pi^-$ and $K_S^0K^+K^-$. The measurement uses the optimal binning scheme for the $D \rightarrow K_S^0\pi^+\pi^-$ mode, with the combined strong-phase inputs from the BESIII [74] and CLEO [73] collaborations published in Ref. [74]. For the $D \rightarrow K_S^0K^+K^-$ channel, the 2-bins scheme is used with the strong-phase parameters measured by the CLEO collaboration [73]. The details of the analysis are presented in Chapter (5), but the overall strategy and a few extensions of the formalism from the previous sections are given here.

Due to the geometry of the LHCb detector, the signal reconstruction efficiency for $B^\pm \rightarrow D(\rightarrow K_S^0h^+h^-)h'^\pm$ decays varies significantly across the D -decay phase space. Denoting the efficiency profile as $\eta(s_-, s_+)$, the yield equations of Eq. (2.25) are therefore modified slightly

$$\begin{aligned} N_i^- &= h^{B^-} \left[F_i + r_B^2 F_{-i} + 2\sqrt{F_i F_{-i}} (c'_i x_- + s'_i y_-) \right], \\ N_i^+ &= h^{B^+} \left[F_{-i} + r_B^2 F_i + 2\sqrt{F_i F_{-i}} (c'_i x_+ - s'_i y_+) \right], \end{aligned} \quad (2.41)$$

where the phase-space integrated quantities now include the efficiency profile

$$F_i = \frac{1}{N_F} \int ds^2 \eta(s_{-+}) |A_S^D(s_{-+})|^2, \quad N_F = \int ds^2 \eta(s_{-+}) |A_S^D(s_{-+})|^2, \quad (2.42)$$

$$c'_i = \frac{\int_i ds^2 \eta(s_{-+}) |A_S^D(s_{-+})| |A_S^D(s_{-+})| \cos[\delta_D(s_{-+})]}{\sqrt{\int_i ds^2 \eta(s_{-+}) |A_S^D(s_{-+})|^2} \sqrt{\int_i ds^2 \eta(s_{-+}) |A_S^D(s_{-+})|^2}}, \quad (2.43)$$

with an analogous definition of s'_i . At leading order, the strong-phase parameters are unaffected by the non-uniform efficiency, and, in addition, the bin definitions favour bins for which $\cos[\delta_D(s_{-+})]$ and $\sin[\delta_D(s_{-+})]$ take on similar values across each bin. Therefore, the c_i and s_i values reported by the charm factories are used directly in the measurement. The impact on the obtained central values is negligible, as described in detail in Section 5.6 where a systematic uncertainty is assigned.

The F_i are significantly different to the K_i due to the experimental acceptance profile in LHCb. Given external inputs for the strong-phase parameters, it is possible to fit the F_i parameters and x_\pm and y_\pm simultaneously in a fit to the LHCb $B^\pm \rightarrow DK^\pm$ data set, in which case the obtained F_i parameters incorporate the correct acceptance profile correction by construction. However, the obtainable precision for the CP observables measured by this procedure is suboptimal. As

an alternative, the first LHCb measurement [71] made a simultaneous analysis of $B^\pm \rightarrow DK^\pm$ and a much larger sample of $B^\pm \rightarrow D\pi^\pm$ decays; since the F_i parameters relate to the D decay, they can effectively be obtained in the $D\pi^\pm$ sample and shared between the two $B^\pm \rightarrow Dh^\pm$ channels. However, there is CP violation present in the $B^\pm \rightarrow D\pi^\pm$ decays, which led to a dominant systematic uncertainty. Later LHCb measurements [3, 82] instead relied on flavour tagged D mesons from $\bar{B}^0 \rightarrow D^{*+}(\rightarrow D^0\pi^+)\mu^-\bar{\nu}_\mu X$ decays to obtain F_i , where no CP violation is possible. However, due to necessarily different triggering paths and selections, the acceptance profile is not exactly identical between semi-leptonic decays and the $B^\pm \rightarrow Dh^\pm$ decays of interest. An efficiency correction based on simulation was therefore applied to obtain the correct F_i , and in this case, the uncertainty related to the correction constituted the largest systematic uncertainty on the measurement.

Both sources of systematic uncertainty can be avoided by making a simultaneous analysis of $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays, where CP -violating observables are measured in *both* channels and the F_i parameters are shared. It is a reasonable assumption that $F_i^{DK} = F_i^{D\pi}$ to a very good approximation, given the similar kinematics of the decays. The assumption is confirmed using simulated decays in Section 5.2.2, for the candidate selection used in the measurement of the thesis. Effectively, the F_i are determined in the high statistics $B^\pm \rightarrow D\pi^\pm$ channel, but with no systematic effect from CP -violation in that channel, since the CP -violation is incorporated in the yield description.

ADD SECTION ON INFORMATION ON GAMMA FROM DPI AND THE TWO SOLUTIONS IN THE LHCb COMBINATION?!

At the start of the work that lead to this thesis, it was not clear to what degree the measured CP -violating observables in $B^\pm \rightarrow D\pi^\pm$ decays were affected by CP violation in the neutral kaon sector. The impact had been shown to scale as $\mathcal{O}(|\epsilon|/r_B)$ [42], which is negligible for the $B^\pm \rightarrow DK^\pm$ channel but suggests potentially large biases in the $B^\pm \rightarrow D\pi^\pm$ channel, where r_B is 20 times smaller. However, the dedicated analysis presented in Chapter 4 has proved the effect on GGSZ measurements to be in fact be *smaller* than $\mathcal{O}(|\epsilon|/r_B)$ and the simultaneous measurement is indeed viable.

The measurement is performed by making extended maximum-likelihood fits to the m_B spectra of $B \rightarrow D(\rightarrow K_S^0 h^+ h^-)h'^\pm$ candidates split by charge and Dalitz bin. The $B^\pm \rightarrow DK^\pm$ signal yields are parameterised using the expressions in Eq. (2.41) directly, thus obtaining values for x_\pm^{DK} and y_\pm^{DK} directly. The Cartesian CP -violating observables x_\pm and y_\pm are employed because they lead to better statistical behaviour than fits to data where the underlying parameters $(\gamma, r_B^{DK^\pm}, \delta_B^{DK^\pm})$ are

determined, at the cost of introducing a fourth degree of freedom. With the addition of the $B^\pm \rightarrow D\pi^\pm$ mode as a true signal channel, two new underlying parameters are introduced, $r_B^{D\pi^\pm}$ and $\delta_B^{D\pi^\pm}$. There is a choice to be made, in terms of how to define the observables that are measured directly. One is to introduce an additional set of four observables, $(x_-^{D\pi}, y_-^{D\pi}, x_+^{D\pi}, y_+^{D\pi})$, that are analogous to the $B^\pm \rightarrow DK^\pm$ parameters. As an alternative, it is possible to introduce only two Cartesian parameters [83], by defining

$$\xi_{D\pi^\pm} = \left(\frac{r_B^{D\pi^\pm}}{r_B^{DK^\pm}} \right) \exp[i(\delta_B^{D\pi^\pm} - \delta_B^{DK^\pm})] \quad (2.44a)$$

and letting

$$x_\xi^{D\pi} = \text{Re}[\xi_{D\pi^\pm}] \quad y_\xi^{D\pi} = \text{Im}[\xi_{D\pi^\pm}]. \quad (2.44b)$$

In terms of these parameters, the usual Cartesian x_\pm and y_\pm are given by

$$x_\pm^{D\pi} = x_\xi^{D\pi} x_\pm^{DK} - y_\xi^{D\pi} y_\pm^{DK}, \quad y_\pm^{D\pi} = x_\xi^{D\pi} y_\pm^{DK} + y_\xi^{D\pi} x_\pm^{DK}. \quad (2.45)$$

Using this expression, the $B^\pm \rightarrow D\pi^\pm$ yields can also be defined via Eq. (2.41) in the maximum-likelihood fit. Note that ξ does not depend on γ : all information on CP asymmetries in both the $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ channels is encoded in x_\pm^{DK} and y_\pm^{DK} . In the thesis, the latter parameterisation is chosen, because it allows for a stable fit for all six x and y parameters and the shared F_i ; the choice is described in much greater detail in Section 5.5.1.

The combined analysis of $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays presents a significant step forward, because it solves the problem of obtaining F_i parameters for the appropriate acceptance profile in a manner that avoids leading systematic uncertainties, and almost all reliance on simulation. This is of great importance, if the large data samples that will be collected by LHCb in the future are to be exploited to their full potential.

3

756

757

The LHCb experiment

758 The LHCb experiment is one of the four large experiments at the Large Hadron
759 Collider (LHC), the World's most powerful accelerator, able to accelerate protons
760 to record centre-of-mass energies of $\sqrt{s} = 13$ TeV in a 27 km long tunnel underneath
761 Geneva. The LHCb experiment is specifically designed to study the large number
762 of particles containing b or c quarks produced in such collisions, which has led
763 to a number of design decisions that make the LHCb unique among the LHC
764 experiments. The LHCb detector is not a full-solid-angle detector like the other
765 three LHC experiments, CMS, ATLAS, and ALICE, but a single-arm spectrometer,
766 instrumented in the forward region where the majority of $b\bar{b}$ pairs are produced.
767 During data-taking the experiment is operated at a lower instantaneous luminosity
768 than the other experiments, leading to far fewer pp interactions. This, in combination
769 with a vertex detector located extremely close to the interaction point, allows for
770 excellent resolution in the reconstruction of primary and secondary vertex locations,
771 crucial to many of the central measurements of the experiment. Finally, dedicated
772 particle-identification detectors allow for very efficient separation of hadron species,
773 absolutely crucial to isolate a number important signal decays (including the
774 $B^\pm \rightarrow DK^\pm$ decay studied in the thesis). Each of these features is described in
775 much greater detail in the sections below.

776 During operation of the LHC, bunches of about $\mathcal{O}(10^{11})$ protons are accelerated
777 to the desired centre-of-mass energy in a series of linear and circular accelerators,
778 the final one being the LHC itself. This is illustrated in Fig. 3.1. The bunches
779 remain in the LHC for the duration of a *fill*, typically about 12 hours, where they are
780 made to collide at four distinct locations, the collision points, each home to one of

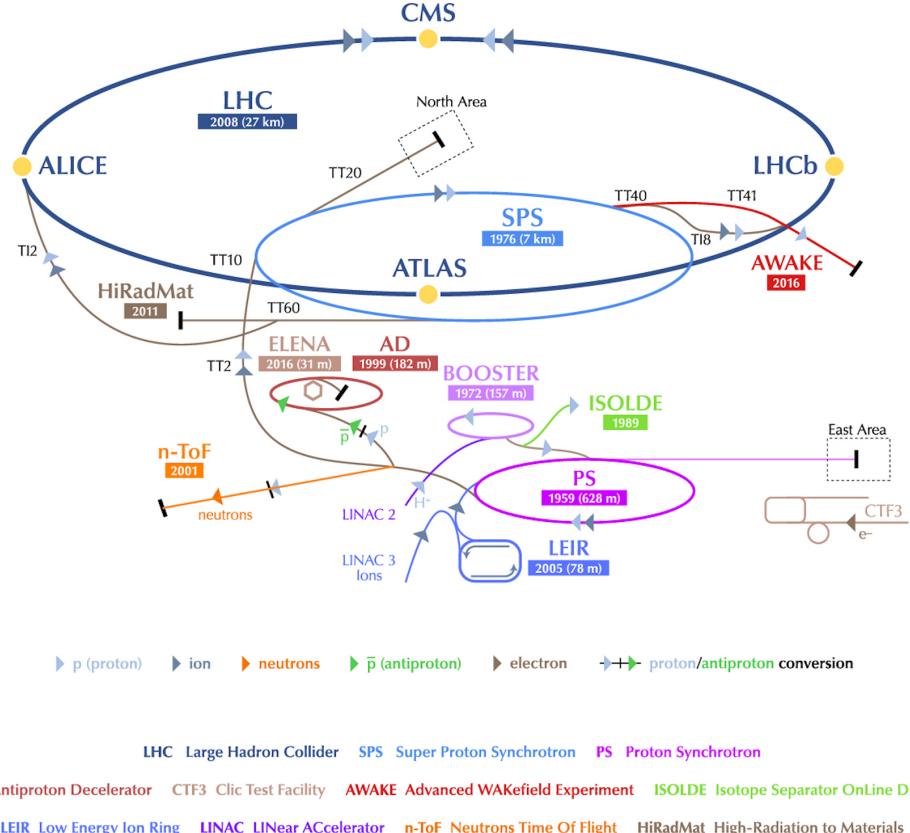


Figure 3.1: The CERN accelerator complex, including the length and construction year for a number of accelerators, not all of which are used in pp operations. During pp operation, the proton acceleration chain is: LINAC 2 → BOOSTER → PS → SPS → LHC. The figure is reproduced from Ref. [84].

the large experiments. The collisions occur with a frequency of up to 40 MHz. A fill ends when the beams are dumped, typically because the average number of protons in the bunches has become too low, after which the whole process begins again.

The LHC has been providing pp collisions during two periods so far: Run 1 during 2011 and 2012, where the centre-of mass energies were $\sqrt{s} = 7 \text{ TeV}$ and 8 TeV respectively, and Run 2 from 2015 to 2018, where $\sqrt{s} = 13 \text{ TeV}$. The instantaneous luminosity at the LHCb collision point has been $4 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$, and has allowed for the collection of data set corresponding to a total of 3 fb^{-1} during Run 1 and 6 fb^{-1} during Run 2. The full data set forms the basis of the thesis. This instantaneous luminosity is significantly lower than at other collision points, for example the peak instantaneous luminosity in the ATLAS detector was about $20 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ in 2018 [85], 50 times higher than in LHCb. The lower luminosity is necessary to limit the number of pp interactions per bunch crossing to an average of about 1.1–1.4 (depending on the data taking period), necessary for a vertex reconstruction with the required precision. The lower luminosity is

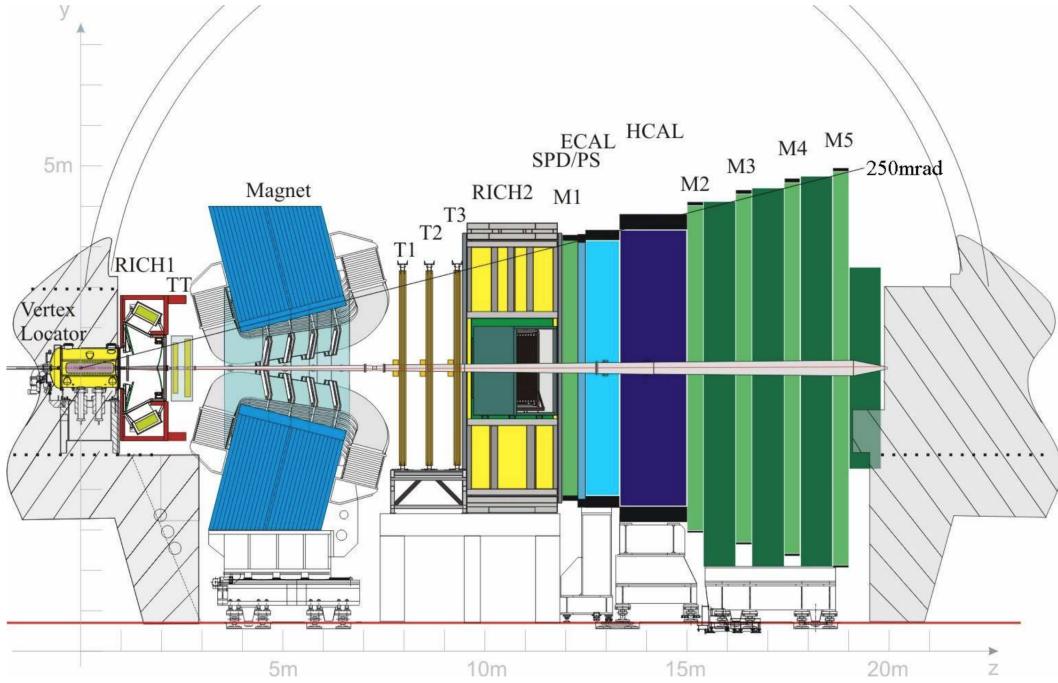


Figure 3.2: Overview of the LHCb detector reproduced from Ref. [86,87]. The individual subdetectors are described in detail in the text.

achieved by colliding the proton beams with an off-set at the LHCb collision point. This has the added benefit that the offset can be continuously adjusted during a fill of the LHC, and thus all data can be taken at the same instantaneous luminosity, allowing for simpler trigger configuration, and simpler subsequent analysis because the detector occupancy is constant. The lower luminosity, of course, comes with the downside that the collected data sample is smaller.

3.1 The LHCb subdetectors

The LHCb detector, shown in Fig. 3.2, is able to detect particles in the forward region $\eta \in [2, 5]$, corresponding to an angle θ with respect to the beam line between 15 and 300/250 mrad in the horizontal/vertical direction. As illustrated in Fig. 3.3, the $b\bar{b}$ production cross section is very large within the LHCb acceptance: even though the acceptance covers less than 2% of the solid angle, 24% of all $b\bar{b}$ pairs created at $\sqrt{s} = 14$ TeV are within the acceptance. The detector is described with a coordinate system, where the z -axis is along the beam line and the x (y) axis is in the horizontal (vertical) directions normal to the beam line. The origin is at the collision point. The experiment consists of a number of sub detectors, located in the region from around the interaction point, and up to a distance of $z = 20$ m along

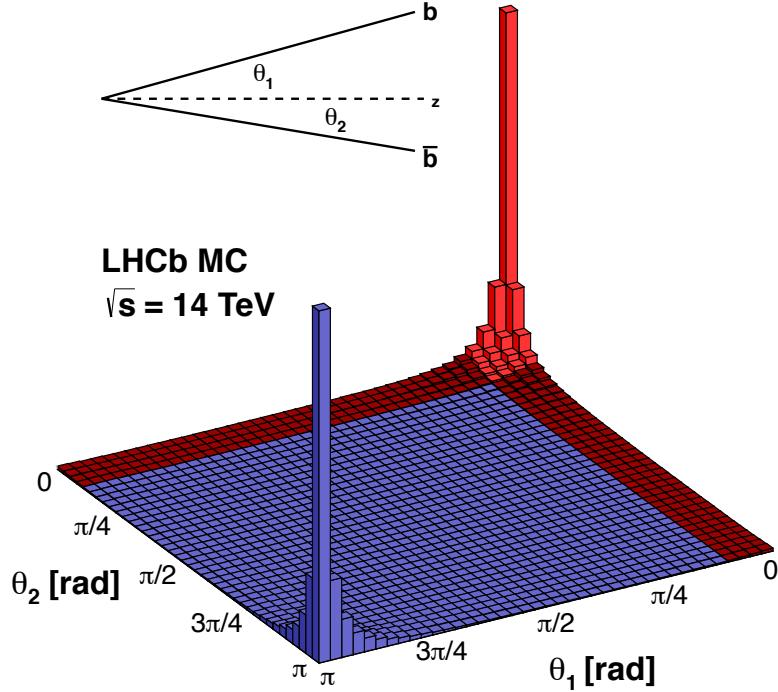


Figure 3.3: Production cross section of $b\bar{b}$ pairs at a centre-of-mass energy of $\sqrt{s} = 14 \text{ TeV}$, as a function of θ_1 and θ_2 , the angle of the b and \bar{b} quark, respectively, with respect to the beam axis z . The LHCb acceptance is marked in red. The cross-section looks very similar for $\sqrt{s} = 7, 8 \text{ TeV}$. The figure is taken from Ref. [88].

the beam line (in the following, the direction from the interaction point towards the sub detectors is denoted *downstream*, and the opposite direction *upstream*).
This section describes each of them in detail.

3.1.1 The VELO

The VErtex LOcator (VELO) [89] is a silicon detector located immediately around the collision point, used to provide precise measurements of the particle track coordinates in the interaction region. These are used to reconstruct the production and decay vertices of beauty and charm hadrons with a very high accuracy, allowing for an accurate reconstruction of their life times, and play an important role in the full track reconstruction. The ability to distinguish tracks originating in secondary vertices also plays a crucial role in efficient triggering, as described further below.

The detector consists of 21 VELO stations positioned along the beam line as illustrated in Fig. 3.4. Each station consists of two *modules*, mounted on each side

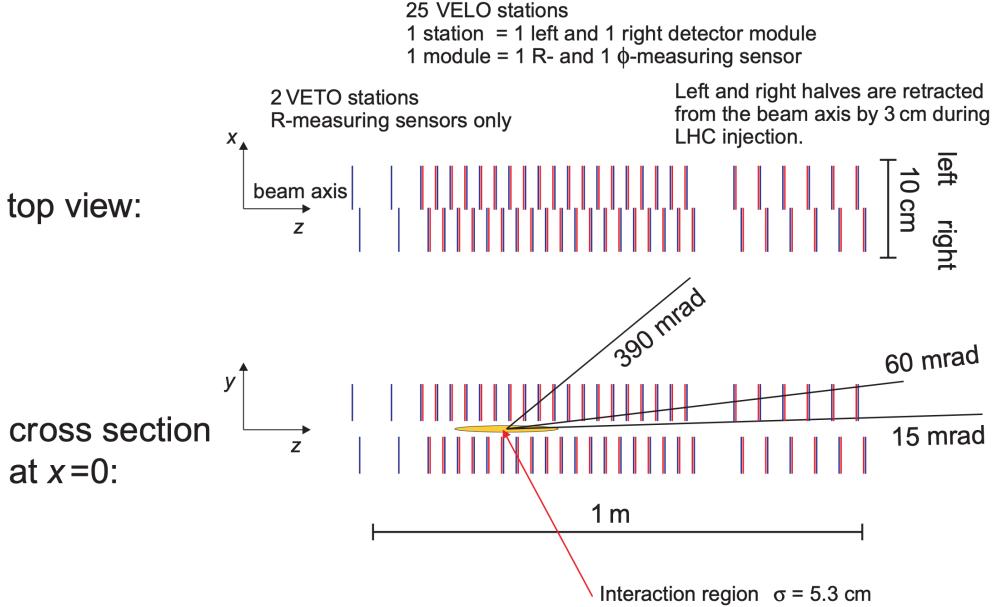


Figure 3.4: Overview of the arrangement of VELO stations from the VELO Technical Design Report (TDR) [89]. The actual detector includes 21 stations instead of 25, but the overall design is identical [90].

of the beam line; each module, in turn, consists of two silicon strip detectors, where the strips are oriented to provide a measurement of r , the radial distance from the beam line, and ϕ , the azimuthal angle, respectively. This is illustrated in Fig. 3.5. The strip pitch varies between 40 and 100 μm depending on the distance from the beam line. The stations are positioned such that all tracks that are within the acceptance region of the downstream detectors and originate at the interaction point are guaranteed to intersect 3 detector stations. During operation, the segments are located only 8 mm from the beam; this is achieved by mounting them on a moving frame that can be retracted during beam commissioning to avoid radiation damage. The detectors are kept in a vacuum, shielded from the beam vacuum by a 0.3 mm thick *RF foil* made of aluminium that also serves to screen the detector from electric fields induced by the proton beam. The silicon sensors were kept at an operating temperature of about -7 $^{\circ}\text{C}$, achieved with a liquid-CO₂ cooling system.

The primary vertex (PV) resolution of the VELO is typically $\sim 10 \mu\text{m}$ in the x and y directions and $\sim 50 \mu\text{m}$ in the z direction, improving with the number of tracks originating at the PV, and deteriorating with the overall number of PVs [90]. The typical uncertainty on the decay length of a B meson is about 230 μm , compared to a typical decay length $O(10)$ mm. The resolution of the *impact parameter*, IP, of a track is well-described by the formula $\sigma_{\text{IP}} = (15 + 29/[p_T/(\text{GeV}/c)]) \mu\text{m}$. This parameter excellently distinguished particles produced in secondary decays, from

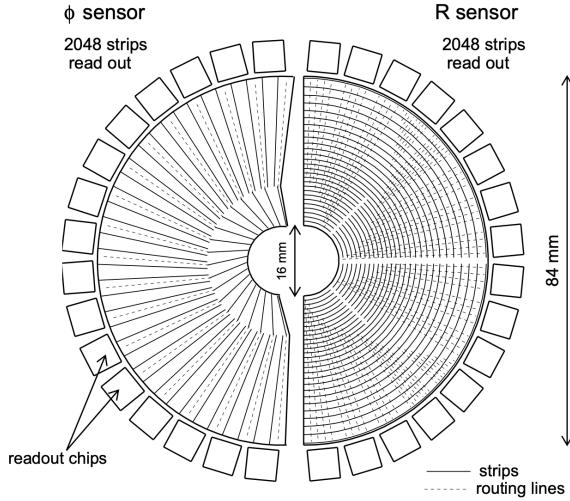


Figure 3.5: Illustration of the silicon strip layout in the VELO modules designed to measure (left) the azimuthal angle, ϕ , of a track, and (right) the radial distance from the beam, r . Reproduced from Ref. [89].

846 those produced in the primary interaction (for which the IP would be zero, were
847 it not for the experimental resolution).

848 3.1.2 Magnet and tracking stations

849 The LHCb experiments uses a warm (non-superconducting) dipole magnet to
850 measure the momentum of charged particles, by providing a maximum magnetic
851 field strength of approximate 1T and a total bending power of about 4 T m over
852 the region where $z \in [2.5, 8]$ m. The magnetic field has been measured to a relative
853 precision of about 4×10^{-4} and is uniform within a percent within the tracking
854 volume. The profile of the magnetic field along the z -axis is shown in Fig. 3.14 on
855 page 41, where the track types within LHCb are defined. The magnet can provide
856 a magnetic field in either vertical direction; over the span of a year of running the
857 experiment approximately equal amounts of data are collected with the magnet in
858 the "Up" and "Down" configurations; this leads to the cancellation of a number of
859 charge-asymmetry effects, significantly reducing potential systematic uncertainties.

860 The tracking system consists of the VELO, and four other tracking stations:
861 the Tracker Turicensis (TT) upstream of the magnet, and the tracking stations
862 1–3 (T1, T2, T3) downstream of the magnet. The downstream tracking stations
863 each consist of an Inner Tracker (IT) based on silicon strips, and an Outer Tracker
864 (OT) that employs drift tubes.

865 Both the TT and IT are based on silicon strip detectors with a pitch of about
866 200 μm ; they were developed as a single project and are collectively known as the

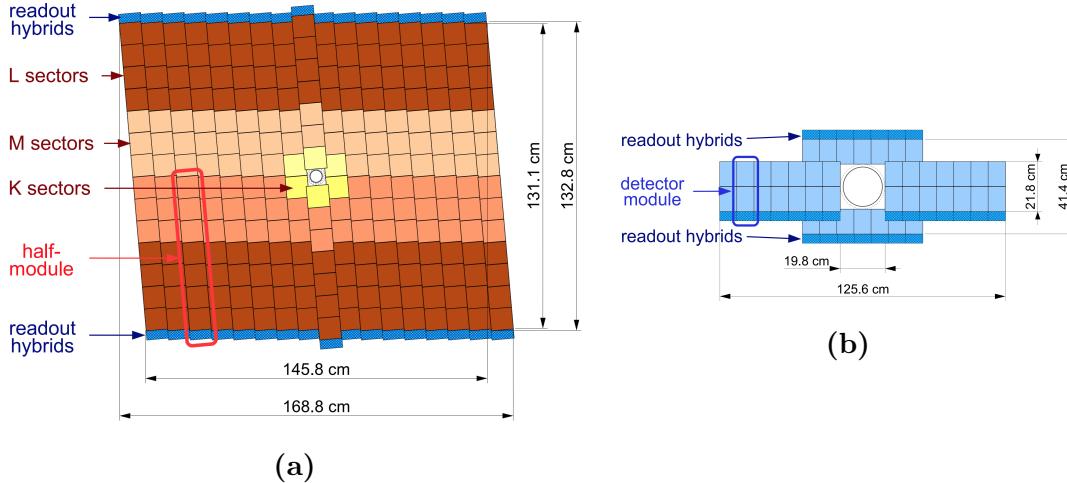


Figure 3.6: Overview of (a) a *v*-layer module of the IT and (b) an *x*-layer module of the IT. Reproduced from Ref. [86]

867 Silicon Tracker (ST). The TT is a 140 cm wide and 130 cm tall planar tracking
 868 station, covering the whole LHCb acceptance. It is shown in Fig. 3.6a. At each
 869 of the T1–T3 stations, the IT consist of four modules, arranged around the beam
 870 pipe as illustrated in Fig. 3.6b. They do now cover the full LHCb acceptance,
 871 only the very-forward region where the number of tracks is largest. Each TT or
 872 IT module comprises of four layers of silicon strips, where the central two layers
 873 are rotated $\pm 5^\circ$ with respect to the first and last layer (an *x-u-v-x* geometry).
 874 The ST has a spatial resolution for a given track of approximately 50 μm , chosen
 875 because the overall momentum resolution is then dominated by multiple-scattering
 876 effects for almost all reconstructed tracks.

877 At the T1–T3 stations, the OT covers the part of the overall acceptance of
 878 300 (250) mrad in the horizontal bending (vertical non-bending) plane that is not
 879 covered by the IT. The OT consists of arrays of gas-tight drift tubes with inner
 880 diameters of 4.9 mm. The OT is shown illustrated in Fig. 3.7. An Ar/CO₂/O₂
 881 (70/28.5/1.5) gas mixture is used to fill the tubes that ensures a drift time below
 882 50 ns and a drift coordinate resolution of 200 μm . The use of a drift-chamber
 883 detector is necessary, because it was not economically feasible to instrument the
 884 whole LHCb acceptance with silicon strip detectors in T1–T3. The condition that
 885 the OT occupancy should not be above 10 % in typical run conditions determined
 886 the boundary between the IT and the OT.

887 The overall relative momentum resolution achieved for most charged tracks in
 888 LHCb is less than a percent, as illustrated in Fig. 3.8, where it has been determined
 889 from a fit to the mass peak in $J/\psi \rightarrow \mu^+\mu^-$ decays in Run 1 data.

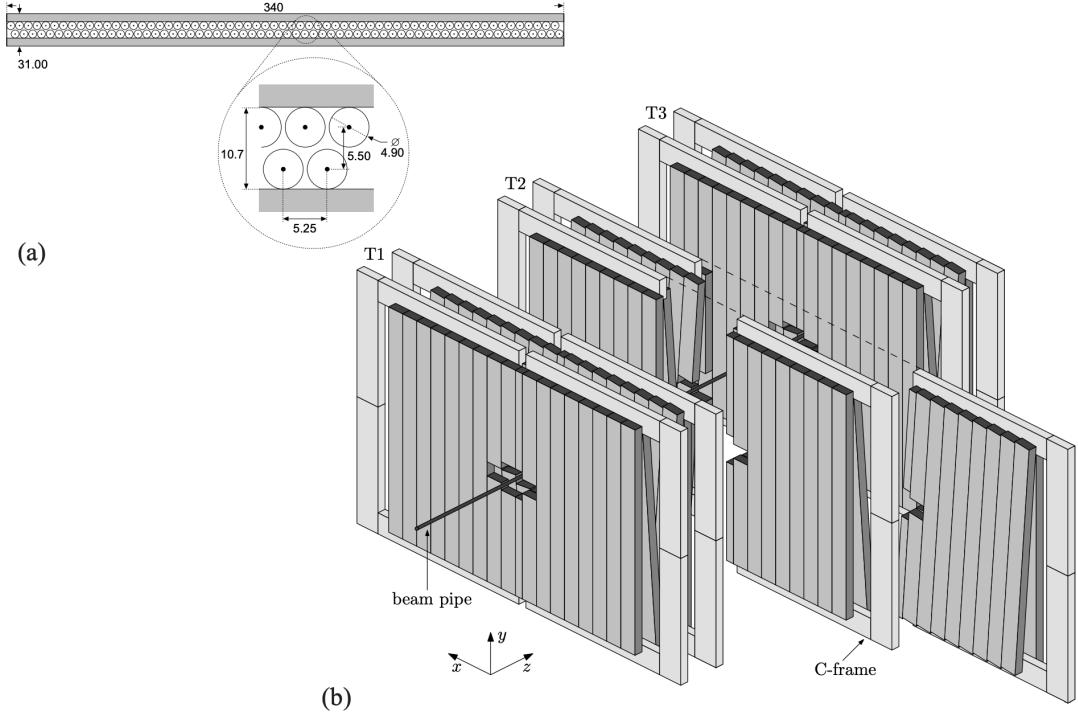


Figure 3.7: (a) Cross section of an OT module. (b) Arrangement of the OT modules in tracking stations. Reproduced from Ref. [91].

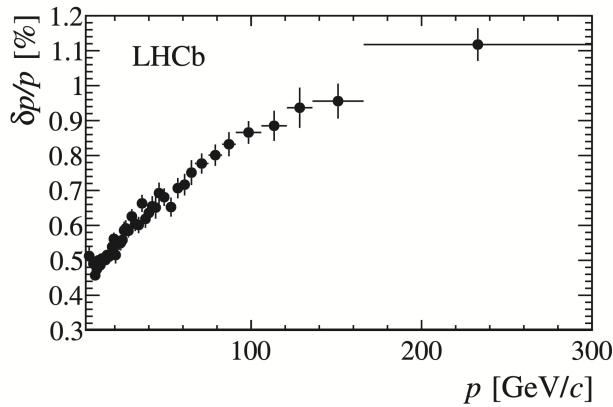


Figure 3.8: Relative uncertainty on the momentum of charged tracks (specifically long tracks, cf. the definitions in Section 3.2) in the LHCb detector, determined via the mass resolution obtained in $J/\psi \rightarrow \mu^+\mu^-$ decays in Run 1 data. Reproduced from Ref. [92]

3.1.3 The RICH detectors

- 891 Two Ring Imaging Cherenkov detectors (RICH) provide crucial information for
- 892 particle identification (PID) in LHCb, in particular the ability to separate pions
- 893 and kaons that is absolutely essential for the measurement presented in the thesis.
- 894 The RICH 1 detector is located upstream of the magnet, in between the VELO
- 895 and the TT tracking station. It is designed to provide PID capability for tracks

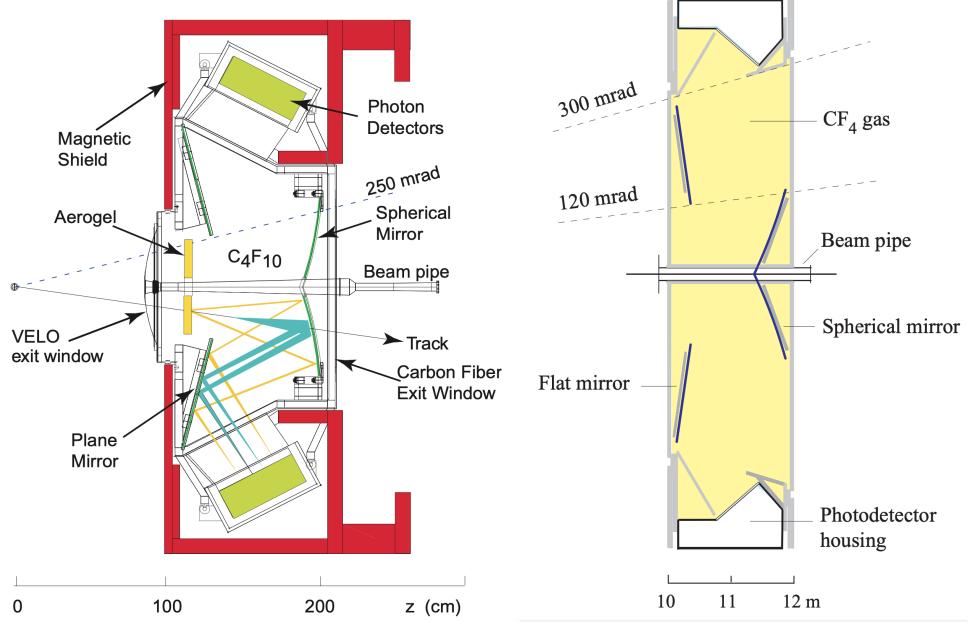


Figure 3.9: Overview of (left) the Rich 1 and (right) the RICH 2 detectors. Reproduced from Ref. [86, 93].

in the momentum range $p \in [1, 60] \text{ GeV}/c$ using a C_4F_{10} radiator, and covers the full LHCb acceptance. During Run 1 the RICH 1 detector also included an Aerogel radiator designed to provide PID for very low momentum particles; however, it was removed before Run 2 because it did not meet the performance requirements during Run 1 [94, 95]. The RICH 2 detector is located downstream of the T1–T3 tracking stations. It is designed to provide PID capabilities for higher momentum tracks in the range $p \in [15, 100] \text{ GeV}/c$ using a CF_4 radiator. It only covers the very forward region where $|\theta| < 120 \text{ mrad}(100 \text{ mrad})$ in the horizontal (vertical) directions, as high momentum particles are produced in that region. In both RICH detectors, mirrors are used to reflect the Cherenkov photons to arrays of Hybrid Photon Detectors (HPDs) located outside the LHCb acceptance. The optics are designed such that photons originating from a given track form rings in the HPD arrays, where the radius is determined by the Cherenkov angle θ_c . The detectors are illustrated in Fig. 3.9.

The resolution on θ_c can be measured by fitting the obtained θ_c distribution in high momentum tracks, where the Cherenkov angle is saturated. It is found to be $1.618 \pm 0.002 \text{ mrad}$ for RICH 1 and $0.68 \pm 0.02 \text{ mrad}$ for RICH 2 in Run 1 data [94], and was essentially unchanged in Run 2 [95]. Figure 3.10 shows the relation between track momentum and θ_c in RICH 1 for *isolated tracks* in Run 1 data; these are tracks where the Cherenkov ring does not overlap with any other

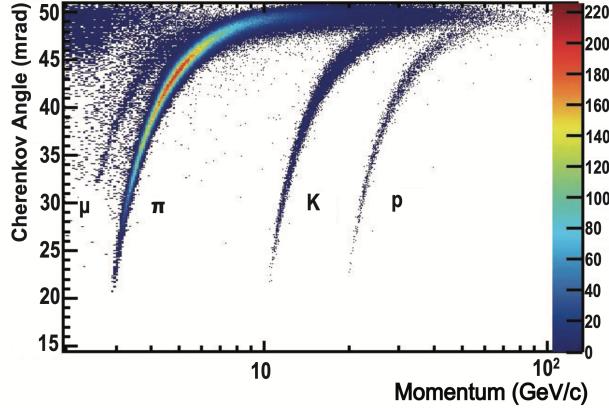


Figure 3.10: Cherenkov angle for isolated tracks in the RICH 1 radiator as a function of track momentum. Reproduced from Ref. [94].

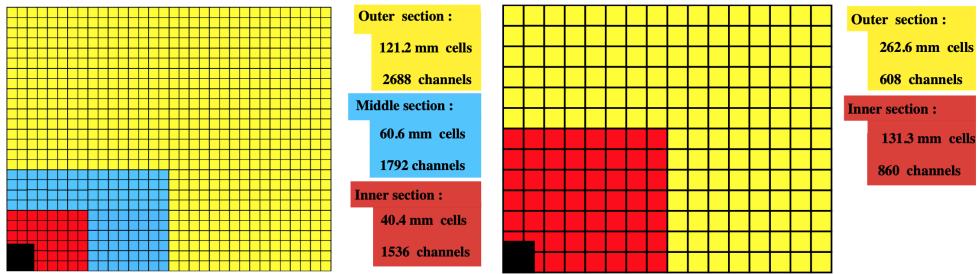


Figure 3.11: Illustration of the calorimeter cell size of (left) the ECAL and (right) the HCAL. Reproduced from Ref. [96].

Cherenkov rings. The bands for each hadron species are clearly visible, and it can be seen that the RICH detector also provide some ability to distinguish muons. The definition of the PID variables used in analysis is discussed in Section 3.2.2, along with the achieved PID performance.

3.1.4 Calorimeters

The calorimeter system of the LHCb detector has four components. Ordered from the interaction point, these are the Scintillating Pad Detector (SPD), the Pre-Shower (PS) detector, an Electromagnetic Calorimeter (ECAL), and a Hadron Calorimeter (HCAL). Information from the calorimeters also provide identification of electrons, photons, and hadrons, and measurements of their energies and positions, and also plays a crucial role in the triggering, as described below. In all four cases, light is produced in organic scintillators and transmitted to Photo Multiplier Tubes (PMTs) via optical fibres [86].

The SPD and PS detectors consist of almost identical planes of rectangular scintillator pads, with a 15 mm thick lead absorber located in between. The presence

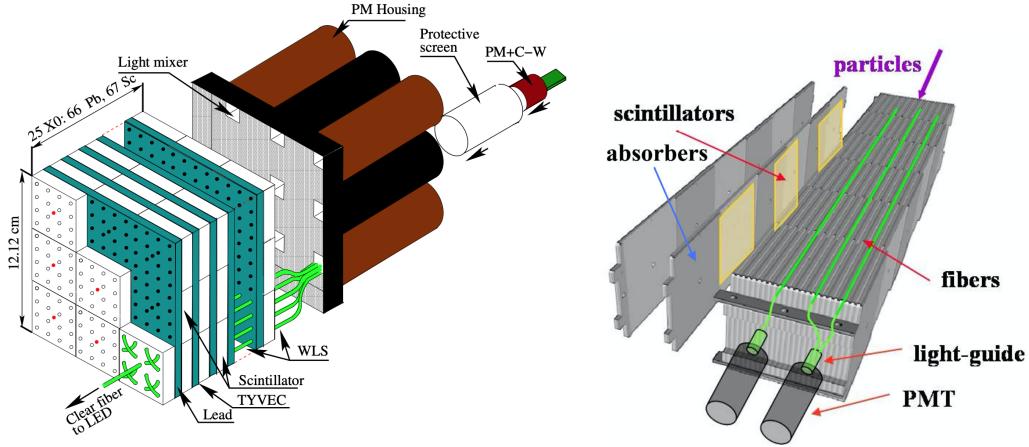


Figure 3.12: Illustration of (left) an ECAL and (right) a HCAL module. Reproduced from Ref. [92, 97].

of the SPD before the first absorption layer allows for the separation of photons and electrons in the trigger, because only electrons cause a signal in the SPD. The PS allows for the separation of pion and electron tracks, as only the latter interact significantly with the thin layer of lead. The cell divisions of the detectors closely follow that of the ECAL, shown in Fig. 3.11, to allow for the matching of energy deposits.

The ECAL has a Shashlik structure, with 66 layers consisting of 2 mm of lead absorber and 4 mm of scintillator; an example of a calorimeter module is shown in Fig. 3.12. Accurate energy measurements require that the full electronic shower is contained in the ECAL, which is achieved since the structure extends for 25 radiation lengths. The scintillators are divided into cells that allow for the determination of the location and shape of energy deposits; the cell dimensions vary as a function of radial distance from the beam pipe as shown in Fig. 3.11, to take into account the varying occupancy. The resolution of the ECAL has been measured to be $\Delta E/E \simeq (9/\sqrt{E} \oplus 0.8)\%$ (E in GeV/c^2) [86].

The HCAL is located downstream of the ECAL, designed to measure the energy of charged hadrons (which leave relatively little energy in the ECAL). It is constructed with layers of 1 cm iron absorbers inter-spaced with scintillators, oriented *along* the beam direction, such that a typical track will traverse 16 mm of iron per 4 mm of scintillator [96]. As for the ECAL, the cell size varies as a function of distance to the beam line, as shown in Fig. 3.11. An example of a module is shown in Fig. 3.12. The energy resolution required for efficient triggering is moderate; therefore, the HCAL only has a length of 5.6 interaction lengths and can measure the hadron energies at a resolution of $\Delta E/E \simeq (69/\sqrt{E} \oplus 9)\%$ (E in GeV/c^2) [86].

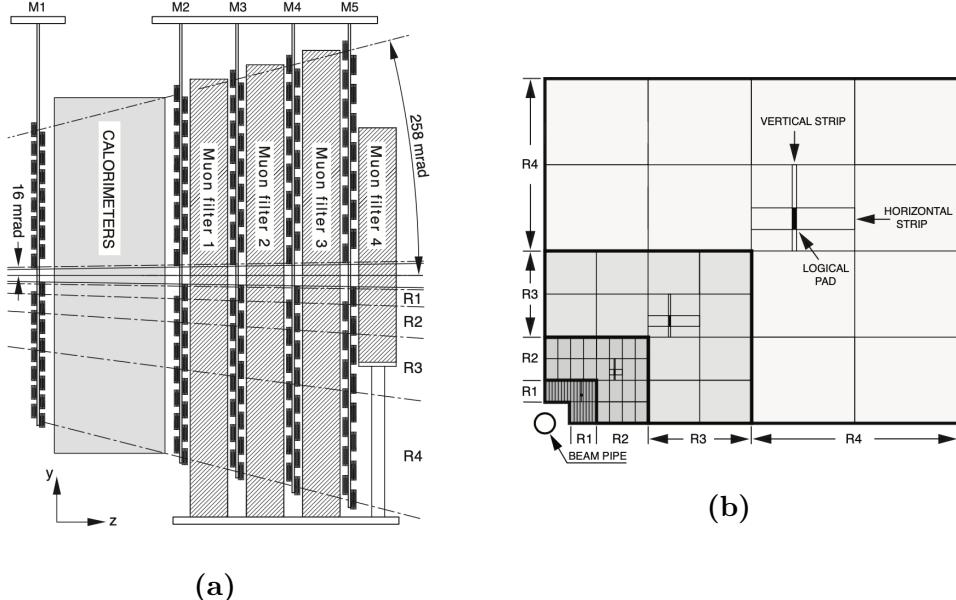


Figure 3.13: Illustration of (a) the location of the muon stations along the z -axis of the experiment, and (b) the geometry of the logical pads of the M3 muon station. Reproduced from Ref. [92].

3.1.5 Muon detectors

Muon identification and triggering is crucial for a range of high-profile LHCb measurements, such as lepton-universality tests [98, 99] or measurements of $B_{(s)}^0 \rightarrow \mu^+ \mu^-$ decays [100]. In the thesis, muon identification plays a role in suppressing a number of backgrounds. The LHCb muon system consists of 5 tracking stations, M1–M5, covering the full LHCb acceptance. M1 is located upstream of the ECAL, whereas M2–M5 are located downstream of the HCAL and inter-spaced with 80 cm thick ion absorbers in order to select penetrating muons. This is illustrated in Fig. 3.13a. The detectors are predominantly multiwire proportional chambers (MWPC), organised into logical pads, the dimensions of which define the (x, y) resolution of the measured spatial points. The exception is the central region of the M1 station, which is a triple gas-electron-multiplier detector, due to the higher track density in that region [101]. As for the calorimeters, the size of the pads vary as a function of the radial distance from the beam pipe, as illustrated in Fig. 3.13b. The resolution is significantly better in the bending plane (x) than in the non-bending plane (y). The resolution is also significantly better in the M1–3 stations than in M4 and M5, which are mostly used to identify penetrating tracks. The muon system can independently measure the p_T of a muon to within 20 %, which allows for efficient triggering.

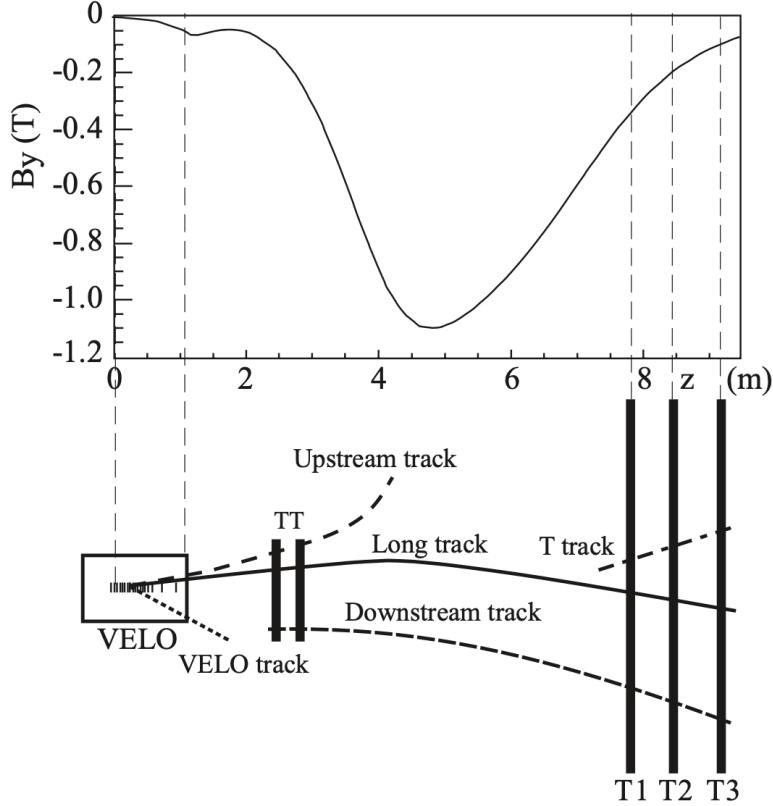


Figure 3.14: Definition of track types within the LHCb detector, depending on which set of tracking detectors the track intersects. The profile of the magnetic field is also shown. Reproduced from Ref. [92].

3.2 Reconstruction

This section describes the reconstruction algorithms that fit the detector hits in the tracking stations to form track candidates, as well as the algorithms used to identify the types of the particles that formed these tracks.

3.2.1 Track reconstruction

The LHCb experiments operates with a number of different particle track types, depending on which sub detectors a track intersects; these are summarised in Fig. 3.14. The two track types that are important for this thesis are *long* tracks, which have hits in the VELO and the TT and T1–T3 tracking stations, and *downstream* tracks that only have hits in the TT and T1–3 tracking stations. The analysis depends on both track types because a number of K_S^0 mesons produced in the signal decay leave the VELO before they decay into the $\pi^+\pi^-$ final state that is reconstructed; hence these pions necessarily form downstream tracks.

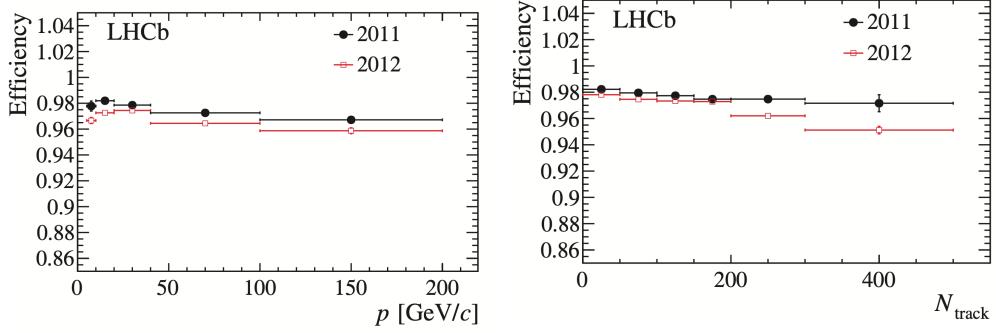


Figure 3.15: The long track reconstruction efficiency as a function of (left) track momentum and (right) the number of charged tracks in the event. The lower efficiency in 2012 than 2011 is partially due to the higher event multiplicity, given the higher centre-of-mass energy. The figure is reproduced from Ref. [92].

987 The first step is to form track candidates from hits in the VELO (VELO tracks)
 988 and T1–3 stations (T tracks) separately; because the magnetic field is low in the
 989 tracking detectors, these tracks are fairly straight. Long tracks are formed using
 990 two separate search strategies: in one, *forward tracking* [102], VELO tracks are
 991 used as seeds and matched with hits in the TT and T1–3 tracking stations by
 992 extrapolation. These are combined to form long tracks that are required to pass
 993 a set of quality conditions. An alternative approach, *track matching* [103, 104],
 994 matches VELO and T tracks by extrapolating both through the bending region,
 995 and deciding if they below together; finally TT hits are added. The union of tracks
 996 found via both approaches is saved, where only the track candidate with the best
 997 fit quality is kept in the case where a track appears twice. Downstream tracks
 998 are formed based on T tracks as seeds, matched with hits in the TT detector in
 999 a search region obtained by extrapolation of the seed [105]. Finally, each track
 1000 is reprocessed using a Kalman filter that takes into account multiple scattering
 1001 and corrects for energy loss due to ionisation [106, 107].

1002 Many of the interesting signal decay channels of LHCb have 4–6 charged final
 1003 state tracks, and therefore it is crucial to have a single-track reconstruction efficiency
 1004 close to 100 %. The single-track reconstruction efficiency is shown in Fig. 3.15 as
 1005 a function of track momentum and the number of tracks in an *event* (an *event*
 1006 denotes a pp collision and all the particles produced therein and in subsequent
 1007 decays). The efficiencies have been obtained in data, using a tag-and-probe method
 1008 in $J/\psi \rightarrow \mu^+\mu^-$ decays [108]. One muon, the *tag*, is fully reconstructed, while the
 1009 other, the *probe* is only partially reconstructed, allowing for the J/ψ invariant mass
 1010 to be reconstructed with reasonable resolution. If the partially reconstructed probe
 1011 track is matched to a full long track, the track is classified as efficient. Similar
 1012 efficiencies have been achieved in Run 2.

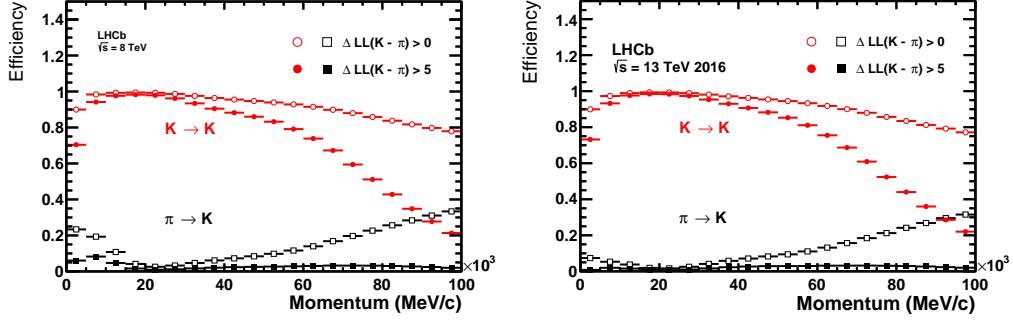


Figure 3.16: The probability to correctly identify a kaon/misidentify a pion as a kaon given two different requirements on $\Delta LL(K)$, as a function of track momentum in (left) Run 1 data from 2012 and (right) Run 2 data from 2016. Reproduced from Ref. [109].

3.2.2 Particle identification

The information from the RICH detectors, the calorimeters, and the muon system is combined for optimal identification of charged tracks as electrons, muons, pions, kaons, or protons. Photons and neutral pions are identified using the ECAL, but play no role in the thesis, and will not be discussed further.

The ability to separate $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays is essential to the measurement presented in this thesis. In LHCb, hadron separation is achieved via information from the RICH detectors, using a likelihood method where the observed pattern of hit pixels in the photo detectors is compared to the expected pattern, given all reconstructed tracks in an event under a given set of particle hypothesis. The likelihood is maximised by varying the particle hypotheses for each track being an electron, muon, pion, kaon, or proton [110]. It is necessary to consider all tracks of an event simultaneously because the Cherenkov rings of different tracks overlap. For each track, the maximum log likelihood of a particle hypothesis, say that the track is a kaon, relative to the hypothesis that it is a pion

$$\Delta LL_{\text{track}_i}(K) = \ln \mathcal{L}_{\max}^{\text{RICH}}(\text{pattern} | \text{track}_i = K) - \ln \mathcal{L}_{\max}^{\text{RICH}}(\text{pattern} | \text{track}_i = \pi), \quad (3.1)$$

is saved to inform PID decisions. In the case of pion-kaon separation, this variable alone is enough to achieve good separation power; in the remainder of the thesis it is denoted PIDK. The PID performance for pion-kaon separation has been measured in calibration data, following a procedure described in Section 5.2.1, and is illustrated in Fig. 3.16.

Muons are identified by extrapolating tracks to the muon stations to define fields-of-interest (FOI). A track is considered as a muon candidate when a minimum number of stations (2–4 depending on the track momentum) have hits in the

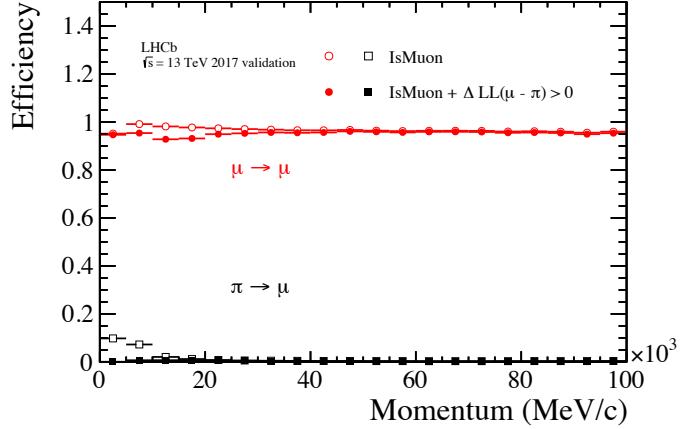


Figure 3.17: The probability to correctly identify a muon/misidentify a pion as a muon given requirements on either `isMuon` or $\Delta LL(\mu)$, as a function of track momentum in Run 2 data from 2017. Reproduced from Ref. [109].

corresponding FOI [111, 112]. This information is encoded in a variable denoted `isMuon` throughout the thesis. Additional information, such as a comparison of the slopes of the track in the main tracker and the muon stations, and the average track-hit distance in the FOI is used to form a $\Delta LL^{\text{muon}}(\mu)$ variable analogous to the one defined in Eq. (3.1) for the RICH detectors; it can be combined with $\Delta LL^{\text{RICH}}(\mu)$ to form a PID variable that takes information from both detectors into account, denoted `PIDmu`. The performance of the muon PID variables is shown in Fig. 3.17 as obtained in data. It can be seen that requiring `isMuon=0` rejects muon tracks efficiently at all momenta; this is used in the analysis to veto a number of semi-leptonic backgrounds.

In similar manner, a potential semi-leptonic background with electrons is also vetoed in the analysis presented in the thesis. In LHCb, electron PID is mainly based on the balance between deposited energy and track momentum in the ECAL [113]. This information is combined with information on photon energy deposits from brehmstrahlung, and energy deposits in the PS and HCAL, as well as information from the RICH and muon detectors, to form yet another ΔLL variable, denoted `PIDE`. As an example of the obtainable performance, an average electron selection efficiency of $(91.9 \pm 1.3\%)$ was achieved in displaced $J/\psi \rightarrow e^+e^-$ decays in Run 1, with a hadron misidentification rate of $(5.54 \pm 0.02)\%$ [92].

3.3 The LHCb trigger system

The collision rate in the LHC is up to 40 MHz, with a visible inelastic collision rate in LHCb of up to 30 MHz. The LHCb uses a multi-stage trigger to reduce rate with

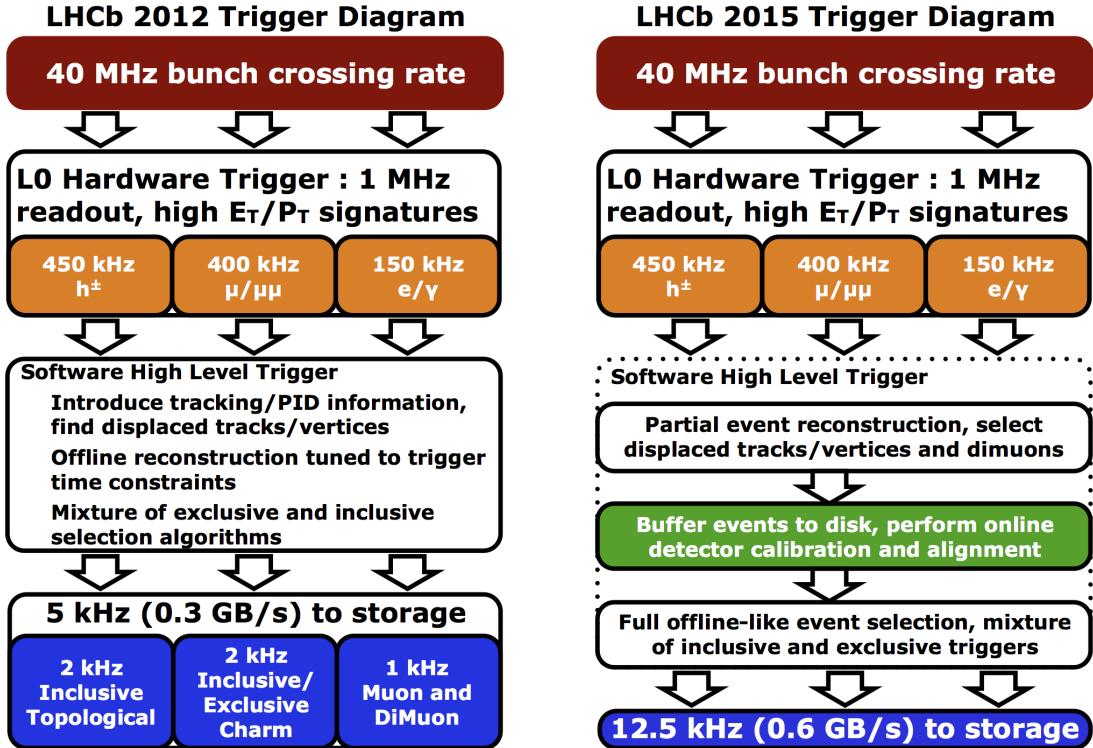


Figure 3.18: Illustration of stages and event processing rates in the LHCb trigger during (left) Run 1 and (right) Run 2.

which events are stored to a manageable level (of eg. 12.5 kHz during Run 2). The first stage consists of a hardware trigger that selects events with high transverse energy in the calorimeters, or hits in the muon detectors. This is followed by two software stages that rely on a reconstruction of tracks in the detector to select events that are likely to include interesting physics. The overall trigger stages were identical in Run 1 and Run 2, however the throughput rate was upgrade significantly between the two data taking period, as was the quality of the reconstruction in the software trigger stages; in Run 2, the final software trigger decisions are in fact based on an event reconstruction that is fully equivalent to the one performed offline [114]. The stages are illustrated in Fig. 3.18, and described in detail in the following.

A further, offline processing and reconstruction step is applied to all events before they are made available to most LHCb analyses, commonly denoted as the *stripping* step. Although the stripping does not form part of the LHCb trigger, it does constitute an additional, centralised filter on the data, and a description is included in Section 3.3.3.

3.3.1 The level-0 hardware trigger

The level-0 (L0) triggers that select physics events are based on the calorimeters and the muon system. The ECAL and HCAL are divided into clusters of 2×2 cells, for which the transverse energy is defined as

$$E_T = \sum_j E_j \sin \theta_j, \quad (3.2)$$

where θ_j is the angle of cell j with respect to the beam axis and the average collision point. The trigger forms a L0Hadron candidate with the highest E_T found in the HCAL, combined with the ECAL cluster in front of it if such a cluster is present. Photon and electron candidates are formed based on clusters in the ECAL, identified by the presence (lack) of hits in the SPD for an electron (photon). The transverse energies of the candidates are compared to a fixed set of thresholds, and events where at least one candidate is above threshold are retained.

The muon trigger searches for straight line tracks in the muon stations, estimating the associated muon p_T based on the track direction. An event is retained if either the largest muon p_T is above a given threshold, or the product of the two highest muon p_T values is above a different threshold.

High-multiplicity events take a long time to process in the subsequent software stage; therefore it is favourable for the overall retention rate of interesting physics decays to put a maximum limit on the event multiplicity at the L0 stage. This is achieved by requiring the number of hits in the SPD detector to be below a threshold value in most L0 lines.

3.3.2 High-level triggers

The events that pass the L0 trigger are passed to a farm of multiprocessor computing node, the Event Filter Farm (EFF), tasked with bringing the rate down from approximately 1 MHz to the $\mathcal{O}(1 - 10)$ kHz rate that can be saved to disk. The EFF consisted of 900 (1700) nodes during Run 1 (Run 2). The software-based filtering proceeds in two stages: a first filter (HLT1) brings the rate down to approximately 40 (110) kHz based on a limited reconstruction of the event, after which a second stage (HLT2) filters the events further based on a more complete reconstruction. Each step executes a number of different algorithms, each of which can allow an event to be accepted; these are denoted *trigger lines*.

During both runs, the HLT1 performed a partial event reconstruction by building long tracks that satisfy a p_T requirement using the forward tracking approach described in Section 3.2.1, and determining the location of PVs using Velo tracks.

In both runs, the HLT1 included an inclusive trigger that selected a high p_T track with significant displacement of all PVs (typical of a b or c decay). This line is denoted `HLT1TrackAllL0` in Run 1 [115]; for Run 2 the track requirements were reoptimised and it is denoted `Hlt1TrackMVA`. Further, an additional inclusive trigger was added that forms a two-prong vertex out of high p_T tracks inconsistent with originating in a PV, and applies a multivariate classifier to determine if it is signal-like based on a number of track and vertex properties. This line is denoted `Hlt1TwoTrackMVA` [114]. These lines triggered all events included in the analysis of the thesis; other lines exist for selecting events that include muons, calibration data, low-multiplicity events, and a number of exclusive lines, for a total of approximately 20 lines during Run 2 [114].

Because the rate of events is reduced significantly by HLT1, the HLT2 decisions can be based in a more complete reconstruction of the event. Indeed, during Run 2 it was based on a complete, fully aligned reconstruction equivalent to the offline reconstruction. During Run 1 the HLT2 reconstruction only included long tracks and did exclude some low momentum tracks; this was a main motivation for the upgrade of the EFF during the shutdown period. The need for full alignment in HLT2 means that it could not be run fully online in Run 2; instead the output events from HLT1 were saved to disk in the EFF, and processed with some delay [114]. The analysis presented in the thesis is based on a number of inclusive "topological" trigger lines, based on combinations of 2, 3, or 4 tracks that satisfy fit quality requirements, have high p_T , are separated from the PVs, and have a distance-of-closes-approach below 0.2 mm. A multivariate classifier [116] is applied to each formed n -body object, to determine if the event should be accepted based on the track momenta, invariant mass, a corrected invariant mass that takes into account missing transverse momentum, distance of closest approach, and the impact parameter and separation with the associated PV. The resulting trigger lines were denoted `Hlt2Topo{2, 3, 4}BodyBBDT` during Run 1 and `Hlt2Topo{2, 3, 4}Body` during Run 2. A large number of other HLT2 lines exist (more than 500 in Run 2), including a significant number of exclusive lines that aim to select specific decays and only save information on the signal decay, not the whole event. This was made possible by the full reconstruction within HLT2 [114], and have allowed for larger signal yields to be collected within the data storage limits.

3.3.3 Offline data filtering: the LHCb stripping

Events that are written to disk are processed with the full detector alignment and calibration. In a further, offline processing step denoted the *stripping*, hundreds of different, dedicated reconstructions are performed; decay candidates for various signal decays are built and a number of requirements are made to reject backgrounds from random track combinations. For example, the $B^\pm \rightarrow D(\rightarrow K_S^0 h'^+ h'^-) h^\pm$ candidates that are analysed in this thesis are built during the stripping stage, as described further in Section 5.1. The stripping is a centralised computing task, executed on the Worldwide LHC Computing Grid [117], and allows the analysts to process much smaller data sets during their individual analysis. Because the stripping is based on data saved to offline storage it can be repeated; however, the processing of data collected during a year of data taking takes many weeks, so this does not happen often.

3.4 Simulation

A centralised LHCb simulation is able to simulate pp collisions with the proper conditions within LHCb, model subsequent secondary decays and the full detector response, and process the output in the full LHCb reconstruction. In this thesis, simulated decays are used to determine the reconstructed invariant-mass distribution of a number of decay modes, as well as a number of relative selection efficiencies. The pp collisions are generated using PYTHIA [118] with a specific configuration specific to LHCb [119]. The time-dependent evolution and decays of unstable particles are described by the EVTGEN [120] package, designed specifically for B physics. Final-state radiation is generated using PHOTOS [121]. The interaction of the generated particles with the detector, and its response, are implemented using the GEANT4 toolkit [122] as described in Ref. [123].

The most significant computational cost of the simulation is due to the detector simulation. A single pp collision produces $\mathcal{O}(100)$ tracks in the detector, out of which only a handful belong to the signal decay under study. Therefore, significant computational resources can be saved by reusing the detector simulation of non-signal tracks a number of times, while redescaying the signal particle, say a B^+ , each time. This approach is called ReDecay [124], and has been relatively widely adopted within LHCb. ReDecay has been used to produce simulation samples corresponding to the conditions in 2017 and 2018 for this thesis. In some cases, the use of ReDecay necessitates special statistical treatment due the correlated detector occupancies between signal candidates, but for the analysis in this thesis the impact is negligible.

1174 A number of sub-dominant backgrounds are investigated using the fast-simulation
1175 package **RapidSim** [125]. This package can decay heavy b and c hadrons with
1176 kinematic distributions similar to those in LHCb pp collisions, or with user defined
1177 input distributions. The decays are typically evenly distributed over phase space,
1178 but can also be handled with **EVTGEN** [120] to take involved spins and resonant
1179 structure into account. Furthermore, a smearing of the obtained momenta can
1180 be applied that is based on the LHCb resolution.



4

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1183

Neutral kaon CP violation and material interaction in BPGGSZ measurements

1184 The presence of a K_S^0 meson in the $D \rightarrow K_S^0 h^+ h^-$ final states introduces a small
1185 bias in BPGGSZ measurements due to CP -violation in the neutral kaon sector
1186 and asymmetries caused by the interaction between the neutral kaons and detector
1187 material. These fundamental physics effects are reviewed in Section 4.1, after which
1188 the chapter presents a detailed analysis of the impact on the LHCb measurement
1189 that is the subject of the thesis, as well as future γ measurements with the Belle II
1190 experiment. Prior to this analysis, the only existing work on the effect on γ
1191 measurements suggested a small effect in $B^\pm \rightarrow D K^\pm$ measurements but potentially
1192 very significant effects in measurements based on $B^\pm \rightarrow D \pi^\pm$ decays [42]. However,
1193 as described in Section 4.1.1, the analysis in Ref. [42] does not take into account
1194 the fundamental aspect of the BPGGSZ method: that it relies on the phase-space
1195 distribution of signal decays, not phase-space integrated asymmetries. Furthermore,
1196 the study only considers the CP -violation effect, not material interaction. Therefore,
1197 a more detailed study was necessary before the $B^\pm \rightarrow D \pi^\pm$ decay mode could
1198 reliably be promoted to a signal channel.

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1200

4.1 CP violation and material interaction of neutral kaons

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1202

A brief review of the general phenomenology of mixing and CP violation in the neutral kaon system is useful, before analysing the impact on γ measurements.

1203 The presentation in this section follows the PDG review of *CP violation in the*
 1204 *quark section* [31]. The general theory considers any pair of neutral mesons $|M^0\rangle$
 1205 and $|\bar{M}^0\rangle$ related by CP conjugation

$$CP|M^0\rangle = e^{i\phi_M}|\bar{M}^0\rangle \quad CP|\bar{M}^0\rangle = e^{-i\phi_M}|M^0\rangle, \quad (4.1a)$$

1206 where ϕ_M is an arbitrary phase. In this thesis, the convention $\phi_M = 0$ is chosen
 1207 to equal zero, so that

$$CP|M^0\rangle = |\bar{M}^0\rangle \quad CP|\bar{M}^0\rangle = |M^0\rangle. \quad (4.1b)$$

1208 A meson state that starts as a general superposition of $|M^0\rangle$ and $|\bar{M}^0\rangle$ states

$$\begin{aligned} \psi_M^0 &\equiv \psi_M(0) = a(0)|M^0\rangle + b(0)|\bar{M}^0\rangle \\ &\equiv \psi_{M^0}^0 + \psi_{\bar{M}^0}^0 \end{aligned} \quad (4.2)$$

1209 will, over time, involve into a state that consists of a different superposition of
 1210 $|M^0\rangle$ and $|\bar{M}^0\rangle$, as well as components for all possible states the meson system
 1211 can decay into

$$\begin{aligned} \psi_M(t) &= a(t)|M^0\rangle + b(t)|\bar{M}^0\rangle + \sum c_i(t)f_i \\ &\equiv \psi_{M^0}(t) + \psi_{\bar{M}^0}(t) + \sum c_i(t)f_i. \end{aligned} \quad (4.3)$$

1212 For time scales that are longer than the typical strong-interaction, the time evolution
 1213 of the $M^0 - \bar{M}^0$ superposition can be described by a 2×2 Hamiltonian

$$\frac{d}{dt} \begin{pmatrix} \psi_{M^0}(t) \\ \psi_{\bar{M}^0}(t) \end{pmatrix} = -i\mathcal{H}_0 \begin{pmatrix} \psi_{M^0}(t) \\ \psi_{\bar{M}^0}(t) \end{pmatrix} \quad (4.4)$$

1214 that is *non-Hermitian* (to allow for decay) but can be parameterised in terms
 1215 of two Hermitian matrices \mathcal{M} and Γ_0

$$\mathcal{H}_0 = \mathcal{M} - \frac{i}{2}\Gamma_0. \quad (4.5)$$

1216 The quantum states with well-defined (real) masses, m_j , and (real) decay widths,
 1217 Γ_j , are the two eigenstates of \mathcal{H}_0 with eigenvalues $\lambda_j = m_j - \frac{i}{2}\Gamma_j$. The eigenstates
 1218 (of course) evolve independently in time, so that

$$\psi_j(t) = e^{-i\lambda_j t}\psi_j^0 = e^{-im_j t - \frac{\Gamma_j}{2}t}\psi_j^0. \quad (4.6)$$

1219 The eigenstates are denoted H and L according to the size of m_j , the real part
 1220 of the eigenvalues, such that $m_H > m_L$. Assuming that \mathcal{H}_0 conserves CPT the
 1221 eigenstates have the general form

$$\begin{aligned} |M_H\rangle &\equiv p|M^0\rangle - q|\bar{M}^0\rangle \\ |M_L\rangle &\equiv p|M^0\rangle + q|\bar{M}^0\rangle \end{aligned} \quad (4.7)$$

where p and q are complex numbers that satisfy $|q|^2 + |p|^2 = 1$. With the convention in Eq. (4.1b) it follows that if \mathcal{H}_0 also conserves CP , so that $|M_H\rangle$ and $|M_L\rangle$ are CP eigenstates, then $p = \pm q$, where the sign depends on which of the heavy and the light meson states is CP even, and which is CP odd.

The eigenstates of the Hamiltonian governing the neutral kaon system are almost, but not exactly, equal to the CP eigenstates

$$|K_1\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}} \quad |K_2\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}, \quad (4.8)$$

which are CP even and odd, respectively. This approximate equality leads to the most prominent feature of the neutral kaon system: the two eigenstates of \mathcal{H}_0 have lifetimes that differ by orders of magnitude. This is best understood by assuming, for a moment, that the states in Eq. (4.8) *do* equal the eigenstates with definite life times. The K_1 state can decay in the CP even $\pi^+\pi^-$ and $\pi^0\pi^0$ modes, and does so almost 100% of the time; these decay modes are not available to the K_2 (in the absence of direct CP violation) which results in a much lower decay rate and much longer life time. Therefore, the eigenstates in the kaon system are labelled the *short-lived* kaon, K_S^0 , which is almost CP even, and the *long-lived* kaon, K_L^0 , which is almost CP odd. The life times are [31]

$$\tau_{K_S^0} = (8.954 \pm 0.004) \times 10^{-11} \text{s} \quad \tau_{K_L^0} = (5.116 \pm 0.021) \times 10^{-8} \text{s}. \quad (4.9)$$

Experimentally, it is found that the K_S^0 corresponds to the light eigenstate, but that the mass splitting [31]

$$\begin{aligned} \Delta m = m_{K_L^0} - m_{K_S^0} &= (0.5289 \pm 0.0009) \times 10^{10} \hbar s^{-1} \\ &\simeq 3.5 \times 10^{-6} \text{ eV} \end{aligned} \quad (4.10)$$

is tiny compared to the neutral kaon masses of $m_{K_S^0} = 497.6 \text{ MeV}/c^2$ [31].

However, the discovery of $K_L^0 \rightarrow \pi\pi$ decays by Kronin and Fitch in 1964 established that the K_S^0 and K_L^0 are *not* exactly equal to the CP eigenstates in Eq. (4.8), because the \mathcal{H}_0 relevant to the kaon system is CP -violating. The CP violation in the kaon sector is conventionally parameterised in terms of the complex parameters ϵ and ϵ' , in terms of which

$$\frac{A(K_L^0 \rightarrow \pi^+\pi^-)}{A(K_S^0 \rightarrow \pi^+\pi^-)} = \epsilon + \epsilon' \quad \frac{A(K_L^0 \rightarrow \pi^0\pi^0)}{A(K_S^0 \rightarrow \pi^0\pi^0)} = \epsilon - 2\epsilon'. \quad (4.11)$$

In these expressions ϵ denotes the contribution from CP violation in mixing and ϵ' the contribution due to direct CP violation in the decays. The ϵ parameter has been measured to be [31]

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}, \quad \arg \epsilon = (43.52 \pm 0.05)^\circ. \quad (4.12)$$

1249 Direct CP violation is ignored for the remainder of the thesis, because ϵ' is measured
1250 to be three orders of magnitude smaller than ϵ . In terms of the CP eigenstates
1251 of Eq. (4.8), the mass eigenstates K_S^0 and K_L^0 are given by

$$\begin{aligned} |K_S^0\rangle &= \frac{|K_1\rangle + \epsilon|K_2\rangle}{\sqrt{1+|\epsilon|^2}} &= \frac{(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}} \\ |K_L^0\rangle &= \frac{|K_2\rangle + \epsilon|K_1\rangle}{\sqrt{1+|\epsilon|^2}} &= \frac{(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}}, \end{aligned} \quad (4.13)$$

1252 corresponding to the definition $p = (1+\epsilon)/\sqrt{2(1+|\epsilon|^2)}$ and $q = (1-\epsilon)/\sqrt{2(1+|\epsilon|^2)}$
1253 in Eq. (4.7).

1254 In an experimental setting, the time evolution of a neutral kaon state is affected
1255 by nuclear interactions with the detector. The interaction is governed by the strong
1256 force, and therefore sensitive to the *flavour* of the kaon state; the interaction
1257 strength is thus different for K^0 and \bar{K}^0 mesons. This difference introduces a
1258 non-zero $K_S^0 \leftrightarrow K_L^0$ transition amplitude for neutral kaons traversing a detector
1259 segment. This effect was predicted early in the history of kaon physics [126] and is
1260 commonly denoted *kaon regeneration*. The effect can be described by including a
1261 material-interaction term in the Hamiltonian that is diagonal in the $(|K^0\rangle, |\bar{K}^0\rangle)$
1262 basis, so that the equation governing the time evolution is [127, 128]

$$\frac{d}{dt} \begin{pmatrix} \psi_{K^0}(t) \\ \psi_{\bar{K}^0}(t) \end{pmatrix} = -i \left[\mathcal{H}_0 + \begin{pmatrix} \chi & 0 \\ 0 & \bar{\chi} \end{pmatrix} \right] \begin{pmatrix} \psi_{K^0}(t) \\ \psi_{\bar{K}^0}(t) \end{pmatrix}. \quad (4.14)$$

1263 The complex parameters χ and $\bar{\chi}$ describe the material interaction of the K^0
1264 and \bar{K}^0 flavour eigenstates and are related to their scattering cross section, as
1265 described further in Section 4.3.4. The solution of Eq. (4.14) for the time evolution
1266 in the K_S^0 and K_L^0 states is [128]

$$\begin{aligned} \psi_S(t) &= e^{-i\Sigma t} \left(\psi_S^0 \cos \Omega t + \frac{i}{2\Omega} (\Delta\lambda\psi_S^0 - \Delta\chi\psi_L^0) \sin \Omega t \right), \\ \psi_L(t) &= e^{-i\Sigma t} \left(\psi_L^0 \cos \Omega t - \frac{i}{2\Omega} (\Delta\lambda\psi_L^0 + \Delta\chi\psi_S^0) \sin \Omega t \right), \end{aligned} \quad (4.15)$$

1267 in terms of the parameters

$$\begin{aligned} \Delta\chi &= \chi - \bar{\chi}, \\ \Delta\lambda &= \lambda_L - \lambda_S = (m_L - m_S) - \frac{i}{2}(\Gamma_L - \Gamma_S), \\ \Sigma &= \frac{1}{2}(\lambda_S + \lambda_L + \chi + \bar{\chi}), \\ \Omega &= \frac{1}{2}\sqrt{\Delta\lambda^2 + \Delta\chi^2}. \end{aligned} \quad (4.16)$$

1268 In the vacuum limit where $\chi = \bar{\chi} = 0$, the expressions in Eq. (4.6) and Eq. (4.15) are
1269 equal.

4.1.1 A first look at the impact on γ measurements

The effects described above have an impact on measurements of CP asymmetries in modes with a neutral kaon in the final state. This was analysed for the first time in relation to γ measurements by Grossman and Savastio in 2014 [42]. The authors point out two sources of corrections to be included:

- the fact that K_S^0 is not an exact CP eigenstate can break potential symmetry relations employed in an analysis, and
- that when the neutral kaon is reconstructed in a $\pi\pi$ final state there will be contributions from both K_S^0 and K_L^0 decays.

The analysis in this chapter considers yet another effect, not treated by Grossman and Savastio, namely that

- material interaction can emulate the effect of neutral kaon CP violation, because it couples the almost- CP -even K_S^0 and the almost- CP -odd K_L^0 states.

Due to the presence of $K_L^0 \rightarrow \pi\pi$ decays, Grossman and Savastio point out that the relevant decay rates to consider in an experimental setting are of the form

$$d\Gamma(t) \propto |\psi_S(t) + \epsilon\psi_L(t)|^2. \quad (4.17)$$

The time dependence of the decay rates considered in Chapter 2 was left out because all terms shared a common time dependence. That is not the case in Eq. (4.17), due to the very different decay rates of the K_S^0 and K_L^0 components of the kaon state. As a consequence, the time-integrated yields have the form

$$N \propto \int dt \eta(t) |\psi_S(t) + \epsilon\psi_L(t)|^2, \quad (4.18)$$

where $\eta(t)$ is the time acceptance in a given experimental setting. Thus, the acceptance is crucial to model in order to correctly estimate the impact of kaon CP -violation effects on a given measurement.

Considering BPGBSZ measurements, the main effect of neutral kaon CP violation is a breakdown of the fundamental Dalitz-plot symmetry that is exploited in the derivation of the bin yield equations. Extending the amplitude definition of Eq. (2.21) to include K_L^0 decays

$$A_{S(L)}^{(\overline{D})}(s_-, s_+) = A((\overline{D})^0 \rightarrow K_{S(L)}^0 \pi^+ \pi^-), \quad (4.19)$$

the authors point out that CP -violation in the K_S^0 system means that the relation $A_S^{(\overline{D})}(s_{-+}) = A_S^D(s_{+-})$ is not exactly true; and in addition, there is now a

dependence on $A_L^D(s_{-+})$ which satisfies a different approximate symmetry, namely $A_L^{\bar{D}}(s_{-+}) \simeq -A_L^D(s_{+-})$. Grossman and Savastio describe these symmetry breaking effects in detail, but do not explicitly derive the corrections to the yield equations of Chapter 2, nor try to quantify the potential bias on γ in a measurement based on the binned yields. Instead, they derive expressions for the bias in a measurement obtained from phase-space integrated CP asymmetries. This is done for both GLW measurements that use $D \rightarrow K_S^0 X$ final states and for the $D \rightarrow K_S^0 h^+ h^-$ final states; however, for their quantitative estimate of $\Delta\gamma$ the authors make an approximation that corresponds to assuming that the $D \rightarrow K_S^0 h^+ h^-$ final state is a CP eigenstate, making the two results identical. The authors find that in this case, assuming a uniform experimental acceptance for all kaon decay times, the asymmetry has the form¹

$$A = \frac{2r_B \sin \gamma \sin \delta_B + 2\text{Re}[\epsilon]}{1 + r_B^2 - 2r_B \cos \gamma \cos \delta_B}, \quad (4.20)$$

If a measured value of A is interpreted to obtain γ without taking the ϵ term into account, it leads to a bias of

$$\Delta\gamma = -\frac{\text{Re}[\epsilon]}{r_B \cos \gamma \sin \delta_B} + O(|\epsilon|). \quad (4.21)$$

The scaling $\Delta\gamma \sim \mathcal{O}(r_B/|\epsilon|)$ is the main result of the analysis by Grossman and Savastio. For $B^\pm \rightarrow D K^\pm$ decays, where $r_B^{DK^\pm} \simeq 0.1$ this suggests a bias at the percent level, which is negligible compared to current experimental uncertainties. However, in the $B^\pm \rightarrow D \pi^\pm$ case, where $r_B^{D\pi^\pm} \simeq 0.005$ [51], their result suggests relative biases that are potentially of $\mathcal{O}(1)$.

The conclusions are lacking on two accounts, however. Firstly, as made clear in Section 2.3.5, the $K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$ states are *far from* CP eigenstates. From the asymmetry expression in that section, it is clear that the bias in a determination of γ based on phase-space asymmetries will in fact scale as

$$\Delta\gamma \sim \mathcal{O}\left(\frac{|\epsilon|}{(2\mathcal{F}_+ - 1)r_B}\right), \quad (4.22)$$

which suggests that Grossman and Savastio severely *underestimates* the potential impact. This is described in detail in Section 4.2.3. More importantly, the analysis of the phase-space integrated asymmetry is in fact *irrelevant* to BPGGSZ measurements as they are currently performed: as described in Section 2.3.5 the information from

¹In fact the expression in Eq. (4.20) is missing a term, as will be clear when an analogous expression is derived in detail in Section 4.2.3.

the global asymmetry is completely discarded. Therefore it is necessary to analyse the effects of kaon CP -violation on a full, binned analysis of $D \rightarrow K_S^0 h^+ h^-$ decays, which is done in detail in the following sections. While the aim is to extend the analysis if Grossman and Savastio, the treatment in the following sections is completely independent of that in Ref. [42].

4.2 Impact on BPGBSZ measurements of γ : principles

The analysis of the impact on BPGBSZ measurements is carried out in two stages. This section treats the leading order effects analytically, and derives the overall order of magnitude of the expected bias in a general setting. Then Section 4.3 presents a detailed numerical study of the expected effect in measurements with the LHCb and Belle II experiments specifically, because these will be crucial to constrain γ during the coming decade [40, 41].

4.2.1 Modified symmetry relations

In order to derive the corrections to the asymmetry relation $A_S^D(s_{-+}) \simeq A_S^{\bar{D}}(s_{+-})$, it is beneficial to express $A_{S(L)}^D$ in terms of the amplitudes

$$A_{1/2}^{\bar{D}} = A(\bar{D}^0 \rightarrow K_{1/2}^0 \pi^+ \pi^-), \quad (4.23)$$

because these amplitude satisfy the exact symmetries $A_1^D(s_{-+}) = A_1^{\bar{D}}(s_{+-})$ and $A_2^D(s_{-+}) = -A_2^{\bar{D}}(s_{+-})$. This approach is different to that of Grossman and Savastio, but the final results are equivalent. After the decay of a D^0 meson to a neutral kaon, the kaon state is

$$\begin{aligned} \psi^0 &= A_1^D |K_1\rangle + A_2^D |K_2\rangle \\ &= N \left[(A_1^D - \epsilon A_2^D) |K_S^0\rangle + (A_2^D - \epsilon A_1^D) |K_L^0\rangle \right], \end{aligned} \quad (4.24)$$

with the normalisation constant $N = \sqrt{1 + |\epsilon|^2}/(1 - \epsilon^2)$. Thus it can be seen that

$$\begin{aligned} A_S^D(s_{-+}) &= N \left[(A_1^D(s_{-+}) - \epsilon A_2^D(s_{-+})) \right], \\ A_L^D(s_{-+}) &= N \left[(A_2^D(s_{-+}) - \epsilon A_1^D(s_{-+})) \right], \end{aligned} \quad (4.25)$$

¹³⁴⁶ with an analogous expression for the \bar{D}^0 decay amplitudes. Therefore, the generalised
¹³⁴⁷ relations between the D^0 and \bar{D}^0 amplitudes are

$$\begin{aligned} A_S^{\bar{D}}(s_{+-}) &= N[A_1^{\bar{D}}(s_{+-}) - \epsilon A_2^{\bar{D}}(s_{+-})] \\ &= N[A_1^D(s_{-+}) + \epsilon A_2^D(s_{-+})] = A_S^D(s_{-+}) + 2N\epsilon A_2^D(s_{-+}), \\ A_L^{\bar{D}}(s_{+-}) &= N[A_2^{\bar{D}}(s_{+-}) - \epsilon A_1^{\bar{D}}(s_{+-})] \\ &= -N[A_2^D(s_{-+}) + \epsilon A_1^D(s_{-+})] = -A_L^D(s_{-+}) - 2N\epsilon A_1^D(s_{-+}). \end{aligned} \quad (4.26)$$

¹³⁴⁸ 4.2.2 Relationship between the K_S^0 and K_L^0 amplitudes

¹³⁴⁹ The decay amplitude $A(D^0 \rightarrow K_S^0 \pi^+ \pi^-)$ has been carefully studied, and a number
¹³⁵⁰ of amplitude models have been published [55, 60–62, 65]. No models have been
¹³⁵¹ published for $D^0 \rightarrow K_L^0 \pi^+ \pi^-$ decays. However, following an approach laid out by
¹³⁵² the CLEO collaboration [73], the two amplitudes can be related. Again, this is
¹³⁵³ most easily done by relating the $A_1^D(s_{+-})$ and $A_2^D(s_{+-})$ amplitudes. In the isobar
¹³⁵⁴ formalism, the decay amplitude $A(D^0 \rightarrow K_1 \pi^+ \pi^-)$ is expressed as a non-resonant
¹³⁵⁵ constant amplitude plus a sum of resonances

$$A(D^0 \rightarrow K_1 \pi^+ \pi^-) = k_{NR} + \sum_{CF} k_i R^i(s_{K\pi^-}) + \sum_{DCS} k_j R^j(s_{K\pi^+}) + \sum_{R_{\pi\pi}} k_k R^k(s_{\pi^+\pi^-}). \quad (4.27)$$

¹³⁵⁶ The resonances are split into Cabibbo-favoured (CF) K^{*-} resonances, doubly
¹³⁵⁷ Cabibbo-suppressed (DCS) K^{*+} resonances and $\pi\pi$ resonances.² The CF resonances
¹³⁵⁸ couple to the \bar{K}^0 component of $K_1 (\propto K^0 + \bar{K}^0)$, and therefore the corresponding
¹³⁵⁹ k_i in the $K_2 (\propto K^0 - \bar{K}^0)$ amplitude will have a relative minus sign. The DCS
¹³⁶⁰ resonances couple to the K^0 component of K_1 , and so the corresponding k_j in the
¹³⁶¹ K_2 amplitude will have a relative plus sign. For the $h^+ h^-$ resonances, there will be
¹³⁶² a coupling to both the K^0 and \bar{K}^0 components, however the coupling to the K^0
¹³⁶³ component is expected to be suppressed with a Cabibbo suppression factor $r_k e^{i\delta_k}$,
¹³⁶⁴ where $r_k \simeq \tan^2 \theta_C \simeq 0.05$ is determined by the Cabibbo angle θ_C and δ_k can take
¹³⁶⁵ any value. Therefore, the k_k for these resonances have a relative $-(1 - 2r_k e^{i\delta_k})$
¹³⁶⁶ factor in the K_2 amplitude. The same effect leads to the differences in decay rates
¹³⁶⁷ between $D^0 \rightarrow K_S^0 \pi^0$ and $D^0 \rightarrow K_L^0 \pi^0$ decays [129, 130]. Thus, given a model of the

²In modern models, the $\pi\pi$ and $K\pi$ S -wave components are modelled via the K -matrix formalism and LASS parametrisations, respectively, instead of sums of individual resonances [55]. This does not alter the arguments below, as the R functions of Eq. (4.27) can equally well represent such terms.

¹³⁶⁸ form in Eq. (4.27), a model for the $A(D^0 \rightarrow K_2\pi^+\pi^-)$ amplitude will have the form

$$A(D^0 \rightarrow K_2\pi^+\pi^-) = k_{NR} + \sum_{CF} (-k_i) R^i(s_{K\pi^-}) + \sum_{DCS} (+k_j) R^j(s_{K\pi^+}) + \sum_{R_{\pi\pi}} (-(1 - 2r_k e^{i\delta_k}) k_k) R^k(s_{\pi^+\pi^-}). \quad (4.28)$$

¹³⁶⁹ An important consequence of these substitution rules is that

$$A_2^D(s_{+-}) = -A_1^D(s_{+-}) + r_A \Delta A(s_{+-}), \quad (4.29)$$

¹³⁷⁰ where $r_A \simeq \tan^2 \theta_C$ and $\Delta A(s_{+-}) \sim A_1^D(s_{+-})$ are of the same order of magnitude
¹³⁷¹ (at least when averaged over the bins used in γ measurements). This relation is
¹³⁷² sufficient to make the qualitative arguments of this section, while the full set of
¹³⁷³ substitution rules above are used in the quantitative studies of Section 4.3.

¹³⁷⁴ 4.2.3 Modification of the BPGBSZ yield equations

¹³⁷⁵ With suitable models to calculate $A_{S(L)}^{\bar{D}}$ (or $A_{1/2}^{\bar{D}}$) and knowledge of $\Delta\chi$ for the
¹³⁷⁶ materials relevant to an experimental setting, the relations derived in the preceding
¹³⁷⁷ sections can be employed to calculate the expected phase-space bin yields, N_i^\pm ,
¹³⁷⁸ including the effects of kaon CP violation and material interaction. The decay
¹³⁷⁹ rates have additional terms compared to those in Eq. (2.24), because the K_L^0
¹³⁸⁰ contribution must be taken into account

$$d\Gamma(t, s_{+-}) \propto |\psi_S(t, s_{+-}) + \epsilon\psi_L(t, s_{+-})|^2, \quad (4.30)$$

¹³⁸¹ where the time-dependence of $\psi_{S/L}(t, s_{+-})$ is governed by Eq. (4.15), and the phase-
¹³⁸² space dependence is included in the state component, by defining $\psi_{S/L}^0$ in terms of
¹³⁸³ $A_{S(L)}^{\bar{D}}(s_{+-})$. For example, for the case of a $B^- \rightarrow DK^-$ decay, the definition is

$$\begin{aligned} \psi_{S/L}^{0,B^-}(s_{+-}) &= A_S^D(s_{+-}) + r_B e^{i(\delta_B - \gamma)} A_S^{\bar{D}}(s_{+-}) \\ &= A_1^D(s_{+-}) - \epsilon A_2^D(s_{+-}) + r_B e^{i(\delta_B - \gamma)} (A_1^{\bar{D}}(s_{+-}) - \epsilon A_2^{\bar{D}}(s_{+-})) \\ &= A_1^D(s_{+-}) - \epsilon A_2^D(s_{+-}) + r_B e^{i(\delta_B - \gamma)} (A_1^D(s_{+-}) + \epsilon A_2^D(s_{+-})). \end{aligned} \quad (4.31)$$

¹³⁸⁴ It is useful to look at the corrections to the BPGBSZ yield expressions in Eq. (2.25)
¹³⁸⁵ to lowest order in ϵ and $r_\chi = \frac{1}{2} \frac{\Delta\chi}{\Delta\lambda}$, the dimensionless parameter governing material
¹³⁸⁶ interactions. For LHCb and Belle II the average $|r_\chi| \simeq 10^{-3}$, as detailed in the
¹³⁸⁷ Section 4.3. To first order in r_χ , the time-dependent kaon states within a material,
¹³⁸⁸ given in Eq. (4.15), simplify to [128]

$$\begin{aligned} \psi_S(t, s_{+-}) &= e^{-\frac{i}{2}(x+\bar{x})t} e^{-i\lambda_{St}} (\psi_S^0(s_{+-}) - r_\chi (1 - e^{-i\Delta\lambda t}) \psi_L^0(s_{+-})), \\ \psi_L(t, s_{+-}) &= e^{-\frac{i}{2}(x+\bar{x})t} e^{-i\lambda_{Lt}} (\psi_L^0(s_{+-}) + r_\chi (1 - e^{+i\Delta\lambda t}) \psi_S^0(s_{+-})). \end{aligned} \quad (4.32)$$

By inserting these expressions into Eq. (4.30) and employing the definition in Eq. (4.31) (and a similar definition for B^+ decays), the binned yields can be calculated by an integration over time and phase space. In the remainder of this section, it is assumed that the experimental time acceptance is $\eta(t) = 1$ for all times and that r_χ is constant at all times; more realistic assumptions are introduced in Section 4.3. In this case, the binned yields are given by the expression

$$\begin{aligned} N_i^- &= h_B^{-'} \left(\hat{K}_{+i} + r_B^2 \hat{K}_{-i} + 2\sqrt{\hat{K}_{+i} \hat{K}_{-i}} (x_- \hat{c}_i + y_- \hat{s}_i) + O(r\epsilon) \right), \\ N_i^+ &= h_B^{+'} \left(\hat{K}_{-i} + r_B^2 \hat{K}_{+i} + 2\sqrt{\hat{K}_{+i} \hat{K}_{-i}} (x_+ \hat{c}_i - y_+ \hat{s}_i) + O(r\epsilon) \right), \end{aligned} \quad (4.33)$$

where a number of new parameters have been defined, and where $O(r\epsilon)$ denotes terms of $O(r_A\epsilon)$, $O(r_B\epsilon)$, $O(r_A r_\chi)$, and $O(r_B r_\chi)$. Since $r_B \sim r_A \sim 10^{-1}$ (in $B^\pm \rightarrow D K^\pm$ decays) and $r_\chi \sim \epsilon \sim 10^{-3}$, these terms are all of the same order of magnitude.

The new normalisation constants $h_B^{\pm'} = h_B^\pm (1 + |\epsilon + r_\chi|^2 \frac{\Gamma_S}{\Gamma_L} \mp \Delta h)$ are defined in terms of

$$\Delta h = 2\text{Re}[\epsilon + r_\chi] - 4 \frac{\Gamma_S}{\Gamma_L + \Gamma_S} \frac{\text{Re}[\epsilon + r_\chi] + \mu \text{Im}[\epsilon + r_\chi]}{1 + \mu^2}, \quad \mu = 2 \frac{m_L - m_S}{\Gamma_L + \Gamma_S}. \quad (4.34)$$

The \hat{K}_i parameters are defined to be

$$\hat{K}_i = \frac{1}{1 + |\epsilon + r_\chi|^2 \frac{\Gamma_S}{\Gamma_L}} \left(K_i^{(1)} + |\epsilon + r_\chi|^2 \frac{\Gamma_S}{\Gamma_L} K_i^{(2)} \right), \quad (4.35)$$

in which the $K_i^{(1/2)}$ parameters are phase-space integrals, defined as in Eq. (2.27) but for $A_{1/2}^D$. To lowest order, the \hat{K}_i correspond to the fractional D^0 decay yield in each bin, as obtained in a measurement that averages D^0 and \bar{D}^0 decays, and assumes the $A_S^D(s_{-+}) = A_S^D(s_{+-})$ symmetry to be exact:

$$K_i^{\text{meas}} \equiv \frac{N_i^D + N_{-i}^{\bar{D}}}{\sum_j N_j^D + N_{-j}^{\bar{D}}} = \hat{K}_i + \mathcal{O}(r\epsilon). \quad (4.36)$$

Here, N_i^D ($N_i^{\bar{D}}$) is the expected yield of flavour tagged D^0 (\bar{D}^0) mesons into bin i of the D decay phase-space.

In similar fashion, the parameters (\hat{c}_i, \hat{s}_i) have been introduced to denote the measured average strong-phases, which are expected to differ from (c_i, s_i) at $O(\epsilon)$, since neutral kaon CP violation is not taken into account in the measurements by CLEO. Thus, any corrections arising if (\hat{c}_i, \hat{s}_i) and (c_i, s_i) are substituted in Eq. (4.33) will appear in the $O(r_B\epsilon)$ terms.

Two observations can be made from the expression in (4.33). The first is that the phase-space distribution is only changed at $O(r\epsilon)$ compared to the expression in

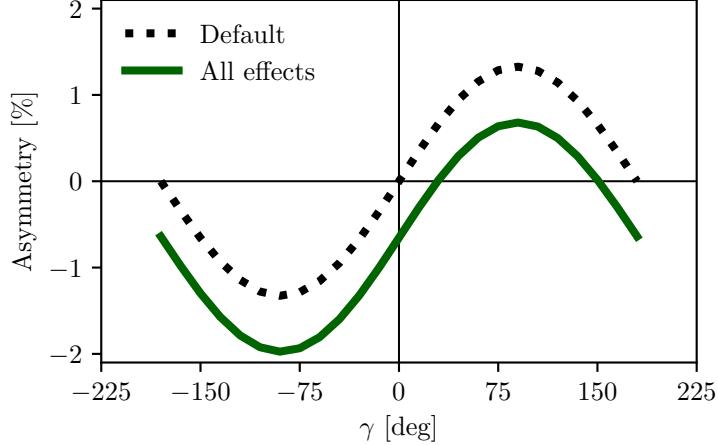


Figure 4.1: The asymmetry A_{total} as a function of γ calculated to $O(\epsilon)$ using Eq. (4.37). The calculation is made using for (black dotted line) the default case where $\Delta h = 0$ and (green) including neutral kaon CP -violation and material interaction with $r_\chi = \epsilon$.

Eq. (2.25), if the measured \hat{K}_i are used in the experimental analysis. This equally true whether the K_i are fitted in the signal channel along with x_\pm and y_\pm , as is the case in the measurement presented in the thesis, or if they are obtained in a control channel with flavour tagged D decays, according to Eq. (4.36). As the $D^0 - \bar{D}^0$ interference term that provides sensitivity to γ enters at order $O(r_B)$, the impact on γ measurements can be expected to be $\Delta\gamma/\gamma \sim O(r\epsilon/r_B)$. For $B \rightarrow DK$ analyses, where $r_B \simeq 0.1$, this is at the permille level, so the induced $\Delta\gamma$ bias can be expected to be smaller than 1° . Even in the case of $B^\pm \rightarrow D\pi^\pm$ decays, this suggests biases that are maximally a few percent. This is the main result of the chapter, because it means that the effect of neutral kaon CP violation and material interaction is small compared to the precision of the measurement that is the main subject of the thesis.

The second observation relates to potential future measurements of γ , which may also include sensitivity from the total, phase-space-integrated yield asymmetry

$$A_{\text{total}} = \frac{N^- - N^+}{N^- + N^+} = \frac{2(2\mathcal{F}_+ - 1)r_B \sin \delta_B \sin \gamma + \Delta h}{1 + r_B^2 + 2(2\mathcal{F}_+ - 1)r_B \cos \delta_B \cos \gamma} + O(r\epsilon), \quad (4.37)$$

where the definition of \mathcal{F}_+ from Section 2.3.5 has been employed. In the limit $r_B \rightarrow 0$ the expression agrees with the result for the analogous asymmetry in $D^\pm \rightarrow \pi^\pm K_S^0$ decays in Ref. [131], evaluated to $O(\epsilon)$ for an infinite and uniform time-acceptance. As hinted at above, the fact that $\mathcal{F}_+ \simeq 0.5$ means that the asymmetry due to γ being non-zero is not $\mathcal{O}(r_B)$, but of approximately the same order of magnitude as the asymmetry due to CP violation in the neutral kaon sector, governed by Δh . This is illustrated in Fig. 4.1, where the expression in Eq. (4.37) is plotted in the default

1434 case where $\Delta h = 0$, using the model in Ref. [55] to calculate K_i and c_i , as well as
1435 including neutral kaon CP violation and material interaction effects, calculated using
1436 $r_\chi = \epsilon$, with ϵ taking the value in Eq. (4.12). The asymmetry changes significantly
1437 when including the latter effects. Therefore, measurements based only on the global
1438 asymmetry will suffer relative biases of tens of degrees, not a few degrees, if neutral
1439 kaon CP violation and material interaction is not taken into account.

1440 4.3 Impact on BPGBGSZ measurements of γ : 1441 LHCb and Belle II measurements

1442 The previous section has established that the bias due to neutral kaon CP violation
1443 and material interaction is at the sub-percent level for measurements based on
1444 $B^\pm \rightarrow DK^\pm$ decays, and just a few percent in $B^\pm \rightarrow D\pi^\pm$ decays. Thus, the effects
1445 only contribute a manageable systematic uncertainty in the measurement that is
1446 the subject of the thesis. However, the expected precision on γ measurements
1447 will increase significantly in the coming decade, as both the LHCb [41] and Belle
1448 II [40] collaborations expect to make BPGBGSZ measurements that measure γ with
1449 a precision of 1–3°. Therefore a deeper understanding of the expected bias for
1450 these specific experiments is important.

1451 This section details a study, where the equations of the previous section are
1452 evaluated numerically to all orders, and care is taken to realistically model the
1453 experiment specific conditions. The scope of the original analysis, published in
1454 Ref. [2], was a stand-alone paper that covers both LHCb and Belle II, and which
1455 therefore does not rely on full detector simulation. Instead the following approaches
1456 are taken to model the necessary input

- 1457 • the experimental time-acceptance is modelled based on the detector geometry
1458 and typical neutral kaon momentum spectrum
- 1459 • the material interaction is included, using the material budget information
1460 available in the technical design reports on each experiment
- 1461 • both the time-acceptance and material interaction depends on the neutral
1462 kaon momentum, for which realistic distributions are estimated using the
1463 `RapidSim` simulation package [125].

1464 Each input is described in detail in the following sections. The study has been
1465 repeated to assign a systematic uncertainty to the LHCb measurement in Chapter 5,
1466 with slight adjustments to match the exact fit setup and with the inputs above
1467 extracted from full LHCb simulation. This is described further in Section 4.3.7.

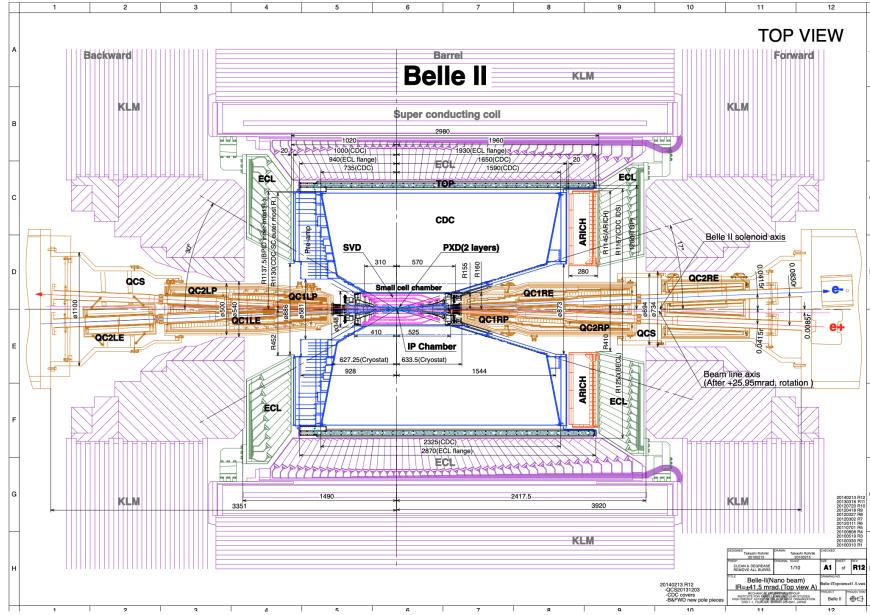


Figure 4.2: Schematic of the Belle II detector, reproduced from Ref. [40].

1468 4.3.1 Detector geometries

1469 The LHCb geometry and sub detectors are described in details in Chapter 3. In the
 1470 LHCb measurement discussed in Chapter 5, the K_S^0 mesons are reconstructed in
 1471 the $\pi^+\pi^-$ final state and two distinct categories of decay are considered, depending
 1472 on where in the detector the K_S^0 decay occurs. The categories have very different
 1473 decay-time acceptance, and therefore two scenarios are considered for LHCb: one
 1474 in which the decay products of the K_S^0 leave reconstructed tracks in both the silicon
 1475 vertex detector and downstream tracking detectors (denoted *long-long* or LL), and
 1476 one in which the decay products of the K_S^0 only leave tracks in the downstream
 1477 tracking detectors (denoted *down-down* or DD).

1478 The Belle II detector is a general purpose spectrometer, built to collect data from asymmetric e^+e^- collisions provided by the SuperKEKB accelerator in
 1479 Japan [132]. A schematic of the detector is shown in Fig. 4.2. The relevant sub
 1480 detectors for the present study are the tracking detectors: a central silicon vertex
 1481 detector, comprised of a total of six layers within 140 mm of the beam, and a large
 1482 volume drift chamber with 56 wire layers, extending to a radius of 1130 mm [40].
 1483 A single scenario is considered for Belle II, because essentially all the K_S^0 mesons
 1484 produced in signal decays in Belle II decay within the tracking volume, with more
 1485 than 90 % decaying in the vertex detector according to the studies described below.
 1486 Thus, three scenarios are considered in total: LL LHCb, DD LHCb, and Belle II.

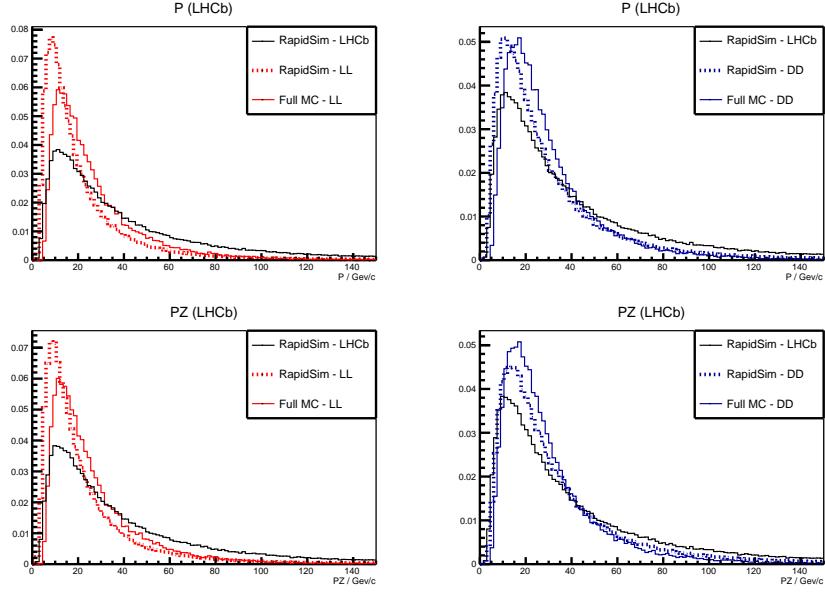


Figure 4.3: Momentum spectra for the K_S^0 meson in LHCb, as generated using **RapidSim** (black lines) directly, as well as reweighted to match decay time acceptance in the (red) LL and (blue) DD data categories of LHCb. The LHCb spectra are compared with the spectra in fully simulated signal decays, for both (dotted red lines) LL and (dotted blue lines) DD data categories.

1488 4.3.2 Kaon momentum distributions

1489 The neutral kaon momentum distributions are obtained using **RapidSim** [125], a
1490 simple tool to generate MC samples. **RapidSim** has an inbuilt capability to generate
1491 decays of B mesons with the kinematic distribution found in LHCb collisions and
1492 falling in the LHCb acceptance. However, the distributions need to be reweighted
1493 to take the kaon-decay-time acceptance into account. After being reweighted, the
1494 **RapidSim** momentum spectra are reasonably close to those found in full LHCb
1495 simulation samples of $B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)K^\pm$ decays, as seen in Fig. 4.3

1496 At Belle II, the signal B mesons stem from decays of $\Upsilon(4S)$ mesons produced in
1497 asymmetric electron-positron collisions. This leads to substantially different decay
1498 kinematics in comparison to those found at LHCb. The momentum distribution in
1499 Belle II is estimated by letting **RapidSim** decay B mesons with a momentum of 1.50
1500 GeV/ c along the z -axis using **RapidSim**, corresponding to the $\gamma\beta = 0.28$ boost of
1501 the centre-of-mass system in Belle II when operated at the $\Upsilon(4S)$ resonance [40]. A
1502 perfect 4π angular acceptance is assumed. It is not necessary to reweigh the Belle II
1503 momentum spectrum to account for the kaon-decay-time acceptance because all
1504 produced K_S^0 mesons decay in the tracking volume.

1505 The resulting momentum distributions for the three types of sample are shown in
1506 Fig. 4.4.

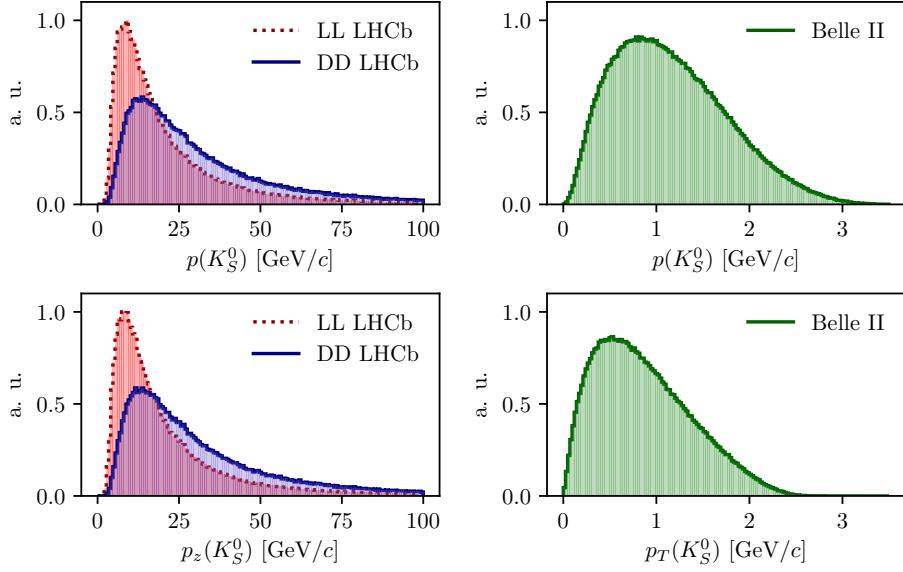


Figure 4.4: Momentum distributions for the LHCb (red dotted line) LL and (blue) DD categories, as well as (green) Belle II, obtained using `RapidSim`.

1507 4.3.3 Experimental time acceptance

1508 In order to model the experimental time acceptance, the time-dependent decay
1509 rates are only integrated over a finite time interval (τ_1, τ_2) . The intervals are
1510 defined for each of the three experimental categories, by requiring that a neutral
1511 kaon, if produced at $x = y = z = 0$ with momentum $p = (p_T, p_z)$, decays within
1512 the relevant part of the corresponding detector. For the LL LHCb category, it is
1513 required that the kaon decays before reaching $z_{max} = 280$ mm, corresponding to
1514 a decay where the decay products traverse at least 3 VELO segments (ignoring
1515 a number of widely spaced VELO segments placed at a distance of up to $z =$
1516 750 mm from the interaction point) [87]. For the DD LHCb category a decay at
1517 $z \in [280, 2350]$ mm is required, corresponding to decay between the LL cut-off and
1518 the first downstream tracking station [133]. The time acceptance has a significant
1519 impact for the LHCb categories, where some 20 % of the kaons escape the tracking
1520 stations completely before decaying.

1521 For Belle II, it is assumed that the K_S^0 reconstruction is similar to the Belle K_S^0
1522 reconstruction, which is based on a neural network and reconstructs K_S^0 decays for
1523 which the decay product leave tracks in both the drift chamber and silicon vertex
1524 detectors, as well as decays that leave tracks in the drift chamber only [134, 135].
1525 Therefore, the K_S^0 decay is required to be within $r_{max} = 1130$ mm of the beam axis,
1526 corresponding to a decay within the outer radius of the drift-chamber. In practice,

1527 most of the kaons decay inside the silicon vertex detector, and requiring a decay
1528 before 1130 mm is essentially equivalent to having no time cut-off.

1529 4.3.4 Detector material budget

1530 The effect of the material interaction is governed by parameter $\Delta\chi$ of Eq. (4.16). The
1531 parameter varies along a given kaon path, as the kaon intersects detector components
1532 made of different materials. In these studies, the calculations are simplified by
1533 using a single average material parameter for each experimental scenario. The
1534 average material parameters can be estimated for a given experimental scenario by
1535 considering the type and length of material traversed by a kaon in the relevant sub-
1536 detector(s). The average value is estimated, by exploiting that $\Delta\chi$ is related to the
1537 forward scattering amplitude f (\bar{f}) of K^0 (\bar{K}^0) mesons in a given material [127, 128]

$$\Delta\chi = -\frac{2\pi\mathcal{N}}{m_K}(f - \bar{f}) = -\frac{2\pi(N_A\rho/A)}{m_K}(f - \bar{f}), \quad (4.38)$$

1538 where $\mathcal{N} = N_A\rho/A$ is the scattering centre density of the material, m_K is the mass
1539 of the kaon state, A and ρ are the nucleon number and density of the material,
1540 and N_A is Avogadro's number. Measurements made for a range of nuclei [136]
1541 show that in the momentum range $p_K \in [20, 140] \text{ GeV}/c$

$$\left| \frac{f - \bar{f}}{p_K} \right| = 2.23 \frac{A^{0.758}}{p_K^{0.614} (\text{GeV}/c)} \text{ mb}, \quad \arg[f - \bar{f}] = -\frac{\pi}{2} (2 - 0.614), \quad (4.39)$$

1542 where the phase of Δf is determined via a phase-power relation [137]. In the
1543 numerical studies presented here, Eq. (4.39) is also used for the low momentum
1544 neutral kaons in the Belle II calculations, as a more detailed modelling of the
1545 low momentum $\Delta\chi$ based on Ref. [138] is found to yield very similar results.
1546 The scattering centre density \mathcal{N} is approximated as being constant, equal to the
1547 average density along a neutral kaon path due to its intersection with different
1548 detector segments. This average is estimated using the simplifying assumption that
1549 the total detector material budget is due to silicon. In practice, $\mathcal{N} = N_A\rho/A$ is
1550 calculated using $A = 28$ and $\rho = f^{\text{Si}}\rho^{\text{Si}}$, where $f^{\text{Si}} < 1$ is the average fraction
1551 of a neutral kaon path length that is inside detector material, estimated via
1552 the known dimensions of the detector, the average nuclear interaction length
1553 seen by a track traversing it cf. the technical design reports [87, 93], and the
1554 nuclear interaction length of silicon $\lambda_I^{\text{Si}} = 465.2 \text{ mm}$ [31]. The average value of
1555 $r_\chi = \frac{1}{2}\frac{\Delta\chi}{\Delta\lambda}$, which governs the size of the matter regeneration effect, can be calculated
1556 for the three considered experimental scenarios and satisfy $|r_\chi^{\text{LL}}| = 2.7 \times 10^{-3}$,
1557 $|r_\chi^{\text{DD}}| = 2.2 \times 10^{-3}$, and $|r_\chi^{\text{Belle II}}| = 1.0 \times 10^{-3}$.

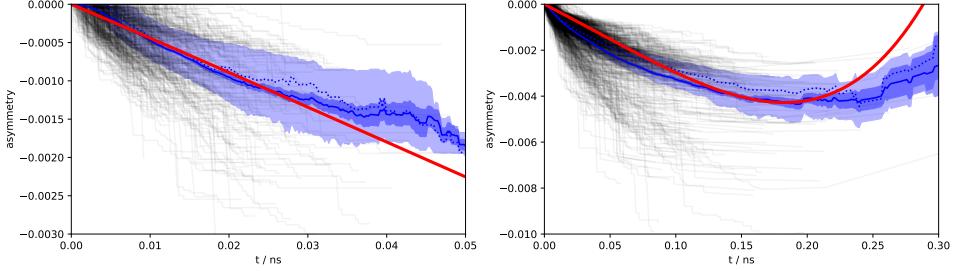


Figure 4.5: The asymmetry in Eq. (4.40) as a function of time for (left) LL and (right) DD K_S^0 tracks in a simulated LHCb sample. The black lines show individual tracks. The light blue area is the central 50 % quantile, the dark blue area is the 1σ uncertainty band on the mean. The red lines are calculated using the average $\Delta\chi$ values that are also used in the calculation of biases in BPGGSZ measurements.

The neutral kaon tracks in LHCb generally pass through somewhere between zero (for a significant amount of the LL tracks) and a hundred (for some DD tracks) distinct detector segments. Therefore it is worth examining the degree to which using a single average $\Delta\chi$ value, obtained following the procedure outlined above, provides a reasonable description of the average material interaction. This can be done using full LHCb simulation, where the kaon state for a simulated track can be evaluated at all times, by applying Eq. (4.15) iteratively for each detector segment the track traverses, using a $\Delta\chi$ value appropriate for that segment. This is done in Fig. 4.5 for a simple observable: the yield asymmetry

$$A_{K^0} = \frac{|\psi_K^0(t)|^2 - |\psi_{\bar{K}^0}(t)|^2}{|\psi_K^0(t)|^2 + |\psi_{\bar{K}^0}(t)|^2}, \quad (4.40)$$

where $\psi_K^0(t)$ ($\psi_{\bar{K}^0}(t)$) is the amplitude for an initial K^0 (\bar{K}^0) to decay to two pions at time t . In this calculation, it is assumed that $\epsilon = 0$ to isolate the material effect with no asymmetry contribution from the inherent CP -violation in the neutral kaon sector. While the track-by-track asymmetries are found to differ significantly depending on the exact detector segments a track intersects, the average asymmetry is seen to evolve smoothly as a function of decay time, and in reasonable agreement with the asymmetry value that is calculated using the average $\Delta\chi$ values estimated above.

The LHCb detector is undergoing a significant upgrade prior to the start of the LHC Run 3. However, the material budget and geometry of the relevant sub-detectors will be similar to the sub-detectors used during Run 1 and 2 [139, 140]. Hence the results of this study will be valid for measurements during the upgrade phases of LHCb, even though the detector parameters presented in this section relate to the original LHCb detector.

1580 **4.3.5 Calculation procedure**

1581 The main idea in the bias study is to calculate the BPGBSZ bin yields including
1582 the full effect of neutral kaon CP violation and material, fit them using the default
1583 equations of Chapter 2, and obtain the bias $\Delta\gamma = \gamma - \gamma^0$ due to the kaon effects not
1584 being considered in the parameter extraction. For the purpose of Ref. [2], a simple
1585 fit setup of a single $B^\pm \rightarrow Dh^\pm$ mode is investigated, where the K_i parameters are
1586 determined in a control channel with the relevant experimental acceptance. This
1587 setup is modified in the study used to assign a systematic uncertainty on the LHCb
1588 measurement of Chapter 5, as described in Section 4.3.7 below.

1589 In practice, the amplitude model for $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays in Ref. [55] is taken
1590 to represent the $A_1(s_{+-})$ amplitude. Then $A_2(s_{+-})$ is obtained as described in
1591 Section 4.2.2. In terms of A_1 and A_2 , the amplitudes $A_{S(L)}^{(\bar{D})}(s_{+-})$ can be expressed
1592 and related via Eqs. (4.25) and (4.26), and the full signal decay amplitudes as a
1593 function of phase-space coordinates, time, and the material interaction parameter
1594 $\Delta\chi$ can be calculated for a given set of input parameters $(\gamma^0, r_B^0, \delta_B^0)$. The squared
1595 decay amplitudes are then integrated over phase space and the kaon decay times
1596 to obtain the binned signal yield.

1597 The signal yields depend on the momentum via the time-acceptance parameters
1598 τ_1 and τ_2 , and because the material interaction parameter $\Delta\chi$ is momentum
1599 dependent. Therefore, the yields are averaged over the K_S^0 momentum distributions
1600 of LHCb and Belle II.

1601 The parameters x_\pm and y_\pm are determined by a maximum likelihood fit to the
1602 calculated yields, after which the fit result and covariance matrix are interpreted
1603 in terms of the physics parameters (γ, r_B, δ_B) using another maximum likelihood
1604 fit [49]. In the fits, the K_i are obtained using the definition $K_i = K_i^{\text{meas}} =$
1605 $(N_i^D + N_{-i}^{\bar{D}})/(\sum_j N_j^D + N_{-j}^{\bar{D}})$, in terms of the expected yields N_i^D ($N_i^{\bar{D}}$) of a flavour-
1606 tagged D^0 (\bar{D}^0) decays in bin i of the D decay phase space, calculated as described
1607 above for $r_B^0 = 0$. This corresponds to experimentally measuring the K_i in a control
1608 channel, and takes the effect of neutral kaon CP violation and material interaction
1609 on K_i measurements into account, as well the experimental time acceptance. The
1610 (c_i, s_i) are calculated using $A_1(s_{+-})$ and the experimental time acceptance is taken
1611 into account in this calculation as well.

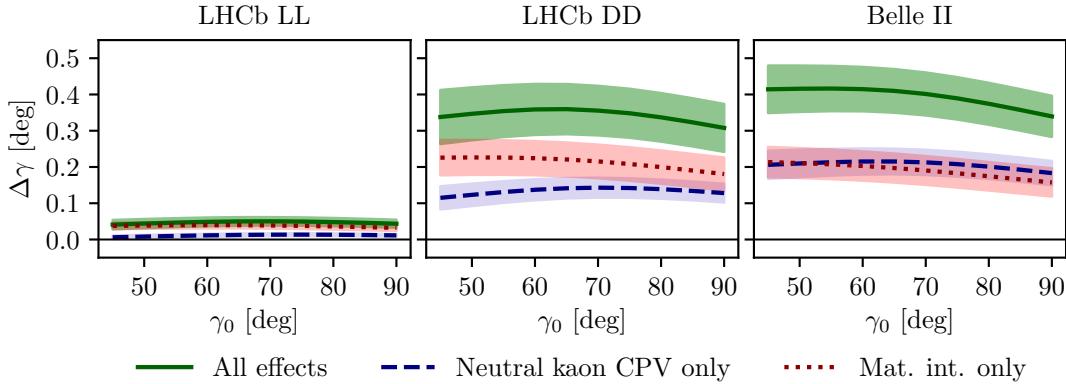


Figure 4.6: The bias $\Delta\gamma$ as a function of input γ_0 for (left) the LL LHCb category, (centre) the DD LHCb category, and (right) Belle II. The bias is calculated due to (blue, dashed line) neutral kaon CP violation alone, (red, dotted line) material interaction alone, and (green line) both effects. The shaded region shows the estimated 1σ uncertainty band.

4.3.6 Results

The obtained bias $\Delta\gamma$ is shown as a function of input γ^0 for the various experimental conditions in Fig. 4.6. The calculations are made using $(r_B^0, \delta_B^0) = (0.1, 130^\circ)$, approximately equal to the physics parameters relevant for $B^\pm \rightarrow DK^\pm$ decays [27, 39]. The bias does not vary significantly with γ^0 in the plotted range, which includes the world average value of direct γ measurements as well as the values obtained in full unitarity-triangle fits [27, 38, 39], and for all cases, the bias is found to be below 0.5° , corresponding to relative biases of about half a percent. Thus the biases are of $O(r\epsilon/r_B)$ as expected, given the arguments of Section 4.2.3. The contributions from the individual K_S^0 CPV and material interaction effects are also shown. It is seen that the neutral kaon CP violation and material interaction effects leads to approximately equal biases in all three cases.

Given the decay-time acceptance and momentum distribution for each experimental category, the mean life time, $\langle\tau\rangle$, of the reconstructed kaons can be calculated. In terms of the K_S^0 lifetime $\tau_{K_S^0} = (0.895 \pm 0.004) \times 10^{-11}$ s [31], $\langle\tau_{LL}\rangle \simeq 0.1\tau_{K_S^0}$ for the LHCb LL category, $\langle\tau_{DD}\rangle \simeq 0.8\tau_{K_S^0}$ for the LHCb DD category, and at Belle II $\langle\tau_{Belle\,II}\rangle \simeq \tau_{K_S^0}$. The difference in average kaon lifetime is reflected in the observed biases, which are found to be larger in the samples with longer lived kaons. The very small effect in the LL category is to be expected because the CP -violation effect due to K_S^0 not being CP -even is approximately cancelled by the CP -violation effect arising from $K_S^0 - K_L^0$ interference for kaons with decay times much smaller than $\tau_{K_S^0}$ [131].

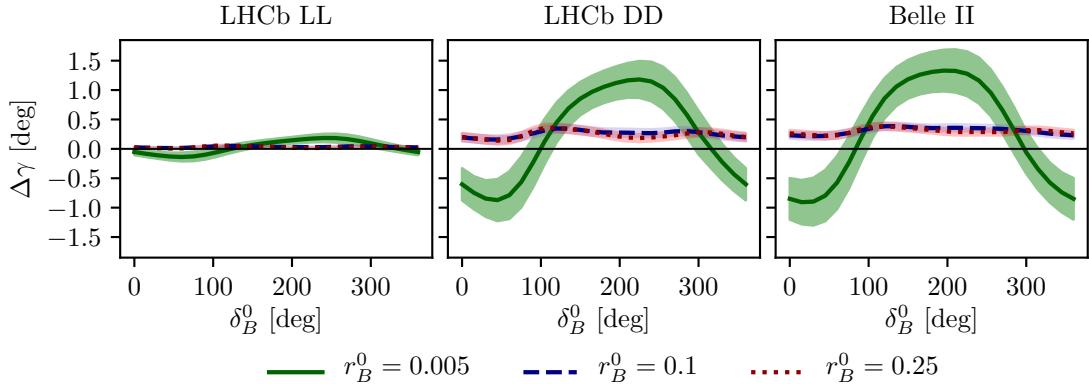


Figure 4.7: The bias $\Delta\gamma$ as a function of input δ_B for (left) the LL LHCb category, (centre) the DD LHCb category, and (right) Belle II. The bias is calculated for $\gamma = 75^\circ$ and (green line) $r_B = 0.005$, (blue, dashed line) $r_B = 0.1$, and (red, dotted line) $r_B = 0.25$. The shaded region shows the estimated 1σ uncertainty band.

1634 The uncertainty bands in Fig. 4.6 are calculated by repeating the study while
1635 varying some of the inputs. The model dependence of the predicted biases is
1636 probed by repeating the study using two other amplitude models as input for
1637 $A_1(s_{+-})$ and $A_2(s_{+-})$: the model published in Ref. [65] and the model included in
1638 EVTGEN [120]. hen defining $A_2(s_{+-})$ in terms of $A_1(s_{+-})$, there is an uncertainty
1639 due to the unknown (r_k, δ_k) parameters used to describe the $\pi\pi$ resonance terms.
1640 This uncertainty is assessed by making the study with several different random
1641 realisations of the parameter set. The studies are repeated while varying the time
1642 acceptances and material densities with $\pm 10\%$. There is an additional uncertainty
1643 due to the use of simulation samples generated with `RapidSim` to describe the kaon
1644 momentum distribution, in lieu of full detector simulations.

1645 There is also an uncertainty from the use of (c_i, s_i) as calculated using $A_1(s_{+-})$.
1646 It is to be expected that the measured values (\hat{c}_i, \hat{s}_i) from the CLEO collaboration
1647 differ by those calculated using $A_1^D(s_-, s_+)$ by terms of $O(\epsilon)$ due to neutral kaon
1648 CP violation, which is not taken into account in the measurement [73]. These
1649 corrections can be calculated via a procedure analogous to the one used to estimate
1650 the corrections on measurements of γ in this paper. However, as these corrections
1651 are much smaller than the experimental uncertainties in the measurement, they
1652 have not been studied further.

1653 For the purpose of this thesis, it is important to consider the bias in measurements
1654 that use $B^\pm \rightarrow D\pi^\pm$ decays as well, and other B decay modes can also be used in
1655 BPFGSZ measurements, such as $B^\pm \rightarrow D^*K^\pm$, $B^\pm \rightarrow DK^{*\pm}$, and $B^0 \rightarrow DK^{*0}$.
1656 For the purpose of the study presented here, the main difference between the decay

channels is that they have different values of r_B and δ_B . Figure 4.7 shows $\Delta\gamma$ as a function of input δ_B^0 , for $\gamma^0 = 75^\circ$ and three different values of r_B^0 . Aside from $r_B^0 = 0.1$, the results are shown for $r_B^0 = 0.005$, which corresponds to the expectation in $B^\pm \rightarrow D\pi^\pm$ decays [51] and $r_B^0 = 0.25$, which corresponds to $B^0 \rightarrow DK^{*0}$ decays [50]. The most notable feature is that the biases are significantly larger in the $B^\pm \rightarrow D\pi^\pm$ case. This is expected: the r_B^0 dependent behaviour is governed by the relative importance of different $O(r\epsilon)$ correction terms to the phase-space distribution. There are terms of both $O(r_A\epsilon)$ and $O(r_B\epsilon)^3$, which lead to expected biases of size $O(r_A\epsilon/r_B)$ and $O(r_B\epsilon/r_B) = O(\epsilon)$, respectively, cf. the discussion of Section 4.2.3. In the $B^\pm \rightarrow D\pi^\pm$ case, the $O(r_A\epsilon)$ correction terms dominate because $r_A/r_B \simeq (0.05/0.005) = 10$. This explains the relatively large bias, as $|r_A\epsilon/r_B^{D\pi}| \simeq 4\%$. The bias is seen to be up to $\pm 1.5^\circ$, but only about $+0.2^\circ$ with the expected value of $\delta_B^{D\pi} \simeq 300^\circ$ [49, 51]. These biases are *much smaller* than the precision on γ that is obtainable in a $B^\pm \rightarrow D\pi^\pm$ analysis with current experimental yields, and do thus not pose a problem. In the $r_B^0 = 0.1$ and $r_B^0 = 0.25$ cases the $O(r_B\epsilon)$ correction terms dominate, and the biases are of $O(\epsilon)$, independent of the r_B^0 value. Therefore both cases have biases of similar size.

Further, it is clear that the biases depend on δ_B^0 and that the oscillation period of the δ_B dependence is different between the $r_B^0 = 0.005$ case and the $r_B^0 \in \{0.1, 0.25\}$ cases. It is to be expected that $\Delta\gamma$ oscillates as a function of δ_B^0 , because δ_B^0 enters the yield equations via $\cos(\delta_B^0 \pm \gamma)$ and $\sin(\delta_B^0 \pm \gamma)$ terms. As explained above, the $O(r_A\epsilon)$ terms dominate the $B^\pm \rightarrow D\pi^\pm$ bias, and these are independent of δ_B^0 . The $O(r_B\epsilon)$ terms, however, are important for the bias corrections for larger r_B values, and the terms include factors of $\cos(\delta_B^0 \pm \gamma)$ and $\sin(\delta_B^0 \pm \gamma)$. This explains the different bias dependence on δ_B^0 .

While the input value of $\gamma^0 = 75^\circ$ was chosen for these studies, there is minimal variation in the results if another value of γ^0 in the range $[60^\circ, 85^\circ]$ is used.

4.3.7 Coupled $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ measurements

The studies presented above have been extended on two accounts in order to assign a systematic uncertainty to the LHCb measurement presented in Chapter 5. Firstly, full LHCb simulation has been used to obtain the momentum distributions, as well as to fit a better description of the time acceptance and the reconstruction efficiency profile over the D -decay phase space. Secondly, the fit setup is modified

³There are similar terms of $O(r_A r_\chi)$ and $O(r_B r_\chi)$, but as ϵ and r_χ are of the same order of magnitude, these terms can be treated completely analogously to the $O(r_A\epsilon)$ and $O(r_B\epsilon)$ terms, and have been left out of the discussion for brevity.

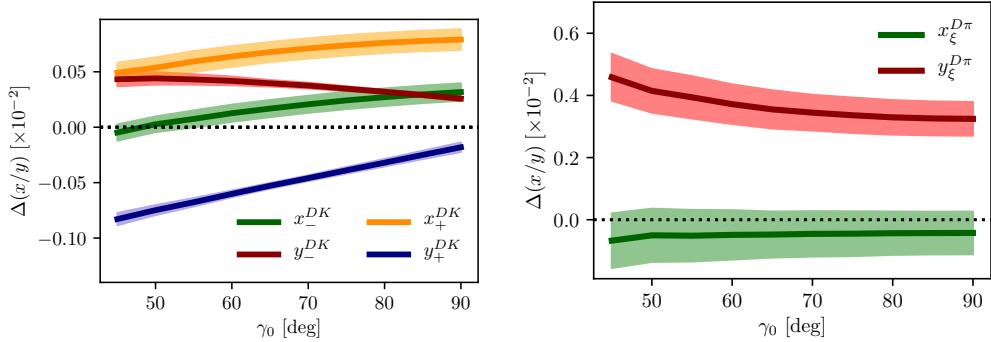


Figure 4.8: The bias on (left) the $B^\pm \rightarrow DK^\pm$ and (right) $B^\pm \rightarrow D\pi^\pm$ CP -violation observables in the LHCb DD category, evaluated in bias studies with inputs based on full LHCb simulation, calculated as a function of input γ_0 .

to correspond to the experimental approach described in Section 2.4 and Chapter 5: the signal yields are calculated for both the $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ channels, and fitted in a combined fit to obtain $(x_\pm^{DK}, y_\pm^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi})$, where the F_i parameters are allowed to float in the fit. The biases obtained for each observable are shown in Fig. 4.8, evaluated using the time-acceptance, momentum distribution, and material budget relevant for the DD category (since the effect in the LL category is much smaller). As will be clear in Chapter 5, these biases are all significantly smaller than the corresponding statistical uncertainties. Thus, the effects of neutral kaon CP violation and material interactions contribute a manageable systematic uncertainty in current BPGGSZ measurements, even if the $B^\pm \rightarrow D\pi^\pm$ channel is promoted to a signal channel.

As the statistical uncertainty becomes comparable with the bias effects described in this chapter, the systematic uncertainty should be assigned by a more accurate study, incorporating the traversed material on a track-by-track basis in full detector simulation. Such a detailed calculations can also be used to apply a bias correction if desired.

4.4 Concluding remarks

The analysis presented in this chapter has shown the expected impact of neutral kaon CP violation and material interaction on current BPGGSZ measurements to be small compared to the statistical uncertainties; first by simple order-of-magnitude estimates and then by a detailed calculation of the expected effect in LHCb and Belle II.

While the calculations were made for the case of $D \rightarrow K_S^0 \pi^+ \pi^-$ decays, the BPGGSZ approach can of course also be applied in other D -decay final states,

such as $D \rightarrow K_S^0 K^+ K^-$ and $D \rightarrow K_S^0 \pi^+ \pi^- \pi^0$. The biases on measurements of γ based the D decay phase-space distributions should be of similar size in these decay channels. The impact on γ measurements based on the phase-space-integrated yield asymmetry can be expected to be tens of degrees for the $D \rightarrow K_S^0 K^+ K^-$ channel, where the yield asymmetry is expected to be around 2 %, for the reasons explained in Section 4.2.3. The $D \rightarrow K_S^0 \pi^+ \pi^- \pi^0$ decay, however, is dominantly CP -odd [141], and the bias in measurements based on the total asymmetry is therefore expected to be $O(\epsilon/r_B)$, ie. a few degrees [42]. More precise calculations of the biases would require a repeat of the study included here, with relevant amplitude models and binning schemes in place.

The chapter focuses on the model-independent, binned approach that is the subject of the thesis. However, the underlying mechanism that determines the scale of the bias, namely that the phase-space *distribution* of signal decays is unaffected at $\mathcal{O}(\epsilon)$ and $\mathcal{O}(r_\chi)$, is independent on the exact measurement approach. Therefore it is expected that amplitude-model-based measurements and measurements made with new unbinned methods such as those in Ref [72] will be similarly biased if kaon CP violation and regeneration are not accounted for.

5

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A BPGBGSZ measurement of γ with $B^\pm \rightarrow Dh^\pm$ decays

1734 This chapter describes a model-independent BPGBGSZ measurement of γ with
1735 $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays where $D \rightarrow K_S^0\pi^+\pi^-$ and $D \rightarrow K_S^0K^+K^-$,
1736 commonly denoted $B^\pm \rightarrow D(\rightarrow K_S^0h^+h^-)h'^\pm$ decays. The measurement is made
1737 with the full LHCb data set collected during Run 1 and 2 of the LHC, corresponding
1738 to an integrated luminosity of about 9 fb^{-1} . The analysis is under review for
1739 publication in the Journal of High Energy Physics at the time of writing [1]
1740 (one can hope). 

1741

5.1 Candidate reconstruction and selection

1742 The $B^\pm \rightarrow D(\rightarrow K_S^0h^+h^-)h'^\pm$ candidates are constructed during the offline *stripping*
1743 stage described in Section 3.3.3. The candidates are defined by first combining
1744 tracks to form a $K_S^0 \rightarrow \pi^+\pi^-$ vertex, then a $D \rightarrow K_S^0h^+h^-$ vertex, and finally
1745 the $B^\pm \rightarrow Dh'^\pm$ candidate. Each final state track is required to satisfy certain
1746 momentum thresholds and track-quality requirements, and to be separated from all
1747 primary interaction vertices. Each decay vertex is required to satisfy a fit-quality
1748 threshold and to be separated from the primary vertex. Momentum thresholds
1749 are applied to the composite particles and they are required to have reconstructed
1750 invariant masses close to their known masses¹ except that the B candidate is
1751 required to have a reconstructed invariant mass in the interval $4750\text{--}7000\text{ MeV}/c^2$.

¹The exact mass window depends on the particle type and reconstruction category; narrower mass windows are applied at a later stage, as described below.

1752 The B candidate is required to satisfy $\chi^2_{\text{IP}} < 25$, where χ^2_{IP} is the difference in χ^2
 1753 value of the primary vertex fit, when the vertex is formed with- and without the B
 1754 candidate. As the final *stripping* stage, a multivariate algorithm is applied to the
 1755 formed B candidate to reduce the amount of random track combinations, denoted
 1756 combinatorial background, even further than the aforementioned requirements.

1757 Two data categories are defined, depending the tracks used to form the K_S^0
 1758 candidate: the LL category where both pions are long tracks, and DD category where
 1759 both pions are downstream tracks, using the track classifications of Section 3.2.1.

1760 Each candidate is re-analysed with the `DecayTreeFitter` (DTF) frame work [142],
 1761 where a simultaneous fit of the full decay chain is made with a number of constraints
 1762 applied: the momenta of the composite D and K_S^0 particles are required to form
 1763 invariant masses exactly equal to the known particle masses [31], and the momentum
 1764 of the B candidate is required to point in the direction defined by the B decay
 1765 vertex and the primary vertex. This refit results in improved resolution of the
 1766 invariant masses of the composite particles and, very importantly, of the Dalitz
 1767 coordinates in the D -decay phase space. It also ensures that all candidates fall in the
 1768 kinematically allowed region of the D -decay phase space. Unless otherwise specified,
 1769 all results in this chapter are based on the refitted track momenta; for reasons
 1770 explained below, some studies have to be based on parameters that are obtained
 1771 without the constraints described above, or with only a subset of them applied.

1772 Following the stripping stage, the further selection of signal candidates is
 1773 performed in three steps: an initial set of requirements to remove candidates
 1774 that are very likely to be background and vetoes a number of specific backgrounds,
 1775 the application of a multivariate analysis algorithm designed to reject combinatorial
 1776 background decays, and finally a set of particle-identification requirements. The
 1777 requirements are summarised in Table 5.1, and each step is described in detail
 1778 in the following sections.

1779 5.1.1 Initial requirements

1780 At the hardware trigger level, it is required that a particle associated with the
 1781 signal decay triggered the hadronic L0 trigger (classifying the event as *Trigger*
 1782 *on Signal*, or TOS), or that the L0 decision was caused by a particle that is not
 1783 associated with the signal decay (*Trigger Independent of Signal*, or TIS). The
 1784 inclusion of the latter category increases the data sample about 50 %. At the
 1785 software trigger level, a particle belonging to the signal decay is required to have
 1786 caused one of each of the inclusive HLT1 and HLT2 lines to accept the events.
 1787 Specifically, the Run 1 events are required to be TOS on the `HLT1TrackAllL0` and

Table 5.1: Summary of requirements applied to data. The base requirements are applied to all data samples before training or applying the BDT.

Base requirements		
Variable	Cut	Comment
Bachelor momentum, p	$< 100 \text{ GeV}/c$	
Bachelor has RICH	<i>true</i>	
K^\pm in D decay: momentum, p	$< 100 \text{ GeV}/c$	In $D \rightarrow K_S^0 K^+ K^-$
K^\pm in D decay: have RICH	<i>true</i>	In $D \rightarrow K_S^0 K^+ K^-$
DecayTreeFit converged	<i>true</i>	
D mass	$m_{D^0} \in m_{D^0}^{PDG} \pm 25 \text{ MeV}/c^2$	From DTF with constrained K_S^0 mass
K_S^0 mass	$m_{K_S^0} \in m_{K_S^0}^{PDG} \pm 15 \text{ MeV}/c^2$	From DTF with constrained D^0 mass
Background suppressing requirements		
Variable	Cut	Comment
K_S^0 flight distance χ^2	> 49	for LL only
$\Delta z_{\text{significance}}^{DB}$	> 0.5	for all candidates
PID requirements		
Channel	Cut	Comment
$B^\pm \rightarrow DK^\pm$	PIDK > 4	for bachelor
$B^\pm \rightarrow D\pi^\pm$	PIDK < 4	for bachelor
$B^\pm \rightarrow Dh^\pm$	IsMuon = 0	for bachelor
$B^\pm \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-)h^\pm$	PIDe < 0 & IsMuon = 0	for charged D decay products
$B^\pm \rightarrow D(\rightarrow K_S^0 K^+ K^-)h^\pm$	PIDK > -5 & IsMuon = 0	for charged D decay products
BDT requirements		
Channel	Cut	Comment
Run 1, DD	> 0.6	
Others	> 0.8	

1788 Hlt2Topo{2, 3, 4}BodyBBDT lines and the Run 2 events are required to be TOS on
 1789 the Hlt1{Track, TwoTrack}MVA and Hlt2Topo{2, 3, 4}Body lines. These trigger
 1790 lines were described in Section 3.3.

1791 Before any processing of the data, a loose preselection is applied to remove
 1792 obvious background candidates. The reconstructed D (K_S^0) mass is required to
 1793 be within 25 (15) MeV/c^2 of the known values [31]. The *companion* particle, the
 1794 pion or kaon produced in the $B^\pm \rightarrow Dh^\pm$ decay, is required to have associated
 1795 RICH information and a momentum less than $100 \text{ GeV}/c$; this ensures good particle-
 1796 identification performance. Finally, all of the DTF fits of the full decay chain
 1797 are required to have converged properly.

1798 Two additional requirements are made at this stage in order to suppress specific
 1799 backgrounds. In order to suppress decays of the type $B^\pm \rightarrow K_S^0 h^+ h^- h'^\pm$ with
 1800 no intermediate D meson, so called *charmless* decays, it is required that the
 1801 significance of the z -separation of the D^0 decay vertex and the B^\pm decay vertex
 1802 is above 0.5. The significance of the z -separation of the D^0 decay vertex and

1803 the B^\pm decay vertex is defined as

$$\Delta z_{\text{significance}}^{D-B} = \frac{z_{vtx}^D - z_{vtx}^B}{\sqrt{\sigma^2(z_{vtx}^D) + \sigma^2(z_{vtx}^B)}}. \quad (5.1)$$

1804 This source of background described further in section 5.3.1. In order to suppress
 1805 a background from $D \rightarrow 4\pi$ and $D \rightarrow \pi\pi KK$ decays, it is required that the K_S^0
 1806 flight distance χ_{FD}^2 is greater than 49, where

$$\chi_{\text{FD}}^2 = \left(\frac{\Delta r}{\sigma(\Delta r)} \right)^2, \quad (5.2)$$

1807 and Δr is the measured flight distance of the K_S^0 meson. This background is
 1808 described in further detail in section 5.3.2.

1809 5.1.2 Boosted decision tree

1810 A Gradient Boosted Decision Tree [143] (abbreviated BDT in the following) is
 1811 applied to classify each candidate on a scale from -1 to $+1$ as signal-like ($+1$)
 1812 or combinatorial-background-like (-1), based on the values of a number of input
 1813 parameters for a candidate in question. The BDT is implemented in the TMVA
 1814 frame work [144].

1815 A boosted decision tree classifier consists of a number of sequentially trained
 1816 decision trees, each of which classify events as either signal or background. Each tree
 1817 bases the decision on an individual subset of variables, out of an overall set of input
 1818 variables. At each training step, the input events are weighted when training a new
 1819 tree, so that events that the already-trained trees classify incorrectly are given a
 1820 higher weight; this is denoted boosting. The term *gradient boosting* denotes a specific
 1821 weight calculation scheme [143]. The final score is the average over all decision trees.

1822 The full set of input variables are given in Table 5.2. It includes the momenta
 1823 of particles in the decay; a number of geometric parameters such a absolute and
 1824 relative vertex positions, and distances of closest approach between tracks; χ_{IP}^2
 1825 values for a number of particles in the decay chain; the χ^2 per degree of freedom
 1826 of the DTF refit; DIRA values, which denote the angle between the fitted particle
 1827 momenta and the vector spanned by it's production ad decay vertices; and finally
 1828 an isolation variable, defined as

$$A_{p_T} = \frac{p_T(B) - \sum p_T(\text{other})}{p_T(B) + \sum p_T(\text{other})} \quad (5.3)$$

1829 where the sum is over all other tracks in a cone around the B -candidate. The cone
 1830 is defined as being within a circle with a radius of 1.5 units around the B candidate

Table 5.2: Input parameter set used in BDT trained to separate signal and combinatorial background, sorted according to importance in the LL classifier.

Variable name	Importance LL/DD (Rank in DD)	Description
<code>log10(1-Ks_DIRA_BPV)</code>	7.2 % / 3.5 % (16)	$\log \cos \theta_{\text{DIRA}}$ for K_S^0
<code>log(Bu_RHO_BPV)</code>	5.7 % / 5.5 % (5)	Radial distance of B vertex to beam line
<code>log(Bach_PT)</code>	5.2 % / 6.9 % (1)	p_T of the bachelor particle
<code>log10(1-DO_DIRA_BPV)</code>	4.9 % / 5.8 % (4)	$\log \cos \theta_{\text{DIRA}}$ for D
<code>log10(1-Bu_DIRA_BPV)</code>	4.9 % / 6.4 % (3)	$\log \cos \theta_{\text{DIRA}}$ for B^\pm
<code>log(DO_RHO_BPV)</code>	4.8 % / 5.3 % (6)	Radial distance of D vertex to beam line
<code>Bu_FTASY_1.5</code>	4.7 % / 4.9 % (7)	Asymmetry parameters of B^\pm
<code>log(DO_PT)</code>	4.7 % / 6.6 % (2)	p_T of the D meson
<code>log(Bu_constDOOKSPV_CHI2NDDF)</code>	4.2 % / 4.5 % (9)	χ^2 /d.o.f of kinematical refit with DecayTreeFitter
<code>log(Bu_FDGHI2_QWNPV)</code>	3.9 % / 4.1 % (11)	Flight distance χ^2 of the B^\pm
<code>log(max_Ksh1h2_IPCHI2_0WNPV)</code>	3.9 % / 3.0 % (20)	Largest χ^2_{IP} of the K_S^0 decay products
<code>log(DO_IPCHI2_QWNPV)</code>	3.8 % / 3.3 % (17)	χ^2_{IP} of the D
<code>log(min_Ksh1h2_IPCHI2_0WNPV)</code>	3.7 % / 0.9 % (26)	Smallest χ^2_{IP} of the K_S^0 decay products
<code>log(Bu_P)</code>	3.7 % / 3.9 % (12)	p of the B_\pm^0 meson
<code>log(Bu_IPCHI2_QWNPV)</code>	3.6 % / 4.6 % (8)	χ^2_{IP} of the B_\pm^0
<code>Bu_MAXDOCA</code>	3.6 % / 3.3 % (18)	"Distance of closest approach" for B^\pm vertex
<code>log(Bach_IPCHI2_0WNPV)</code>	3.3 % / 4.3 % (10)	χ^2_{IP} of the bachelor particle
<code>log(Bu_PT)</code>	3.3 % / 3.7 % (14)	p_T of the B^\pm meson
<code>log(max_h1h2_IPCHI2_0WNPV)</code>	3.1 % / 3.8 % (13)	Largest χ^2_{IP} of the D decay products
<code>log(min_h1h2_IPCHI2_0WNPV)</code>	3.0 % / 3.4 % (19)	Smallest χ^2_{IP} of the D decay products
<code>log(Ks_VTXCHI2D0F)</code>	2.9 % / 2.3 % (21)	χ^2 of vertex fit for K_S^0
<code>DO_MAXDOCA</code>	2.9 % / 1.0 % (25)	"Distance of closest approach" for D vertex
<code>log(DO_VTXCHI2D0F)</code>	2.7 % / 1.6 % (24)	χ^2 of vertex fit for D
<code>log(DO_P)</code>	2.7 % / 1.8 % (22)	p of the D meson
<code>log(Bach_P)</code>	2.2 % / 3.6 % (15)	p of the bachelor particle
<code>log(Bu_VTXCHI2D0F)</code>	1.8 % / 1.7 % (23)	χ^2 of vertex fit for B^\pm

in the (η, ϕ_{azim}) -plane. This variable is highly efficient in rejecting combinatorial background. Two algorithms are trained, one for the LL category of K_S^0 mesons and one for the DD category, because some input parameters relate to the K_S^0 meson and have very different distributions between the two categories.

The BDTs are trained and tested with input samples representing typical signal and background decay candidates: a signal sample that consists of simulated $B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)\pi^\pm$ decays corresponding to the LHCb running conditions for the years 2012–2018, and a sample of combinatorial background candidates from real data, where the reconstructed invariant mass of the B meson is larger than 5800 MeV/ c^2 . The candidates in both samples were required to have passed the initial requirements described in the preceding section. The input-parameter distributions in the signal and background training samples are shown in Figs. 5.1 and 5.2. The signal and background samples are each split into two before the training stage: one sub sample, the training sample, is used to train the BDT, after which the trained algorithm is applied to the other sub sample, the test sample. The classifier is found to perform well on the test sample, not just the training sample, which ensures that it does not suffer significant overtraining. The BDT output distribution are shown for both test and training samples in Fig. 5.3, where it is clear that the classifier very effectively separates signal and background candidates.

Each candidate in data is classified using the BDT, and candidates that are assigned a score below some threshold value are discarded. The threshold values are chosen in a set of pseudo experiments, such that the expected sensitivity to γ is maximised. This is done by performing preliminary fits to the data set for a range of different BDT threshold values, then generating many pseudo data sets with the obtained yields, and applying the full fit and interpretation procedure described in Sections 5.4–5.7 to each data set. Thus, the expected uncertainty on γ is obtained for a range of threshold values. The procedure is applied independently for the LL and DD categories, as well as for the Run 1 and Run 2 data sets, because some parameter distributions differ slightly between the two runs. The optimal threshold values are found to be 0.8 in all situations, except for LL candidates in Run 1 where it is 0.6. This is illustrated in Fig. 5.4 where the results of the threshold scans are shown. The same classifier is applied to both $B^\pm \rightarrow D\pi^\pm$ and $B^\pm \rightarrow DK^\pm$ candidates, and both D final state categories. While the classifiers were trained using samples of $B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)\pi^\pm$ simulation and data, the decays are similar enough that no significant improvement in performance was obtained when considering a more elaborate setup. Across all categories, the requirement on the BDT output is found to remove approximately 98 % of the combinatorial background, while being approximately 93 % efficient on signal.

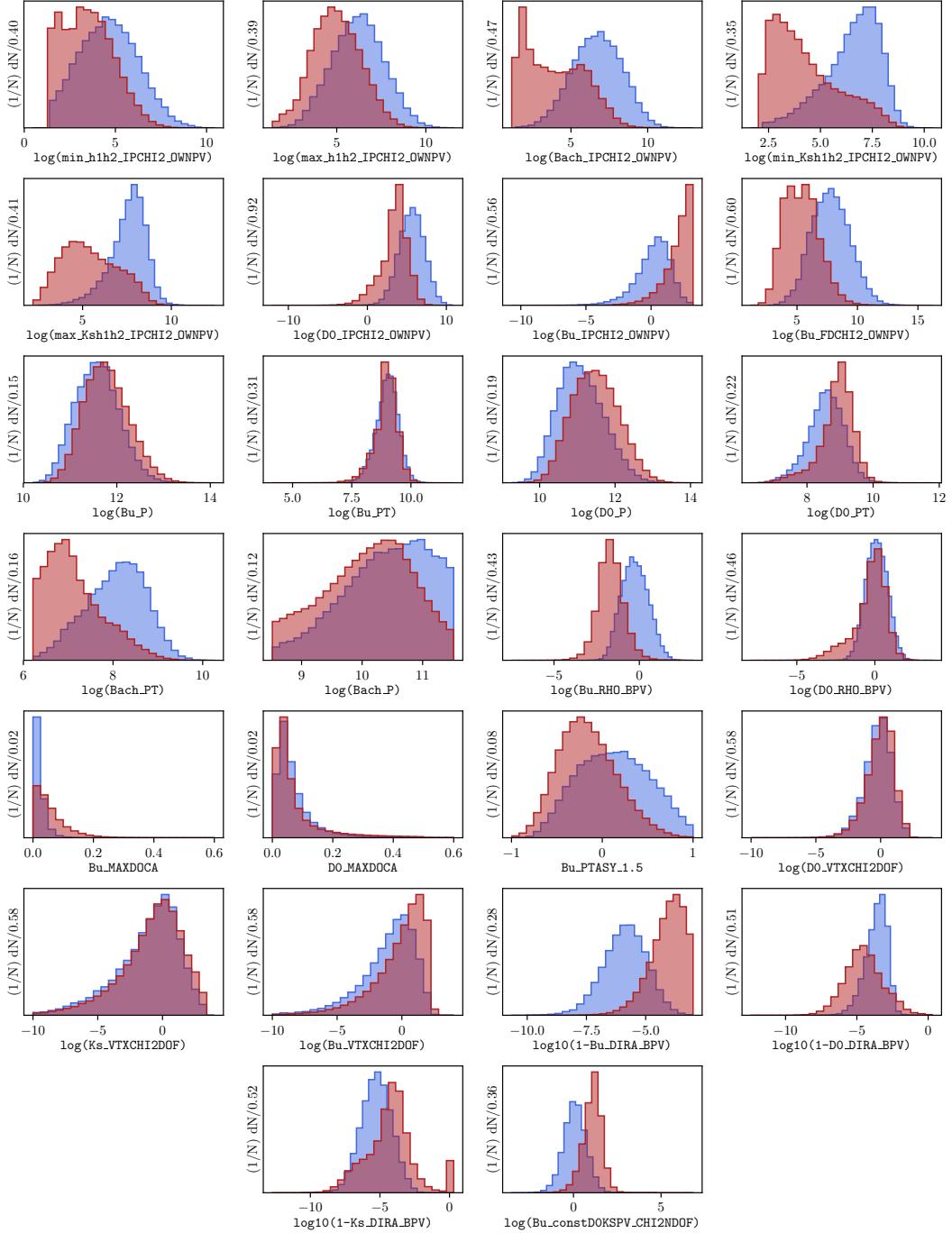


Figure 5.1: Distribution of input parameters in the LL training samples of (blue) signal decays from simulation and (red) background decays from the upper B sideband. The variable names are defined in Table 5.2.

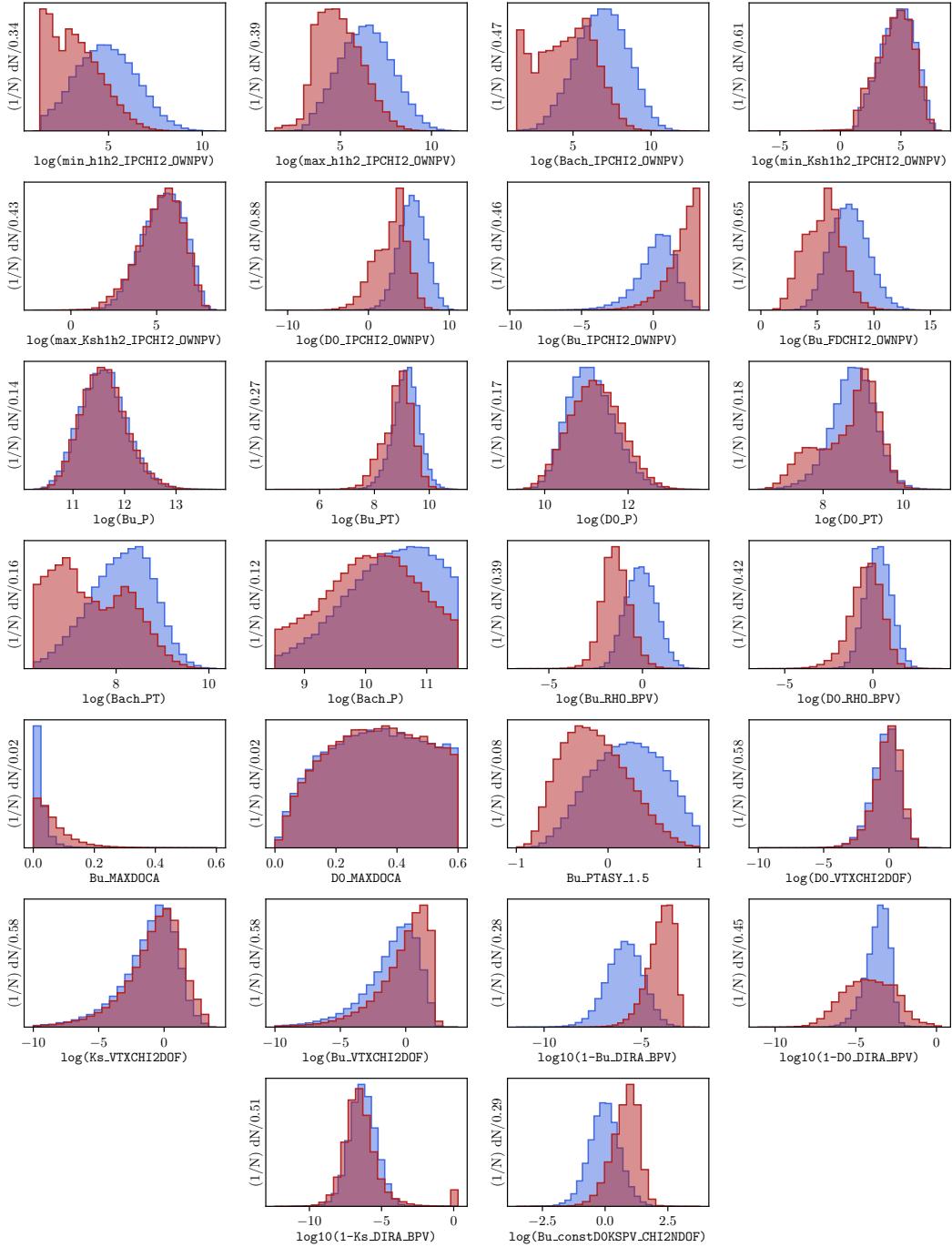


Figure 5.2: Distribution of input parameters in the DD training samples of (blue) signal decays from simulation and (red) background decays from the upper B sideband. The variable names are defined in Table 5.2.

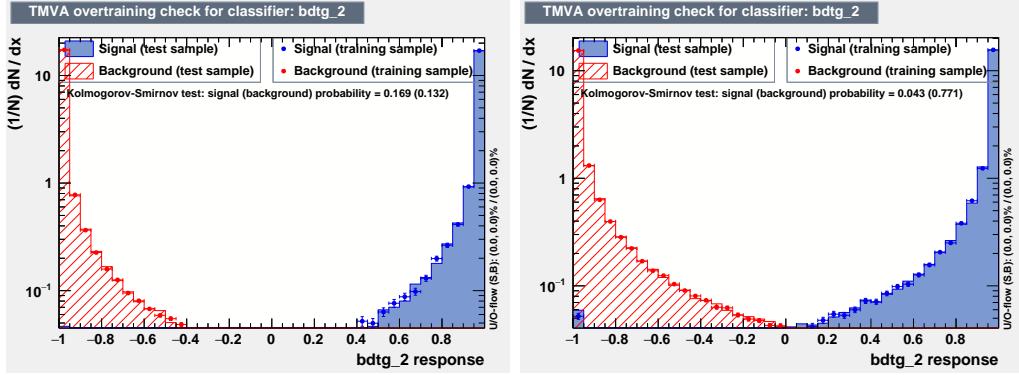


Figure 5.3: Distribution of BDT variable on test and training samples for (left) the LL and (right) the DD category, with logarithmic y -scale.

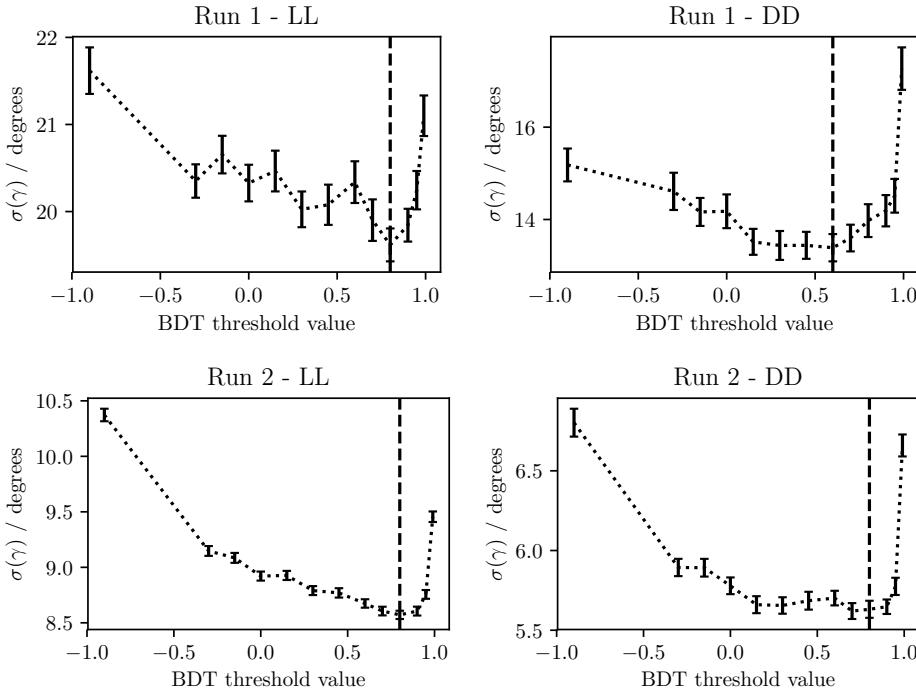


Figure 5.4: The mean uncertainty on γ in toy studies, performed with the signal and background yields corresponding to a given BDT requirement, using (top) the Run 1 and (bottom) Run 2 datasets, using only candidates in (left) the LL category and (right) the DD category. The dashed line shows the threshold value employed to discard background-like candidates in the selection.

5.1.3 Particle-identification requirements

A PID requirement is made to separate $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ candidates in the data sample, by requiring that the PIDK of the companion particle satisfies $PIDK < 4$ for $B^\pm \rightarrow D\pi^\pm$ candidates and $PIDK > 4$ for $B^\pm \rightarrow DK^\pm$ candidates. The PIDK variable was defined in Section 3.1.3. This ensures that any given candidates is selected into only one of these samples.

Further to the requirement on the companion, PID requirements are made to suppress semi-leptonic backgrounds as well as decays where a final state particle decays in flight, and a loose PID requirement is made in the $D \rightarrow K_S^0 K^+ K^-$ channels where it leads to a higher signal purity:

- the companion particle is required to satisfy `IsMuon` = 0.
- For the $B \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-)h^\pm$ samples it is required that the charged pion track from the D decay with opposite charge to the companion satisfies `PIDe < 0 & IsMuon = 0`, and for the other charged pion that `IsMuon` = 0. A very loose requirement of `PIDK < 20` is applied to both pions from the D -decay in the stripping stage.
- For the $B \rightarrow D(\rightarrow K_S^0 K^+ K^-)h^\pm$ samples it is required that the charged kaon tracks from the D decay have RICH information, a momentum less than 100 GeV/c and `PIDK > -5 & IsMuon = 0`.

These backgrounds are described in Section 5.3.3.

5.1.4 Final requirements

For a small fraction of candidates in the final sample, it is the case that two or more candidates originate in the same pp collision. In order to make sure that all candidates are completely independent, a single, arbitrary candidate from each pp collision is kept for these collisions, and the other candidates discarded. This requirement results in the removal of less than 0.7% of candidates in each data category.

Furthermore, the D mass used to define the binning schemes described in Ref. [73] differs slightly from the mass used in the DTF refit. Therefore a few of the decays are reconstructed with Dalitz coordinates outside the allowed kinematic region. Because this problem only concerns a handful of candidates, they are simply discarded.

5.1.5 Selected candidates

In total, about 47,000 $B^\pm \rightarrow DK^\pm$ candidates and 400,000 $B^\pm \rightarrow D\pi^\pm$ candidates are selected, as summarised in Table 5.3. An example of the B mass distribution in one of the data categories is shown in Fig. 5.5; it is clear that a significant number of these candidates are background decays. The Dalitz plots for candidates in the signal region where $m_B \in [5249, 5309] \text{ MeV}/c^2$ are shown in Fig. 5.6 and 5.7. Due to the large yields in the full Run 1 and 2 LHCb data set, the asymmetries between the B^+ and B^- distributions are visible to the eye in the $B^\pm \rightarrow DK^\pm$ plots.

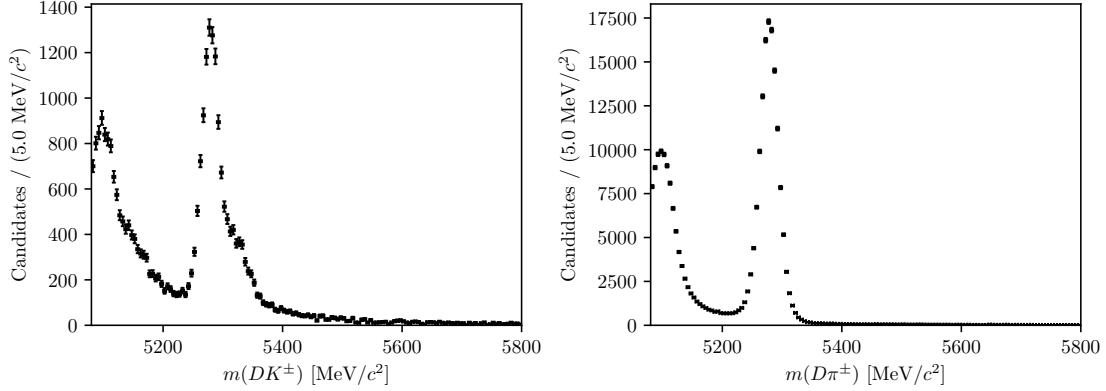


Figure 5.5: The spectrum of m_B in the (left) $B^\pm \rightarrow DK^\pm$ and (right) $B^\pm \rightarrow D\pi^\pm$ samples where $D \rightarrow K_S^0\pi^+\pi^-$ and the K_S^0 meson is reconstructed in the DD category, after the full selection has been applied.

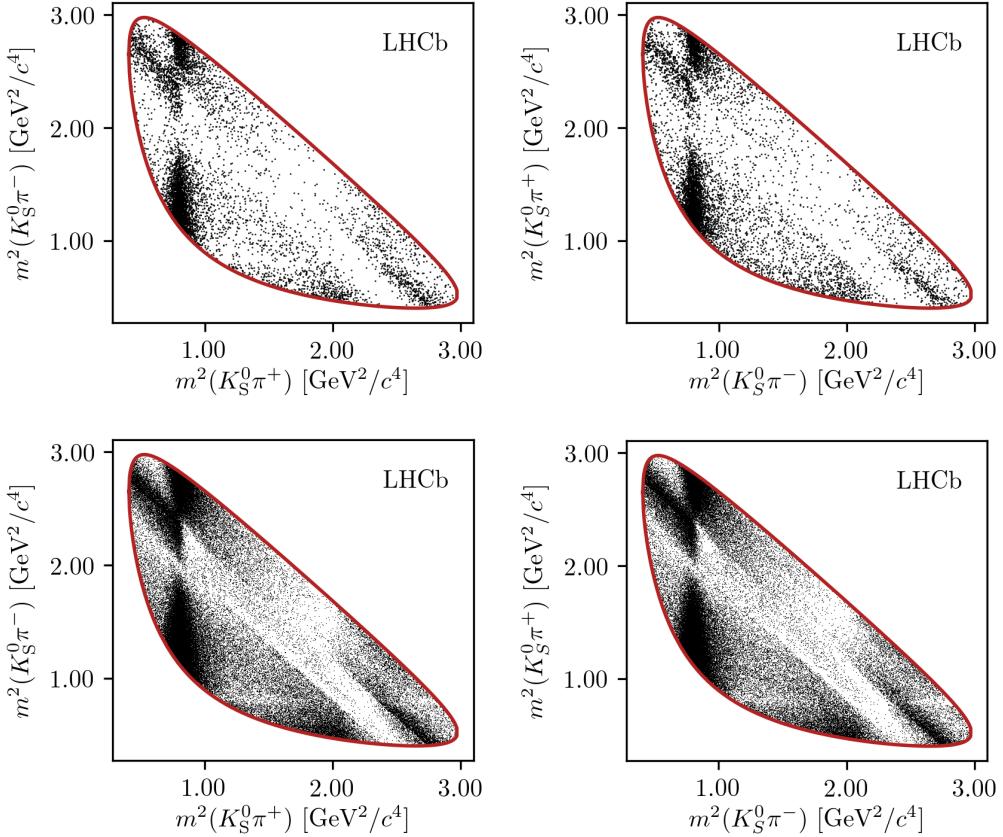


Figure 5.6: Dalitz plots of (left) $B^+ \rightarrow Dh^+$ and (right) $B^- \rightarrow Dh^-$ candidates in the signal region, in the (top) $B^\pm \rightarrow DK^\pm$ and (bottom) $B^\pm \rightarrow D\pi^\pm$ channels where $D \rightarrow K_S^0\pi^+\pi^-$. The LL and DD categories have been combined.

Table 5.3: Final candidate yield in each data category after the full selection has been applied, including removing candidates outside the region $m_B \in [5080, 5800] \text{ MeV}/c^2$.

B Decay	D final state	K_S^0 type	Run 1	Run 2	Total
$B^\pm \rightarrow DK^\pm$	$K_S^0\pi^+\pi^-$	LL	2275	10525	12800
		DD	5097	23508	28605
	$K_S^0K^+K^-$	LL	383	1610	1993
		DD	772	3397	4169
$B^\pm \rightarrow D\pi^\pm$	$K_S^0\pi^+\pi^-$	LL	18209	90509	108718
		DD	40167	205807	245974
	$K_S^0K^+K^-$	LL	2879	13757	16636
		DD	6033	29790	35823

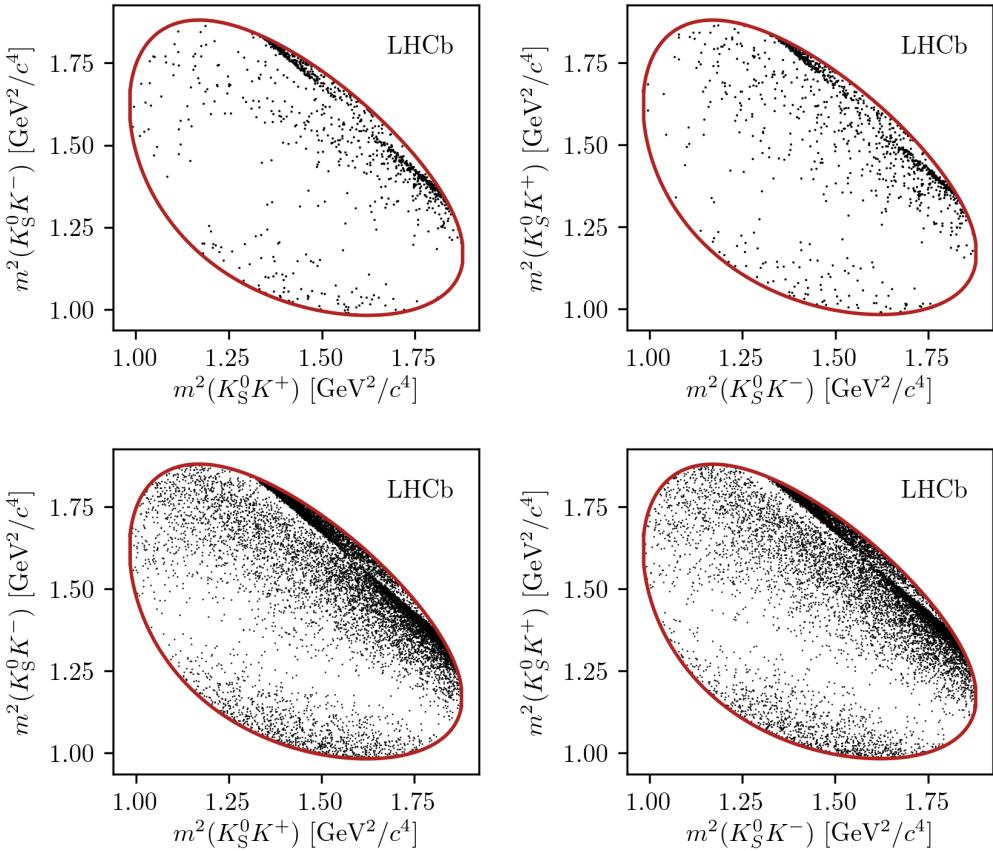


Figure 5.7: Dalitz plots of (left) $B^+ \rightarrow Dh^+$ and (right) $B^- \rightarrow Dh^-$ candidates in the signal region, in the (top) $B^\pm \rightarrow DK^\pm$ and (bottom) $B^\pm \rightarrow D\pi^\pm$ channels where $D \rightarrow K_S^0 K^+ K^-$. The LL and DD categories have been combined.

1908 5.2 Signal selection efficiencies

1909 The efficiency of each step of the selection on signal decays can be investigated
 1910 using simulated decays. In the $B^\pm \rightarrow D\pi^\pm$ channel, only decays that were placed
 1911 in the "test" sample when training the BDT are used, in order to avoid over-
 1912 estimating the efficiency.

1913 In general, the total selection efficiency up until the PID requirements, including
 1914 the offline stage and the effect of the geometrical LHCb acceptance, is about 1
 1915 permille, slightly higher for $B^\pm \rightarrow DK^\pm$ than $B^\pm \rightarrow D\pi^\pm$ decays, slightly higher
 1916 for $D \rightarrow K_S^0 K^+ K^-$ than $D \rightarrow K_S^0 \pi^+ \pi^-$ decays, and somewhat higher in the Run 2
 1917 than in Run 1 due to improvements in the trigger. The PID requirements are
 1918 investigated separately in Section 5.2.1 below, using samples of calibration data.
 1919 The overall selection efficiency does not impact the measurement at all, because
 1920 the observables of interest are sensitive *only* to the distribution of decays over the
 1921 Dalitz plot (except, of course, in the sense that a higher signal efficiency is desirable
 1922 because it leads to larger signal yields). Likewise, it makes no difference that the
 1923 overall selection efficiencies differ slightly between $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$
 1924 decays, as long as the efficiency profile over the Dalitz plot is identical between the
 1925 two decay channels. This is confirmed separately in Section 5.2.2 below.

1926 The efficiencies of each individual selection step are shown in Fig. 5.8, obtained
 1927 using simulated $B^\pm \rightarrow D\pi^\pm$ decays. The main reason that some signal decays
 1928 do not survive the base requirement is the $p_{\text{companion}} < 100 \text{ GeV}/c$ requirement,
 1929 which is in place to ensure that the PID performance for the companion is good.
 1930 For decays with $p_{\text{companion}} > 100 \text{ GeV}/c$, only about 60 % of $B^\pm \rightarrow DK^\pm$ decays
 1931 survive the subsequent $\text{PID}_K > 4$ requirement and the cross-feed from misidentified
 1932 $B^\pm \rightarrow D\pi^\pm$ decays is 50 % larger than in the current selection. Thus, loosening
 1933 this requirement leads to little statistical gain, while leading to larger systematic
 1934 effects from the crossfeed background.

1935 An equivalent plot for the combinatorial background is shown in Fig. 5.9, using
 1936 $B^\pm \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-) \pi^\pm$ candidates in data with a reconstructed B mass above
 1937 $5600 \text{ MeV}/c^2$; it can be seen that the BDT is very efficient at rejecting combinatorial
 1938 background, but that the base requirements and the requirement on the K_S^0 flight
 1939 distance also remove a decent amount of background.

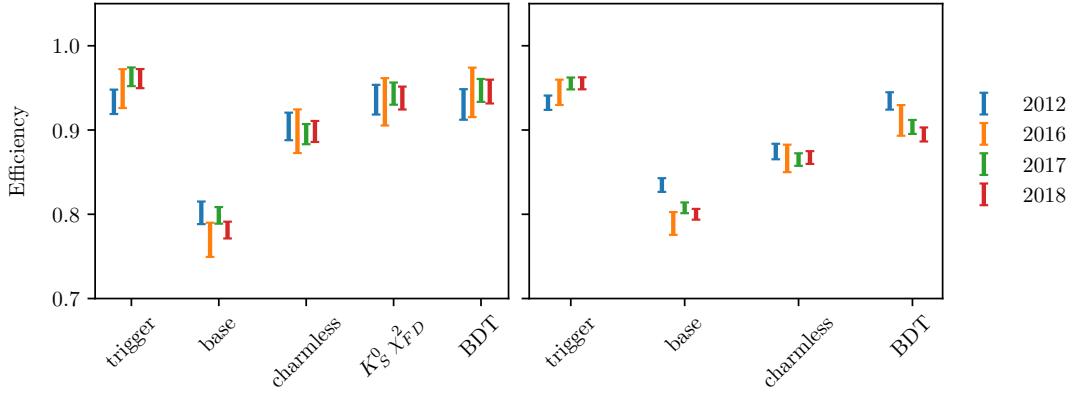


Figure 5.8: The efficiency of each selection step in samples of simulated $B^\pm \rightarrow D(\rightarrow K_0^0 \pi^+ \pi^-) \pi^\pm$ signal decays in the (left) LL and (right) DD categories. The selection steps are applied on top of each other, from left to right on the horizontal axis. The samples are split by year.

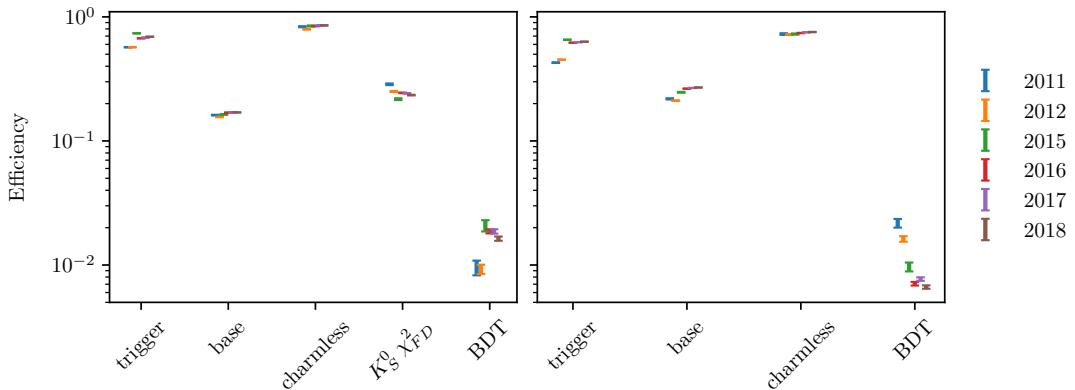


Figure 5.9: The efficiency of each selection step in samples of $B^\pm \rightarrow D(\rightarrow K_0^0 \pi^+ \pi^-) \pi^\pm$ candidates in data where the reconstructed B mass is above $5600 \text{ MeV}/c^2$, meaning the candidates stem from combinatorial background. The efficiency is shown for candidates in the (left) LL and (right) DD categories. The selection steps are applied on top of each other, from left to right on the horizontal axis. The samples are split by year. Notice the logarithmic scale on the vertical axis.

1940 5.2.1 Efficiency of the PID requirements

1941 The efficiencies of the PID requirements on the companion enter the yield parame-
 1942 terisations of the mass fits in Section 5.4 and 5.5 and must therefore be known.
 1943 They are determined using samples of calibration data selected without relying on
 1944 PID variables, as implemented in the `PIDCalib` frame work [145]. Reasonably pure
 1945 samples of pion and kaon tracks are obtained from $D^0 \rightarrow K^- \pi^+$ decays, where the
 1946 D meson originates in a $D^{*+} \rightarrow D^0 \pi^+$ decay and can therefore be flavour tagged.
 1947 The remaining background is subtracted via the `sPlot` [146] procedure, based on

Table 5.4: PID efficiencies obtained with the `PIDCalib` tool. The uncertainty incorporates statistical uncertainty due to the size of the reference sample, the systematic uncertainty due to the choice of binning scheme in `PIDCalib`, and a systematic uncertainty due to the `sWeight` calculation in `PIDCalib` of 0.1 %.

Efficiency	Particle	D final state	$\varepsilon_{\text{PID}} (\%)$	
			LL	DD
Run I and II				
Correct ID	Kaon	$D \rightarrow K_S^0 \pi^+ \pi^-$	86.74 ± 0.13	86.90 ± 0.22
		$D \rightarrow K_S^0 K^+ K^-$	86.22 ± 0.26	86.56 ± 0.30
	Pion	$D \rightarrow K_S^0 \pi^+ \pi^-$	97.11 ± 0.11	97.17 ± 0.13
		$D \rightarrow K_S^0 K^+ K^-$	97.07 ± 0.11	97.16 ± 0.14

1948 a two-dimensional fit of the $m(K^-\pi^+)$ and $m(D^0\pi^+) - m(D^0)$ distributions. The
 1949 obtained weights are employed to calculate the average efficiency of the requirement
 1950 on `PIDK` for a number of bins in the momentum and pseudorapidity of the calibration
 1951 tracks, and the number of charged tracks in the detector, thus constructing a three-
 1952 dimensional efficiency lookup table. The procedure is carried out for each PID
 1953 requirement, companion species, data-taking year, track charge, and magnet polarity.
 1954 Based on these tables, expected PID efficiencies for the $B^\pm \rightarrow D\pi^\pm$ and $B^\pm \rightarrow DK^\pm$
 1955 signal decays are calculated that take the kinematical distribution and detector
 1956 occupancy in the *BPGGSZ* data samples into account, by using the high-purity
 1957 sample of $B^\pm \rightarrow D\pi^\pm$ candidates in the signal region as a reference. The dominating
 1958 uncertainty on the efficiencies is statistical in nature, due to the finite size of the
 1959 reference sample. In addition, systematic uncertainties are included due to the
 1960 `sPlot` procedure, estimated at 0.1 % [145], and due to the choice of binning scheme,
 1961 estimated by repeating the procedure using a number of alternative binning schemes.
 1962 The final estimates of the correct-ID efficiencies, ε_{PID} , are shown in Table 5.4,
 1963 including all sources of uncertainty. Note that the probability to misidentify a decay
 1964 satisfies $\varepsilon_{\text{mis-ID}} = 1 - \varepsilon_{\text{PID}}$ by construction, due to the the definition of the `PIDK`
 1965 variable (given in Section 3.2.2) and the chosen PID requirement.

1966 5.2.2 Efficiency profile over the Dalitz plot

1967 The analysis strategy depends on sharing the F_i parameters between the $B \rightarrow D\pi$
 1968 and $B \rightarrow DK$ channels. This is reasonable, since the phase-space dependence of the
 1969 reconstruction efficiency is expected to be very similar between the two decays, given
 1970 the similar kinematics; an assumption that is verified using samples of simulated
 1971 decays. The full selection is applied to the samples. The $B \rightarrow D\pi$ sample of LL (DD)
 1972 candidates includes about 63,000 (146,000) simulated decays, and the $B \rightarrow DK$

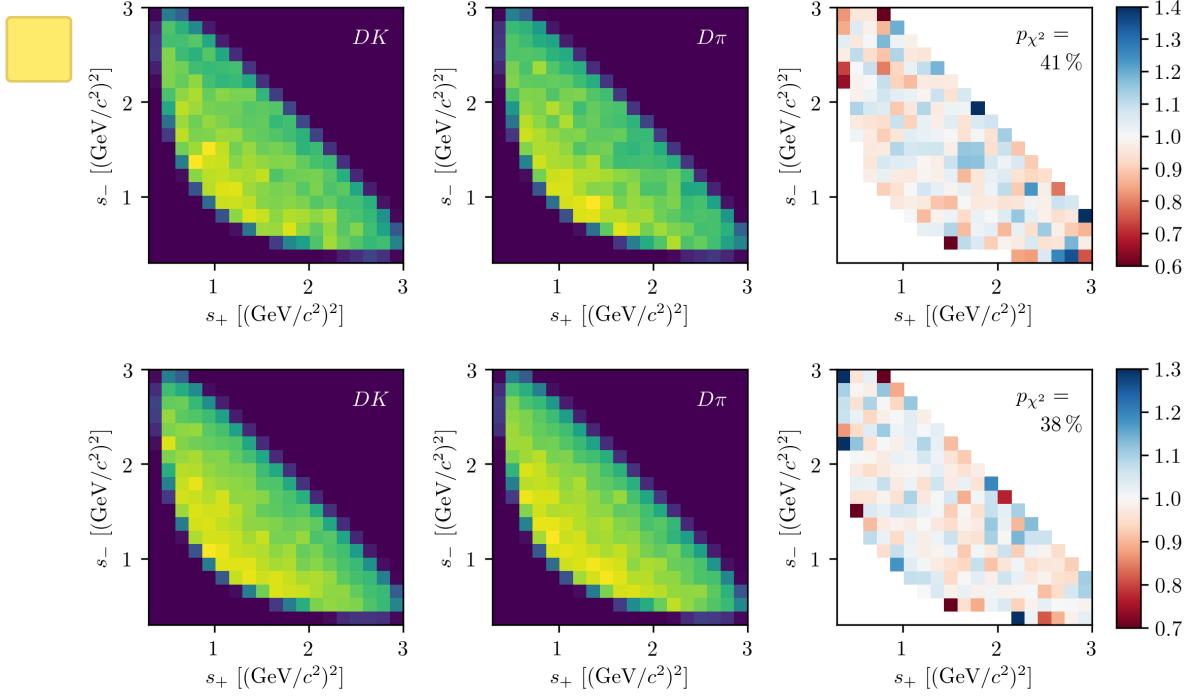


Figure 5.10: The (s_+, s_-) distribution in simulated samples of (left) $B \rightarrow DK$ decays and (center) $B \rightarrow D\pi$ decays where $D \rightarrow K_S^0 \pi^+ \pi^-$, as well as (right) the ratio between the two histograms (corrected for differences in sample sizes). The plots are shown for candidates in the (top) LL and (bottom) DD categories. The p values are the results of χ^2 compatibility tests between the two histograms.

samples include 60,000 (142,000) simulated decays. For the $B \rightarrow D\pi$ mode, this is approximately equal to the number of decays in the full Run 1+2 data sample, and for $B \rightarrow DK$ this is a factor of about 12 larger than the data sample. The decays were simulated with an equal decay probability across the D -decay phase space, so that any non-uniform distribution of reconstructed decays is completely determined by a phase-space dependent reconstruction and selection efficiency. Therefore the assumption that the phase-space dependence is identical between the $B \rightarrow D\pi$ and $B \rightarrow DK$ channels is verified by seeing if the Dalitz coordinates are distributed differently between the samples of simulated $B \rightarrow D\pi$ and $B \rightarrow DK$ decays.

This is investigated with two statistical tests. The first is a χ^2 comparison of 2D histograms of the distribution of $m^2(K_S^0 h^+)$ and $m^2(K_S^0 h^-)$ in the different $B \rightarrow D\pi$ and $B \rightarrow DK$ channels. These histograms, and the ratio between them, are shown in Figs. 5.10 and 5.11, along with the p -values from the χ^2 tests. It can be seen that, in all cases, the probability of obtaining the two histograms assuming that they share the same underlying distribution has a reasonable value, and that there is no clear trend in the ratio plots. The second test is a Kolmogorov-Smirnov

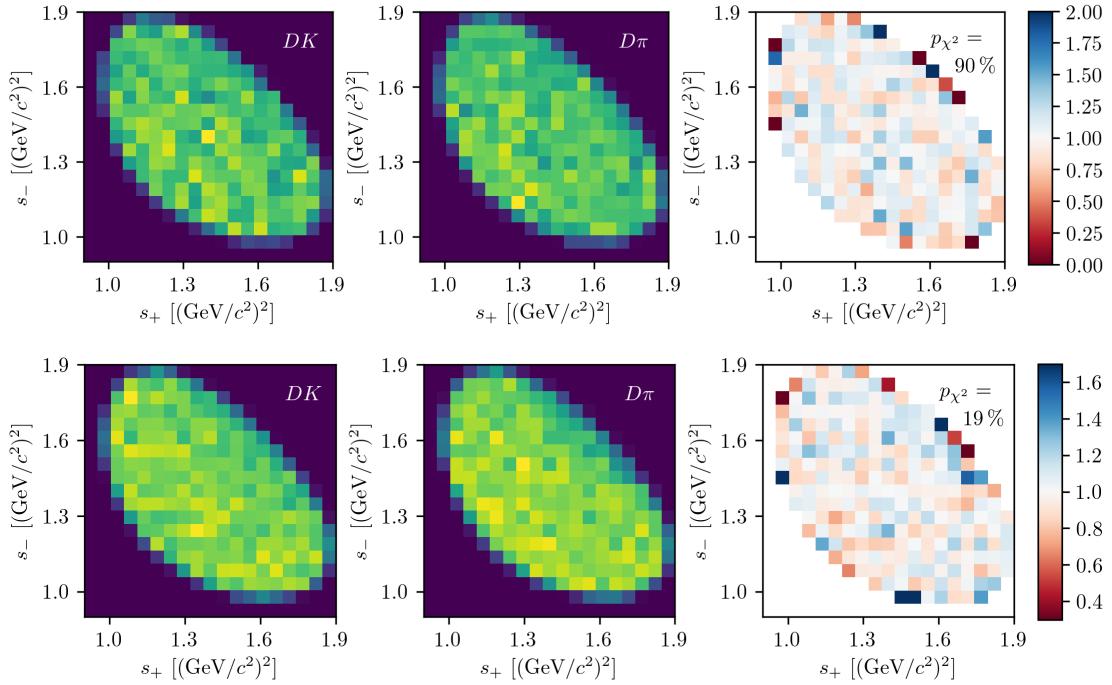


Figure 5.11: The (s_+, s_-) distribution in simulated samples of (left) $B \rightarrow DK$ decays and (center) $B \rightarrow D\pi$ decays where $D \rightarrow K_S^0 K^+ K^-$, as well as (right) the ratio between the two histograms (corrected for differences in sample sizes). The plots are shown for for candidates in the (top) LL and (bottom) DD categories. The p values are the results of χ^2 compatibility tests between the two histograms.

1989 test [147] of the compatibility of the one-dimensional distributions of $m^2(K_S^0 h^+)$,
 1990 $m^2(K_S^0 h^-)$, and $m^2(h^+ h^-)$. These distributions, and the corresponding p -values,
 1991 are shown in Fig. 5.12 and 5.13. Again, all the p values are reasonable. Therefore,
 1992 it is concluded that there are no statistically significant differences between the
 1993 phase-space dependence of the reconstruction and selection efficiency between the
 1994 $B \rightarrow D\pi$ and $B \rightarrow DK$ channels, given the present sample sizes. Because the
 1995 simulation samples have approximately the same amount of decays as data (or
 1996 significantly more, in the $B \rightarrow DK$ case), any potential differences will be negligible
 1997 with data yields. Thus, sharing the F_i parameters between the $B \rightarrow D\pi$ and
 1998 $B \rightarrow DK$ channels is viable, and no efficiency correction is necessary.

1999 5.3 Background studies

2000 A wide range of backgrounds can potentially pollute the sample of signal candidates.
 2001 The backgrounds group into three categories depending on how they are treated
 2002 in the analysis:

- 2003 • Backgrounds that can be effectively removed in the selection

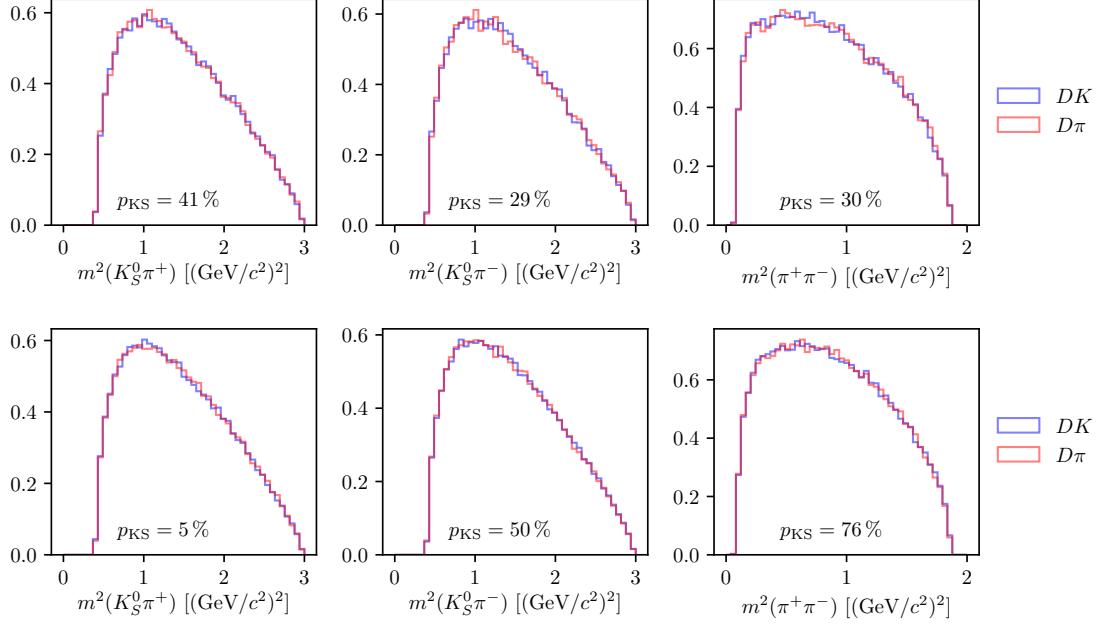


Figure 5.12: One-dimensional distributions of $m^2(K_S^0\pi^+)$, $m^2(K_S^0\pi^-)$, and $m^2(\pi^+\pi^-)$ in simulated (blue) $B^\pm \rightarrow DK^\pm$ and (red) $B^\pm \rightarrow D\pi^\pm$ decays where $D \rightarrow K_S^0\pi^+\pi^-$ in the (top) LL and (bottom) DD categories. The p values are the results of Kolmogorov-Smirnov compatibility tests between the distributions.

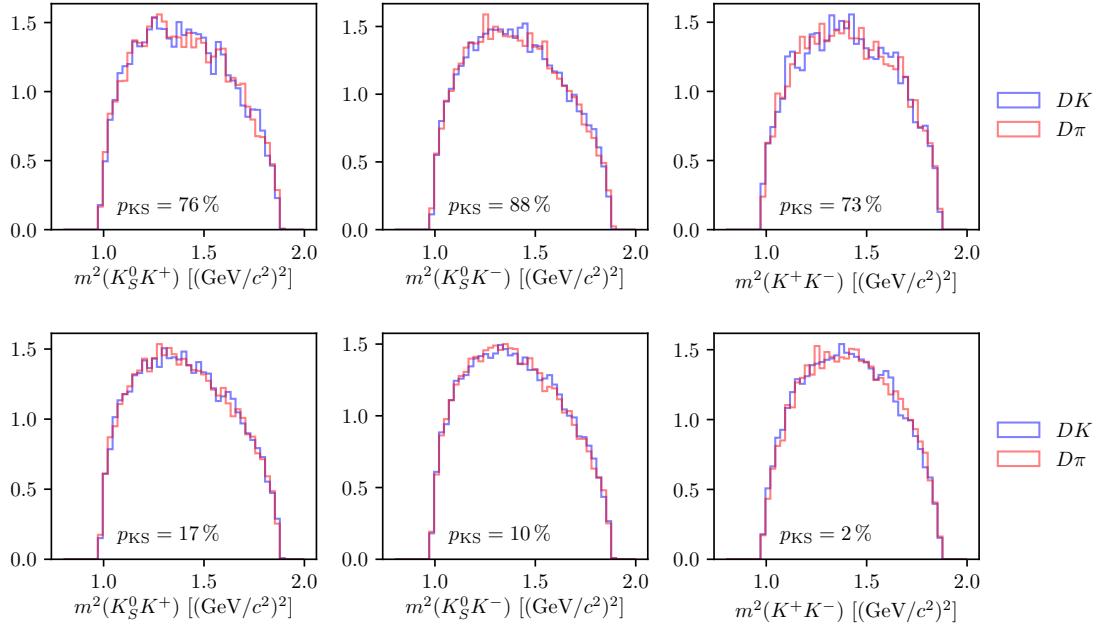


Figure 5.13: One-dimensional distributions of $m^2(K_S^0K^+)$, $m^2(K_S^0K^-)$, and $m^2(\pi^+\pi^-)$ in simulated (blue) $B^\pm \rightarrow DK^\pm$ and (red) $B^\pm \rightarrow D\pi^\pm$ decays where $D \rightarrow K_S^0K^+K^-$ in the (top) LL and (bottom) DD categories. The p values are the results of Kolmogorov-Smirnov compatibility tests between the distributions.

- 2004 • Backgrounds that are only present at a level where the impact on the
2005 measurement result is small, and which do therefore not have to be modelled
 - 2006 • Backgrounds that are present at a level where they have to be modelled in
2007 the fit to data, and cannot effectively be rejected further in the selection
- 2008 The latter category comprises of combinatorial background, which remains present
2009 at a non-negligible level after the application of the BDT described in Section 5.1.2;
2010 contributions from a number of partly reconstructed $B \rightarrow Dh^\pm X$ decays, where
2011 X denotes a pion or photon that is not included in the reconstructed decay, and
2012 which can only be separated from signal decays by their $m(Dh)$ distribution; and
2013 finally $B^\pm \rightarrow D\pi^\pm$ decays that are categorised as $B^\pm \rightarrow DK^\pm$ decays in the particle-
2014 identification step and vice-versa. These background sources are described in detail
2015 in Section 5.4. This section focuses on backgrounds that led to specific requirements
2016 in the selection or proved to be small enough to not merit special treatment.

2017 5.3.1 Charmless decays

2018 There is potentially a so-called *charmless* background present in data, consisting
2019 of $B^\pm \rightarrow K_S^0 h^+ h^- h^\pm$ decays. These have the same final state as the signal decay,
2020 but no intermediate D meson. Because all final state particles are reconstructed,
2021 this background peaks in the B mass spectrum. This background is suppressed
2022 by requiring the reconstructed B and D decay vertices to be separated in the
2023 z direction; specifically by requiring that $\Delta z_{\text{significance}}^{D-B} > 0.5$, where $\Delta z_{\text{significance}}^{D-B}$
2024 was defined in Eq. (5.1). The remaining background level can be investigated
2025 by investigating the D mass sidebands.

2026 However, the use of the `DecayTreeFitter` χ^2_{DTF} as an input variable in the BDT
2027 removes essentially all of the D (and K_S^0) sideband, due to the mass constraints in
2028 the decay chain fit. Therefore separate BDT's are trained for LL and DD candidates
2029 without the χ^2_{DTF} as an input variable, and used when selecting candidates for
2030 the background studies presented in this section, and the following. In a similar
2031 manner, all mass window requirements are made on the *default* reconstructed
2032 masses, obtained with no use of `DecayTreeFitter`. The overlap of the two sets
2033 of selected candidates in the signal B -mass window is above 95 %.

2034 The reconstructed B mass spectrum is shown for $B^\pm \rightarrow D\pi^\pm$ candidates in
2035 the D sidebands in Fig. 5.14, both before and after making a requirement on
2036 $\Delta z_{\text{significance}}^{D-B}$. The check is based on the upper D sideband for $D \rightarrow K_S^0 \pi^+ \pi^-$ decays
2037 and the lower D sideband for $D \rightarrow K_S^0 K^+ K^-$ decays to avoid contamination
2038 from real $B^\pm \rightarrow Dh^\pm$ decays with subsequent $D \rightarrow K_S^0 K^\pm \pi^\mp$ decays, or crossfeed

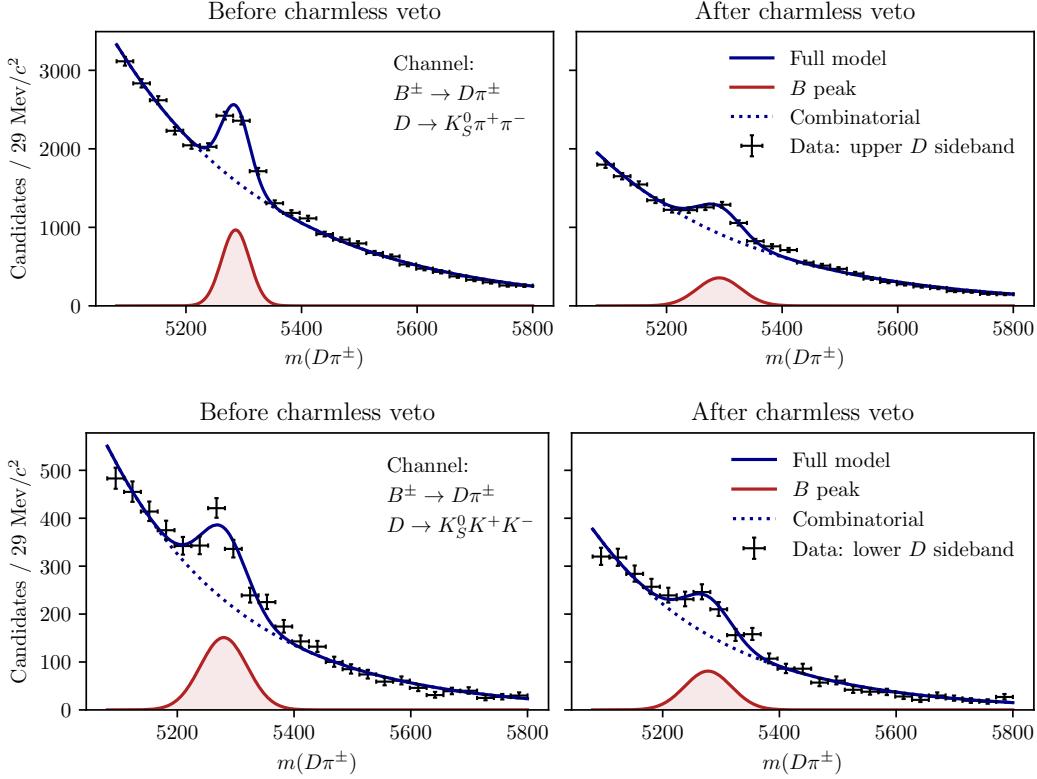


Figure 5.14: The B mass distribution of (top) $B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)\pi^\pm$ and (bottom) $B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)\pi^\pm$ candidates reconstructed in both the LL and DD categories, residing in the upper D mass sideband $m_D \in [1910, 1960] \text{ MeV}/c^2$ for $D \rightarrow K_S^0\pi^+\pi^-$ and in the lower sideband $m_D \in [1910, 1960] \text{ MeV}/c^2$ for $D \rightarrow K_S^0K^+K^-$, with (left) no requirement on $\Delta z_{\text{significance}}^{BD}$ and (right) after a requirement of $\Delta z_{\text{significance}}^{BD} > 0.5$.

2039 between the two signal D -decay modes. A peak is clearly visible, the size of
 2040 which is reduced by the requirement. This peak is partly due to a contribution
 2041 from $B^\pm \rightarrow K_S^0\pi^+\pi^-\pi^\pm$ decays ($B^\pm \rightarrow K_S^0K^+K^-\pi^\pm$ decays) in the $D \rightarrow K_S^0\pi^+\pi^-$
 2042 ($D \rightarrow K_S^0K^+K^-$) channel, and partly due to real signal decays that leak into the
 2043 D sidebands. The number of real signal decays can be calculated from the yield
 2044 obtained in the fit of Section 5.4, and the reconstructed m_D distribution in simulated
 2045 signal decays. Subtracting this contribution, it is estimated that approximately 450
 2046 (200) charmless decays are present in the $K_S^0\pi^+\pi^-$ ($K_S^0K^+K^-$) data samples. In
 2047 similar fashion, Fig. 5.15 shows the m_B spectra for $B^\pm \rightarrow DK^\pm$ candidates in the
 2048 D sidebands. In these plots, the peaks are at m_B values that are lower (higher)
 2049 than the B mass in the $K_S^0\pi^+\pi^-$ ($K_S^0K^+K^-$) categories, because they stem from
 2050 real $B^\pm \rightarrow K_S^0K^+K^-\pi^\pm$ decays where a kaon is mis-reconstructed as a pion or a
 2051 pion is misreconstructed as a kaon, respectively. The total contribution of charmless
 2052 decays in the $B^\pm \rightarrow DK^\pm$ data samples is estimated to be about 200 decays. As
 2053 described further in Section 5.6.11, the presence of a charmless background at these

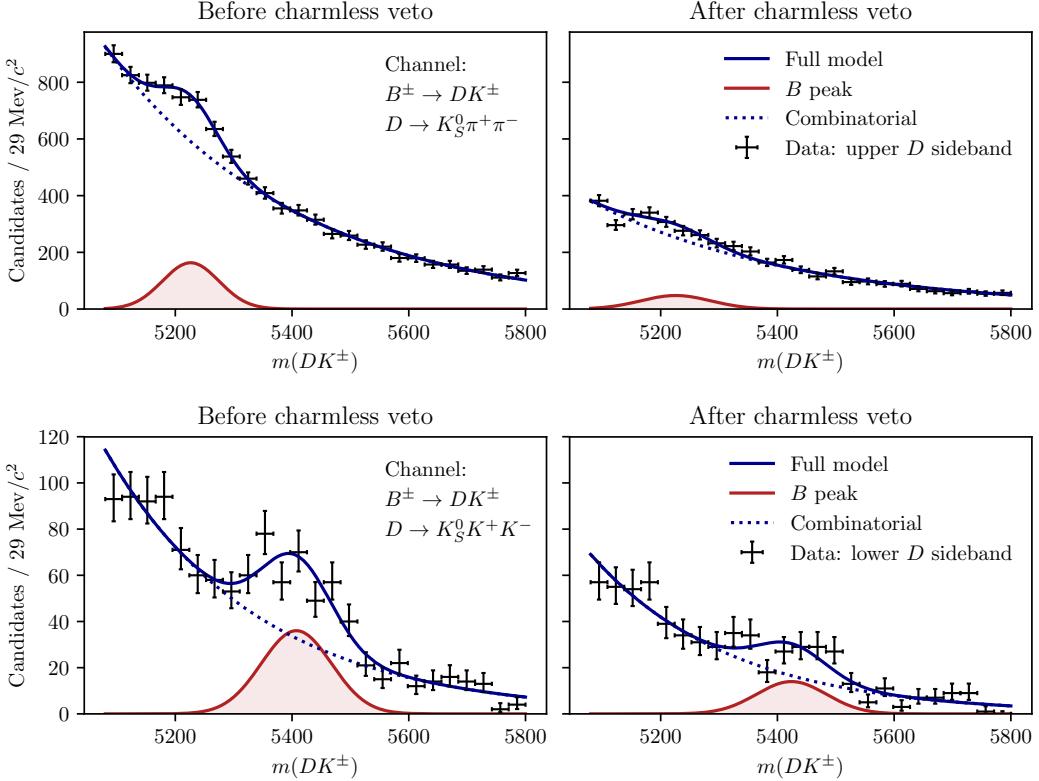


Figure 5.15: The B mass distribution of (top) $B^\pm \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-)K^\pm$ and (bottom) $B^\pm \rightarrow D(\rightarrow K_S^0 K^+ K^-)K^\pm$ candidates reconstructed in both the LL and DD categories, residing in the upper D mass sideband $m_D \in [1910, 1960] \text{ MeV}/c^2$ for $D \rightarrow K_S^0 \pi^+ \pi^-$ and in the lower sideband $m_D \in [1910, 1960] \text{ MeV}/c^2$ for $D \rightarrow K_S^0 K^+ K^-$, with (left) no requirement on $\Delta z_{\text{significance}}^{BD}$ and (right) after a requirement of $\Delta z_{\text{significance}}^{BD} > 0.5$.

2054 levels has a negligible impact on the measurement results. It is not favourable to
2055 tighten the requirement further, as it does remove some real signal decays.

2056 5.3.2 Background from four-body D decays

2057 A similar potential background is from real $B^\pm \rightarrow Dh^\pm$ decays where the D meson
2058 decays directly to the $\pi^+ \pi^- h^+ h^-$ final state, without an intermediate K_S^0 meson.
2059 This background can be investigated by looking for a peak in the B mass spectrum
2060 for candidates in the K_S^0 sideband, as illustrated in Fig. 5.16. The figure shows
2061 the spectrum in the final data sample, illustrating the significant effect of making
2062 the requirement on the K_S^0 flight distance that was discussed in Section 5.1.1. The
2063 BDT that does *not* rely on the DTF χ^2 has been used to suppress combinatorial
2064 background. The remaining peak after requiring $\chi_{\text{FD}}^2 > 49$ is completely accounted
2065 for by real signal decays that leak into the K_S^0 sideband. The requirement is
2066 made for candidates in the LL category only; if the pions stemming from a K_S^0

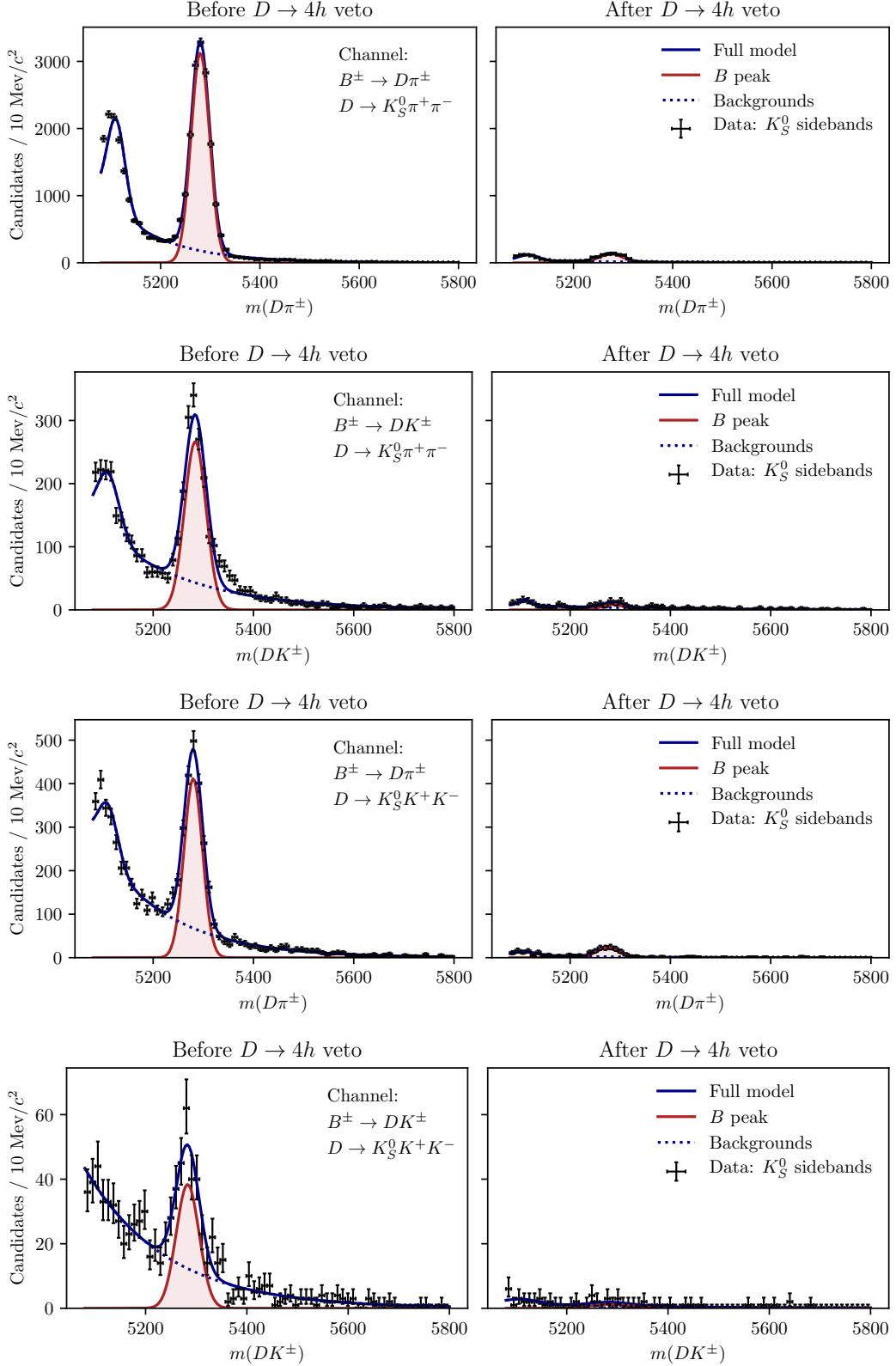


Figure 5.16: The B mass spectrum in the K_S^0 sideband where $m_{K_S^0} \in [467, 482] \text{ MeV}/c^2$ or $m_{K_S^0} \in [512, 527] \text{ MeV}/c^2$ (left) without a requirement on the K_S^0 flight distance significance, and (right) after the requirement implemented in the analysis.

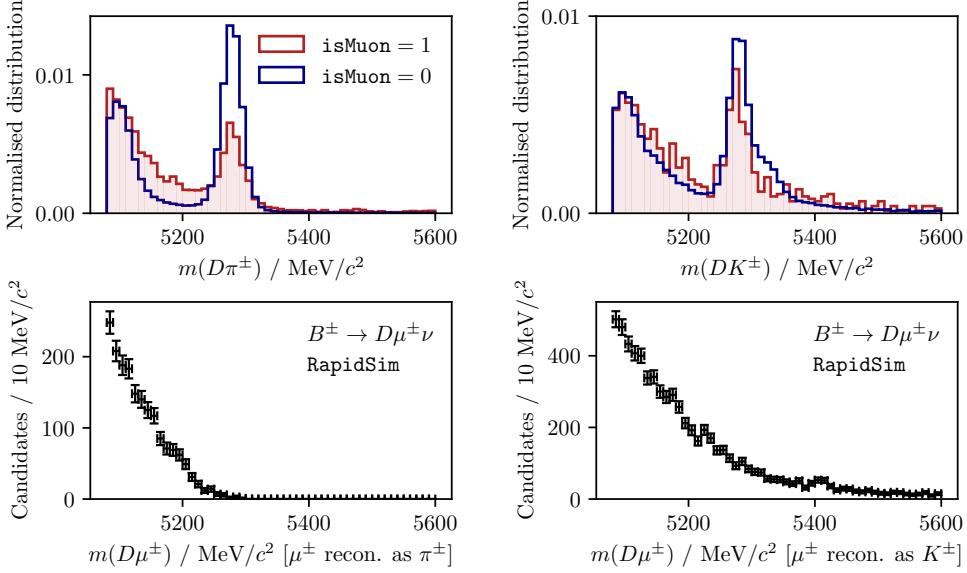


Figure 5.17: (Top) The m_B spectra in data split by the value of `isMuon` for the companion particle, in (left) the $D\pi^\pm$ and (right) the DK^\pm samples where $D \rightarrow K_S^0 \pi^+ \pi^-$. The two histograms are normalised independently, so that the distributions can be compared. The fractions of candidates in data (with $m_B \in [5080, 5800] \text{ MeV}/c^2$) where the companion satisfies `isMuon=1` are 1.6 % and 1.8 % for the $D\pi^\pm$ and DK^\pm channels respectively. (Bottom) the `RapidSim` mass spectra for $B^\pm \rightarrow D^0 \mu^\pm \nu_\mu$ decays reconstructed in the (left) $D\pi^\pm$ and (right) DK^\pm categories.

candidate are reconstructed as downstream tracks it implies that the K_S^0 has travelled from the interaction region.

5.3.3 Semi-leptonic backgrounds

The data sample has a minor background from $B \rightarrow D\mu\nu_\mu X$ decays, visible in the B mass spectrum when the companion is required to satisfy `isMuon=1`. This is shown in Fig. 5.17 for both the $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ channels where $D \rightarrow K_S^0 \pi^+ \pi^-$. The B mass spectra for simulated $B^\pm \rightarrow D\mu^\pm \nu_\mu$ decays reconstructed in each category are also shown, from simulation samples produced via `RapidSim`. The background is very efficiently vetoed by requiring `IsMuon=0` on the companion. This requirement removes approximately 85 % of the background decays, as estimated using the `PIDCalib` calibration samples and the (p, p_T) distribution for the muon in the `RapidSim` samples. The fraction of signal candidates for which the companion satisfies `IsMuon=1` in simulated signal samples is $\leq 0.9\%$ so the impact on signal yield is small.

The analogous $B \rightarrow D e \nu_e X$ background is investigated by inspecting the B mass spectra after making requirements on `PIDE` for the companion candidate, but

2083 a presence of the semi-leptonic background in data is not visible and no electron
 2084 veto is applied to the companion.

2085 **Background from semi-leptonic D decays**

2086 There is a potential background from real $B^\pm \rightarrow Dh^\pm$ decays where the D
 2087 meson decays semi-leptonically: $D^0 \rightarrow K_S^0\pi^-\ell^+\nu_\ell$. This background is particularly
 2088 dangerous because it peaks at the B mass, when the D -mass requirement is applied
 2089 and it is reconstructed in the $D \rightarrow K_S^0\pi^+\pi^-$ category. This is illustrated in
 2090 Fig. 5.18 using **RapidSim** samples of $B^\pm \rightarrow D(\rightarrow K^{*-}(\rightarrow K_S^0\pi^-)\ell^+\nu_\ell)h^\pm$ decays
 2091 for $K^* \in \{K^*(892), K_0^*(1430), K_2^*(1430), K^*(1680)\}$. The respective spin of each
 2092 resonance is taken into account in generation, by handling the decay via **EVTGEN**.
 2093 The expected background yields relative to signal can be estimated by applying the B
 2094 and D mass cuts to decays in the **RapidSim** samples, and using the relative branching
 2095 ratios. Only the $D^0 \rightarrow K^{*-}(892)\ell\nu_\ell$ branching fractions have been measured [31],
 2096 but there is no reason to expect that higher K^* resonances should not contribute.
 2097 To estimate their potential contribution, the branching ratios are approximated by

$$\text{BR}[D^0 \rightarrow K^{*-}(X)(\rightarrow K_S^0\pi^-)\ell\nu_\ell] \simeq \frac{\text{BR}[D^0 \rightarrow K^{*-}(X)(\rightarrow K_S^0\pi^-)\pi^+]}{\text{BR}[D^0 \rightarrow K^{*-}(892)(\rightarrow K_S^0\pi^-)\pi^+]}\text{BR}[D^0 \rightarrow K^{*-}(892)(\rightarrow K_S^0\pi^-)\ell\nu_\ell]$$

2098 because all the relevant $D^0 \rightarrow K^{*-}(\rightarrow K_S^0\pi^-)\pi^+$ branching fractions are known [31].
 2099 The efficiencies and branching ratios relative to the signal channel are given in
 2100 Table 5.5. It is clear that the higher K^* resonances are important: the smaller
 2101 branching ratios are compensated for by a higher selection efficiency, due to the
 2102 smaller phase-space of the missed neutrino. The total background yield is 1.1 %
 2103 of the signal yield in both the $B^\pm \rightarrow D\pi^\pm$ and $B^\pm \rightarrow DK^\pm$ channels. However,
 2104 there will be an additional contribution in the $B^\pm \rightarrow DK^\pm$ channel from real
 2105 $B^\pm \rightarrow D\pi^\pm$ decays with semi-leptonic D decays and a mis-identification of the
 2106 companion. This background also peaks, and the yield is approximately 0.4 %
 2107 of the $B^\pm \rightarrow DK^\pm$ signal yield.

2108 The potential impact from the presence of the background is estimated by

- 2109 1. calculating the expected $B^\pm \rightarrow D\pi^\pm$ and $B^\pm \rightarrow DK^\pm$ signal yields in each
 2110 bin for physics parameters similar to the world average values
- 2111 2. then calculating the background bin yields in each bin, using the relative
 2112 branching fractions and efficiencies described above and taking the bin-
 2113 distribution from the **RapidSim** samples. The **RapidSim** samples are produced
 2114 using the **ISGW2** model in **EVTGEN** [120], yielding the Dalitz distributions in
 2115 Fig. 5.19.

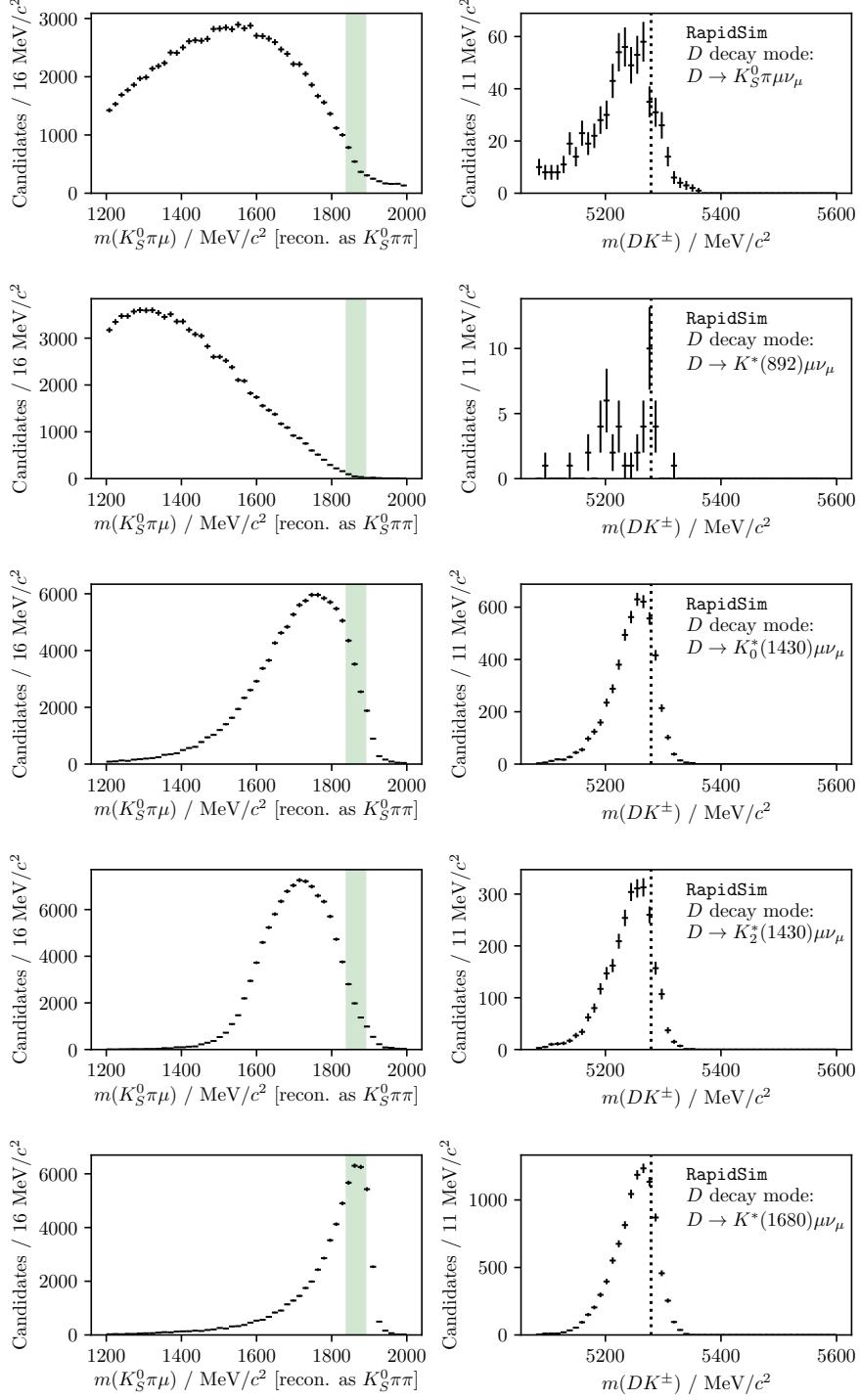


Figure 5.18: The reconstructed (left) $m(K_S^0 \pi^+ \pi^-)$ and (right) $m(Dh)$ distributions in **RapidSim** samples of $B^\pm \rightarrow DK^\pm$ decays where $D^0 \rightarrow K_S^0 \pi^- \mu^+ \nu_\mu$. The top plot is for decays that were uniformly distributed over phase space, and the following plots show the distribution where the $K_S^0 \pi^-$ originate in the resonances $K^{*-}(892)$, $K_0^{*-}(1430)$, $K_2^{*-}(1430)$, and $K^{*-}(1680)$. The shapes for the $D^0 \rightarrow K_S^0 \pi^- e^+ \nu_e$ case are almost identical.

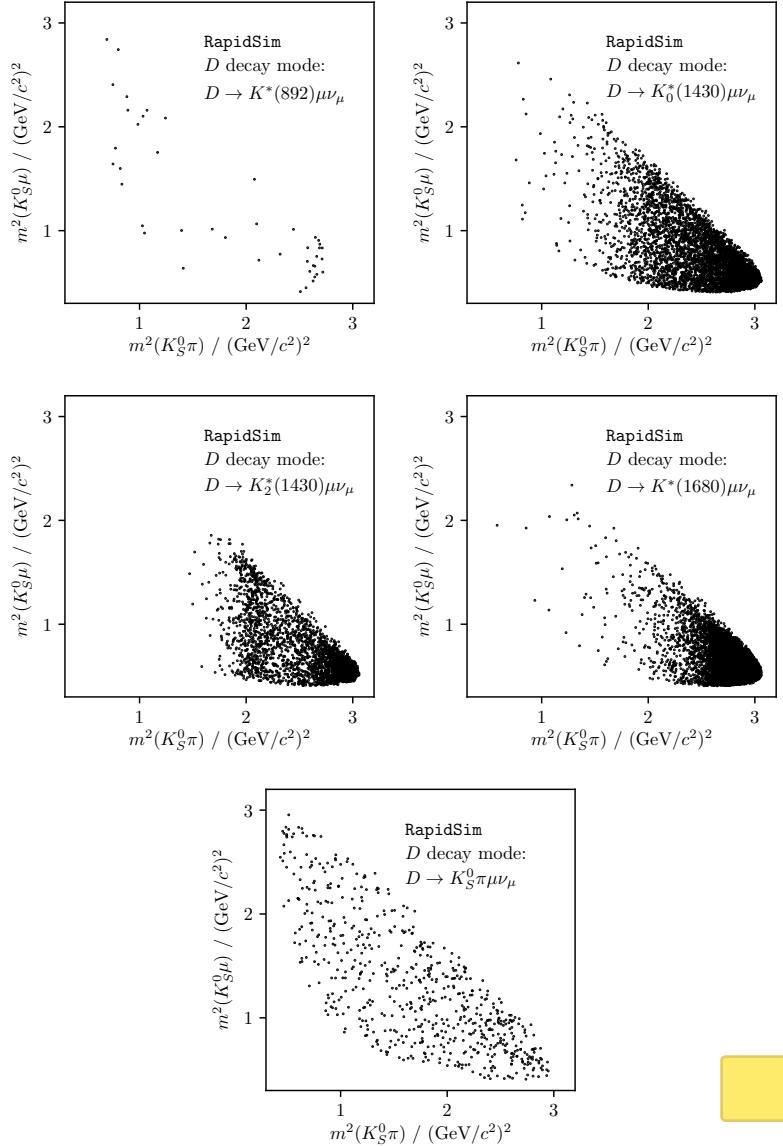


Figure 5.19: Dalitz distribution for $D \rightarrow K_S^0\pi\mu\nu_\mu$ decays in *RapidSim*, where the $K_S^0\pi^-$ originate in the resonances $K^{*-}(892)$, $K^{*-}_0(1430)$, $K^{*-}_2(1430)$, and $K^{*-}(1680)$, as well as for a uniform distribution over phase space.

2116 3. adding the signal and background yields, and fitting the new $B^\pm \rightarrow D\pi^\pm$ and
 2117 $B^\pm \rightarrow DK^\pm$ yields back with the default signal-yield expressions (including a
 2118 fit of the F_i parameters)

2119 The obtained biases are shown in Fig. 5.20, where they are calculated a number
 2120 of times, each time varying the efficiencies within statistical uncertainties and the
 2121 relevant branching fractions within the measurement uncertainties. The systematic
 2122 uncertainty due to the unknown branching fractions and the use of *RapidSim* in
 2123 lieu of full simulation is not included, but is of course significant. Nevertheless

Table 5.5: The selection efficiencies of $B^\pm \rightarrow DK^\pm$ decays where $D^0 \rightarrow K_S^0\pi^-\ell^+\nu_\ell$ when reconstructed in the $D \rightarrow K_S^0\pi^+\pi^-$ mode in **RapidSim** relative to the signal selection efficiencies, for a number of decay modes: PHSP as well as resonant production where the $K_S^0\pi^-$ pair originates in one of several K^* resonances. The relative branching ratios are also shown, calculated as explained in the main text, as well as the predicted relative yields.

Mode	$\epsilon_{bkg}/\epsilon_{signal}$ (%)	$\Gamma_{bkg}/\Gamma_{signal}$ (%)	N_{bkg}/N_{signal} (%)
$D \rightarrow K_S^0\pi^-\mu^+\nu_\mu$ (PHSP)	0.92 ± 0.05	18.3 ± 14.8	0.17 ± 0.14
$D \rightarrow (K_S^0\pi^-)_{K^{*-}(892)}\mu^+\nu_\mu$	0.06 ± 0.01	22.3 ± 3.2	0.013 ± 0.003
$D \rightarrow (K_S^0\pi^-)_{K_0^{*-}(1430)}\mu^+\nu_\mu$	7.3 ± 0.1	3.7 ± 0.8	0.27 ± 0.06
$D \rightarrow (K_S^0\pi^-)_{K_2^{*-}(1430)}\mu^+\nu_\mu$	3.7 ± 0.1	0.5 ± 0.3	0.02 ± 0.01
$D \rightarrow (K_S^0\pi^-)_{K^{*-}(1680)}\mu^+\nu_\mu$	24.4 ± 0.3	0.6 ± 0.5	0.15 ± 0.12
$D \rightarrow K_S^0\pi^-e^+\nu_e$ (PHSP)	0.53 ± 0.02	20.8 ± 16.3	0.11 ± 0.09
$D \rightarrow (K_S^0\pi^-)_{K^{*-}(892)}e^+\nu_e$	0.15 ± 0.02	25.6 ± 2.5	0.04 ± 0.01
$D \rightarrow (K_S^0\pi^-)_{K_0^{*-}(1430)}e^+\nu_e$	6.3 ± 0.1	4.2 ± 0.8	0.26 ± 0.05
$D \rightarrow (K_S^0\pi^-)_{K_2^{*-}(1430)}e^+\nu_e$	4.12 ± 0.08	0.5 ± 0.3	0.02 ± 0.01
$D \rightarrow (K_S^0\pi^-)_{K^{*-}(1680)}e^+\nu_e$	10.0 ± 0.2	0.7 ± 0.5	0.07 ± 0.05
Total	-	-	1.1 ± 0.4

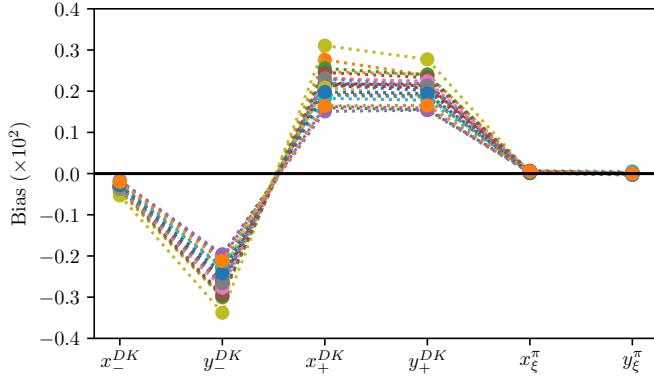


Figure 5.20: Estimated biases on the measured observables due to the presence of $D \rightarrow K_S^0\pi\ell\nu_\ell$ backgrounds, calculated while varying efficiencies and branching ratios within uncertainties.

it is clear that the potential biases are significant compared to the size of the systematic uncertainties of the analysis presented in Section 5.6. Therefore the backgrounds are vetoed by requiring `IsMuon=0` and `PIDe < 0` on the pions from the D -decay with opposite charge to the bachelor in the $D \rightarrow K_S^0\pi^+\pi^-$ channel. This requirement removes 88 % of the muonic background and 99 % of the electron background, according to PID efficiencies obtained via the `PIDCalib` package, using the (p, p_T) distribution for the muon/electron in the **RapidSim** samples. The

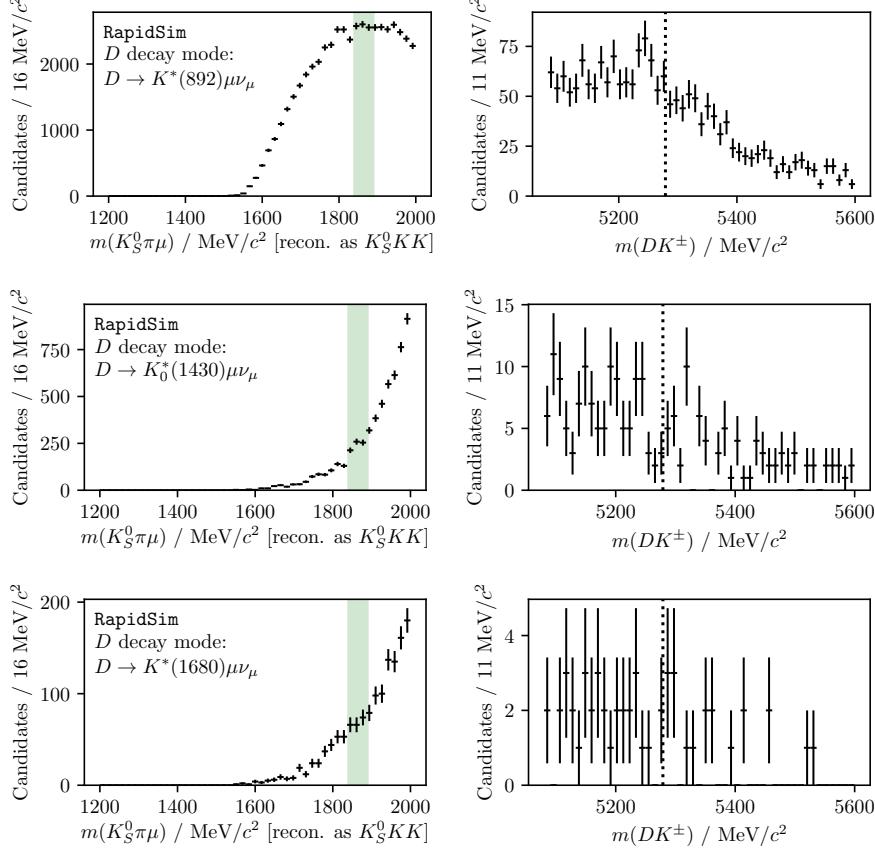


Figure 5.21: The reconstructed (left) $m(K_S^0 K^+ K^-)$ and (right) $m(Dh)$ distributions in **RapidSim** samples of $B^\pm \rightarrow DK^\pm$ decays where $D^0 \rightarrow K_S^0 \pi^- \mu^+ \nu_\mu$, where the $K_S^0 \pi^-$ originate in (top to bottom) the resonances $K^{*-}(892)$, $K_0^{*-}(1430)$, and $K^{*-}(1680)$. The shapes for the $D^0 \rightarrow K_S^0 \pi^- e^+ \nu_e$ case are almost identical.

survival rate for signal decays in full simulation is 94 %, so the impact on the obtainable precision is only about 3 %. A systematic uncertainty is assigned to account for the potential remaining background.

In the $D \rightarrow K_S^0 K^+ K^-$ channel an analogous study shows the relative yields to be similar. The selection efficiencies are higher, as are the relative branching ratios due to the lower $D \rightarrow K_S^0 K^+ K^-$ branching fraction, but in this mode the PIDK > -5 requirement placed on the pion and lepton removes approximately 90 % of the background, leaving the relative rate similar to in $D \rightarrow K_S^0 \pi^+ \pi^-$. However, importantly, *the background is not peaking*, as shown in Fig. 5.21. The presence of a percent-level, *non-peaking* background in the $D \rightarrow K_S^0 K^+ K^-$ channel is safe to ignore and thus no veto is applied in the $D \rightarrow K_S^0 K^+ K^-$ channel. 

The muon-veto for the semi-leptonic background does remove some signal decays, where an original pion or kaon results in hits in the muon detectors. A significant contribution is from particles that decay in flight. The track quality

of these decays is worse than for nominal decays, which affects the resolution on the reconstructed Dalitz coordinates. In simulated signal decays the standard deviation of $\Delta m_\pm^2 = m_{reco}^2(K_S^0\pi^\pm) - m_{TRUE}^2(K_S^0\pi^\pm)$ is 50 % larger for decays where one of the D -decay products has `IsMuon=1` than in decays where this is not the case. This can lead to systematic biases on the observables, as described further in Section 5.6.7. The overall effect is small, as evidenced by the systematic uncertainty described in that section; nevertheless this fact motivates removing decay-in-flight decays of the D -decay products. Therefore it is also required that `IsMuon=0` for the D -decay pion with the same charge as the companion in the $D \rightarrow K_S^0\pi^+\pi^-$ channels, and on the D -decay kaons in the $D \rightarrow K_S^0K^+K^-$ channels. This veto removes about 2 % of signal candidates in simulation that survive the lepton vetoes described in the previous sections.

5.3.4 Cross-feed from other $D \rightarrow K_S^0h^+h'^-$ decays

Misidentification of a D decay product can lead to background from cross-feed between the $B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)h^\pm$ and $B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)h^\pm$ signal channels, or cross-feed from $B^\pm \rightarrow D(\rightarrow K_S^0K\pi)h^\pm$ decays into either of the signal channels. However, this background is very highly suppressed by the employed requirement on the D mass. This is illustrated in Fig. 5.22, where the D mass distribution in samples of simulated $B^\pm \rightarrow D(\rightarrow K_S^0K\pi)K^\pm$ decays are shown, when reconstructed as $D \rightarrow K_S^0\pi^+\pi^-$ and $D \rightarrow K_S^0K^+K^-$ decays. Essentially no decays that fall in the selected D mass window survive the full selection. Therefore this background is not considered further. Neither is the background due to cross-feed between $B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)h^\pm$ and $B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)h^\pm$, since it involves two misidentified particles, and therefore will result in reconstructed D masses even further away from the selected mass window. A very loose PID requirement on the charged D decay products is nonetheless included in the $D \rightarrow K_S^0K^+K^-$ channel, because it helps reduce the level of combinatorial background.

5.3.5 Swapped-track backgrounds

A possible peaking background stems from real $B \rightarrow DhX$ decays with the same final state tracks as in the signal case, but where some tracks are misassigned in the reconstruction. Examples are $B^+ \rightarrow (K_S^0h^+h'^-)Dh^+$ decays where the companion and the D -decay product with the same charge are swapped, or $B^\pm \rightarrow (K^-\pi^+)DK_S^0h^\pm$ decays, where the K_S^0 is assigned to the D decay and the real companion is swapped with the D -decay product of the same charge. The

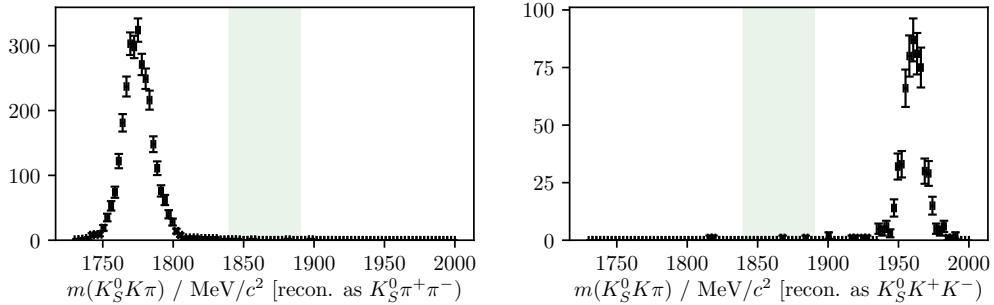


Figure 5.22: Simulated samples of $B^\pm \rightarrow D(\rightarrow K_S^0 K\pi)\pi^\pm$ decays reconstructed in the (left) $D \rightarrow K_S^0 \pi^+ \pi^-$ and (right) $D \rightarrow K_S^0 K^+ K^-$ channels, combining the LL and DD categories. The D -mass region included in the selection of signal decays is illustrated with the green band. The plots in the $B^\pm \rightarrow DK^\pm$ channels look almost identical.

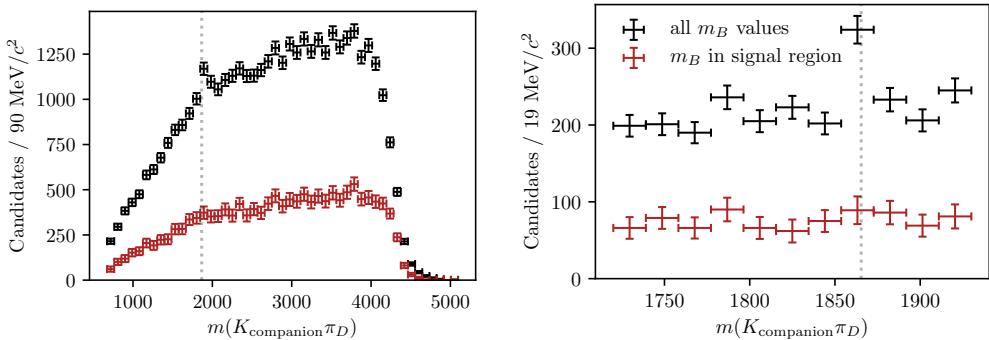


Figure 5.23: Invariant mass spectrum of the $m^2(K^\pm \pi^\mp)$ combination in the $B^\pm \rightarrow (K_S^0 \pi^+ \pi^-)K^\pm$ data sample for (black) all candidates and (red) candidates for which $m_B \in m_B^{PDG} \pm 30 \text{ MeV}/c^2$. The LL and DD categories are combined. The only difference between the left and right plots is the $m(K\pi)$ mass range on the horizontal axis. The dotted line indicated the known D mass [31].

signature of this background type is a peak at the D mass, when the invariant mass corresponding to the companion track and some subset of the D -decay tracks is formed. The presence of the background has been investigated by forming all such combinations, for all data categories, after the full selection has been applied. Only in a single channel is a peak visible: the $B^\pm \rightarrow (K_S^0 \pi^+ \pi^-)K^\pm$ channel, where $m(K^\pm \pi^\pm)$ has a peak, as shown in Fig. 5.23. Thus, a background is present from the favoured two-body D decay $B^\pm \rightarrow (K^\pm \pi^\mp)_D K_S^0 \pi^\pm$, where the K^\mp is reconstructed as the companion, and the K_S^0 meson and both pions are assigned to the D decay. Is is not favourable to veto this background, because a requirement on the invariant mass of a track combination that includes the companion track would impact the Dalitz-plot acceptance differently in the DK^\pm and $D\pi^\pm$ channels. Thus it would break a fundamental underlying feature of the measurement: the identical

selection efficiency profile between these modes. However, the yield excess in the $m(K^\pm_{\text{companion}}\pi^\mp_D)$ range around m_D , attributed to the background, corresponds to only about 0.5 % of the signal yield. A background at this level does not lead to a limiting systematic uncertainty on the measurement, as described in Section 5.6.9.

5.4 Signal and background mass shapes

The measurement employs *extended maximum-likelihood fits* [148] to the $m(Dh^\pm)$ distribution of signal candidates to determine the observables of interest. The analysis implements a two-step fit procedure: first the data samples are analysed without separating the candidates by B charge or Dalitz bin, in order to determine appropriate parametrisations of the $m(Dh^\pm)$ distribution of the signal and relevant background components. The parameterisations are then kept fixed in a subsequent fit of the observables of interest, where the candidates are split by B charge and Dalitz bin. This section describes the first step, whereas the latter fit is the subject of Section 5.5.

In both steps, the candidates are split in 8 categories depending on whether the companion is categorised as a kaon or pion, whether the K_S^0 meson is in the LL or DD category, and by whether the D meson is reconstructed in the $K_S^0\pi^+\pi^-$ or $K_S^0K^+K^-$ final state. In the remainder of this text, these categories are indexed with the letter c . For each category, c , the expected number of observed decays at a given B mass, $F^c(m)$, is given by the sum of a signal contribution and a number of background distributions

$$F^c(m|\theta) = N_s^c(\theta)f_s^c(m|\theta) + \sum_b N_b^c(\theta)f_b^c(m|\theta), \quad (5.4)$$

where θ denotes a set of parameters that describe the mass shapes and expected yields, in which some parameters are shared between categories. The distributions $f_{s/b}^c$ are normalised to integrate to unity, and the expected signal (background) yields are denoted N_s^c (N_b^c). A total, normalised distribution can then be defined

$$f^c(m|\theta) = \frac{1}{N_{\text{tot}}^c(\theta)}F^c(m|\theta), \quad N_{\text{tot}}^c(\theta) = N_s^c(\theta) + \sum_b N_b^c(\theta). \quad (5.5)$$

Given a set of N_{observed}^c measured B masses, $\{m_i^c\}$, in a given category, the extended log-likelihood function is defined

$$\ln \mathcal{L}_c(\theta|\{m_i^c\}) \equiv \sum_i \ln f^c(m_i^c|\theta) + \ln \text{Poisson}(N_{\text{tot}}^c(\theta), N_{\text{observed}}^c) \quad (5.6)$$

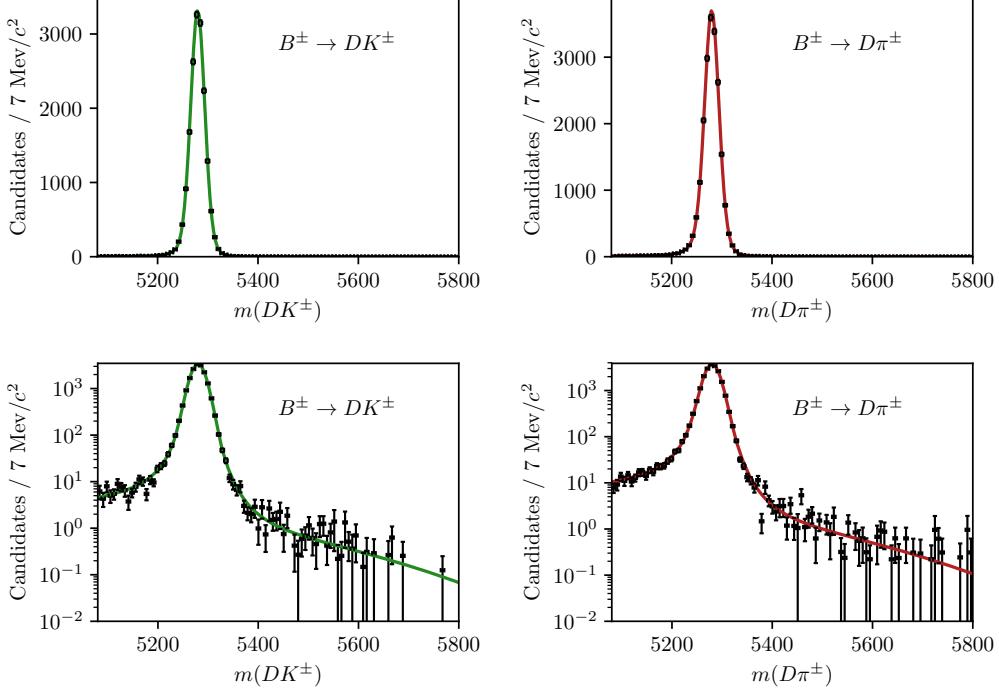


Figure 5.24: Fit projection of the signal shape to simulated $B^\pm \rightarrow D(\rightarrow K_0^0\pi^+\pi^-)h^\pm$ samples reconstructed in the LL category. (Left) shows DK shapes, and (right) shows $D\pi$ shapes. The shapes are shown with both linear and logarithmic y -axis scales.

In a simultaneous fit the total, negative log-likelihood is $-\ln \mathcal{L} = -\sum_c \mathcal{L}_c$, and this function can be minimised to find the maximum-likelihood estimates of the parameters in θ , as well as their confidence regions and correlation coefficients. This is handled with the `RooFit` package [149].

Apart from signal decays, the fit includes components that describe combinatorial background, backgrounds from decays where a companion pion is misidentified as a kaon or vice versa, and partially reconstructed backgrounds. Each of these components are described in detail in the following, before the results of the first-stage fit are presented in Section 5.4.5.

5.4.1 Signal decays

The signal component is modelled with a sum of a Gaussian density function, $f_G(m|m_B, \sigma)$, and a modified Gaussian distribution with the parameterisation

$$f_C(m|m_B, \sigma, \alpha_L, \alpha_R, \beta) \propto \begin{cases} \exp\left[\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_L\Delta m^2}\right], & \Delta m = m - m_B < 0 \\ \exp\left[\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_R\Delta m^2}\right], & \Delta m = m - m_B > 0, \end{cases}$$

which is Gaussian when $\Delta m^2 \ll \sigma^2/\alpha_{L/R}$ or $\Delta m^2 \gg \beta^{-1}$ (with widths of σ and $\sqrt{\alpha_{L/R}/\beta}$ respectively), with an exponential-like transition that is able to

model the effect of the experimental resolution of LHCb very well. For the case $\beta = 0$ the shape is denoted the *Cruijff* shape; however, in this case it tends to a uniform distribution for large Δm^2 values, and cannot model the tails of the signal distribution. Thus, the full density function is

$$f_s(m|m_B, \sigma, \alpha_L, \alpha_R, \beta) = k_C f_C(m|m_B, \sigma, \alpha_L, \alpha_R, \beta) + (1 - k_C) f_G(m|m_B, \sigma). \quad (5.8)$$

The tail parameters $(\alpha_{L/R}, \beta)$ and the constant k_C are determined in fits to simulated signal decays that have passed the full selection. The parameters are shared between the $K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$ channels, but otherwise independent in the fit categories. An example of a fit to simulated $B^\pm \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-) h^\pm$ decays is given in Fig. 5.24. The resolution parameters σ are determined in the fit to actual data. Separate parameters are determined in the LL and DD categories, because the LL category has a better resolution on the K_S^0 momentum, and therefore a narrow peak in reconstructed B mass. Likewise, separate resolution parameters are used for $B^\pm \rightarrow D\pi^\pm$ and $B^\pm \rightarrow DK^\pm$ decays, because the smaller Q value in the latter case leads to smaller momenta of the decay products, and a correspondingly better resolution.

The signal yields are determined independently in each $B^\pm \rightarrow D\pi^\pm$ category. The yields in the $B^\pm \rightarrow DK^\pm$ categories are then parameterised in terms of a single yield-ratio $\mathcal{R}_{K/\pi}$, and ϵ^c , the corresponding selection efficiency for a given category

$$N_{DK^\pm}^c = \mathcal{R}_{K/\pi} \times N_{D\pi^\pm}^c \times \frac{\epsilon_{DK^\pm}^c}{\epsilon_{D\pi^\pm}^c}. \quad (5.9)$$

The selection efficiency is obtained in simulation, except for the PID efficiencies which are obtained in calibration data as described in Section 5.1.3. The parameter $\mathcal{R}_{K/\pi}$ is shared between all categories, and corresponds to the branching ratio between $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays. Therefore, it can be compared to the known branching ratio [31], which serves as an important cross check of the determination of relative efficiencies.

5.4.2 Cross-feed between $B^\pm \rightarrow Dh^\pm$ channels

There is a cross-feed between the $B^\pm \rightarrow D\pi^\pm$ and $B^\pm \rightarrow DK^\pm$ channels, where real $B^\pm \rightarrow D\pi^\pm$ decays are reconstructed as $B^\pm \rightarrow DK^\pm$ decays, or where $B^\pm \rightarrow DK^\pm$ decays are reconstructed as $B^\pm \rightarrow D\pi^\pm$ decays. Due to relative branching fractions the former contribution is by far the most important, but both are modelled.

The cross-feed shapes are obtained in a data-driven manner using the sPlot method [146], and fixed in the fit to data. Separate shapes are determined for each category, using the following steps:

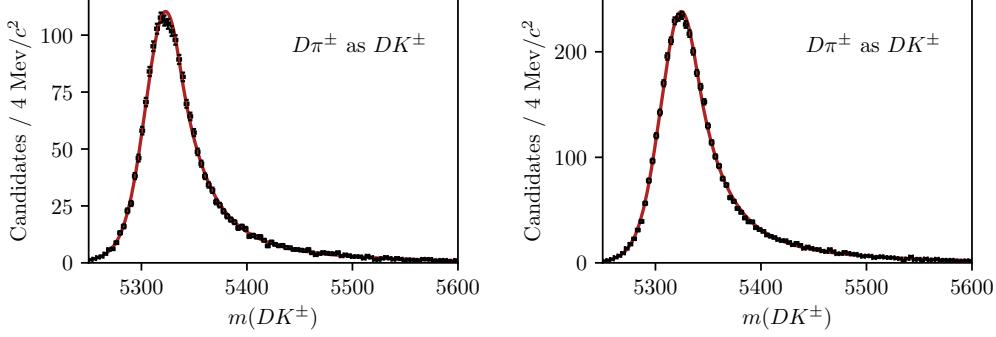


Figure 5.25: Fitted shape of the B^\pm invariant mass spectrum for $B^\pm \rightarrow D\pi^\pm$ decays misidentified as $B^\pm \rightarrow DK^\pm$ decays for (left) LL and (right) DD candidates in the $D \rightarrow K_S^0 \pi^+ \pi^-$ mode.

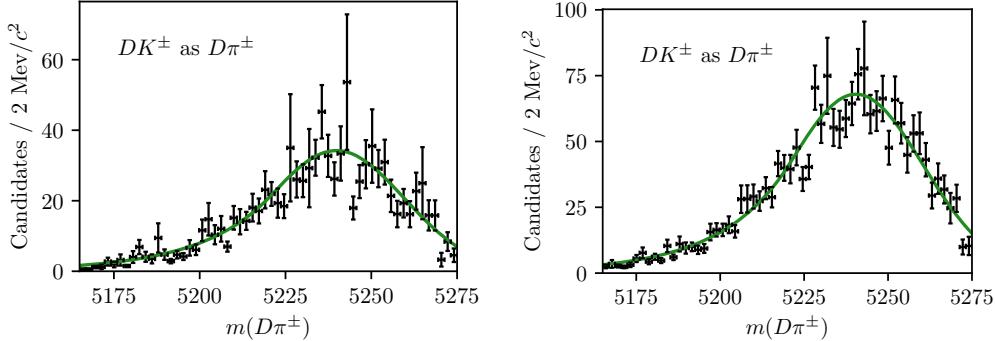


Figure 5.26: Fitted shape of the B^\pm invariant mass spectrum for $B^\pm \rightarrow DK^\pm$ decays misidentified as $B^\pm \rightarrow D\pi^\pm$ decays for (left) LL and (right) DD candidates in the $D \rightarrow K_S^0 \pi^+ \pi^-$ mode.

- The procedure is based on the reasonably pure $B^\pm \rightarrow D\pi^\pm$ sample obtained when the full selection is applied. A simple mass fit is performed to the invariant mass spectrum and the sPlot method [146] is used to obtain the sWeights, w_s , for the signal component. The mass fit uses the same components for signal, low mass shape, and combinatorial background as described in this section.
- A set of weights are defined, based on the candidate-by-candidate PID efficiencies obtained as described in Section 5.1.3:
 - The extracted PID efficiencies of the $\text{PIDK} < 4$ cut $\epsilon_{D\pi \rightarrow D\pi}(p, \eta, n_{\text{tracks}})$ are used to reverse-weight the $B^\pm \rightarrow D\pi^\pm$ sample, in order to obtain the bachelor kinematic distributions before the $\text{PIDK} < 4$ cut is applied.

- 2275 – The extracted PID efficiencies of the $\text{PIDK} > 4$ cut $\epsilon_{D\pi \rightarrow DK}(p, \eta, n_{\text{tracks}})$
 2276 are used to obtain the bachelor kinematic distribution of the $B^\pm \rightarrow D\pi^\pm$
 2277 candidates mis-identified as $B^\pm \rightarrow DK^\pm$.
- 2278 • The raw distribution of the invariant mass of B^\pm particles with a misidentified
 2279 bachelor, $m_B^{\text{mis-ID}}$, is produced by also doing the **DecayTreeFitter** kinematic
 2280 refit while swapping the companion mass hypothesis of each $B^\pm \rightarrow D\pi^\pm$
 2281 candidate to a kaon hypothesis.
- 2282 • Each candidate is reweighted by the overall weight $w = w_s^{\text{cand.}} / \epsilon_{D\pi \rightarrow D\pi}^{\text{cand.}}$.
 2283 $\epsilon_{D\pi \rightarrow DK}^{\text{cand.}}$, and the reweighed $m_B^{\text{mis-ID}}$ distribution is fitted to obtain the cross-
 2284 feed mass distribution function.

2285 The distributions are modelled with a sum of two Crystal Ball density functions,
 2286 each defined by the parameterisation [150]

$$f_{\text{CB}}(m, \mu, \sigma, \alpha, n) \propto \begin{cases} \exp\left[-\frac{1}{2}\left(\frac{m-\mu}{\sigma}\right)^2\right] & \text{if } (m - \mu)/\sigma > -\alpha \\ A\left(B - \frac{m-\mu}{\sigma}\right)^{-n} & \text{otherwise,} \end{cases} \quad (5.10)$$

2287 where $\alpha > 0$, and

$$A = \left(\frac{n}{\alpha}\right)^n \exp[-\alpha^2/2], \quad B = \frac{n}{\alpha} - \alpha. \quad (5.11)$$

2288 The obtained $m_B^{\text{mis-ID}}$ spectrum and obtained mass shape is given in Fig. 5.25
 2289 for the $D \rightarrow K_S^0 \pi^+ \pi^-$ category; the $D \rightarrow K_S^0 K^+ K^-$ shapes are very similar. An
 2290 analogous procedure is used to obtain the mass distribution of $B^\pm \rightarrow DK^\pm$ decays
 2291 reconstructed in the $B^\pm \rightarrow D\pi^\pm$ category. In the first stage where sPlots are
 2292 extracted by a fit to the $B^\pm \rightarrow DK^\pm$ mass spectrum, the cross-feed component
 2293 determined as described above is included. An example of one of the resulting
 2294 shapes is given in Fig. 5.26. The shape obtained in this manner performs better
 2295 than one obtained using simulated decays, because the precision of the momentum
 2296 determination is slightly overestimated in simulation.

2297 The yield of cross-feed from $B^\pm \rightarrow D\pi^\pm$ decays in a given $B^\pm \rightarrow DK^\pm$ category
 2298 is parameterised in terms of the yield of correctly identified $B^\pm \rightarrow D\pi^\pm$ decays and
 2299 the mis-identification probability extracted from calibration samples as described
 2300 in Section 5.1.3. Denoting the rate at which a pion is reconstructed as a kaon
 2301 by $\epsilon_{\pi \rightarrow K}^c$ in a given category, c , the yield is

$$N_{\pi \rightarrow K}^c = N_{D\pi^\pm}^c \frac{\epsilon_{\pi \rightarrow K}^c}{1 - \epsilon_{\pi \rightarrow K}^c}, \quad (5.12)$$

2302 with an analogous definition of the yield of the cross-feed component from
 2303 $B^\pm \rightarrow DK^\pm$ decays in the $B^\pm \rightarrow D\pi^\pm$ spectrum.

2304 5.4.3 Partially reconstructed backgrounds

2305 A number of background candidates stem from partly reconstructed B decays of the
 2306 type $B \rightarrow DhX$, where X denotes a photon or a pion that is not reconstructed. It
 2307 is not possible to reject these decays in the selection, due to the similarity to signal
 2308 decays. The missing momentum results in reconstructed B masses below the actual
 2309 B mass, and therefore the backgrounds are also denoted *lowmass* backgrounds.
 2310 These mass distributions are modelled with analytic shapes, derived based on two
 2311 principles. Firstly, the kinematic endpoints of the distributions are fully defined by
 2312 the particle masses in the decay. Secondly, the angular distribution of the missing
 2313 particle has a one-to-one relation to the missing momentum, and therefore to the
 2314 reconstructed B mass. Depending on the spin-parity of the particles and resonances
 2315 involved in the decay, two different mass distributions arise.

2316 In B decays where the missing particle is a scalar that is produced in the
 2317 decay of a vector resonance (eg. $B^\pm \rightarrow D^{*0}(\rightarrow D^0\pi^0)\pi^\pm$ decays where the π^0 is
 2318 not reconstructed), the $m(D^0\pi^\pm)$ distribution has a double-peak structure. The
 2319 D^{*0} helicity angle θ is defined as the angle between the π^0 momentum vector in
 2320 the D^{*0} rest frame and the D^{*0} boost vector in the B rest frame. The helicity
 2321 of the D^{*0} meson means that the π^0 will travel predominantly in the direction
 2322 where $\theta = 0$ or $\theta = \pi$. When $\theta = 0$ the fraction of momentum carried by the
 2323 missing π^0 is lower, leading to a higher reconstructed $m(D^0\pi^\pm)$. When $\theta = \pi$
 2324 the converse occurs. The resulting B mass distribution is a parabola $f_{\text{HORNS}}^0(m)$
 2325 peaking near both kinematic endpoints a and b

$$f_{\text{HORNS}}^0(m) = \begin{cases} (m - \frac{a+b}{2})^2, & \text{if } a < m < b \\ 0, & \text{otherwise.} \end{cases} \quad (5.13)$$

2326 Due to the double-peaking structure, and the fact that was developed by Paolo
 2327 Gandini for the two-body ADS/GLW analyses [151], this shape is denoted a
 2328 *HORNSdini* shape when convolved with a resolution function as described below.

2329 The second relevant decay situation is where the missing particle is a vector,
 2330 again produced via the intermediate decay of a vector resonance (eg. $B^\pm \rightarrow$
 2331 $D^{*0}(\rightarrow D^0\gamma)\pi^\pm$ decays where the photon is not reconstructed). In this case, the
 2332 spin-parity of the photon (1^-) means that it will decay preferentially in the $\theta = \frac{\pi}{2}$
 2333 or $\theta = \frac{3\pi}{2}$ directions, and so a double-peak structure is not seen. In this case
 2334 the parabolic distribution $f_{\text{HILL}}^0(m)$ with kinematic endpoints a, b has negative
 2335 curvature and can be described by

$$f_{\text{HILL}}^0(m) = \begin{cases} -(m - a)(m - b), & \text{if } a < m < b \\ 0, & \text{otherwise.} \end{cases} \quad (5.14)$$

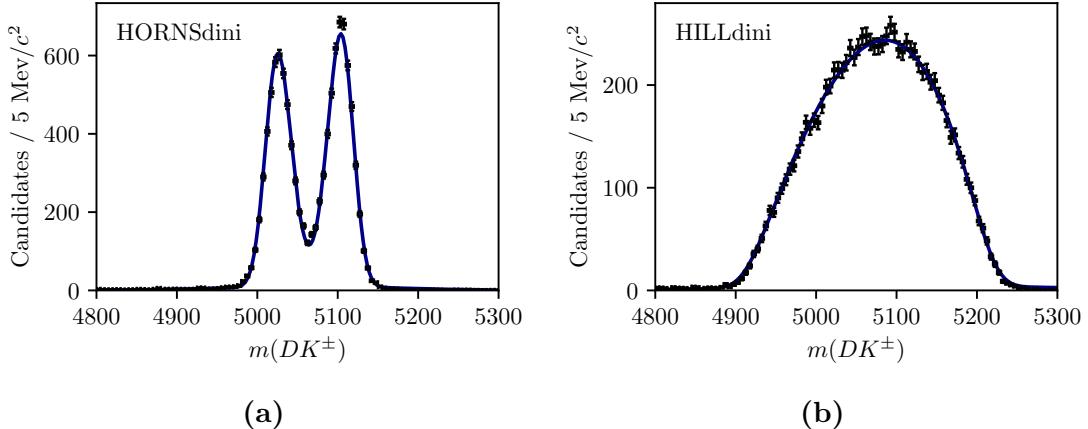


Figure 5.27: Examples of (a) the *HORNSdini* distribution fit to simulated $B^\pm \rightarrow (D^{*0} \rightarrow D^0[\pi^0])K^\pm$ decays, and (b) the *HILLSdini* distribution fit to simulated $B^\pm \rightarrow (D^{*0} \rightarrow D^0[\gamma])K^\pm$ decays. The fits in this figure are made to illustrate the features of each shape, but do not enter the actual fit to data.

This shape is denoted a *HILLdini* shape when convolved with a resolution function. A convolution is applied to take into the non-perfect resolution in the momentum determination. The resolution function is chosen to be a sum of two Gaussians. For a single Gaussian shape $f_G(x|\mu, \sigma)$ with mean μ and width σ , the double Gaussian is expressed as

$$f_{DG}(x) = f_G(x|\mu, \sigma) + k_G f_G(x|\mu, R_\sigma \sigma). \quad (5.15)$$

where σ is the width of the first Gaussian, k_G is the relative fractions between the two Gaussians, and R_σ is their relative widths. Further, selection effects can distort the horns shape such that one of the peaks is higher than the other. This is taken into account by introducing a linear polynomial with slope parameter ξ . As $\xi \rightarrow 0$, the left hand peak decreases in size relative to the right hand peak. The resulting *HORN**Sdini* and *HILL**dini* distributions are therefore

$$f_{\text{HORNS/HILL}}(m) = \int_a^b dx f_{\text{HORNS/HILL}}^0(x) f_{DG}(m|x, \sigma, k_G, R_\sigma) \left(\frac{1-\xi}{b-a}x + \frac{b\xi - a}{b-a} \right). \quad (5.16)$$

²³⁴⁷ Examples of the shapes are given in Fig. 5.27. These shapes are used to fit all
²³⁴⁸ partially reconstructed backgrounds, as described in the following section.

Determination of the partially reconstructed background distributions

In both the $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ categories, components are included to describe contributions from the partially reconstructed decays

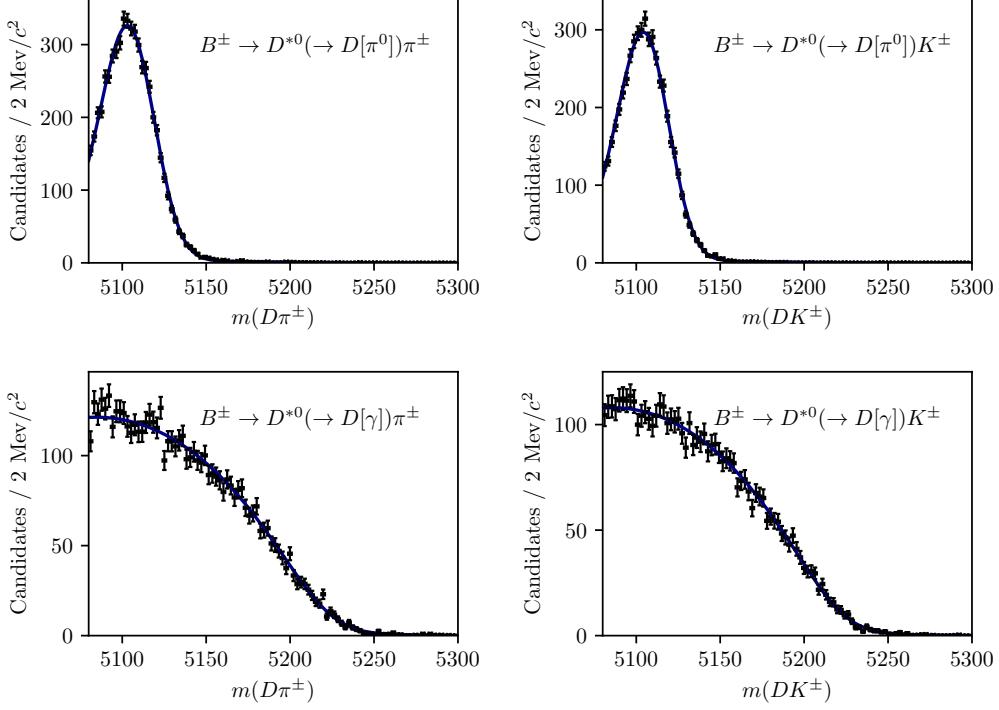


Figure 5.28: Fit projection of the fit to (top) simulated $B^+ \rightarrow D^{*0}(\rightarrow D^0[\pi^0])h^\pm$ decays and (bottom) simulated $B^+ \rightarrow D^{*0}(\rightarrow D^0[\gamma])h^\pm$ decays, all reconstructed in the DD category. Both the (left) DK and (right) $D\pi$ shapes are shown.

- $B^\pm \rightarrow (D^{*0} \rightarrow D^0[\pi^0])h^\pm$, described using a *HORNsdini* distribution,
- $B^\pm \rightarrow (D^{*0} \rightarrow D^0[\gamma])h^\pm$, described using a *HILLdini* distribution
- $B^0 \rightarrow (D^{*\pm} \rightarrow D^0[\pi^\pm])h^\mp$, described using a *HORNsdini* distribution,
- $B^{\pm(0)} \rightarrow D^0 h^\pm [\pi^{0(\mp)}]$, described using a *HORNsdini* distribution,

where the particle in square brackets is not reconstructed. The mass distributions of all the $B \rightarrow D^*h^\pm$ contributions are obtained from fits to samples of full LHCb simulation. Examples of these fits are shown in Fig. 5.28. All shape parameters are kept fixed in the fit to data, except for the parameter σ of the resolution function in Eq. (5.15) which is allowed to obtain the value preferred by data.

The mass distribution of $B^\pm \rightarrow D^0 h^\pm [\pi^0]$ and $B^0 \rightarrow D^0 h^\pm [\pi^\mp]$ decays reconstructed in the $B^\pm \rightarrow D\pi^\pm$ categories is obtained from full LHCb simulation samples of $B^\pm \rightarrow D^0\rho^\pm$ and $B^0 \rightarrow D^0\rho^0$ decays. The shapes were compared to those predicted by an amplitude model for $B^0 \rightarrow D^0\pi^\pm\pi^\mp$ decays developed by LHCb [152], but found to be very similar for the $m(D\pi^\pm)$ range relevant to this analysis. The obtained shapes are shown in Fig. 5.29.

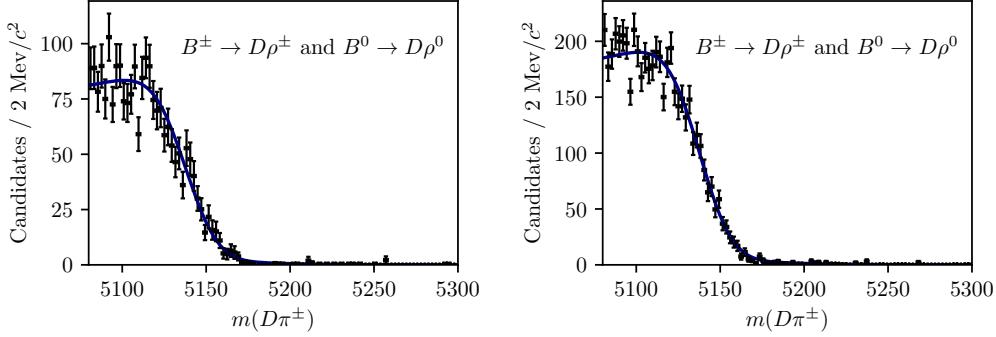


Figure 5.29: Projections of the fit to simulated $B^\pm \rightarrow D\rho^\pm$ and $B^0 \rightarrow D\rho^0$ samples reconstructed as $B^\pm \rightarrow D\pi^\pm$ decays for the (left) LL and (right) DD categories.

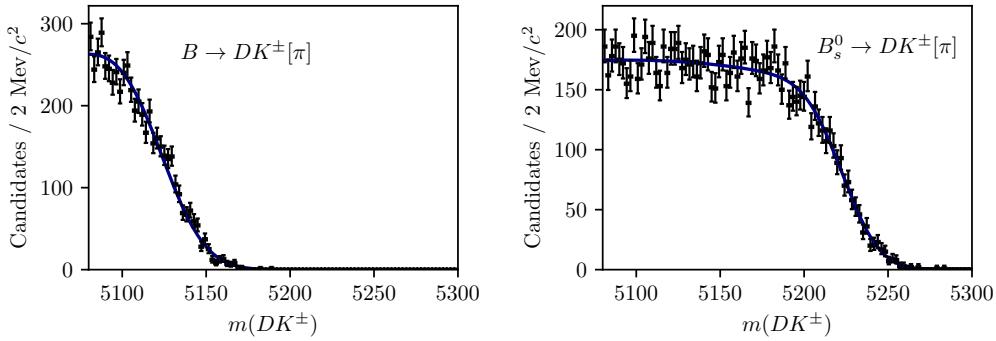


Figure 5.30: Fit projection for the fit used to obtain a shape for the partly reconstructed background from (left) $B \rightarrow DK\pi$ decays and (right) $B_s^0 \rightarrow DKh^-$ decays where a pion is not reconstructed.

The mass distribution of $B^\pm \rightarrow D^0 K^\pm [\pi^0]$ and $B^0 \rightarrow D^0 K^\pm [\pi^-]$ decays reconstructed in the $B^\pm \rightarrow DK^\pm$ categories, on the other hand, is obtained from a sample of signal decays, generated via a an amplitude model for $B^0 \rightarrow D^0 K^+ \pi^-$ decays developed by LHCb [153] and smeared to take the LHCb resolution into account. This follows an approach developed in the context of a GLW analysis based on partially reconstructed decays made within LHCb [154]. The obtained shape is shown in Fig. 5.30.

The background yields of these backgrounds are parameterised in terms of one total yield parameter, accounting for all partially reconstructed B^\pm and B^0 decays, and a number of parameters that describe the relative rates of the different contributions. In the $B^\pm \rightarrow D\pi^\pm$ channels, the relative rates of the $B^\pm \rightarrow (D^{*0} \rightarrow D^0[\pi^0])h^\pm$ and $B^0 \rightarrow (D^{*\pm} \rightarrow D^0[\pi^\pm])h^\mp$ backgrounds are fixed from the known branching fractions, and relative selection efficiencies in simulation. These backgrounds have almost identical mass distributions and it is not possible to determine the ratio in the fit to data. The relative yield of $B^\pm \rightarrow D^*(\rightarrow D^0[\gamma])\pi^\pm$

2382 compared to the $B \rightarrow D^*(\rightarrow D^0[\pi])\pi^\pm$ is denoted $f_{D^*\gamma}^{D\pi}$ and is floated in the fit to
 2383 data, as is the relative yield of $B \rightarrow D^0\pi^\pm[\pi]$ decays compared to the $B \rightarrow D^*\pi$
 2384 modes, denoted $f_{D\pi\pi}^{D\pi}$. In the $B^\pm \rightarrow DK^\pm$ channels, all the relative background
 2385 rates are fixed via known branching fractions and relative selection efficiencies; this
 2386 is necessary to obtain a stable fit, due to the lower yields.

2387 In the $B^\pm \rightarrow DK^\pm$ categories, an additional partially reconstructed background
 2388 is considered from $B_s^0 \rightarrow \bar{D}^0[\pi^+]K^-$ (and conjugate) decays. The mass shape is
 2389 obtained from simulated decays, generated using an amplitude model published by
 2390 LHCb [155] and smeared to account for the experimental resolution. The obtained
 2391 shape is shown in Fig. 5.30. The yield of this background component is fixed relative
 2392 to the signal yields in the corresponding $B^\pm \rightarrow D\pi^\pm$ category, taking the relative
 2393 efficiencies, branching ratios and hadronisation factors into account [31, 156].

2394 In the $B^\pm \rightarrow DK^\pm$ channels there is a contribution from partially reconstructed
 2395 $B \rightarrow D^*\pi^\pm X$ decays where the companion pion is misidentified as a kaon. The
 2396 reverse contribution is negligible due to the relative branching fractions, and the
 2397 fact that the $K \rightarrow \pi$ misidentification shifts most of these background decays
 2398 below the mass range of the fit. These are modelled using analytic, empirical
 2399 mass distributions (essentially sums of a number of regular *HORN*/*HILLdini*
 2400 distributions), with parameters that are determined in fits to simulated $B \rightarrow D^*\pi^\pm$
 2401 and $B \rightarrow D\rho$ decays where the pion is reconstructed with the kaon mass hypothesis.
 2402 The shapes are fixed in the fit to data.

2403

2404 Partially reconstructed backgrounds that are not modelled

2405 It was considered whether a background from $\Lambda_b^0 \rightarrow D^0 p \pi^-$ decays where a pion is
 2406 not reconstructed, and the proton is misidentified as the companion, can be expected
 2407 to contribute significantly. This background has been investigated using full LHCb
 2408 simulation samples for the D final state $K_S^0\pi^+\pi^-$. Taking into account the selection
 2409 efficiencies, branching fractions, and hadronisation fraction of this background, the
 2410 expected relative yield of the Λ_b^0 background compared to signal of 0.03 % in the
 2411 $B^\pm \rightarrow D\pi^\pm$ channel, which is completely negligible. In the $B^\pm \rightarrow DK^\pm$ channel
 2412 the yield relative to signal is about 1.2 %, for total of about 200 decays. However,
 2413 most of these lie at B masses smaller than the signal peak, and their impact is
 2414 small. Therefore it is not necessary to model the background in the nominal fit; a
 2415 systematic uncertainty is assigned that accounts for the small potential impact.

2416 In the analogous case of $\Lambda_b^0 \rightarrow D^0 p K^-$ decays, the missing energy of the non-
 2417 reconstructed kaon results in a reconstructed B mass below the fit range.

2418 It has also been investigated whether a background from $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$ or
 2419 $\Lambda_b^0 \rightarrow \Lambda_c^+ K^-$ decays can be expected, where $\Lambda_c^+ \rightarrow p K_S^0 \pi^+ \pi^-$, a pion is missed
 2420 and the proton is misidentified as a pion or kaon from the D decay. In practice,
 2421 the background is sufficiently suppressed from the applied D mass requirement to
 2422 have no significant impact, and is therefore not modelled. A systematic uncertainty
 2423 is assigned that accounts for any potential impact on the measurement due
 2424 to this choice.

2425 5.4.4 Combinatorial background

2426 The combinatorial background is modelled with an exponentially falling density
 2427 function, where both the yield and exponential slope are determined independently
 2428 for each category. This shape is found to model the combinatorial well in all
 2429 categories, most evident in the high- m_B regions where this background dominates.

2430 5.4.5 Fit results

2431 The fit range is chosen to be $m_B \in [5080, 5800] \text{ MeV}/c^2$. The low end of this
 2432 interval includes the higher mass peak of the double-peak structure in the partially
 2433 reconstructed background, which helps the fit constrain the relative contributions
 2434 of backgrounds in the lowmass region. A number of additional backgrounds exist at
 2435 even lower m_B values, thus extending the fit range to lower masses would necessitates
 2436 an extended model, but not benefit the description of the signal region. The high
 2437 end of the interval includes enough combinatorial background to allow the fit to
 2438 determine the exponential slope parameter accurately.

2439 A large number of pseudoexperiments are carried out to verify that the fit
 2440 procedure is self-consistent, in which toy data sets are generated according to
 2441 the expected B mass distributions, and then fitted. None of the parameters
 2442 obtained in the fit exhibit a mean bias different from zero. For most parameters
 2443 the uncertainties are well estimated. This is the case for the signal yields, and
 2444 the DK^\pm - $D\pi^\pm$ yield ratio \mathcal{R} , as evidenced by the pull plots in Fig. 5.31. The
 2445 fit underestimates the uncertainty by 10-20 % for some of the parameters related
 2446 to the partly reconstructed backgrounds, as shown in Fig. 5.32, but this is taken
 2447 into account when the uncertainties are propagated to the observables in the
 2448 second-stage fit, as described in Section 5.6.3.

2449 The projections of the fit to data are shown in Figs. 5.33 and 5.34, for the
 2450 $D \rightarrow K_S^0 \pi^+ \pi^-$ and $D \rightarrow K_S^0 K^+ K^-$ data sets, respectively. The obtained yields for
 2451 each fit component are given in Table 5.6. The total yield of $B^\pm \rightarrow D\pi^\pm$ decays is

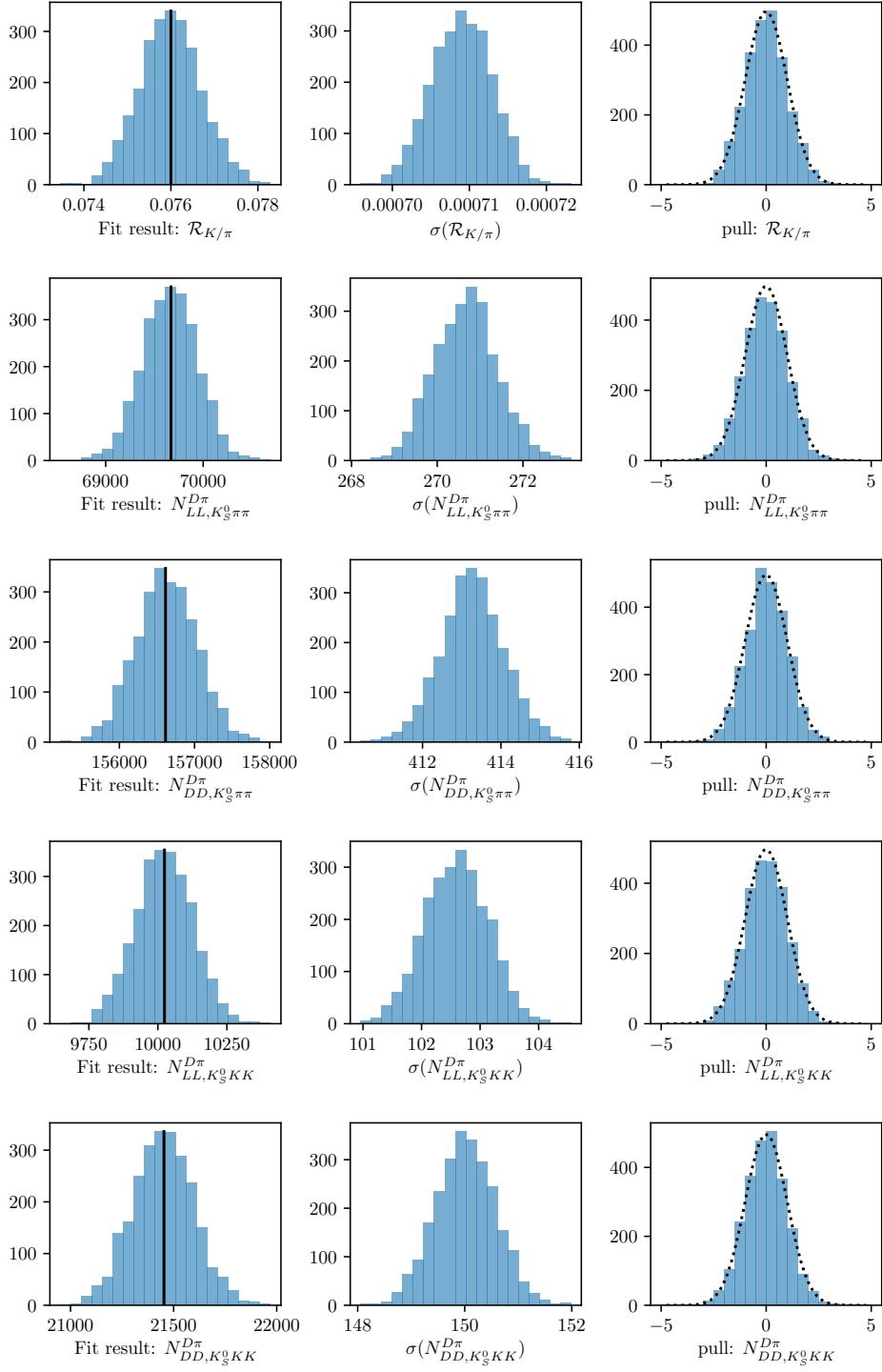


Figure 5.31: The (left) fitted value, (centre) estimated statistical uncertainty, and (right) pull plots for the signal yield parameters, as obtained in a number of pseudo experiments. The black line on the left shows the value used to generate the pseudo data sets; the dotted line on the right shows a Gaussian distribution with mean equal to zero and a standard deviation equal to unity.

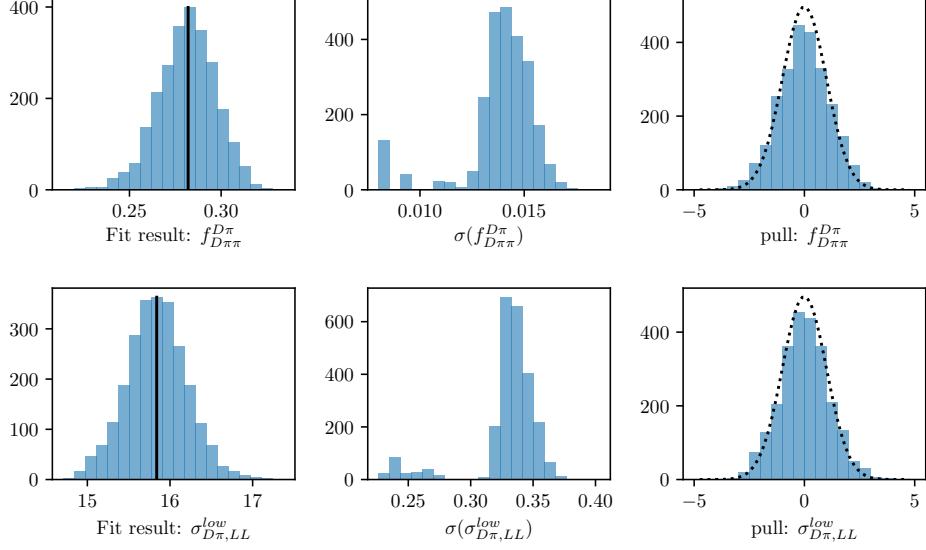


Figure 5.32: The (left) fitted value, (centre) estimated statistical uncertainty, and (right) pulls obtained in a number of pseudo experiments for two examples of parameters relating to the partially reconstructed backgrounds, where the uncertainties are slightly underestimated on average. The standard deviation of the pull distributions is approximately 1.15 in both cases.

approximately 230,000 across all channels. The obtained value of the yield ratio is $\mathcal{R}_{K/\pi} = (7.7 \pm 0.1)\%$, corresponding to a total $B^\pm \rightarrow DK^\pm$ yield of 16,500, of which about 14,300 pass the PID requirement and are reconstructed in the $B^\pm \rightarrow DK^\pm$ category. This value of \mathcal{R} is in excellent agreement with expectation from the known branching fractions [31], which predict $\mathcal{R}_{K/\pi}^{\text{PDG}} = (7.8 \pm 0.3)\%$.² The shape parameters determined in the fit to data are summarised in Table 5.7.

²While it would seem this measurement thus determines the yield ratio $\mathcal{R}_{K/\pi}$ with a much better precision than the current world average uncertainty, that is because the result quoted here does not include any systematic uncertainties; it is only included to serve as a, successfully passed, cross check.

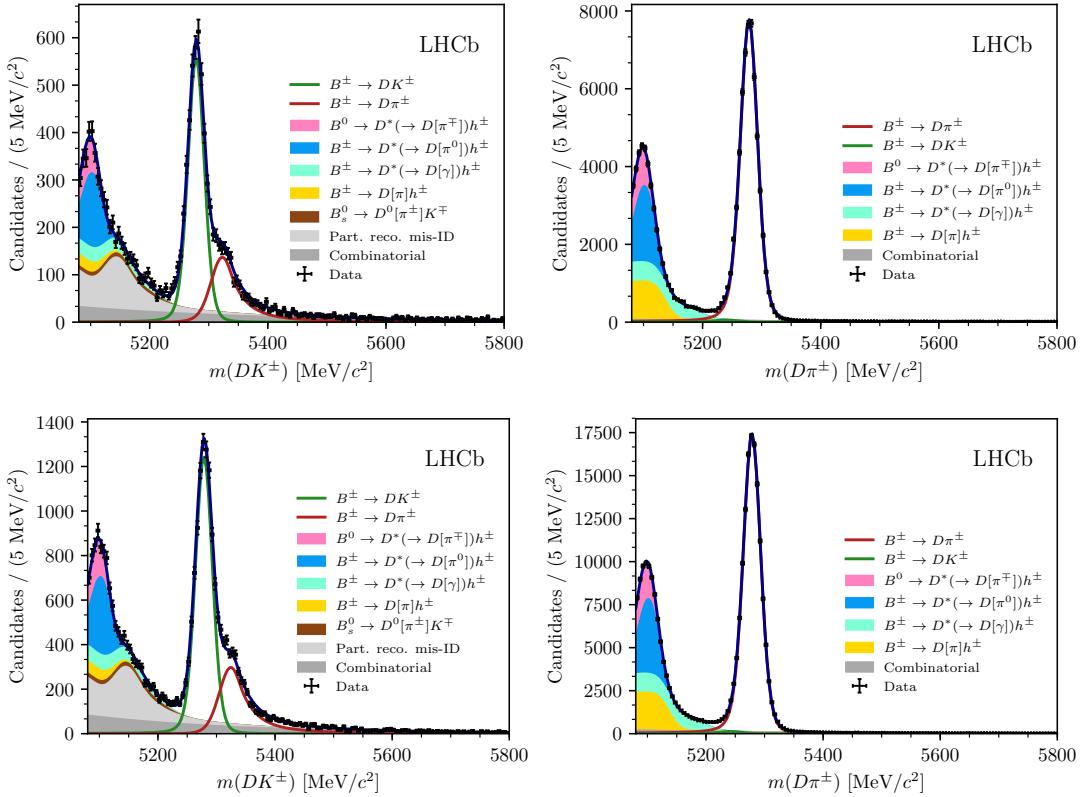


Figure 5.33: The invariant mass distribution for the (left) $B^\pm \rightarrow DK^\pm$ channel and (right) $B^\pm \rightarrow D\pi^\pm$ channel, where $D \rightarrow K_S^0\pi^+\pi^-$ and the K_S^0 is in the (top) LL and (bottom) the DD categories. The particle within square brackets in the legend denotes the particle that has not been reconstructed.

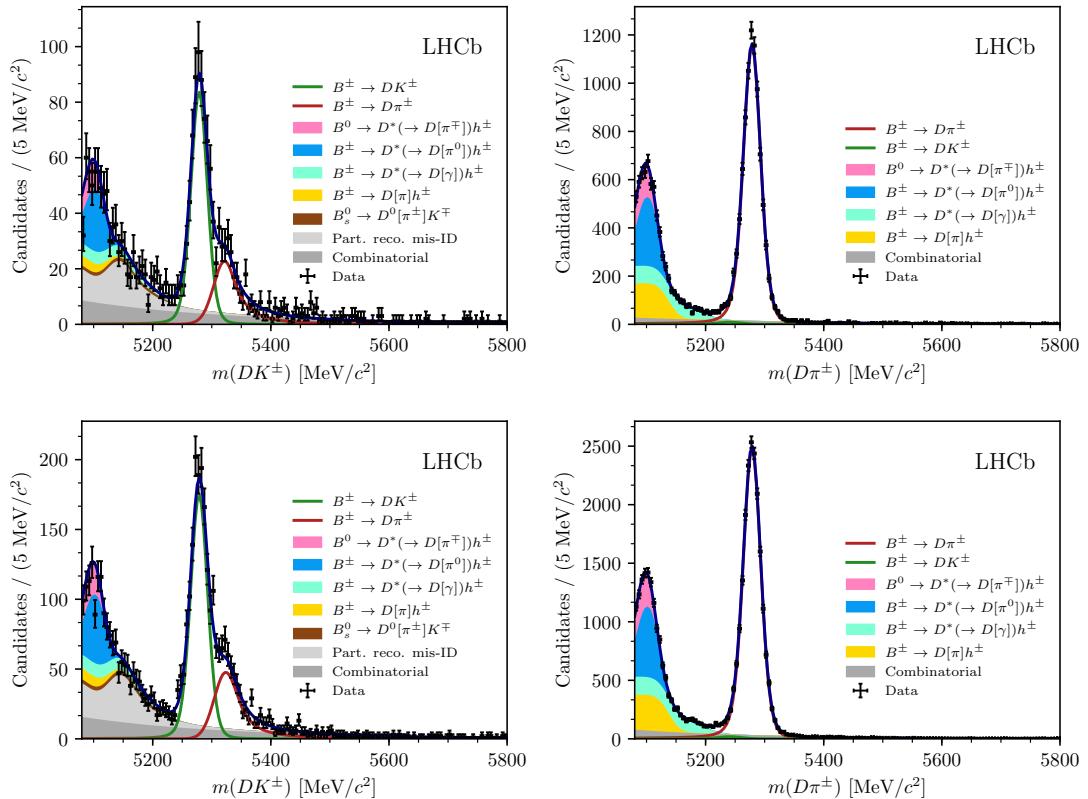


Figure 5.34: The invariant mass distribution for the (left) $B^\pm \rightarrow DK^\pm$ channel and (right) $B^\pm \rightarrow D\pi^\pm$ channel, where $D \rightarrow K_S^0 K^+ K^-$ and the K_S^0 is in the (top) LL and (bottom) the DD categories. The particle within square brackets in the legend denotes the particle that has not been reconstructed.

Table 5.6: Fitted total candidate yields. The quoted signal yields are for the number of candidates that survive the respective PID cut, whereas the DK^\pm - $D\pi^\pm$ ratio is corrected for PID and selection efficiencies so that it corresponds to the branching ratio.

Component	LL	DD
Signal		
$B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)\pi^\pm$	$61,573 \pm 254$	$139,080 \pm 389$
$B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)\pi^\pm$	$9,160 \pm 98$	$19,910 \pm 144$
$R_{K/\pi} = n(DK)/n(D\pi)$ (%)		7.72 ± 0.08
Combinatorial background		
$B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)\pi^\pm$	$3,479 \pm 198$	$9,928 \pm 376$
$B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)\pi^\pm$	$1,103 \pm 94$	$2,545 \pm 155$
$B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)K^\pm$	$1,826 \pm 107$	$3,987 \pm 177$
$B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)K^\pm$	380 ± 39	655 ± 58
Partially reconstructed background		
$B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)\pi^\pm$	$43,004 \pm 242$	$95,452 \pm 403$
$B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)\pi^\pm$	$6,247 \pm 99$	$13,241 \pm 157$
$R_{K/\pi}^{low} = n_{low}(DK)/n_{low}(D\pi)$ (%)		6.65 ± 0.12

Table 5.7: Fitted parameter values.

	LL	DD
$\sigma_{D\pi}$ (MeV/ c^2)	14.27 ± 0.05	14.58 ± 0.04
σ_{DK} (MeV/ c^2)	13.61 ± 0.24	14.19 ± 0.17
μ (MeV/ c^2)		5278.60 ± 0.04
Combinatorial Slopes		
Decay mode	Slope ($10 \times 10^{-3} GeV^{-1}c^2$)	
$B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)\pi^\pm$	-3.1 ± 0.2	-4.0 ± 0.1
$B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)\pi^\pm$	-4.1 ± 0.4	-5.5 ± 0.3
$B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)K^\pm$	-3.2 ± 0.2	-3.9 ± 0.2
$B^\pm \rightarrow D(\rightarrow K_S^0K^+K^-)K^\pm$	-4.2 ± 0.4	-4.3 ± 0.4
Part. Reco.		
$\sigma_{D\pi}^{low}$ (MeV/ c^2)	13.73 ± 0.33	13.78 ± 0.28
$f_{D\pi\pi}^{D\pi}$		0.268 ± 0.013
$f_{D^*\gamma}^{D\pi}$		0.317 ± 0.005

2458 5.5 Measurement of the CP-violation observables

2459 The section describes the second fit stage, in which the *CP*-violation observables
 2460 of interest are determined. Compared to the first fit stage, the candidates are
 2461 further split by *B* charge, and by the assigned Dalitz bin number, making for a
 2462 total of 160 subcategories.³ Another extended maximum-likelihood fit is carried
 2463 out, in which shape parameters of all signal and background components are fixed
 2464 to those determined in the first fit stage, and all floating parameters relate to
 2465 the signal and background yields. The signal yields are expressed in terms of the
 2466 observables of interest, $(x_\pm^{DK}, y_\pm^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi})$, allowing the fit to determine their
 2467 optimal values. The details of the fit setup are summarised in the following section,
 2468 along with a number of studies that lead to the specific setup being chosen. The
 2469 results are presented in Section 5.5.2, and a wide range of consistency checks
 2470 are described in Section 5.5.3.

2471 5.5.1 Fit setup

2472 The basic principle of the measurement is that the signal yields in each bin
 2473 (in a given category) are defined using the equations of Chapter 2, in order to
 2474 allow for the determination of the *CP*-violation observables. In practice, a set
 2475 of variables are defined

$$Y_{c,i}^- = F_{c,-i} + [(x_-^c)^2 + (y_-^c)^2]F_{c,-i} + 2\sqrt{F_{c,i}F_{c,-i}}(c_i^cx_-^c + s_i^cy_-^c), \quad (5.17)$$

$$Y_{c,i}^+ = F_{c,-i} + [(x_+^c)^2 + (y_+^c)^2]F_{c,-i} + 2\sqrt{F_{c,i}F_{c,-i}}(c_i^cx_+^c - s_i^cy_+^c), \quad (5.18)$$

2476 for each data category, *c*, in terms of which the bin yields that enter the like-
 2477 lihood are given by

$$N_{c,i}^\pm = \frac{Y_{c,i}^\pm}{\sum_j Y_{c,j}^\pm} \times N_{c,\text{total}}^\pm. \quad (5.19)$$

2478 This parameterisation is essentially identical to the expressions in Section 2.4,
 2479 slightly modified so that the phase-space-integrated yields of B^+ and B^- decays
 2480 in a given category are determined directly, in lieu of the normalisation constants
 2481 h^\pm of that section. As discussed briefly in Section 2.4, there are choices to be
 2482 made in terms of how the *x* and *y* are parameterised in the $B^\pm \rightarrow D\pi^\pm$ channel,
 2483 and how the *F*_{*i*} parameters are determined. A series of feasibility studies were
 2484 carried out to determine the optimal setup; these are presented in the following
 2485 section, before the final fit setup is described in detail.

³In the thesis, the word *category* is used for the 8-way split of data by companion species, K_S^0 track type, and *D*-decay mode, indexed with a *c*; the word *bin* denotes the 16 (4) regions of the $D \rightarrow K_S^0\pi^+\pi^-$ ($D \rightarrow K_S^0K^+K^-$) Dalitz plots, indexed with an *i*; the simultaneous grouping by *category*, *bin*, and *B* charge is denoted a *subcategory*, of which there are $4 \times 2 \times (16 + 4) = 160$.

2486 **Feasibility of alternative fit setups**

2487 The motivation for promoting the $B^\pm \rightarrow D\pi^\pm$ channel to a signal channel is two-fold:
2488 one aim is to extract the information on γ from the $B^\pm \rightarrow D\pi^\pm$ data, even the
2489 precision gain is limited, and another is to be able to the F_i parameters directly
2490 from the $B^\pm \rightarrow Dh^\pm$ channels, to avoid the need for a control channel and a
2491 simulation-reliant efficiency correction. Two different sets of observables can be
2492 defined to describe the CP -violation effects in the $B^\pm \rightarrow D\pi^\pm$ channel:

- 2493 • one option, defined the 8-parameters setup below, is to define a new set of four
2494 Cartesian for the $B^\pm \rightarrow D\pi^\pm$ mode, $(x_-^{D\pi}, y_-^{D\pi}, x_+^{D\pi}, y_+^{D\pi})$, defined analogously
2495 to the $B^\pm \rightarrow DK^\pm$ observables

$$x_\pm^{D\pi} = r_B^{D\pi} \cos(\delta_B^{D\pi} \pm \gamma), \quad y_\pm^{D\pi} = r_B^{D\pi} \sin(\delta_B^{D\pi} \pm \gamma), \quad (5.20)$$

- 2496 • another, proposed in Refs. [83, 157], is to introduce the parameter

$$\xi_{D\pi^\pm} = \left(\frac{r_B^{D\pi^\pm}}{r_B^{DK^\pm}} \right) \exp[i(\delta_B^{D\pi^\pm} - \delta_B^{DK^\pm})] \quad (5.21a)$$

2497 and then determine the observables

$$x_\xi^{D\pi} = \text{Re}[\xi_{D\pi^\pm}] \quad y_\xi^{D\pi} = \text{Im}[\xi_{D\pi^\pm}]. \quad (5.21b)$$

2498 This is denoted the 6-parameters setup below. In terms of $x_\xi^{D\pi}$ and $y_\xi^{D\pi}$, the
2499 usual Cartesian x_\pm and y_\pm are given by

$$x_\pm^{D\pi} = x_\xi^{D\pi} x_\pm^{DK} - y_\xi^{D\pi} y_\pm^{DK}, \quad y_\pm^{D\pi} = x_\xi^{D\pi} y_\pm^{DK} + y_\xi^{D\pi} x_\pm^{DK}. \quad (5.22)$$

2500 The former parameterisation has the benefit that information on γ from the
2501 $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ channels is encoded in separate sets of observables,
2502 whereas the latter parameterisation encodes information on CP violation from both
2503 channel in the (x_\pm^{DK}, y_\pm^{DK}) parameters. In combinations of many measurements, it
2504 is a useful cross check to be able to compare constraints obtained from individual
2505 decay modes; a good example is the LHCb combination from 2016 [49] where both
2506 $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow Dh^\pm$ combinations are made and compared in detail. This
2507 is only possible with the former parameterisation. On the other hand, the latter
2508 parameterisation avoids the introduction of two non-physical degrees of freedom,
2509 which, as seen below, leads to better statistical behaviour.

2510 In order to inform the choice of parameterisation, a series of pseudo experiments
2511 has been carried out to compare the obtainable precision on γ (these studies were

performed, and discussed within LHCb, prior to the publication of Ref. [157]; thus, the results presented here constitute independent work, even if there is some overlap in scope and conclusions with that reference). Many simulated data sets were generated, constituting of a number signal yields approximately equal to the expected yields in the full Run 1 and 2 LHCb data set: approximately 15,000 $B^\pm \rightarrow DK^\pm$ decays and 210,000 $B^\pm \rightarrow D\pi^\pm$ decays.⁴ The signal decays were distributed between Dalitz bins according to $(\gamma, r_B^{DK}, \delta_B^{DK}) = (75^\circ, 0.1, 130^\circ)$ in the $B^\pm \rightarrow DK^\pm$ mode, which is to the world average values of direct γ measurements at the time. In the $B^\pm \rightarrow D\pi^\pm$ mode, the behaviour is investigated for different sets of input values; of most importance is the case $(r_B^{D\pi}, \delta_B^{D\pi}) = (0.005, 300^\circ)$, because it corresponds to the solution in the LHCb combination [49] that is in agreement with the theoretical expectation $r_B^{D\pi} \simeq 0.005$ [51]. The behaviour at larger $r_B^{D\pi}$ values is also investigated. For each generated data set

1. the observables are measured in a fit to the data set, using both the 6-parameter and 8-parameter setups
2. the obtained observables are then fitted to obtain the underlying physics parameters $(\gamma, r_B^{DK}, \delta_B^{DK}, r_B^{D\pi}, \delta_B^{D\pi})$ using a maximum-likelihood fit, essentially following the procedure outlined in Section 5.7.1.

In the 8-parameter setup it is possible to determine γ using the results in either the $B^\pm \rightarrow DK^\pm$ or $B^\pm \rightarrow D\pi^\pm$ channels separately, or consider the combined results; in the 6-parameter setup only the latter option is available. The studies are performed in two modes: with the F_i floating in the fit, emulating a realistic fit to data, as well as with the F_i fixed to the input values used in data generation. The latter studies emulate a setup where the F_i parameters are determined in an ultra-high statistics control channel, and perfect efficiency corrections are applied. In all cases, a single set of F_i parameters is shared between the $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ modes.

The 6-parameter setup shows significantly better statistical performance than the 8-parameter setup in the realistic case where the F_i parameters are determined in the fit and $r_B^{D\pi} \sim 0.005$.⁵ The fits that employ the 6-parameter setup behave well in this case, whereas the additional degrees of freedom in the 8-parameter fit leads to essentially all parameters being 100% (anti-)correlated, and a significant number of fits not converging. For the fits that do converge, the uncertainties

⁴No backgrounds were included in these studies, and thus the quoted uncertainties on γ are better than what is obtainable in the final measurement; a similar study including realistic backgrounds is presented for the final setup below.

⁵For larger, non-physical values of $r_B^{D\pi} > 0.03$ both fit setups behave well.

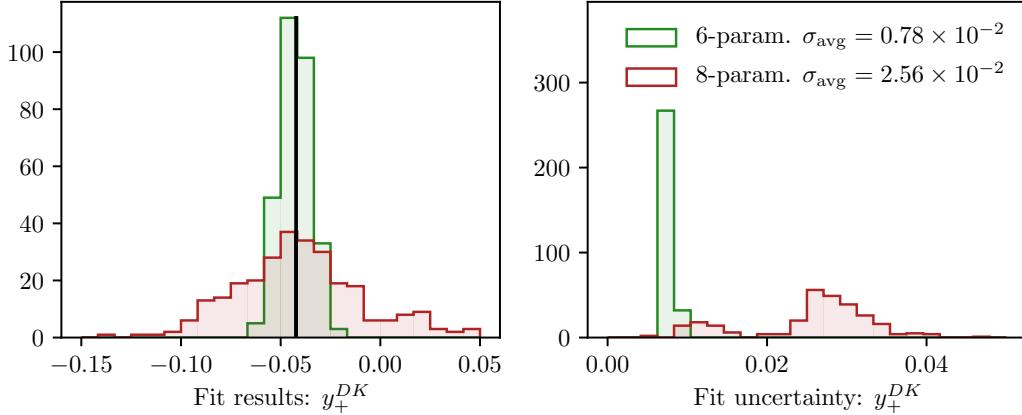


Figure 5.35: The distribution of (left) fit values and (right) statistical uncertainty estimate for y_+^{DK} in a series of pseudo experiments, for both the (green) 6-parameter and (red) 8-parameter setups.

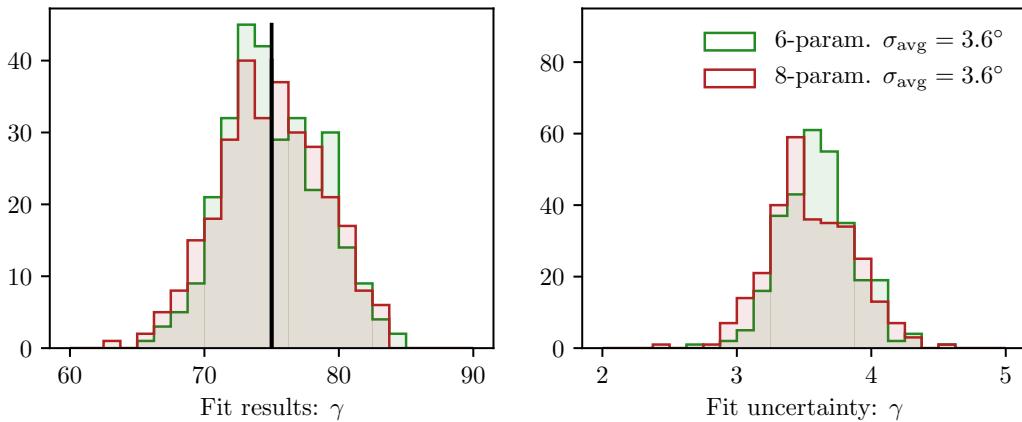


Figure 5.36: The distribution of (left) fit values and (right) statistical uncertainty estimate for γ in a series of pseudo experiments, for both the (green) 6-parameter and (red) 8-parameter setups.

on the observables are significantly larger due to the large correlations, as shown exemplified with the case of y_+^{DK} in Fig. 5.35. This essentially determines the choice of parameterisation: it is possible to reliably model CP violation in the $B^\pm \rightarrow D\pi^\pm$ channel and simultaneously determine the F_i parameters by using the 6-parameter setup, but not by using the 8-parameter setup.

Interestingly, when the constraints on γ are compared, both setup lead to similar precision; in spite of the large uncertainties on the individual observables in the 8-parameter setups, the constraints on γ are tight. This is illustrated in Fig. 5.36. Nevertheless, it remains true that the 8-parameter setup is ruled out due the statistical behaviour in the determination of the observables.

Furthermore, both the 6- and 8-parameter setups lead to fits that behave well in the studies where the F_i parameters are kept fixed, and the resulting uncertainties on the Cartesian observables and γ are essentially identical. Thus, the 6-parameter setup does not inherently lead to a gain in precision over the 8-parameter setup; the strength of the parameterisation is that it allows for a reliable, simultaneous determination of the F_i parameters and the observables of interest. This conclusion differs somewhat from the one drawn in Ref. [157].

The fixed- F_i studies allow for an assessment of the gain in precision on γ due to the inclusion of the $B^\pm \rightarrow D\pi^\pm$ mode, by comparing the precision obtained in the simultaneous fits with that obtained when γ is constrained using only information from the $B^\pm \rightarrow DK^\pm$ channel. In the realistic case where $r_B^{D\pi} = 0.005$, the gain in precision is about 0.1° . The reason for the small impact, in spite of the yield being approximately 14 times larger in the $B^\pm \rightarrow D\pi^\pm$ channel than in the $B^\pm \rightarrow DK^\pm$ channel, is that r_B is 20 times smaller, and the CP asymmetries are proportional to r_B . Thus, the main improvement to the analysis from including $B^\pm \rightarrow D\pi^\pm$ as a signal channel comes from the ability to determine the F_i parameters without adding a large systematic uncertainty.⁶

Finally, it is worth considering whether any precision can be gained by including information on the F_i parameters from a control channel, even if the fit is well behaved without external information. The potential yield in the $\bar{B}^0 \rightarrow D^{*+}(\rightarrow D^0\pi^+)\mu^-\bar{\nu}_\mu X$ control channel is approximately three times larger than in the $B^\pm \rightarrow D\pi^\pm$ channel, and it does therefore offer a better statistical handle on the F_i values (at the significant cost of having to worry about efficiency corrections). This question can be answered by comparing the obtained precision on γ in the fits where F_i parameters were floating, to the precision in the case where they were kept fixed. Such a comparison is shown for the 6-parameter setup in Fig. 5.37 for the realistic scenario where $r_B^{D\pi} = 0.005$. The difference in the average $\sigma(\gamma)$ is *less than* 0.05° , which is of course completely negligible. Therefore, no gain in precision can be obtained by including the control channel in the analysis, and it is not considered further.

⁶If this comparison is made using the parameter set $(r_B^{D\pi}, \delta_B^{D\pi}) = (0.03, 330^\circ)$, which corresponds to the alternative, non-physical solution in the LHCb combination [49], the gain in precision is 1.3° instead; this fact made the statistical interpretation of the $B^\pm \rightarrow Dh^\pm$ combination in Ref. [49] non-trivial.

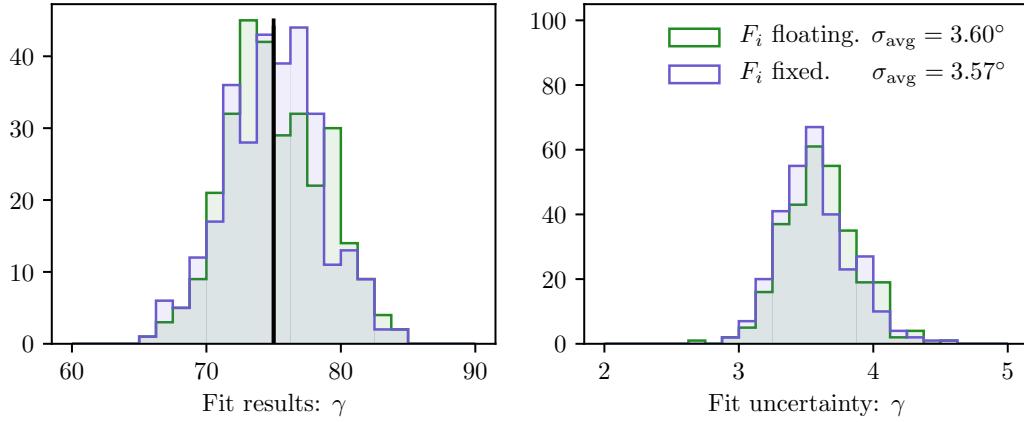


Figure 5.37: The distribution of (left) central values and (right) statistical uncertainty estimate for γ in a series of pseudo experiments that use the 6-parameter setup, where (green) the F_i parameters are determined in the fit and (blue) where they are kept fixed at their input values.

2584 Final choice of observables and the determination of the F_i parameters

2585 In the chosen setup, a single set of four parameters, $(x_-^{DK}, y_-^{DK}, x_+^{DK}, y_+^{DK})$, are
 2586 shared between *all* $B^\pm \rightarrow DK^\pm$ categories; they enter the expressions of Eq. (5.17)
 2587 directly, and are thus determined in the fit. In the $B^\pm \rightarrow D\pi^\pm$ categories, the
 2588 four corresponding parameters, $(x_-^{D\pi}, y_-^{D\pi}, x_+^{D\pi}, y_+^{D\pi})$, are parameterised in terms
 2589 of (x_\pm^{DK}, y_\pm^{DK}) and the additional two observables $(x_\xi^{D\pi}, y_\xi^{D\pi})$. The F_i parameters
 2590 are determined in the fit, being shared between the $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$
 2591 channels. However, separate parameter sets are determined for the LL and DD
 2592 categories because the acceptance profile over the Dalitz plot differs between them.

2593 The F_i parameters are subject to the constraint that $\sum_{i=-N}^N F_i^c = 1$, for each
 2594 category, c . Therefore, it is beneficial to introduce a reparameterisation in the
 2595 likelihood function, where the F_i parameters are expressed in terms of a set of
 2596 recursive fractions

$$\mathcal{R}_i = \begin{cases} F_i & , \quad i = -N \\ F_i / (\sum_{j \geq i} F_j) & , \quad -N < i < +N \end{cases} \quad (5.23)$$

2597 for which the constraint is much simpler, namely that each individual \mathcal{R}_i parameter
 2598 lies in the interval $[0, 1]$. This parameterisation leads to well behaved fits, where
 2599 the \mathcal{R}_i parameters do not suffer from significant correlations.

2600 Strong-phase inputs

2601 The strong-phase parameters (c_i, s_i) are fixed in the fit to data. In the $D \rightarrow$
 2602 $K_S^0 \pi^+ \pi^-$ channels, the combined CLEO [73] and BESIII [74] measurement results

2603 are used, as reported in Ref. [74]. The $D \rightarrow K_S^0 K^+ K^-$ categories also use combined
 2604 CLEO [74] and BESIII results [78], which are reported in Ref. [78]. The experimental
 2605 uncertainty on these measurements is propagated to the measured CP -violation
 2606 observables as part of the systematic uncertainties in Section 5.6.1.

2607 Treatment of backgrounds

2608 The yield of combinatorial background decays is determined independently in each
 2609 bin. A single, overall bin yield of partially reconstructed background from B^\pm and
 2610 B^0 decays is determined in each of the 160 subcategories; the relative contribution
 2611 from each individual background is fixed from the results of the first-stage fit,
 2612 corrected for the different fit region (a systematic uncertainty is assigned due to this
 2613 choice). In the $B^\pm \rightarrow DK^\pm$ channels, the bin yields of the partially reconstructed
 2614 background from $B_s^0 \rightarrow \bar{D}^0[\pi^+]K^-$ decays are expressed via the F_i , exploiting that
 2615 a positive companion particle is always produced along with a \bar{D}^0 meson (and vice
 2616 versa). The overall yield is fixed from the results of the first stage fit. Finally,
 2617 the yield of the $D\pi^\pm \leftrightarrow DK^\pm$ cross-feed components in each bin are determined
 2618 via the obtained yield of correctly identified decays in the corresponding bin, and
 2619 the known PID efficiencies. This is true for both fully and partially reconstructed
 2620 decays, although only a $D\pi^\pm \rightarrow DK^\pm$ component is included in the latter case.

2621 The choice of fit range

2622 The fit range is decreased to $m_B \in [5150, 5800] \text{ MeV}/c^2$. The information from
 2623 candidates with lower reconstructed B masses was useful in determining the relative
 2624 rates and free mass shape parameters of the partially reconstructed background
 2625 components in the first-stage fit; however, with these fixed in the second-stage
 2626 fit, this is no longer the case. Furthermore, the setup assumes that the shape of
 2627 the partially reconstructed background is identical across the Dalitz bins. This
 2628 assumption is not perfectly true, but the impact is minimal when the lower limit of
 2629 the fit range is taken to be $5150 \text{ MeV}/c^2$, as described further in Section 5.6.3.

2630 Self-consistency check

2631 In order to establish the fit stability and investigate a potential bias, a series of
 2632 pseudo experiments are run, in which data sets are generated using the model, and
 2633 then fitted back. The total yields are taken from the first-stage fit. The signal
 2634 yields are distributed between Dalitz bins using input physics parameters that
 2635 approximately equal the values obtained in Section 5.5.2 from the results of the fit

Table 5.8: Mean biases and pulls for the observables of interest in the final, binned fit, obtained in a large number of pseudoexperiments.

Parameter	Mean bias ($\times 10^{-2}$)	Mean pull	Pull width
x_-^{DK}	-0.018 ± 0.022	-0.01 ± 0.02	1.01 ± 0.02
y_-^{DK}	-0.014 ± 0.026	-0.00 ± 0.02	0.99 ± 0.02
x_+^{DK}	-0.018 ± 0.022	-0.01 ± 0.02	1.00 ± 0.02
y_+^{DK}	-0.016 ± 0.028	0.01 ± 0.02	1.00 ± 0.02
$x_\xi^{D\pi}$	0.029 ± 0.052	0.06 ± 0.02	1.00 ± 0.02
$y_\xi^{D\pi}$	0.000 ± 0.060	0.01 ± 0.02	1.00 ± 0.02

to data. The F_i parameters are taken from a fit to data. The partly reconstructed background is distributed as " D^0 -like", ie. in the B^\pm channels $N_i^\pm \propto F_{\mp i}$, except for the B_s^0 background, which is " \bar{D}^0 -like" ($N_i^\pm \propto F_{\pm i}$). The combinatorial background includes real D mesons paired with a random bachelor, as well as fake D mesons that are themselves made up of random tracks. The former is distributed as 50/50 D^0 -like and \bar{D}^0 -like in the toy generation, whereas the latter is assumed to be evenly distributed over the Dalitz plot (ie. the bin yield is proportional to the bin area).

A set of 2000 pseudo experiments has been run, out of which 98.8 % converged properly. The pull plots for the observables of interest are shown in Figs. 5.38 and 5.39; the mean biases and pulls are summarised in Table 5.8. No biases are statistically significant, and the uncertainties are seen to be well estimated.

5.5.2 Main results

The values and statistical uncertainties of observables obtained in the fit are

$$\begin{aligned} x_-^{DK} &= (-5.68 \pm 0.96) \times 10^{-2}, & y_-^{DK} &= (-6.55 \pm 1.14) \times 10^{-2}, \\ x_+^{DK} &= (-9.30 \pm 0.98) \times 10^{-2}, & y_+^{DK} &= (-1.25 \pm 1.23) \times 10^{-2}, \\ x_\xi^{D\pi} &= (-5.47 \pm 1.99) \times 10^{-2}, & y_\xi^{D\pi} &= (0.71 \pm 2.33) \times 10^{-2}. \end{aligned} \quad (5.24)$$

The statistical correlation matrix for the observables is given in Table 5.9. None of the correlations are larger than 15 % and the values of both uncertainties and correlation coefficients are similar to those obtained in the feasibility studies. The 2D log-likelihood profile for the observables is shown in Fig. 5.40, based on a full likelihood scan, where the fit is repeated with the observables fixed to a range of values around the optimal solution. It can be seen that the likelihood profile obtained in the scan is very well modelled by the Gaussian approximation, based on the Hessian matrix at maximum likelihood.

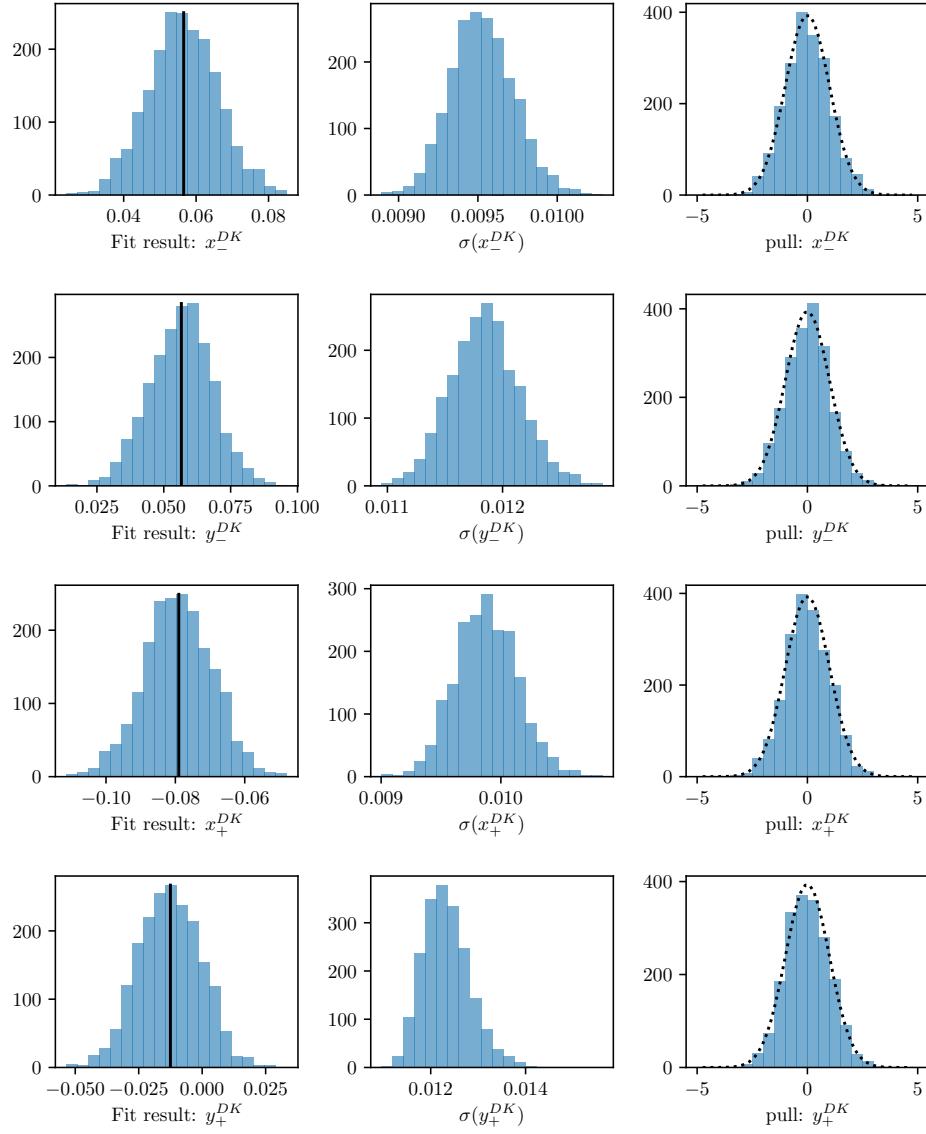


Figure 5.38: The (left) fitted value, (centre) estimated statistical uncertainty, and (right) pulls for the $B^\pm \rightarrow DK^\pm$ observables, as obtained in a number of pseudo experiments. The black line on the left shows the value used to generate the pseudo data sets; the dotted line on the right shows a Gaussian distribution with mean equal to zero and a standard deviation equal to unity.

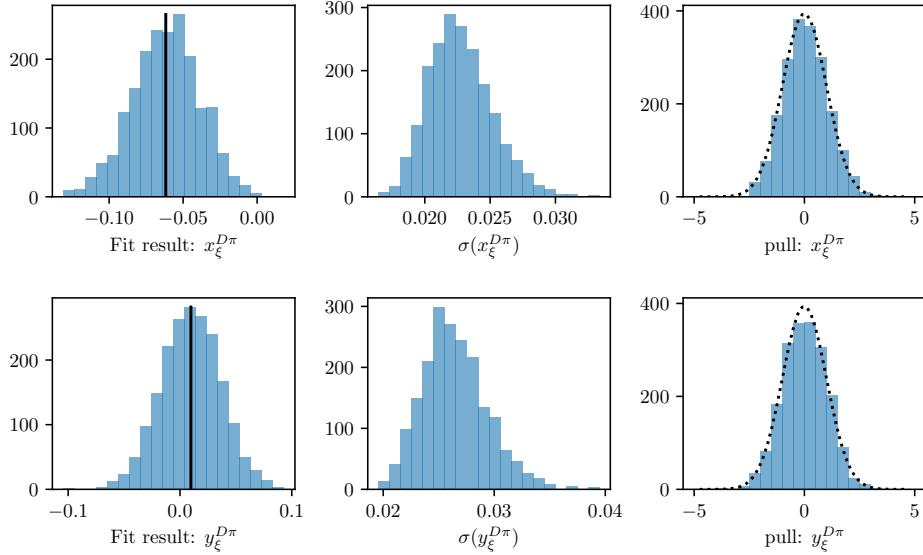


Figure 5.39: The (left) fitted value, (centre) estimated statistical uncertainty, and (right) pulls for the $B^\pm \rightarrow D\pi^\pm$ observables, as obtained in a number of pseudo experiments. The black line on the left shows the value used to generate the pseudo data sets; the dotted line on the right shows a Gaussian distribution with mean equal to zero and a standard deviation equal to unity.

The full set of fit projections in all 160 subcategories is included in Appendix A. While the CP asymmetry of the phase-space integrated yield is small, this is not the case for all individual bin-pairs. This is shown in Fig. 5.41 where, as an example, the fit projections for the $B^+ \rightarrow DK^+$ decays in bin +2 and the $B^- \rightarrow DK^-$ decays in bin -2 of the $D \rightarrow K_S^0 \pi^+ \pi^-$ Dalitz plot are compared. The presence of CP violation is clearly visible.

The obtained F_i parameter values are shown in Table 5.10. These parameters can be useful in other BPGBS measurements made within the LHCb collaboration: it is expected that the systematic uncertainty due to differences between the Dalitz-plot acceptance profile in $B^\pm \rightarrow Dh^\pm$ decays and, say, $B \rightarrow D^*K$ or $B \rightarrow DK^*$ decays is smaller than the systematic arising from extracting the efficiency profile from simulated decays. Therefore, the obtain central values and uncertainties have been made public [1], including a set of systematic uncertainties discussed in Section 5.6.12.⁷

5.5.3 Cross checks

A series of cross checks are performed to verify that the fit to data is behaving as expected.

⁷In practice, it is the obtained \mathcal{R}_i values that are made public, related to the F_i parameters via Eq. (5.23).

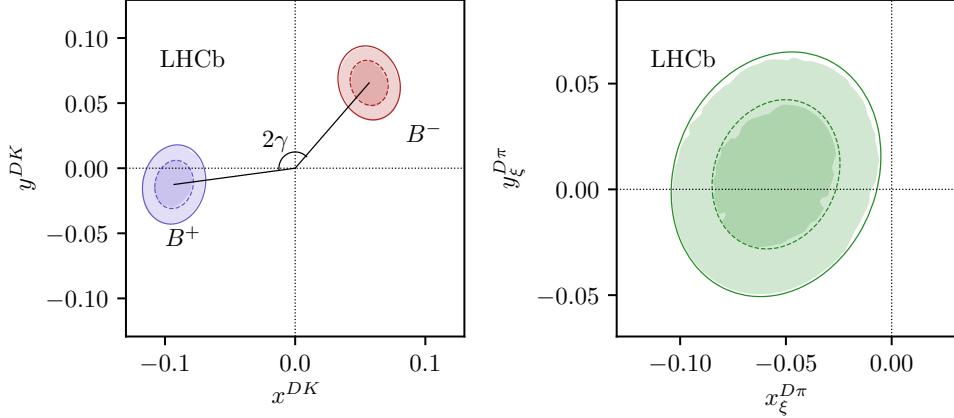


Figure 5.40: The 68 % and 95 % confidence regions for the fitted observables. The lines show the regions estimated from the covariance matrix returned by the default fit. The shaded areas are obtained in a likelihood scan, where the binned fit is run many times with all observables held at fixed values, scanning pairs of observables over the relevant ranges. The scan is made separately for the three pairs (x_{-}^{DK}, y_{-}^{DK}) , (x_{+}^{DK}, y_{+}^{DK}) , and $(x_{\xi}^{D\pi}, y_{\xi}^{D\pi})$, holding the four other parameters fixed at their default-fit central values during a given scan. Then the minimum log-likelihood is related to a χ^2 via $\mathcal{L}_{\min} = \frac{1}{2}\chi^2$ (discarding an irrelevant constant), and the confidence region limits placed at $\chi^2 = 2.30$ and $\chi^2 = 6.18$, yielding the relevant percentiles for a χ^2 distribution with 2 degrees of freedom.

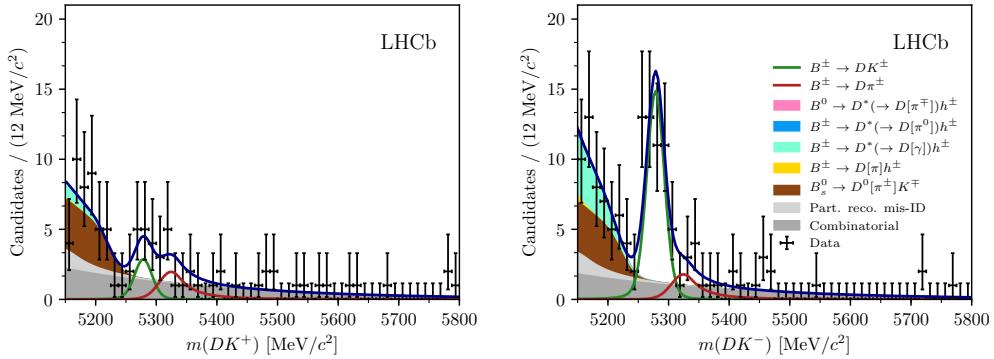


Figure 5.41: The invariant mass distribution for the (left) $B^+ \rightarrow DK^+$ candidates in bin -2 and (right) the $B^- \rightarrow DK^-$ candidates in bin +2, where $D \rightarrow K_S^0\pi^+\pi^-$ and the K_S^0 is reconstructed in the DD category.

2674 Comparison to results of earlier analyses

2675 It is confirmed that the results obtained in fits of the Run 1 or 2015+16 data
 2676 sets in isolation are compatible with the results obtained in the original LHCb
 2677 analyses of those data sets [3, 82]. In order to do so, the whole analysis procedure
 2678 is carried out using only the relevant subset of data, and the strong-phase inputs
 2679 from the CLEO collaboration are used in the fit. Two effects need to be taken

Table 5.9: Statistical uncertainties and correlation matrix for the fit to data.

Uncertainty ($\times 10^{-2}$)						
	$x_-^{DK^\pm}$	$y_-^{DK^\pm}$	$x_+^{DK^\pm}$	$y_+^{DK^\pm}$	$x_\xi^{D\pi^\pm}$	$y_\xi^{D\pi^\pm}$
σ	0.96	1.14	0.96	1.20	1.99	2.34

Correlations						
	$x_-^{DK^\pm}$	$y_-^{DK^\pm}$	$x_+^{DK^\pm}$	$y_+^{DK^\pm}$	$x_\xi^{D\pi^\pm}$	$y_\xi^{D\pi^\pm}$
$x_-^{DK^\pm}$	1.000	-0.125	-0.013	0.019	0.028	-0.165
$y_-^{DK^\pm}$		1.000	-0.011	-0.009	0.105	0.030
$x_+^{DK^\pm}$			1.000	0.088	-0.099	0.038
$y_+^{DK^\pm}$				1.000	-0.076	-0.141
$x_\xi^{D\pi^\pm}$					1.000	0.146
$y_\xi^{D\pi^\pm}$						1.000

2680 into account when comparing the central values.

2681 Firstly, the overlap between the data samples is not 100 % due to changes in
 2682 the candidate selection. The overlap between the new selection and the data set
 2683 of the original analysis of Run 1 data is about 70 %, whereas is it about 90 % for
 2684 the 2015+16 data set. In order to determine the expected difference between the
 2685 observables fitted from data sets with significant overlap, a large number of toy data
 2686 sets were generated in sets of two, where 70 (90) % of decays were shared between the
 2687 data sets. Both data sets were fitted and the difference between the obtained central
 2688 values for each observable tabulated; the standard deviation of these distributions
 2689 are used to calculate the pulls between the old analysis results and the new fits to
 2690 data. This check does not take into account that the semi-leptonic PID cuts were
 2691 introduced to remove a potential peaking background, which may have had a small
 2692 systematic effect on the earlier measurement results. Thus the expected differences
 2693 are likely to be slightly underestimated and the check is conservative.

2694 Furthermore, the F_i parameters were determined in a semi-leptonic control
 2695 channel in the earlier analyses. Therefore, the expected difference obtained above is
 2696 adjusted by adding the F_i -related systematic uncertainty of the original analysis
 2697 in quadrature, when comparing the old results to those in new fits to the Run 1
 2698 and 15+16 data sets. No further corrections have been made to the expected
 2699 differences, which effectively assumes all other systematic uncertainties to be 100 %
 2700 correlated. Also for this reason can the check be considered conservative. As can
 2701 be seen in Tables 5.11 and 5.12, neither the Run 1 and 2015+16 comparisons show
 2702 unreasonable differences in central values.

Table 5.10: The fitted F_i values including statistical uncertainties. The associated systematic uncertainties are negligible, as discussed in Section 5.6.12.

F_i values: $D \rightarrow K_S^0 \pi^+ \pi^-$		
bin	LL	DD
-8	0.024 ± 0.001	0.024 ± 0.000
-7	0.127 ± 0.001	0.133 ± 0.001
-6	0.062 ± 0.001	0.056 ± 0.001
-5	0.046 ± 0.001	0.042 ± 0.001
-4	0.095 ± 0.001	0.095 ± 0.001
-3	0.160 ± 0.001	0.160 ± 0.001
-2	0.153 ± 0.001	0.153 ± 0.001
-1	0.095 ± 0.001	0.097 ± 0.001
1	0.022 ± 0.001	0.020 ± 0.000
2	0.005 ± 0.000	0.005 ± 0.000
3	0.004 ± 0.000	0.004 ± 0.000
4	0.055 ± 0.001	0.056 ± 0.001
5	0.027 ± 0.001	0.022 ± 0.000
6	0.004 ± 0.000	0.003 ± 0.000
7	0.055 ± 0.001	0.057 ± 0.001
8	0.067 ± 0.001	0.072 ± 0.001

F_i values: $D \rightarrow K_S^0 K^+ K^-$		
bin	LL	DD
-2	0.207 ± 0.004	0.202 ± 0.003
-1	0.222 ± 0.004	0.230 ± 0.003
1	0.290 ± 0.005	0.296 ± 0.003
2	0.281 ± 0.005	0.271 ± 0.003

Table 5.11: Comparison between the results on the Run 1 analysis [82] and the central values obtained when fitting the Run 1 dataset with the selection and fit setup described in this note. The pull is calculated using the 1σ expected difference, which takes the sample overlap and the systematic uncertainty on the F_i parameters in the previous analysis into account, but assumes all other systematic uncertainties to be perfectly correlated. The new fits are performed using the CLEO strong-phase inputs.

Observable	Run 1 result [82]	New Fit (central value $\times 10^{-2}$)	Pull
x_-^{DK}	2.50	4.04	0.85
y_-^{DK}	7.50	9.14	1.02
x_+^{DK}	-7.70	-9.40	-0.91
y_+^{DK}	-2.20	0.80	1.77
<i>p</i> -value: 0.057			

Table 5.12: Comparison between the results on the 2015+16 analysis [3] and the central values obtained when fitting the 2015+16 dataset with the selection and fit setup described in this note. The pull is calculated using the 1σ expected difference, which takes the sample overlap and the systematic uncertainty on the F_i parameters in the previous analysis into account, but assumes all other systematic uncertainties to be perfectly correlated. The new fits are performed using the CLEO strong-phase inputs.

Observable	15+16 result [3] (central value $\times 10^{-2}$)	New Fit	Pull
x_-^{DK}	9.00	8.36	-0.50
y_-^{DK}	2.10	1.16	-0.62
x_+^{DK}	-7.70	-8.58	-0.56
y_+^{DK}	-1.00	-2.82	-1.39
<i>p</i> -value: 0.239			

2703 Directly fitting the signal yields

2704 As a cross-check, the fit is run in an alternative mode, in which the signal yields of
 2705 each bin are independent parameters. The obtained yields are compared to those
 2706 predicted from the results of the default fit in Fig. 5.42. The yields are shown for
 2707 each "effective bin", where effective bin i is defined as bin $+i$ for B^+ decays and bin
 2708 $-i$ for B^- decays; in the CP symmetric case, these bins are expected to have equal
 2709 yields (modulo production and detection asymmetries). The agreement between the
 2710 two fit set-ups is seen to be excellent. The normalised yield asymmetries, defined as

$$A^i \equiv \frac{N_{-i}^- - N_i^+}{N_{-i}^- + N_i^+} \quad (5.25)$$

2711 are shown in Fig. 5.43 for all decay channels. Again, the agreement between the
 2712 nominal fit and the alternative fit with independent yields is found to be excellent for
 2713 both $B^\pm \rightarrow Dh^\pm$ decays. It is also clear how, in the case of $B^\pm \rightarrow DK^\pm$ decays, the
 2714 asymmetry is significantly different from zero for a number of bin pairs. The reason
 2715 that the yield asymmetry is not zero in the fit that does not allow for CP violation,
 2716 is that the overall yield of B^+ and B^- decays are allowed to differ in this fit; only the
 2717 phase-space distribution of the decays is enforced to be CP -symmetric. The observed
 2718 asymmetry is due to a combination of production and detection asymmetries,
 2719 CP -violation and material interaction of the K_S^0 meson, and CP violation in the
 2720 $B^\pm \rightarrow Dh^\pm$ decays, but no attempt is made to disentangle the effects.

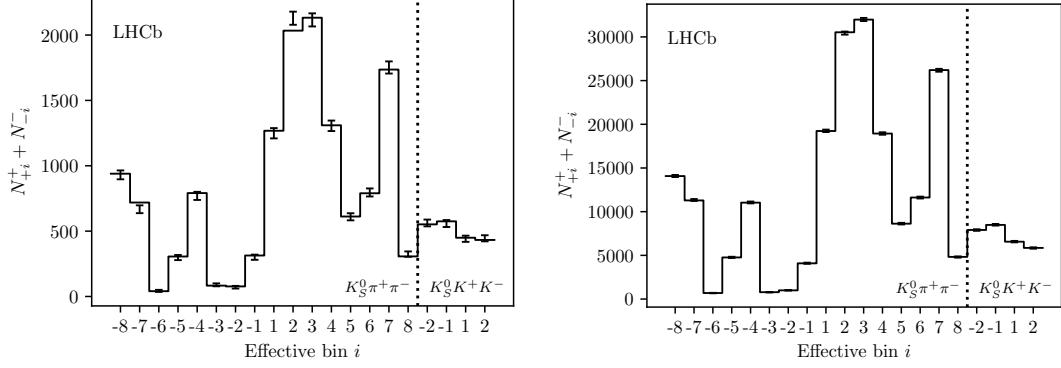


Figure 5.42: Comparison of (lines) the predicted yield given the determined CP observables and (error bars) the yield obtained in fits to data where each yield is an independent parameter. The yields are shown for (left) $B^\pm \rightarrow DK^\pm$ decays and (right) $B^\pm \rightarrow D\pi^\pm$ decays. The LL and DD categories have been combined, as has the B^+ and B^- yields for each effective Dalitz bin, defined as bin $+i$ for B^+ decays and bin $-i$ for B^- decays.

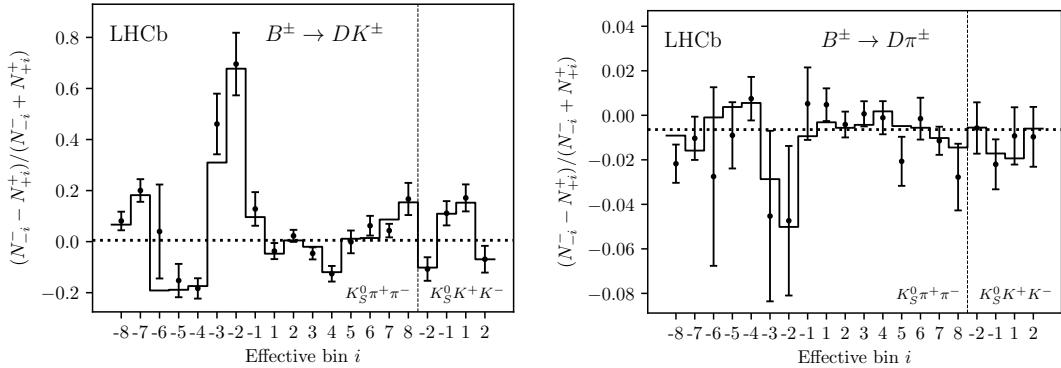


Figure 5.43: The bin-by-bin asymmetries $(N_{-i} - N_{+i})/(N_{-i} + N_{+i})$ for each Dalitz-plot bin number for (left) $B^\pm \rightarrow DK^\pm$ decays and (right) $B^\pm \rightarrow D\pi^\pm$ decays. The prediction from the central values of the CP -violation observables is shown with a solid line and the asymmetries obtained in fits with independent bin yields are shown with the error bars. The predicted asymmetries in a fit that does not allow for CP violation are shown with a dotted line.

2721 Fitting subsets of the data separately

2722 One cross check is carrying out, by determining the CP observables using a number
 2723 of independent sub samples of the data set separately. This is done for the
 2724 following following data splits

- 2725 • Fig. 5.44a shows the same plots, comparing the fits to the data set split by
 2726 K_S^0 track type.

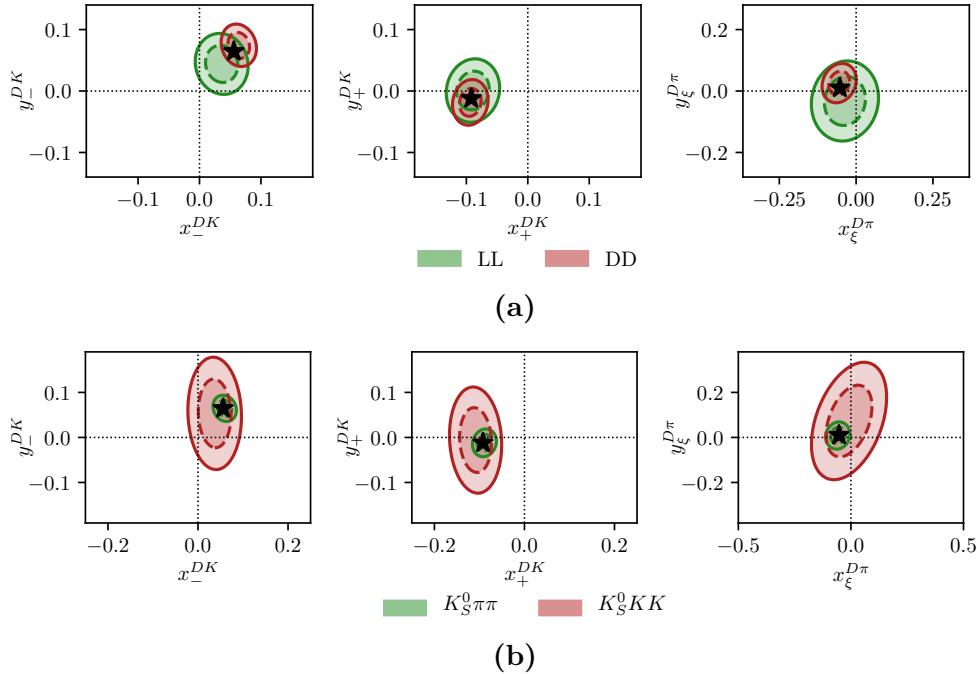


Figure 5.44: Comparison of the 68 % and 95 % confidence regions for (left) (x_{-}^{DK}, y_{-}^{DK}) , (centre) (x_{+}^{DK}, y_{+}^{DK}) , and (right) $(x_{\xi}^{D\pi}, y_{\xi}^{D\pi})$ obtained from fits to sub sets of the data set. The uncertainties are statistical only. The central values of the default fit are shown with a black star. The dataset is split by (a) LL and DD K_S^0 types and (b) D decay mode.

- Fig. 5.44b shows the same plots, comparing the fits to the data set split by whether the D meson decays to the $K_S^0\pi^+\pi^-$ or $K_S^0K^+K^-$ final state.
- Fig. 5.45a shows the two dimensional log likelihood contours for the observables for fits to the Run 1, 2015+16, 2017 and 2018 datasets separately
- Fig. 5.45b shows the same plots, comparing the fits to the data set split by whether the candidate event was triggered by one of the signal particles at the hardware level (TOS), or by another particle in the underlying event (TIS).
- Fig. 5.45c shows the same plots, comparing the fits to the data set split the magnet polarity during data taking.
- All figures show the Gaussian likelihood contours corresponding to the statistical uncertainties. There is good agreement between the results in all cases, given that in each cases the sub datasets are independent and therefore the statistical errors are uncorrelated.

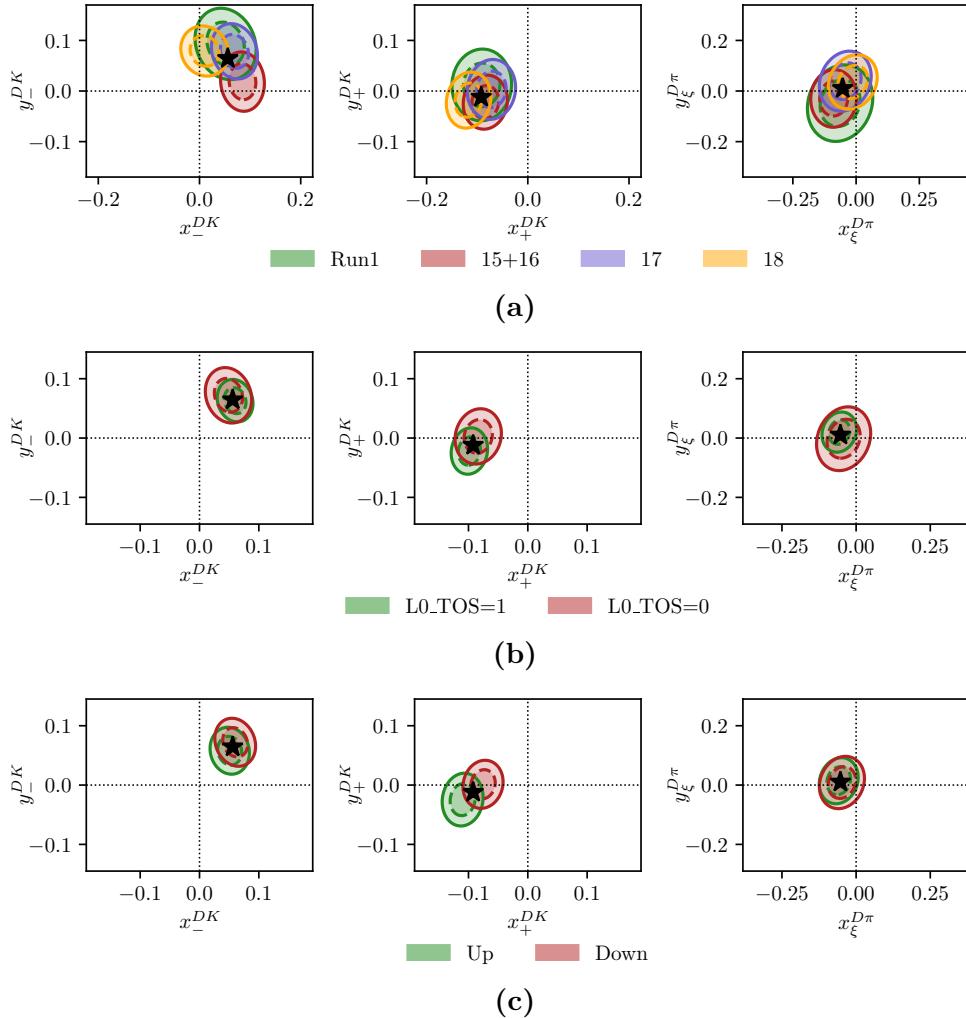


Figure 5.45: Comparison of the 68 % and 95 % confidence regions for (left) (x_-^{DK}, y_-^{DK}) , (centre) (x_+^{DK}, y_+^{DK}) , and (right) $(x_\xi^{D\pi}, y_\xi^{D\pi})$ obtained from fits to sub sets of the data set. The uncertainties are statistical only. The central values of the default fit are shown with a black star. The dataset is split by (a) data taking year, (b) trigger category, and (c) magnet polarity.

2740 Constraints from a subset of bins

2741 An alternative way to subdivide the data is to examine the constraints from a subset
 2742 of bins individually; this forms as a cross check because the observables favoured by
 2743 each sub set should be compatible, and also serves as a useful illustration of the
 2744 features of the BPGBGSZ method. Likelihood contours for (x_\pm^{DK}, y_\pm^{DK}) are shown in
 2745 Fig. 5.46, obtained using the binned yields in the $D \rightarrow K_S^0 \pi^+ \pi^-$ bins, determined
 2746 in the fits of individual bin yields described in Section 5.5.3. The bins are split by
 2747 whether they are ADS-like, GLW-like, or Odd-even according to the classification in
 2748 Section 2.3.5. It is clear that the likelihood regions show a reasonable overlap, and

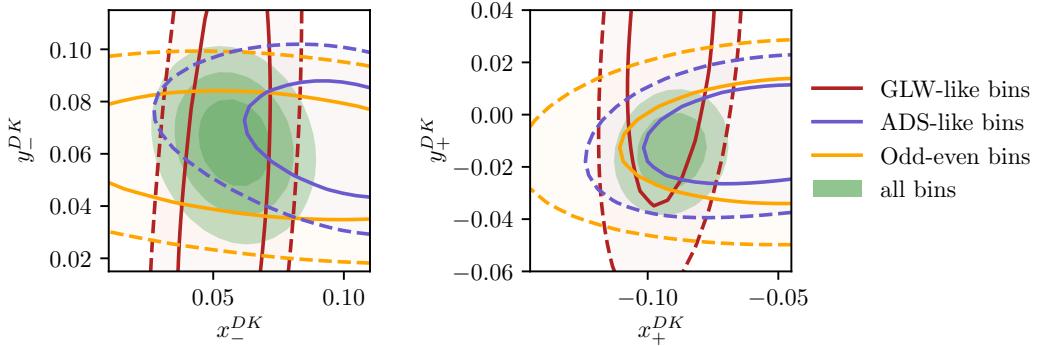


Figure 5.46: Constraints on the $B^\pm \rightarrow DK^\pm$ observables from the signal yields of different subsets the $D \rightarrow K_S^0\pi^+\pi^-$ Dalitz bins, using the bin categorisation developed in Section 2.3.5.

also how it is the GLW bins that constrain the x_\pm parameter, while the Odd-even and ADS-like bins provide the ability to constrain the y_\pm parameters.

Significantly reducing the $B^\pm \rightarrow D\pi^\pm$ to $B^\pm \rightarrow DK^\pm$ cross feed

One of the dominant backgrounds in the signal region of the $B^\pm \rightarrow DK^\pm$ channel is from partly reconstructed $B \rightarrow D\pi X$ decays where the bachelor pion is misidentified as a kaon. The background mode is well described by the included shape component, and included in all relevant systematic studies. Nevertheless, an additional cross check is carried out to ensure that it is not having a significant effect on the fit: the analysis is repeated with PID requirement of $\text{PID}_K > 12$ required to place a candidate in the $B^\pm \rightarrow DK^\pm$ category, instead of $\text{PID}_K > 4$. With this requirement 99.7 % of $B^\pm \rightarrow D\pi^\pm$ decays are correctly identified, making the cross-feed component in the $B^\pm \rightarrow DK^\pm$ channels significantly smaller than in the default fit. This is clearly visible in Fig. 5.47, where the fit projections for the global fit of the $D \rightarrow K_S^0\pi^+\pi^-$ modes are shown. In return, the probability of correctly identifying a kaon companion drops to about 68–69 %, resulting in a smaller effective signal yield.

The measurement results are compared in Table 5.13, where the differences in central value are seen to be reasonably small. It is not trivial to determine whether the difference is statistically significant or not: the same candidates are analysed in both cases, the difference being that a number of candidates that are placed in the $B^\pm \rightarrow DK^\pm$ category in the nominal fit are placed in the $B^\pm \rightarrow D\pi^\pm$ category in the alternative fit. The uncertainty will not be 100 % correlated because signal events that move from the DK to $D\pi$ category are placed in a region with high background; however, this is somewhat compensated for by candidates that remain in the DK category gaining statistical power due to the increased purity.

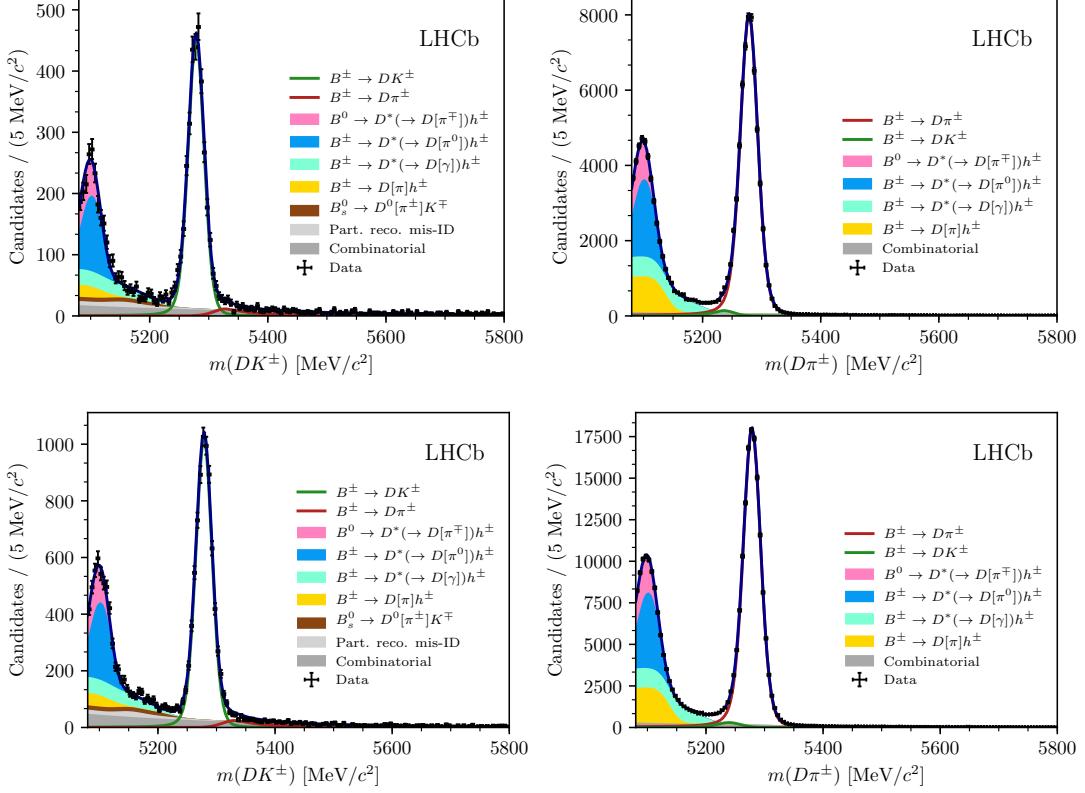


Figure 5.47: Fit projections for fits to the $D \rightarrow K_S^0 \pi^+ \pi^-$ candidates with a companion PIDK requirement at 12 instead of 4 used to split into (left) $B^\pm \rightarrow DK^\pm$ and (right) $B^\pm \rightarrow D\pi^\pm$ candidates, for the (top) LL and (bottom) DD categories.

An estimate of the expected statistical fluctuation can be determined by taking the difference of the statistical uncertainties in quadrature. Using this estimate, the observed shifts are found to be consistent with statistical fluctuation, and thus there is no sign of the background from $D\pi^\pm \rightarrow DK^\pm$ cross-feed causing issues.

2777 Compare results obtained with different strong-phase inputs

It is interesting to compare the results obtained with different strong-phase inputs. This is done in Fig. 5.48, where the default fit results are compared to those obtained if the *CP* fit is done with the CLEO-only inputs [73], and with the model predictions from the 2018 Belle model [55] and the 2008 BaBar model [61]. For the measurements, only the strong-phase-related uncertainties are included in the plot, since the statistical uncertainties are correlated. All results are found to agree well.

Table 5.13: Results of running the measurement with the default PIDK requirement at 4 used to separate $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ candidates, as well as with an alternative PIDK requirement at 12, resulting in much lower cross-feed from misidentified $B^\pm \rightarrow D\pi^\pm$ decays. We also show the pulls, defined as $\Delta x / \sqrt{|\sigma_{PIDK>12}^2 - \sigma_{PIDK>4}^2|}$ as described in the main text body. The comparison was made before the BESIII measurement of the $D \rightarrow K_S^0 K^+ K^-$ strong-phase inputs became available; therefore the fits use the CLEO-only results [73] for this mode, which explains why the results quoted for $PIDK > 4$ differ slightly from the nominal fit results.

Parameter	PIDK > 4	PIDK > 12	$\sigma = \sqrt{\sigma_{PIDK>12}^2 - \sigma_{PIDK>4}^2}$	Pull
x_-^{DK}	5.59 ± 0.96	5.82 ± 1.01	0.30	0.77
y_-^{DK}	6.45 ± 1.14	6.86 ± 1.19	0.36	1.13
x_+^{DK}	-9.21 ± 0.96	-8.94 ± 1.01	0.30	0.93
y_+^{DK}	-1.21 ± 1.20	-0.94 ± 1.26	0.37	0.71
$x_\xi^{D\pi}$	-5.30 ± 1.99	-5.13 ± 2.02	0.32	0.52
$y_\xi^{D\pi}$	1.03 ± 2.34	1.71 ± 2.33	0.28	2.40

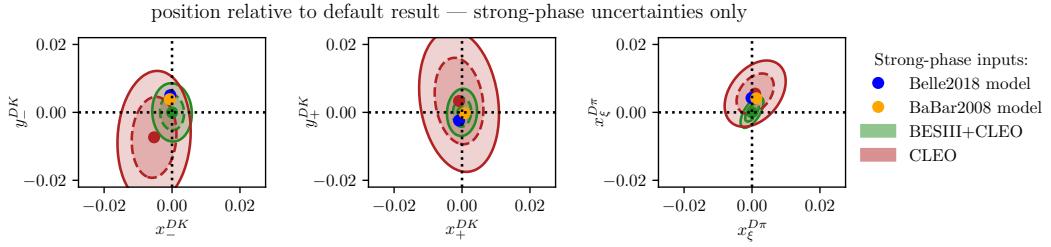


Figure 5.48: Fit results for (left) (x_-^{DK}, y_-^{DK}) , (centre) (x_+^{DK}, y_+^{DK}) , and (right) $(x_\xi^{D\pi}, y_\xi^{D\pi})$ depending on strong-phase inputs, shown relative to the default fit results. The included results are based on (green) the BESIII-CLEO combination, which is the default, (red) the CLEO-only results, (blue dot) the 2018 Belle model [55] and (orange dot) the 2008 BaBar model [61]. For the measurements, only strong-phase related uncertainties are included in the plotted confidence regions.

5.6 Systematic uncertainties

The following sections cover the suite of systematic uncertainties on the measurement that has been considered. All uncertainties are summarised in Section 5.6.12.

5.6.1 Strong phase uncertainties

The observables x_\pm^{DK} , y_\pm^{DK} , $x_\xi^{D\pi}$ and $y_\xi^{D\pi}$ are extracted using the central values of c_i and s_i from the BESIII-CLEO combinations [73, 74, 78]. Subsequently, the measurement uncertainty on these inputs is propagated to the observables by performing a large set of fits to data that use alternative values of c_i and s_i . The new c_i and s_i values are obtained by smearing the central values by their measured

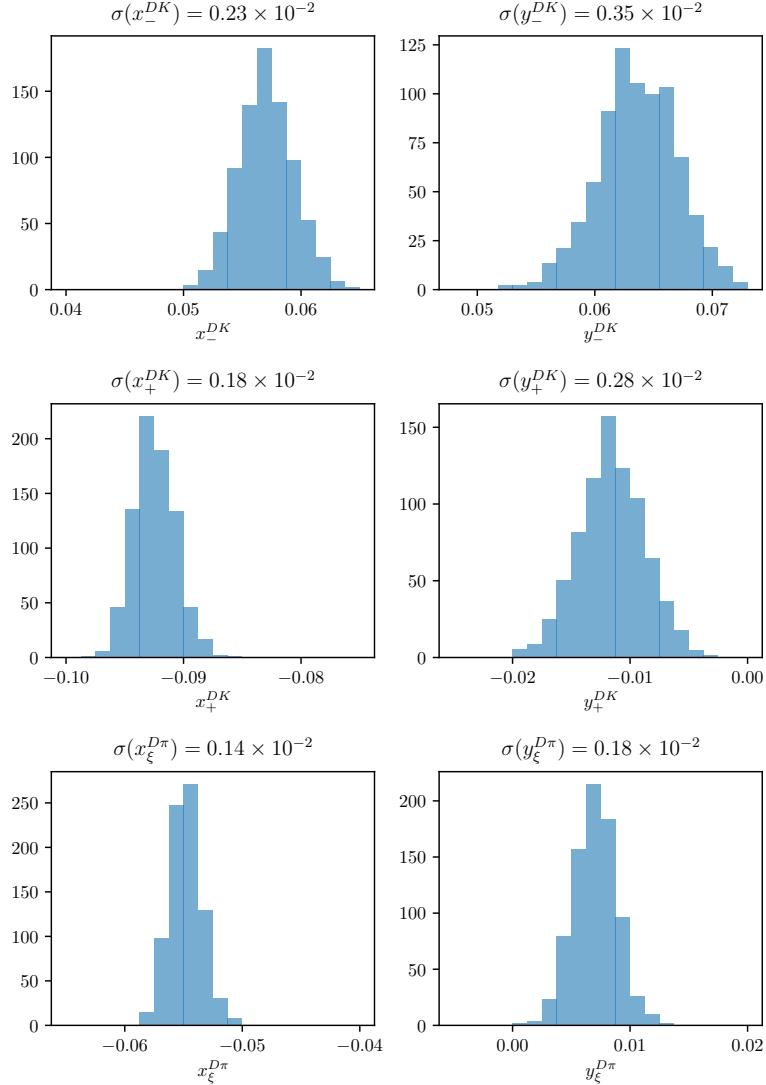


Figure 5.49: Spread of central values for the fitted observables when the input c_i and s_i from the BESIII+CLEO combination are varied according to their uncertainties and correlations.

statistical and systematic uncertainties while taking into account their correlations.
 The use of different c_i and s_i values changes the extracted x_\pm^{DK} , y_\pm^{DK} , $x_\xi^{D\pi}$ and $y_\xi^{D\pi}$ values. The standard deviation of the distributions of central values extracted from 1000 data fits are assigned as a systematic uncertainty. The distributions are shown in Fig. 5.49 and the assigned uncertainties are summarised in Table 5.14. The correlation matrix related to the strong-phase uncertainty can be obtained from the correlations observed between observables in the fits, and is also given in the table.
 The set of (c_i, s_i) that was employed in this analysis will be used in a series of future BPGBGSZ measurements, both with additional B decay modes within the LHCb collaboration and by the Belle II collaboration. This introduces some

Table 5.14: Systematic uncertainties and correlation matrix due to strong-phase inputs.

Uncertainty ($\times 10^{-2}$)						
	$x_-^{DK\pm}$	$y_-^{DK\pm}$	$x_+^{DK\pm}$	$y_+^{DK\pm}$	$x_\xi^{D\pi\pm}$	$y_\xi^{D\pi\pm}$
σ	0.23	0.35	0.18	0.28	0.14	0.18

Correlations						
	$x_-^{DK\pm}$	$y_-^{DK\pm}$	$x_+^{DK\pm}$	$y_+^{DK\pm}$	$x_\xi^{D\pi\pm}$	$y_\xi^{D\pi\pm}$
$x_-^{DK\pm}$	1.000	-0.047	-0.490	0.322	0.189	0.144
$y_-^{DK\pm}$		1.000	0.059	-0.237	-0.116	-0.117
$x_+^{DK\pm}$			1.000	0.061	0.004	-0.139
$y_+^{DK\pm}$				1.000	0.127	-0.199
$x_\xi^{D\pi\pm}$					1.000	0.638
$y_\xi^{D\pi\pm}$						1.000

correlation between the measurement results. In order to allow for an estimate of the degree of correlation by future analysts, the 1000 samples (c_i, s_i) values and the corresponding fit results for $(y_\pm^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi})$ have been made public as supplementary material to Ref. [1].

5.6.2 Efficiency-profile-related systematic uncertainties

The non-trivial efficiency profile over the Dalitz plot can have a range of effects, considered in the sections below.

2810 The assumption that $\eta^{DK}(s_-, s_+) = \eta^{D\pi}(s_-, s_+)$

2811 The assumption that the acceptance profile over the D -decay Dalitz plot is identical
2812 between $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays was examined in detail in Section 5.2.2.
2813 It was found that with signal yields similar to those in the data set, no statistically
2814 significant difference between the efficiency profiles $\eta^{DK}(s_-, s_+)$ and $\eta^{D\pi}(s_-, s_+)$
2815 was discernible, and no additional uncertainty due to this assumption is assigned.

2816 The assumption that $\eta(s_-, s_+) = \eta(s_+, s_-)$

2817 The measurement is sensitive to effects that break the assumption $\eta(s_-, s_+) =$
2818 $\eta(s_+, s_-)$. Such a breakdown would mean that opposite points on the Dalitz plot

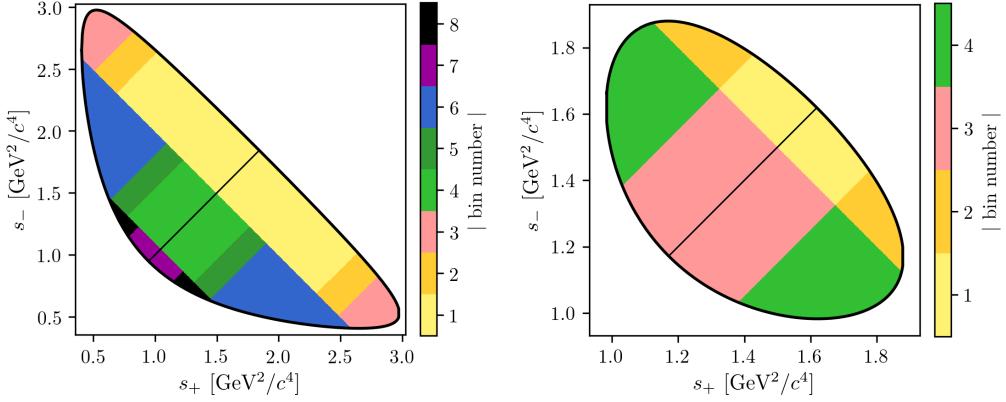


Figure 5.50: The rectangular binning schemes used to group candidates in (left) the $D \rightarrow K_S^0\pi^+\pi^-$ and (right) the $D \rightarrow K_S^0K^+K^-$ categories in a number of systematic uncertainty studies.

have different efficiencies and can only arise through a charge detection asymmetry (e.g that it is more likely to detect a K^+ in the detector rather than a K^-).⁸

The potential size of such an asymmetry can be studied in simulation where the D decay has a uniform distribution over the allowed phase space; in such simulated samples, it would manifest itself as an observation different fractional yields of B^- decays in bin i and B^+ decays in bin $-i$. This effect has been looked for using the large samples of $B^\pm \rightarrow D\pi^\pm$ decays that were generated for the analysis of 2015 and 2016 data. The study is performed using the rectangular binning schemes shown in Fig. 5.50, because this scheme is most sensitive to effects that vary smoothly over phase space. The comparison plots are shown in Fig. 5.51, where it can be seen that the p values for the hypothesis that there is no asymmetry all take on reasonable values. Hence no further systematic uncertainty is considered.

Effect of phase-space efficiency profile on c_i and s_i

As discussed in Section 2.4 there is a small bias, because the c_i and s_i values that are used correspond to the definition

$$c_i = \frac{\int_i ds^2 |A_S^D(s_{-+})| |A_S^D(s_{+-})| \cos[\Delta\delta_D(s_{-+})]}{\sqrt{\int_i ds^2 |A_S^D(s_{-+})|^2} \sqrt{\int_i ds^2 |A_S^D(s_{+-})|^2}}, \quad (\text{and equivalent for } s_i,) \quad (5.26)$$

⁸Note that the measurement is insensitive to any asymmetry in the reconstruction of the companion track.

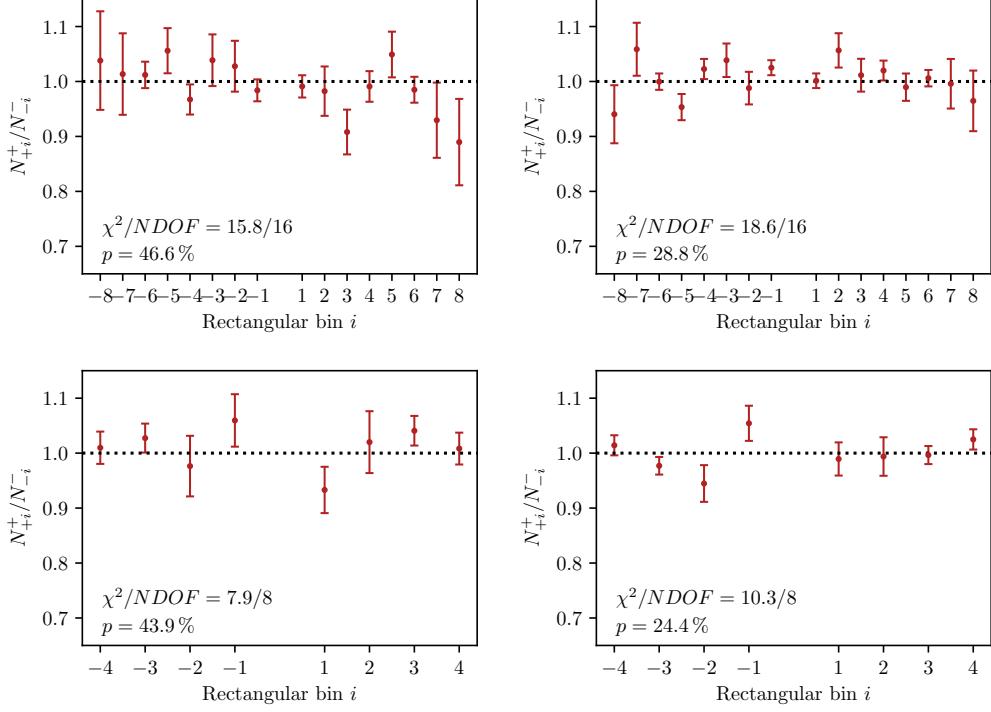


Figure 5.51: Comparison of the ratio of B^+ decays reconstructed in bin $+i$ to B^- decays reconstructed in bin $-i$ for simulated $B^\pm \rightarrow D\pi^\pm$ decays where (top) $D \rightarrow K_S^0 \pi^+ \pi^-$ and (bottom) $D \rightarrow K_S^0 K^+ K^-$, also split into (left) the LL and (right) the DD categories, using the rectangular binning schemes in Fig. 5.50. Calculated p values for the hypothesis that the ratio is flat are also shown, all of them being at least 20 %.

whereas the non-flat efficiency profile in LHCb, $\eta(s_-, s_+) \equiv \eta(s_{-+})$, means that the appropriate c_i' and s_i' entering the exact yield expressions are

$$c_i^{\text{eff}} = \frac{\int_i ds^2 \eta(s_{-+}) |A_S^D(s_{-+})| |A_S^D(s_{+-})| \cos[\Delta\delta_D(s_{-+})]}{\sqrt{\int_i ds^2 |A_S^D(s_{-+})|^2} \sqrt{\int_i ds^2 |A_S^D(s_{+-})|^2}}, \quad (\text{and equivalent for } s_i^{\text{eff}}.) \quad (5.27)$$

The shifts $\Delta c_i = c_i^{\text{eff}} - c_i$, $\Delta s_i = s_i^{\text{eff}} - s_i$ can be estimated using the efficiency profile in simulation and the latest amplitude models: the 2018 Belle model [55] for $D \rightarrow K_S^0 \pi^+ \pi^-$ and the 2010 BaBar model [62] for $D \rightarrow K_S^0 K^+ K^-$. The strong-phase parameters are first calculated assuming a uniform reconstruction efficiency over phase space according to Eq. (5.26), obtaining a set of values $\{c_i^{\text{model}}, s_i^{\text{model}}\}$. Then, an alternative set is calculated, $\{c_i^{\text{eff}}, s_i^{\text{eff}}\}$, using the same model, and the reconstruction efficiency profile found in full LHCb simulation. The LHCb reconstruction efficiency at a given point in phase-space is taken to be proportional to the yield in simulation, as the simulated decays were generated with a uniform distribution over phase space. The efficiency is averaged over the LL and DD

Table 5.15: The c_i and s_i values for $D \rightarrow K_S^0 \pi^+ \pi^-$ decays calculated via the 2018 Belle model [55] in two cases: assuming a uniform reconstruction efficiency over phase space, denoted $(c/s)_i^{\text{model}}$, and including the LHCb efficiency profile as obtained in simulation, averaged for LL and DD, denoted $(c/s)_i^{\text{eff}}$. The change due to including the efficiency is also tabulated.

Bin	c_i^{model}	c_i^{eff}	Δc_i	s_i^{model}	s_i^{eff}	Δs_i
1	-0.027	-0.007	0.019	0.812	0.794	-0.018
2	0.837	0.859	0.022	0.164	0.152	-0.012
3	0.163	0.163	-0.000	0.872	0.880	0.008
4	-0.914	-0.915	-0.001	0.076	0.082	0.006
5	-0.149	-0.170	-0.021	-0.856	-0.854	0.002
6	0.373	0.362	-0.011	-0.782	-0.805	-0.023
7	0.863	0.862	-0.000	-0.203	-0.202	0.002
8	0.860	0.862	0.002	0.330	0.336	0.006

Table 5.16: The c_i and s_i values for $D \rightarrow K_S^0 K^+ K^-$ decays calculated via the 2010 BaBar model [62] in two cases: assuming a uniform reconstruction efficiency over phase space, denoted $(c/s)_i^{\text{model}}$, and including the LHCb efficiency profile as obtained in simulation, averaged for LL and DD, denoted $(c/s)_i^{\text{eff}}$. The change due to including the efficiency is also tabulated.

Bin	c_i^{model}	c_i^{eff}	Δc_i	s_i^{model}	s_i^{eff}	Δs_i
1	0.738	0.735	-0.002	0.266	0.263	-0.003
2	-0.697	-0.744	-0.046	0.332	0.329	-0.003

2846 categories in the calculation, according to the relative signal yields in these categories.

2847 The results, as well as their differences, are tabulated in Tables 5.15 and 5.16.

2848 A systematic uncertainty due to employing the measured c_i and s_i directly
 2849 in the fit is assigned by generating a large number of toy data sets where the
 2850 signal yields are calculated using $(c_i^{\text{eff}}, s_i^{\text{eff}})$, and then fitting the data sets using
 2851 $(c_i^{\text{model}}, s_i^{\text{model}})$. The mean bias of each observable in these toys is assigned as the
 2852 systematic uncertainty, and is determined to be 0.1×10^{-2} or less for all observables.
 2853 The smallness of the effect is the reason no effort is made to correct the c_i and
 2854 s_i values in the nominal measurement.

2855 5.6.3 Mass shapes

2856 A number of uncertainties relate to the mass distributions that enter the fit model.

2857 Each is described in detail the sections below.

2858 **Determination of shape parameters**

2859 The statistical uncertainties on the shape parameters that are obtained in fits
 2860 to simulated decays and in the first stage fit to data need to be propagated to
 2861 the uncertainty on the obtained parameters. This is done via a bootstrapping
 2862 procedure, repeating these steps many times:

- 2863 • Each of the data sets used to determine parameters of the signal, crossfeed,
 2864 and lowmass shapes that are fixed in the first-stage fit to data of Section 5.4
 2865 are re-sampled with replacement, drawing a number of events equal to the
 2866 original data-set size. These are from simulation for signal and lowmass
 2867 shapes, and real data for the crossfeed shapes. All of the shapes are fit again,
 2868 on the re-sampled data sets.
- 2869 • The real dataset is re-sampled with replacement, drawing a number of events
 2870 equal to the original data-set size. Then, the first-stage fit of Section 5.4 is
 2871 repeated with the shapes obtained as described above, obtaining values for
 2872 the remaining shape parameters.
- 2873 • Finally, the CP fit is repeated using the shape parameters determined in the
 2874 preceding steps, but *without* re-sampling the dataset (to avoid a statistical
 2875 spread in the obtained central values that is independent of the shape
 2876 parameters).

2877 The uncertainty on each observable is taken to be the standard deviation of the
 2878 set of central values obtained as described above. This procedure propagates the
 2879 statistical uncertainty on the fixed parameters to the observables, in a way that
 2880 takes correlations into account, and which does not rely on the uncertainty estimates
 2881 in the preliminary fits being accurate. The uncertainties are less than 0.1×10^{-2}
 2882 for all DK^\pm observables and less than 0.2×10^{-2} for all $D\pi^\pm$ observables.

2883 A potential bias arises due the use of the sPlot method when obtaining the
 2884 mass distribution of decays where a $\pi \leftrightarrow K$ misidentification has taken place.
 2885 This is because the $m_{\text{swap}}(Dh^\pm)$ mass that is calculated while assuming a swapped
 2886 companion hypothesis and the nominal $m_{\text{default}}(Dh^\pm)$ mass are correlated (it is
 2887 always the case that $m_{\text{swap}} > m_{\text{default}}$ for a $\pi \rightarrow K$ misidentification ,for example).
 2888 Thus, the assumptions of the sPlot method are not satisfied [146]. The correlation
 2889 coefficient in the signal region is about 20 % for simulated signal decays. In order to
 2890 assess the potential impact, an alternative mass distribution for $(B^\pm \rightarrow D\pi^\pm) \rightarrow$
 2891 $(B^\pm \rightarrow DK^\pm)$ cross-feed is derived that does not rely on sWeights. Instead of

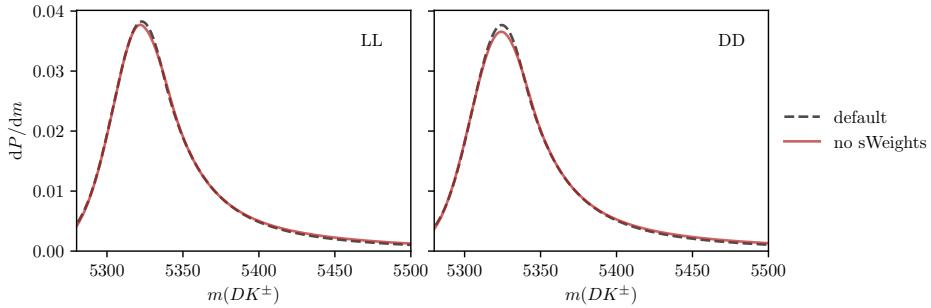


Figure 5.52: Comparison of the default and no-sWeights alternative shape for the $D\pi \rightarrow DK$ cross-feed component in the (left) LL and (right) DD categories where $D \rightarrow K_S^0 \pi^+ \pi^-$. The binned fit obtains essentially the same central values for the CP -violation observables, independently of which shape is used.

fitting $B^\pm \rightarrow D\pi^\pm$ sample in the whole fit range and assigning sWeights before recalculating the B mass under the kaon companion hypothesis, the shape is obtained using $B^\pm \rightarrow D\pi^\pm$ candidates in the signal region. This is possible because the $B^\pm \rightarrow D\pi^\pm$ sample is very pure. The shapes are compared in Fig. 5.52 and are seen to be almost identical. Thus the sWeights do successfully subtract the contribution of combinatorial and partially reconstructed backgrounds in the default setup. The impact on the obtained CP -violation observables of using one or the other shape in the fits is negligible, and no further systematic uncertainty is assigned due to this effect.

Using the same mass shapes in all Dalitz bins

The mass shapes obtained the first-stage fit where all Dalitz bins are combined, are used in each individual bin of the subsequent binned fit. However, there could be some variation in the shape over the D -decay phase space, due to correlations between the phase-space coordinates and particle kinematics. The potential effect is investigated in pseudoexperiments, where toy data sets are generated with alternative signal, crossfeed, and combinatorial-background shapes that are allowed to differ between bins, and fitted with the default shapes. The partially reconstructed background is treated in a separate study, because further physics effects contribute to bin-by-bin variation, as described in the following section.

The alternative signal and cross-feed mass shapes are fitted independently in each bin, following identical procedures to those outlined in Sections 5.4.1 and 5.4.2. Examples of the obtained shapes are compared in Figs. 5.53 and 5.54.

The shape of the combinatorial background can also vary over the D decay phase-space; for example will the relative amount of fake D candidates versus real

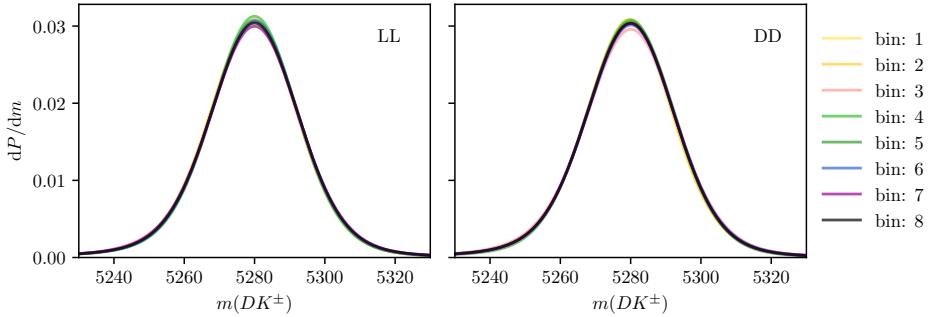


Figure 5.53: Signal shapes obtained in fits simulated $B^\pm \rightarrow D\pi^\pm$ decays for individual Dalitz bins in the optimal binning scheme, for (left) LL and (right) DD candidates in the $B^\pm \rightarrow D(\rightarrow K_S^0\pi^+\pi^-)\pi^\pm$ category.

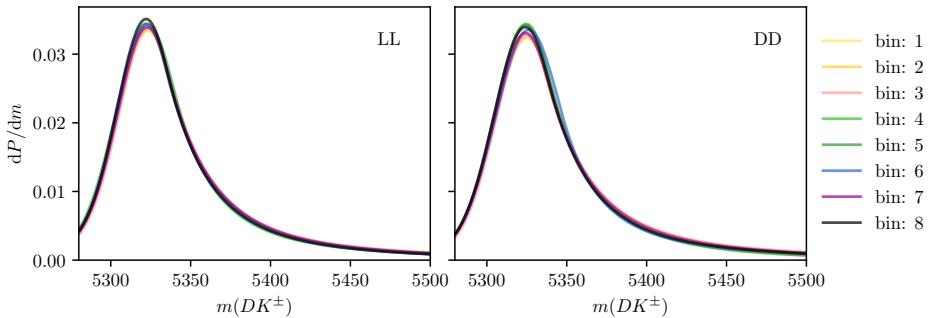


Figure 5.54: Mass shapes for $D\pi \rightarrow DK$ cross feed obtained for individual Dalitz bins in the optimal binning scheme, for (left) LL and (right) DD candidates in the $D \rightarrow K_S^0\pi^+\pi^-$ category.

2916 D decays paired with a random bachelor certainly depend on the real D decay
 2917 amplitude for a given phase-space region. The effect is investigated in the high
 2918 B -mass sideband $m_B \in [5600, 6500] \text{ MeV}/c^2$, in which the $m(Dh^\pm)$ distribution is
 2919 fitted with a single exponential distribution, in bins of the Dalitz plot. The fits
 2920 combine B^+ and B^- candidates and merge bins $+i$ and $-i$, and are carried out
 2921 for both the *optimal* binning scheme of Fig. 2.7 (on page 21) and a *rectangular*
 2922 binning scheme, shown in Fig. 5.50, which better captures continuous trends over
 2923 the Dalitz plot. The study is done for $D \rightarrow K_S^0\pi^+\pi^-$ only due to available statistics.
 2924 The DD category of $B^\pm \rightarrow D\pi^\pm$ decays has the largest statistics and shows the
 2925 largest variation, and the fitted slopes for this channel are shown in Fig. 5.55. Two
 2926 effects are visible: 1) there is some variation in the slope as a function of the Dalitz
 2927 bin, especially visible for the rectangular scheme, and 2) the exponential slope is
 2928 larger in general in the high B -mass sideband. The latter effect does not pose a
 2929 problem, since the employed exponential is found to provide an excellent fit in
 2930 the default fit region. It does however need to be taken into account when when

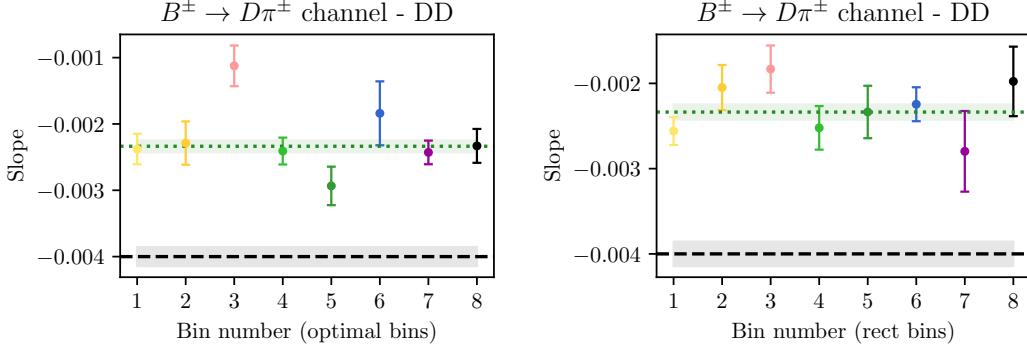


Figure 5.55: Plot of (dots) combinatorial slope in the high B mass background for each bin in the (left) the optimal binning scheme and (right) the rectangular binning scheme, for the DD $B^\pm \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-) \pi^\pm$ category. The slope when all bins are combined (green, dashed line) is also shown, and compared with (black dashed line) the slope in the default fit region.

2931 deriving alternative, bin-dependent combinatorial slopes relevant for the default
 2932 fit region. In order to do so, the alternative slope for bin i is defined

$$\alpha_{\text{default-range}}^i = \frac{\alpha_{\text{high-}m_B}^i}{\alpha_{\text{high-}m_B}^{\text{all-}DP}} \times \alpha_{\text{default-range}}^{\text{all-}DP}, \quad (5.28)$$

2933 and used when generating the combinatorial-background component of the toy
 2934 data sets for the study.

2935 The average bias obtained for each observable in the ensemble of pseudo-
 2936 experiments is assigned as a systematic uncertainty, found to be about 0.1×10^{-2}
 2937 for each observable.

2938 Ignoring physics effects in the lowmass background

2939 In the CP fit, the same relative fractions of partly reconstructed B^\pm and B^0
 2940 backgrounds are used in each bin, as determined in the first-stage fit described
 2941 in Section 5.4 (whereas the partly reconstructed $B_s^0 \rightarrow \bar{D}^0[\pi^+]K^-$ background is
 2942 treated separately). However the distribution over the Dalitz plot depends on
 2943 whether the partly reconstructed decays occur via an intermediate D^0 meson, a \bar{D}^0
 2944 meson, or and admixture of both. Consider a decay reconstructed as $B^- \rightarrow DK^-$
 2945 but which is actually a partially-reconstructed background. There are then four
 2946 types of background that should be considered:

- 2947 • Decays in which the D -meson in the true decay is a D^0 -meson. An example of
 2948 this is $B^- \rightarrow D^{*0}(D^0\pi^0)\pi^-$ for which the π^0 from the D^{*0} decay is missed and

2949 the π^- is misidentified as the companion K^- . These are denoted ' D^0 -like'.⁹

- 2950 • Decays in which the D -meson in the true decay is a \bar{D}^0 -meson. An example of
 2951 this is $B_s^0 \rightarrow \bar{D}^0\pi^+K^-$ for which the π^+ is missed and the K^- is reconstructed
 2952 as the companion K^- . These are denoted ' \bar{D}^0 -like'.
- 2953 • Decays in which the D meson in the true decay can be either flavour, and
 2954 both D flavours contribute to the decay amplitude. An example of this is
 2955 $B^- \rightarrow D^*K^-$ for which the total decay amplitude into a D final state has
 2956 contributions from both D^{*0} (decaying to D^0) and \bar{D}^{*0} (decaying to \bar{D}^0). The
 2957 relative amplitude magnitude and phase between the two possible B decays
 2958 are denoted $r_B^{D^*}$ and $\delta_B^{D^*}$ respectively. These are denoted ' r_B -like'.
- 2959 • Decays that can be reconstructed as both D^0 - and \bar{D}^0 -like but where there is
 2960 no quantum-mechanical interference. An example is $\bar{B}^0 \rightarrow D^0\pi^+\pi^-$ decays
 2961 where either the π^+ or π^- can be reconstructed as the bachelor. These are
 2962 denoted 50/50 D^0 -like and \bar{D}^0 -like.

2963 For $B^+ \rightarrow DK^+$ decays everything is CP conjugated. The Dalitz-plot distribution
 2964 for each of these cases is:

- 2965 • D^0 decays (' D^0 -like')

$$\begin{aligned} N_{\pm i}(B^-) &\propto F_{\pm i} \\ N_{\pm i}(B^+) &\propto F_{\mp i} \end{aligned} \tag{5.29}$$

- 2966 • \bar{D}^0 decays (' \bar{D}^0 -like'):

$$\begin{aligned} N_{\pm i}(B^-) &\propto F_{\mp i} \\ N_{\pm i}(B^+) &\propto F_{\pm i} \end{aligned} \tag{5.30}$$

- 2967 • Decays with a quantum-mechanical admixture of D^0 and \bar{D}^0 (' r_B -like'):

$$\begin{aligned} N_{\pm i}(B^-) &\propto F_{\pm i} + (r_B^*)^2 F_{\mp i} + 2\sqrt{F_{+i}F_{-i}}[x_-^*c_{\pm i} + y_-^*s_{\pm i}] \\ N_{\pm i}(B^+) &\propto F_{\mp i} + (r_B^*)^2 F_{\pm i} + 2\sqrt{F_{+i}F_{-i}}[x_+^*c_{\pm i} - y_+^*s_{\pm i}] \end{aligned} \tag{5.31}$$

2968 where (x_\pm^*, y_\pm^*) are defined analogously to the standard $B^\pm \rightarrow DK^\pm$ case.

⁹The naming convention is defined in terms of the D present in candidates reconstructed as B^- decays. For the charge conjugate case this decay would of course happen via a \bar{D}^0 , but is still denoted D^0 -like.

2969 • 50/50 D^0 -like and \bar{D}^0 -like:

$$\begin{aligned} N_{\pm i}(B^-) &\propto F_{\pm i} + F_{\mp i} \\ N_{\pm i}(B^+) &\propto F_{\pm i} + F_{\mp i} \end{aligned} \quad (5.32)$$

2970 The use of a single background shape across all bins may therefore introduce biases
 2971 because such a shape has no sensitivity to the bin-to-bin variations that will arise
 2972 if an admixture of these backgrounds is present.

2973 In the $D\pi$ channel, the dominant backgrounds are all D^0 -like ($\bar{B}^0 \rightarrow D^{*-}\pi^0$,
 2974 $B^- \rightarrow D^0\rho^-$, $B^- \rightarrow D^{*0}\pi^-$). There is a small contribution from $\bar{B}^0 \rightarrow D^0\rho(\rightarrow$
 2975 $\pi^+\pi^-$) decays where either the π^+ or π^- from the ρ^0 decay can be assigned as the
 2976 bachelor, and thus this background is 50/50 D^0 -like and \bar{D}^0 -like. The background
 2977 only corresponds to about 0.5 % of the total partially reconstructed background and
 2978 thus the impact is small. Nevertheless it is considered in the study described below.

2979 In the DK channel all categories of background appear. In the mass region
 2980 of the CP fit approximately 75.5% of backgrounds are D^0 -like ($\bar{B}^0 \rightarrow D^{*-}K^-$,
 2981 mis-identified $B^- \rightarrow D^{*0}\pi^-$, and mis-identified $B^- \rightarrow D^0\rho^-$), 7.5 % are \bar{D}^0 -like
 2982 ($B_s^0 \rightarrow \bar{D}^0\pi^+K^-$), 1% is 50/50 D^0 - \bar{D}^0 -like (mis-identified $B^0 \rightarrow D\rho^0$), and 16% are
 2983 r_B -like ($B^- \rightarrow D^{*}K^-$, $B^0 \rightarrow DK^{*0}$, and $B^- \rightarrow DK^{*-}$).

2984 In order to estimate the bias due to ignoring this effect, a large number of toy
 2985 data sets are generated using the default low mass shapes and total yields from
 2986 the first-stage fit in Section 5.4, but distributing each of them individually over
 2987 the Dalitz-bins according to Eqs. (5.29)-(5.31). When calculating the distribution
 2988 of $B^+ \rightarrow D^{*0}K^+$ decays over the Dalitz plot, the values [50]

$$r_B^{D^*} = 0.191 \quad \delta_B^{D^*} = 331.6^\circ \quad (5.33)$$

2989 are used. When calculating the distribution of $B^+ \rightarrow D^0K^{*+}$ decays over the
 2990 Dalitz plot the values [50]

$$r_B^{K^*} = 0.092 \quad \delta_B^{K^*} = 40^\circ. \quad (5.34)$$

2991 are used. The toy data sets are then fit with the default set up, and the observed
 2992 mean bias assigned as the corresponding uncertainty. The corresponding uncer-
 2993 tainties were found to be about 0.1×10^{-2} for all uncertainties. The variation in
 2994 the shapes is rather small in the mass range included in the fit, which explains
 2995 the small impact.

2996 If the B_s^0 background is *not* treated separately in the default fit, but instead
 2997 included in a single lowmass background shape along with the B^0 and B^\pm contribu-
 2998 tions, the systematic uncertainty is an order of magnitude larger when evaluated
 2999 as described above, and would be a dominating systematic. This motivates the
 3000 separate treatment of the B_s^0 background.

3001 5.6.4 *CP* violation and material interaction of the K_S^0

3002 A systematic uncertainty due to *CP*-violation effects and material interaction of
 3003 the K_S^0 is assigned using the results obtained in Section 4.3.7. In that section, the
 3004 expected bias on each of the observables in a combined $B^\pm \rightarrow Dh^\pm$ measurement
 3005 was evaluated for the detector geometry and particle kinematics of the LHCb
 3006 experiment. The calculation was made for $(r_B^{DK^\pm/D\pi^\pm}, \delta_B^{DK^\pm/D\pi^\pm})$ values close to
 3007 the world averages, and a number of γ values; the results were summarised in Fig. 4.8.
 3008 The systematic uncertainty is taken to be the largest absolute bias observed for each
 3009 parameter in the study. The largest uncertainty (on $y_\xi^{D\pi}$ where it is 0.46×10^{-2}) is
 3010 still an order of magnitude smaller than the statistical uncertainty.

3011 5.6.5 Impact of D mixing

3012 The effect of D -mixing is not accounted for in the measurement, which leads to
 3013 a small bias. Earlier studies have shown this to lead to a sub-degree bias on
 3014 measurements of γ in $B^\pm \rightarrow DK^\pm$ decays, in the case where the F_i parameters
 3015 are determined experimentally under the same experimental conditions as the γ
 3016 measurement [158]. A number of pseudoexperiments are carried out to verify that
 3017 this is also the case for the combined $DK^\pm-D\pi^\pm$ setup employed in the thesis.
 3018 They are performed following the same procedure described in Section 5.6.4 for
 3019 the case of neutral-kaon *CP* violation. The yields are calculated while taking D
 3020 mixing into account, using the mixing parameter values $x = (0.39^{+0.11}_{-0.12})\%$ and
 3021 $y = (0.65^{+0.06}_{-0.07})\%$ [31], and then fitted back assuming no D mixing. The biases are
 3022 found to be small, as expected, all of them smaller than 0.05×10^{-2} . The largest
 3023 relative biases are on the $B^\pm \rightarrow D\pi^\pm$ parameters, but even for those the relative
 3024 effect is less than 2 %. In agreement with Ref. [158], it is found that the biases
 3025 increase with an order of magnitude if the F_i parameters are fixed to the expected
 3026 values with no D -mixing, instead of being determined as part of the fit.

3027 5.6.6 PID efficiencies

3028 The uncertainty related to PID efficiencies is assessed by repeating the full two-stage
 3029 fit procedure a number of times, each time varying the PID efficiencies within
 3030 the uncertainties. The used uncertainty includes both a statistical and systematic
 3031 component, as described in detail in Section 5.1.3. The standard deviations of
 3032 the central values obtained for each observable are assigned as the systematic
 3033 uncertainty. The uncertainties come out below 0.1×10^{-2} for all observables.

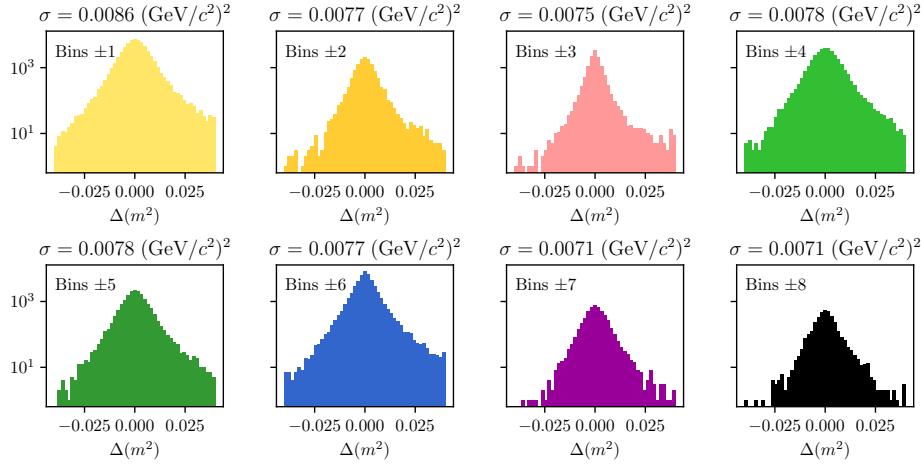


Figure 5.56: Distribution of the biases $\Delta(m^2) = m_{true}^2 - m_{reconstructed}^2$ in simulation for $m^2(K_S^0\pi^\pm)$ in bins of the rectangular binning scheme of Fig. 5.50.

5.6.7 Dalitz-coordinate resolution

There is a small systematic uncertainty related to Dalitz-plot-bin migration, where the non-perfect resolution on the momentum measurement means that a candidate is assigned to a different bin than it truly belongs to. This leads to non-negligible net migration between bins that share a border in a region of phase space where the amplitude varies rapidly. However, since the F_i are measured in the data set, all leading order effects of migration are inherently taken into account. The measurement is only sensitive to differences in migration between the DK^\pm and $D\pi^\pm$ channels and the effect is small.

The systematic uncertainty due to this effect is assigned using pseudoexperiments. The study is made for the $D \rightarrow K_S^0\pi^+\pi^-$ mode only, which is sufficient since it completely dominates the overall sensitivity.

1. Signal $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays are generated continuously over phase space, according to the expected distribution obtained with the latest amplitude model from the Belle collaboration [55], assuming values of γ and $(r_B^{DK^\pm/D\pi^\pm}, \delta_B^{DK^\pm/D\pi^\pm})$ close to the current world averages.
2. The Dalitz coordinates of each candidate are then smeared using the experiment resolution obtained in simulation. This is described further below.
3. Finally, the generated candidates are binned and fitted back using the default setup.



3054 The resolution is obtained via simulation, by comparing the reconstructed
 3055 phase-space coordinates with those calculated from the true momenta in samples of
 3056 simulated $D \rightarrow K_S^0 \pi^+ \pi^-$ decays. As can be seen in Fig. 5.56, the resolution is found
 3057 to vary over phase space and the distribution of shifts has significant exponential
 3058 tails. In order to take both effects into account, the smearing is done by shifting each
 3059 generated decay with a realised coordinate shift in full simulation, for a simulated
 3060 decay that took place at approximately the same place in the Dalitz plot. The
 3061 shift is multiplied with 120 % to take into account that the resolution is generally
 3062 better in simulation than data. If the shift results in Dalitz coordinates outside the
 3063 kinematically allowed region, a different shift is applied randomly instead.

3064 The average bias seen in the pseudoexperiments is assigned as the systematic
 3065 uncertainty. The uncertainties come out at about $(0.1 - 0.2) \times 10^{-2}$ for all parameters.
 3066 It is found that the bias is towards a smaller value of r_B^{DK} for all four DK^\pm
 3067 parameters ; this is to be expected, as bin migration washes out the asymmetries
 3068 in different areas of the Dalitz plot.

3069 5.6.8 The fixed yield fractions

3070 A number of relative yields are fixed from efficiencies in simulation and branching
 3071 fractions. In the DK^\pm modes, this is the case for all the relative yields of the
 3072 partially reconstructed backgrounds, including partially reconstructed $B \rightarrow D\pi[X]$
 3073 decays where the pion is misidentified as a kaon, and the yield of the $B_s^0 \rightarrow DK\pi$
 3074 background relative to the $B \rightarrow D\pi$ yield. In the $B^\pm \rightarrow D\pi^\pm$ channel, the only fixed
 3075 yield ratio is that of the partially reconstructed $B^\pm \rightarrow D^{*0}(\rightarrow D\pi^0)\pi^\pm$ and $B^0 \rightarrow$
 3076 $D^{*\mp}(\rightarrow D\pi^\mp)\pi^\pm$ modes. The uncertainty on the observables due to uncertainties
 3077 on these fixed fractions is assessed by repeating the two-stage fit procedure many
 3078 times, each time shifting the yield ratios randomly within their uncertainties. The
 3079 uncertainty on each observable is taken to be the standard deviation of the set of
 3080 central values obtained in these fits. These are all smaller than 0.1×10^{-2} .

3081 5.6.9 Systematic uncertainty due to backgrounds that are 3082 not modelled in fit

3083 There are a number of backgrounds that are expected to be present at a small
 3084 level, but which are not modelled in the fits to data because their impact on the fit
 3085 results is minimal. Instead, a systematic uncertainty is assigned. Each contribution
 3086 is described in the following sections and the related systematic uncertainties
 3087 are summarised in Table 5.17.



Table 5.17: Summary of systematic uncertainties due to backgrounds that are potentially present with a small yield, but not included in the mass fit.

All uncertainties are quoted with implicit: $\times 10^{-2}$						
Mode	$\sigma(x_-^{DK^\pm})$	$\sigma(y_-^{DK^\pm})$	$\sigma(x_+^{DK^\pm})$	$\sigma(y_+^{DK^\pm})$	$\sigma(x_\xi^{D\pi^\pm})$	$\sigma(y_\xi^{D\pi^\pm})$
Λ_b^0 backgrounds	0.04	0.05	0.04	0.06	0.08	0.13
$B \rightarrow D\mu\nu X$	0.04	0.07	0.04	0.05	0.10	0.11
$B^\pm \rightarrow D(\rightarrow K_S^0\pi\mu\nu)h^\pm$	0.00	0.03	0.02	0.02	0.00	0.00
Swapped tracks	0.10	0.13	0.12	0.08	0.00	0.01
Total	0.11	0.16	0.13	0.12	0.08	0.13

Background from Λ_b decays

This section considers the possible impact of the two potential backgrounds from Λ_b^0 decays described in Section 5.4.3: $\Lambda_b^0 \rightarrow D^0 p\pi^-$ decays where the pion is not included in the candidate reconstruction and the proton assigned as the companion, and $\Lambda_b^0 \rightarrow \Lambda_c^+(\rightarrow pK_S^0\pi^+\pi^-)\pi^-$ decays where a pion in the Λ_c^+ decay is not reconstructed and the proton reconstructed as one of the D decay products. The impact of not including these in the default fit is assessed by generating toy data sets where the backgrounds are included in the generation step, which are then fitted back with the default model. The former background is distributed over the Dalitz plot as \bar{D}^0 -like, cf. the terminology of Section 5.6.3, since a positive bachelor is produced along with a D^0 meson. The latter is also distributed as \bar{D}^0 -like in the study; the exact distribution is unknown, but a \bar{D}^0 -like background is likely to have the largest effect and thus this is a conservative choice. The total yields are taken relative to the signal yields, using the yield ratios discussed in Section 5.4.3. The $m(Dh^\pm)$ distributions are obtained using simulated samples, produced with **RapidSim**. The mean biases come out to be less than 0.1×10^{-2} for each CP -violation observable, which is assigned as a systematic uncertainty.

Semi-leptonic backgrounds

The impact of remnant $B \rightarrow D\mu\nu_\mu$ decays after requiring `isMuon=0` on the bachelor is assessed in pseudoexperiments. Toy datasets are generated where the background is added in the generation step, which are then fitted with the default model. The background yield relative to signal and the mass shape are obtained from a sample of fully simulated decays for conditions corresponding to the run conditions in 2012. The obtained bias in the toys is assigned as the systematic uncertainties: it is below 0.1×10^{-2} for all parameters.

3113 The systematic uncertainty relating to the presence of $D^0 \rightarrow K_S^0 \pi^- \mu^+ \nu_\mu$ is
 3114 estimated by repeating the bias studies of Section 5.3.3, but scaling the background
 3115 yields to 10% to take into account the lepton veto on the D decay products. All
 3116 biases are less than 0.05×10^{-2} in this case.

3117 **Swapped tracks**

3118 There is a peaking background present from $B^\pm \rightarrow D(\rightarrow K^\mp \pi^\pm) K_S^0 \pi^\pm$ decays where
 3119 the kaon is reconstructed as the companion and the K_S^0 is assigned to the D decay.
 3120 The yield of this background is determined to be 0.5% of the signal yield in the
 3121 $B^\pm \rightarrow DK^\pm$ channel in Section 5.3.5. The potential impact from the presence
 3122 of the background is estimated by

- 3123 1. Calculating the expected $B^\pm \rightarrow D\pi^\pm$ and $B^\pm \rightarrow DK^\pm$ signal yields in each
 3124 bin for physics parameters similar to the world average values.
 - 3125 2. Then calculating the background bin yields in each $B^\pm \rightarrow DK^\pm$ bin, using
 3126 a total yield equal to 0.5% of the signal yield, and the bin distribution
 3127 from simulated samples of $B^\pm \rightarrow D(\rightarrow K^\mp \pi^\pm) K_S^0 \pi^\pm$ decays, produced via
 3128 **RapidSim**. The study is carried out for multiple simulated samples, including
 3129 decays where the $K_S^0 \pi$ pair in the B decay originate in different K^* resonances
 3130 (generated with EvtGen and the proper resonance-spin models), as well as B
 3131 decays that are evenly distributed over the allowed phase space.
 - 3132 3. For each sample, the signal and background yields are added, and the new
 3133 $B^\pm \rightarrow D\pi^\pm$ and $B^\pm \rightarrow DK^\pm$ yields are fitted back with the default signal-
 3134 yield expressions (including a fit of the F_i parameters).
- 3135 For each parameter, the most significant bias seen across the different **RapidSim**
 3136 samples is taken as the related systematic uncertainty. The uncertainty is below
 3137 0.15×10^{-2} for all parameters.

3138 **5.6.10 Bias correction**

3139 In the default sensitivity study, the bias was found to be compatible with zero.
 3140 However, the size of a potential bias can vary depending on the input parame-
 3141 ters. The size of the bias has been investigated with alternate input values of
 3142 $(\gamma, r_B^{DK^\pm}, \delta_B^{DK^\pm}, r_B^{D\pi^\pm}, \delta_B^{D\pi^\pm})$, obtaining the results in Table 5.18. A systematic
 3143 uncertainty due to a potential, small bias is calculated as the difference between
 3144 the maximum and minimum bias for a given parameter. The uncertainty assigned
 3145 in this way is very small in general, and less than 0.1×10^{-2} for all parameters.

Table 5.18: Biases observed with alternative input parameters and the systematic uncertainty assigned for the bias correction. All numbers are quoted with an implicit $\times 10^{-2}$.

Input ($\gamma, r_B^{DK^\pm}, \delta_B^{DK^\pm}, r_B^{D\pi^\pm}, \delta_B^{D\pi^\pm}$)	$x_-^{DK^\pm}$	$y_-^{DK^\pm}$	$x_+^{DK^\pm}$	$y_+^{DK^\pm}$	$x_\xi^{D\pi^\pm}$	$y_\xi^{D\pi^\pm}$
(72, 0.080, 117, 0.005, 288)	-0.02	-0.01	-0.02	-0.02	0.03	0.00
(75, 0.100, 130, 0.005, 300)	-0.03	-0.04	-0.00	0.02	0.01	-0.03
(82, 0.112, 144, 0.005, 330)	0.00	-0.01	0.00	0.03	-0.03	0.02
(71, 0.099, 129, 0.005, 300)	-0.02	-0.04	-0.00	-0.00	0.05	-0.00
Syst. uncertainty	0.04	0.03	0.02	0.04	0.09	0.05

3146 5.6.11 Charmless backgrounds

3147 As discussed in Section 5.3.1, a small number of charmless background decays
 3148 survive the D flight distance cut. In this section the systematic uncertainty related
 3149 to those is assessed, in a series of pseudoexperiments. Toy datasets are generated,
 3150 where a charmless background component is included, using the yields and shapes
 3151 obtained in the studies of Section 5.3.1. The Dalitz-bin distribution is obtained
 3152 by repeating the fits of that section for each bin individually. These datasets are
 3153 subsequently fitted back using the default model, which does not include a charmless
 3154 component. No statistically significant bias is observed.

3155 The study described above does not allow for charge-asymmetries in the charmless
 3156 backgrounds, in terms of overall yields and phase-space distributions. These
 3157 asymmetries are likely to be present, due to large local CP -violation in regions of
 3158 phase space in B^\pm decays to hadrons [159, 160]. The yields in the data-driven studies
 3159 of Section 5.3.1 are not large enough to assess asymmetries, let alone asymmetric
 3160 bin distributions with any degree of statistical precision. Instead, an extreme-case
 3161 scenario is investigated, where *all* the charmless background is added to either the B^+
 3162 or B^- data sample in generation. In both cases, no statistically significant biases are
 3163 observed, and it is concluded that the impact of charmless background is negligible.

3164 5.6.12 Summary of systematic uncertainties

3165 The complete set of included systematic uncertainties are summarised in Table 5.19.
 3166 It can be seen that the measurement is statistically limited. The correlation matrix
 3167 pertaining to the LHCb related systematics is given in Table 5.20. For studies where
 3168 the systematic uncertainty is obtained by repeating fits to data multiple times while
 3169 varying some input, the correlation matrix from the correlations of the fitted central
 3170 values. For studies that are based on generating a large number of toy datasets and

Table 5.19: Overview of all sources of uncertainty on the measurement.

Source	All uncertainties are quoted with implicit: $\times 10^{-2}$					
	$\sigma(x_-^{DK\pm})$	$\sigma(y_-^{DK\pm})$	$\sigma(x_+^{DK\pm})$	$\sigma(y_+^{DK\pm})$	$\sigma(x_\xi^{D\pi\pm})$	$\sigma(y_\xi^{D\pi\pm})$
Statistical	0.96	1.14	0.96	1.20	1.99	2.34
Strong-Phase inputs	0.23	0.35	0.18	0.28	0.14	0.18
Efficiency correction of (c_i, s_i)	0.11	0.05	0.05	0.10	0.08	0.09
Mass-shape parameters	0.05	0.08	0.03	0.08	0.16	0.17
Mass-shape bin dependence	0.05	0.07	0.04	0.08	0.07	0.09
Lowmass physics effects	0.04	0.10	0.15	0.05	0.10	0.09
CP violation of K_S^0	0.03	0.04	0.08	0.08	0.09	0.46
D mixing	0.04	0.01	0.00	0.02	0.02	0.01
PID efficiencies	0.03	0.03	0.01	0.05	0.02	0.02
Fixed yield ratios	0.05	0.06	0.03	0.06	0.02	0.02
Dalitz-bin migration	0.04	0.08	0.08	0.11	0.18	0.10
Bias correction	0.04	0.03	0.02	0.04	0.09	0.05
Small backgrounds	0.11	0.16	0.13	0.12	0.08	0.13
Total LHCb systematic	0.20	0.25	0.24	0.26	0.32	0.54
Total systematic	0.31	0.43	0.30	0.38	0.35	0.57

Table 5.20: Total LHCb-related systematic uncertainties and their correlation matrix.

Uncertainty ($\times 10^{-2}$)						
	$x_-^{DK\pm}$	$y_-^{DK\pm}$	$x_+^{DK\pm}$	$y_+^{DK\pm}$	$x_\xi^{D\pi\pm}$	$y_\xi^{D\pi\pm}$
σ	0.20	0.25	0.24	0.26	0.32	0.54

Correlations						
	$x_-^{DK\pm}$	$y_-^{DK\pm}$	$x_+^{DK\pm}$	$y_+^{DK\pm}$	$x_\xi^{D\pi\pm}$	$y_\xi^{D\pi\pm}$
$x_-^{DK\pm}$	1.000	0.864	0.734	0.897	0.349	0.318
$y_-^{DK\pm}$		1.000	0.874	0.903	0.408	0.362
$x_+^{DK\pm}$			1.000	0.771	0.563	0.447
$y_+^{DK\pm}$				1.000	0.507	0.451
$x_\xi^{D\pi\pm}$					1.000	0.484
$y_\xi^{D\pi\pm}$						1.000

³¹⁷¹ determining the average bias, the correlation of a systematic on two observables
³¹⁷² is taken to be +100 % if the biases are in the same direction, and -100 % if they
³¹⁷³ are in opposite directions. The total systematic correlation matrix, including both
³¹⁷⁴ LHCb-related systematics and that of the strong-phase inputs, is given in Table 5.21.

³¹⁷⁵ The studies described in this section also allow for an estimate of the systematic
³¹⁷⁶ uncertainties on the \mathcal{R}_i parameters of Eq. (5.23) or, equivalently the F_i parameters,
³¹⁷⁷ in a completely analogous manner to how the uncertainty on the CP -violation
³¹⁷⁸ observables was assigned. In all cases, however, the systematic uncertainty is found
³¹⁷⁹ to be much small than the statistical uncertainties that were given in Table 5.10.

Table 5.21: Total systematic uncertainties and their correlation matrix, including contributions due to strong-phase inputs as well as LHCb-related uncertainties.

Uncertainty ($\times 10^{-2}$)						
	$x_-^{DK\pm}$	$y_-^{DK\pm}$	$x_+^{DK\pm}$	$y_+^{DK\pm}$	$x_\xi^{D\pi\pm}$	$y_\xi^{D\pi\pm}$
σ	0.31	0.43	0.30	0.38	0.35	0.57

Correlations						
	$x_-^{DK\pm}$	$y_-^{DK\pm}$	$x_+^{DK\pm}$	$y_+^{DK\pm}$	$x_\xi^{D\pi\pm}$	$y_\xi^{D\pi\pm}$
$x_-^{DK\pm}$	1.000	0.301	0.156	0.576	0.265	0.231
$y_-^{DK\pm}$		1.000	0.437	0.218	0.183	0.170
$x_+^{DK\pm}$			1.000	0.445	0.414	0.310
$y_+^{DK\pm}$				1.000	0.353	0.243
$x_\xi^{D\pi\pm}$					1.000	0.502
$y_\xi^{D\pi\pm}$						1.000

3180 The central values, statistical, and systematic uncertainties of the \mathcal{R}_i parameters
3181 have been made public in Ref. [1] because they can be employed in future LHCb
3182 measurements, as discussed in Section 5.5.2.

3183 5.7 Obtained constraints on γ

3184 The measured values of $(x_\pm^{DK}, y_\pm^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi})$ can be used to put constraints on the
3185 possible values of the CKM angle γ and the hadronic nuisance parameters $r_B^{DK\pm}$,
3186 $\delta_B^{DK\pm}$, $r_B^{D\pi\pm}$, and $\delta_B^{D\pi\pm}$. This is handled using the `gammacombo` package, which is also
3187 used to combine all measurements of γ made by the LHCb collaboration [49, 161].

3188 5.7.1 Statistical approach

3189 The optimal central values determined in a maximum likelihood fit. The set of
3190 all observables for which a measurement has been made is denoted A , and the set
3191 of underlying physics parameters is denoted θ . The physics parameters of course
3192 determine the probability density function of measurement results of A , $f(A|\theta)$.
3193 Given a specific set of measurement results, A_{obs} , a likelihood function is defined

$$\mathcal{L}(\theta|A_{\text{obs}}) = f(A_{\text{obs}}|\theta) \quad (5.35)$$

3194 and the estimate of θ is the set of parameters that maximize the likelihood

$$\hat{\theta} = \arg \max_{\theta} \mathcal{L}(\theta|A_{\text{obs}}). \quad (5.36)$$

3195 In practice, a χ^2 function is defined

$$\chi^2(\theta|A_{\text{obs}}) = -2 \ln \mathcal{L}(\theta|A_{\text{obs}}) \quad (5.37)$$

3196 and minimized instead. In the specific case where the likelihood profile is Gaussian,
3197 it can be described by the simple expression

$$\chi^2(\theta|A_{\text{obs}}) = (A_{\text{obs}} - A(\theta))^T \Sigma_{A_{\text{obs}}}^{-1} (A_{\text{obs}} - A(\theta)) + c, \quad (5.38)$$

3198 where $\Sigma_{A_{\text{obs}}}$ is the *measured* covariance matrix for the observables (used as an
3199 estimate of the true, underlying covariance matrix), $A(\theta)$ denotes the value of the
3200 observables expressed in terms of the underlying physics parameters, and c is a
3201 constant that is independent of θ . In the specific case considered here

$$\begin{aligned} A &= (x_-^{DK}, y_-^{DK}, x_+^{DK}, y_+^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi}) \\ \theta &= (\gamma, r_B^{DK\pm}, \delta_B^{DK\pm}, r_B^{D\pi\pm}, \delta_B^{D\pi\pm}). \end{aligned} \quad (5.39)$$

3202 The likelihood scan presented in Section 5.5.2 proved that the Gaussian expression
3203 in Eq. (5.38) provides an excellent description of the likelihood profile of the
3204 measurement, when $\Sigma_{A_{\text{obs}}}$ is taken to be the covariance matrix obtained in that
3205 section. Thus, the χ^2 function defined in Eq. (5.38) is minimised to determine
3206 the best estimate of γ .

3207 Two different methods are employed to construct confidence regions for the
3208 observables of interest, known within the `gammacombo` framework as the PROB and
3209 PLUGIN methods. Both methods aim to construct confidence regions for some
3210 subset, ϕ , of the full parameter set θ . The remaining parameters, dubbed nuisance
3211 parameters below, are denoted $\eta = \theta \setminus \phi$. In practice, ϕ most often denotes a single
3212 parameter, and of special interest is of course the case where $\phi = \gamma$. Both methods
3213 aim to solve the problem that due to the number of parameters in θ (five in the case
3214 considered here, but up to 40 in the latest LHCb combination [50]), it is not feasible
3215 to derive the confidence regions from a full-fledged Neumann construction [162].
3216 Under assumptions discussed below, the methods achieve reasonable coverage
3217 nonetheless, ie. had the measurement been repeated many times, the confidence
3218 region is expected to cover the true parameter(s) with a probability at least at large
3219 as the quoted confidence level (CL), independently of the true parameter value.
3220 The presentation follows the `gammacombo` manual [161].

3221 The PROB method is a simple profile-likelihood method. The minimum value
3222 of the χ^2 function is denoted $\chi^2_{\min} \equiv \chi^2(\hat{\theta}|A_{\text{obs}})$. To evaluate the CL for a specific
3223 value (set of values) of ϕ_0 , the χ^2 function is again minimised, this time under

3224 the constraint that $\phi = \phi_0$, resulting in a new minimum $\hat{\theta}' = (\phi_0, \hat{\eta}')$. In the
3225 approximation that all likelihoods are exactly Gaussian, the variable

$$\Delta\chi^2(\phi_0|A_{\text{obs}}) = \chi^2(\hat{\theta}'|A_{\text{obs}}) - \chi^2_{\min} \quad (5.40)$$

3226 follows a χ^2 distribution with n degrees of freedom, where n is the number of
3227 parameters in ϕ [31]. This can be used to evaluate CL at that point as

$$CL(\phi_0|A_{\text{obs}}) = F_n(\Delta\chi^2(\phi_0|A_{\text{obs}})) \quad (5.41)$$

3228 where F_n is the cumulative distribution function of a χ^2 distribution with n degrees
3229 of freedom. The method takes its colloquial name from the fact that this function
3230 is named **Prob** in the **ROOT** package. Confidence regions can be defined by scanning
3231 the values of ϕ_0 over a region of interest. These confidence regions assume that
3232 the estimates $\hat{\theta}$ follow a Gaussian distribution centred on the true values, which is
3233 the case for maximum likelihood estimates in asymptotically large samples [163];
3234 in other cases they may not have good coverage properties. Given the Gaussian
3235 shape obtained in the likelihood scan of Section 5.5.2 the confidence regions are
3236 likely to be well behaved in the case considered here.

3237 However, for the purpose of comparing to the combination of several LHCb
3238 measurements in Section 5.5.3 below, the **PLUGIN** method is necessary. The method
3239 is described in Ref. [164], based on the hybrid resampling method presented in
3240 Ref. [165]. It foregoes the assumption that $\Delta\chi^2$ follows a χ^2 distribution, and
3241 instead estimates the distribution in a bootstrapping scheme. The procedure is as
3242 follows: the values of $\hat{\theta}$, $\hat{\theta}'$, and $\Delta\chi^2(\phi_0|A_{\text{obs}})$ are determined as described above;
3243 then the following steps are carried out a number, N_{toys} , of times

- 3244 1. Generate a "toy" result, A_{toy}^i , following the distribution $f(A|\hat{\theta}')$
- 3245 2. Determine $\Delta\chi^2(\phi_0|A_{\text{toy}}^i)$ by minimising the χ^2 function for the results A_{toy}^i
3246 twice, once where all parameters in θ are free, and once where $\phi = \phi_0$ is
3247 enforced

3248 Then the CL is defined by

$$CL(\phi_0) = 1 - \frac{N(\Delta\chi^2(\phi_0|A_{\text{obs}}) < \Delta\chi^2(\phi_0|A_{\text{toy}}^i))}{N_{\text{toys}}}. \quad (5.42)$$

3249 While the coverage properties are not proven, evidence is presented in terms of
3250 asymptotic results and simulation studies in Refs. [164, 165]. The coverage properties
3251 have also been investigated in relation to LHCb combinations, and the intervals
3252 were found to perform well in most cases [49].

3253 **5.7.2 Interpretation results**

3254 The central values and confidence regions obtained for the physics parameters are

$$\begin{aligned}\gamma &= (68.7^{+5.2}_{-5.1})^\circ, \\ r_B^{DK^\pm} &= 0.0904^{+0.0077}_{-0.0075}, \\ \delta_B^{DK^\pm} &= (118.3^{+5.5}_{-5.6})^\circ, \\ r_B^{D\pi^\pm} &= 0.0050^{+0.0017}_{-0.0017}, \\ \delta_B^{D\pi^\pm} &= (291^{+24}_{-26})^\circ,\end{aligned}\tag{5.43}$$

3255 where the quoted uncertainties are obtained via the PLUGIN method. The one-
 3256 dimensional CL plots are shown in Fig. 5.57. It is also clear that the PROB and
 3257 PLUGIN confidence regions agree well; this is expected given the Gaussian likelihood.
 3258 A series of two-dimensional confidence regions are shown in Fig. 5.58, where it
 3259 can be seen that the observables define a single solution for γ as expected. It
 3260 is worth noticing that the uncertainty of this measurement alone is on par with
 3261 the current world average, due to the increased data sample, and the significant
 3262 reduction of systematic uncertainties due to the new measurement strategy and
 3263 updated inputs from BESIII.

3264 The contribution to the uncertainty on γ from each of the statistical, strong-
 3265 phase-related, and LHCb-related uncertainties in isolation can be estimated by
 3266 repeating the interpretation while only including subsets of the uncertainties on
 3267 the input parameters. Such studies have been performed using the PROB method.
 3268 Running with statistical uncertainties only yields an uncertainty on γ of 5.05° .
 3269 Including only the statistical and LHCb-related systematic uncertainties yields an
 3270 uncertainty on γ of 5.08° , suggesting that the LHCb-related systematics contribute
 3271 an uncertainty of 0.6° . This is a reduction compared to earlier analyses, where
 3272 the contribution was about 2° . A significant contribution to the improvement is
 3273 the efficiency-related systematic that has been avoided by promoting $B^\pm \rightarrow D\pi^\pm$
 3274 to a signal channel. Including only the statistical and the strong-phase-related
 3275 uncertainties leads to an uncertainty on γ of 5.09° , showing the strong-phase-related
 3276 uncertainty to be 0.6° , somewhat lower than the expectation of 1.2° presented in
 3277 Ref. [74]. This is partly because the uncertainty estimate of that paper does not
 3278 take into account the use of the $D \rightarrow K_S^0 K^+ K^-$ channel, and partly because the
 3279 uncertainty estimate depends on the specific central values.

3280 The obtained statistical uncertainty on γ is in excellent agreement with the
 3281 expectation from pseudoexperiments. The interpretation procedure outlined above
 3282 has been performed for each of the pseudoexperiments performed to establish the

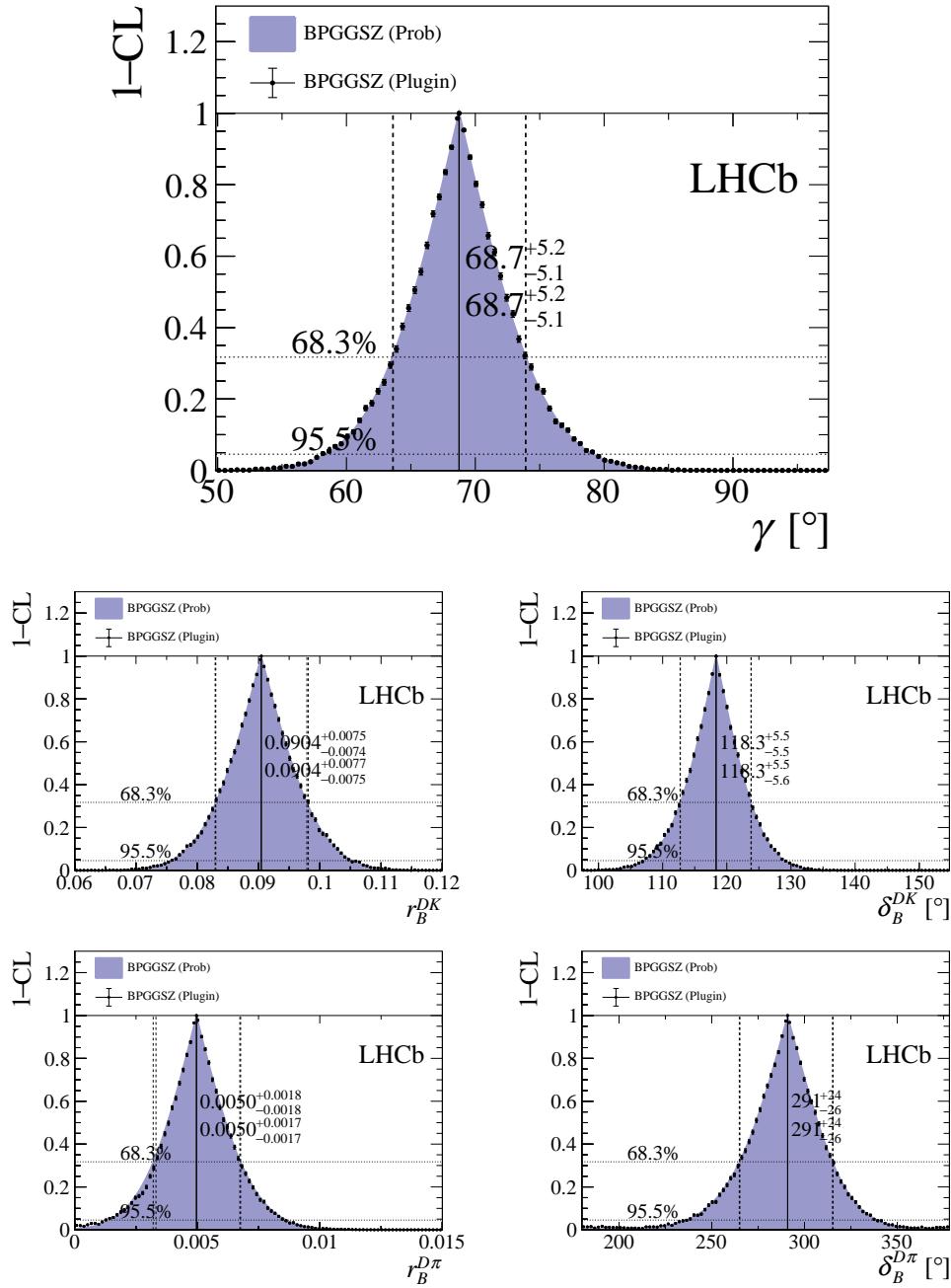


Figure 5.57: Confidence levels for the physics parameters of interest. The solutions are written on the plots, where the top number is given with PROB uncertainties and the bottom number with PLUGIN uncertainties.

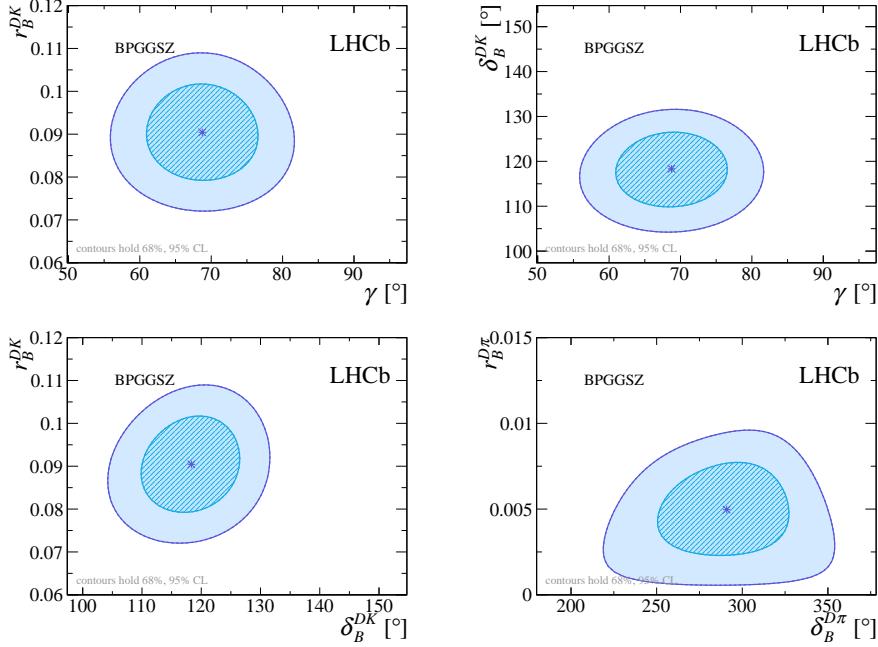


Figure 5.58: The 68 % and 95 % confidence regions for combinations of the physics parameters of interest, as obtained from the results of this measurement. The regions are calculated via the PROB method of `gammacombo`.

feasibility of the CP fit in Section 5.5.1 (including only statistical uncertainties on the observables) and the central 90 % interval of the obtained uncertainties is $[4.4^\circ, 6.0^\circ]$. Furthermore, interpretation of γ is found to be unbiased, and the uncertainties on γ show the expected coverage properties. Similar studies have been carried out where no background decays are included in the generated toy data sets. In this case, the precision on γ is improved by about 30 %. These pseudo experiments do suggest small biases on r_B^{DK} and $r_B^{D\pi}$ in the interpretation step, of 10 % and 25 % of the statistical uncertainty, respectively. In absolute terms, this corresponds to positive biases of $\Delta r_B^{DK} = +0.0008$ (ie. 1 % of the central value) and $\Delta r_B^{D\pi} = +0.0004$ (about 8 % of the central value). These biases have been observed before [49], and have been determined to arise due to the constraint that $r_B > 0$. The results quoted in Eq. (5.43) have *not* been corrected for the bias and should be interpreted accordingly. The results from this thesis that will be used in combinations of γ measurements by LHCb (and others) is the measured observables, $(x_\pm^{DK}, y_\pm^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi})$, and these were shown to be determined without a bias in Section 5.5.

5.7.3 Compatibility with other measurements

It is worth comparing the obtained constraints on the physics parameters with the information available from other measurements, made at the B factories and by

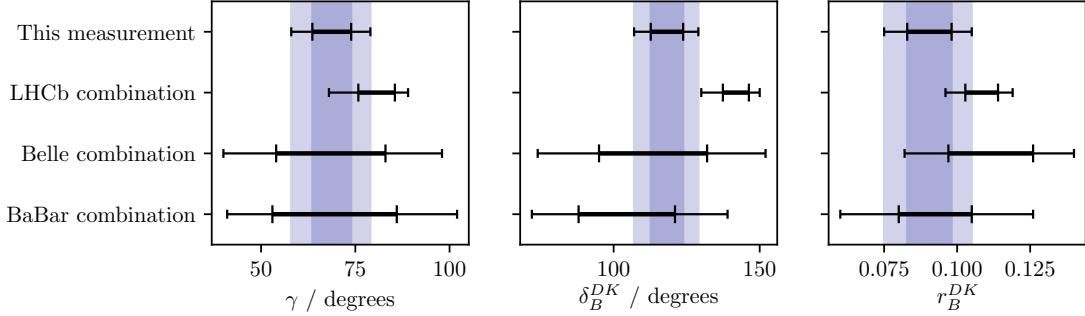


Figure 5.59: Comparison of the 1σ and 2σ confidence intervals obtained results for γ and the physics parameters relating to $B^\pm \rightarrow DK^\pm$ decays, with those from the combinations of γ measurements by the Belle [47] and BaBar [48] collaborations, and the 2018 combination of LHCb results [50] where the BPGGSZ measurements have been excluded.

the LHCb collaboration using other decay channels. This comparison is made for γ and the hadronic parameters in the $B^\pm \rightarrow DK^\pm$ decay in Fig. 5.59, comparing to the results of the combinations of γ measurements by the Belle [47] and BaBar [48] collaborations presented in 2013, and the 2018 combination of LHCb results [50]. For this purpose, the LHCb combination is re-performed, removing the input from earlier BPGGSZ measurements that use $B^\pm \rightarrow DK^\pm$ decays, because they were made using data that is re-analysed in the present thesis; thus they need to be excluded to make the results that are compared independent. The combination employs the same statistical method outlined above, with the exception that the likelihood now depends on observables measured in a number of different analyses. The included measurements are summarised in Table 5.22. It can be seen in Fig. 5.59 that the results obtained in this thesis agree well with the Belle and BaBar results, but are in some tension with the 2018 LHCb combination, especially for the δ_B^{DK} parameter.

The level of compatibility can be quantified by calculating the three-dimensional χ^2 of the BPGGSZ results and those of the LHCb combination (without the earlier BPGGSZ measurements), with respect to the best fit values of $(\gamma, r_B^{DK\pm}, \delta_B^{DK\pm})$ when all measurements are combined. The two-dimensional confidence regions obtained in these three cases are compared in Fig. 5.60, where some tension in $r_B^{DK\pm}$ and $\delta_B^{DK\pm}$ is visible again. The calculation is based on the PLUGIN uncertainties; for the LHCb combination these uncertainty estimates are slightly larger than the ones obtained via the PROB method. One obtains $\chi^2 = \chi_{GGSZ}^2 + \chi_{LHCb}^2 = 0.7 + 9.1 = 9.8$, which for 3 degrees of freedom correspond to a p -value of 2 %, or a 2.3σ deviation. However, this tension is expected to be reduced when other measurements in the LHCb combination are updated to include results based on the full Run 1 and 2 data

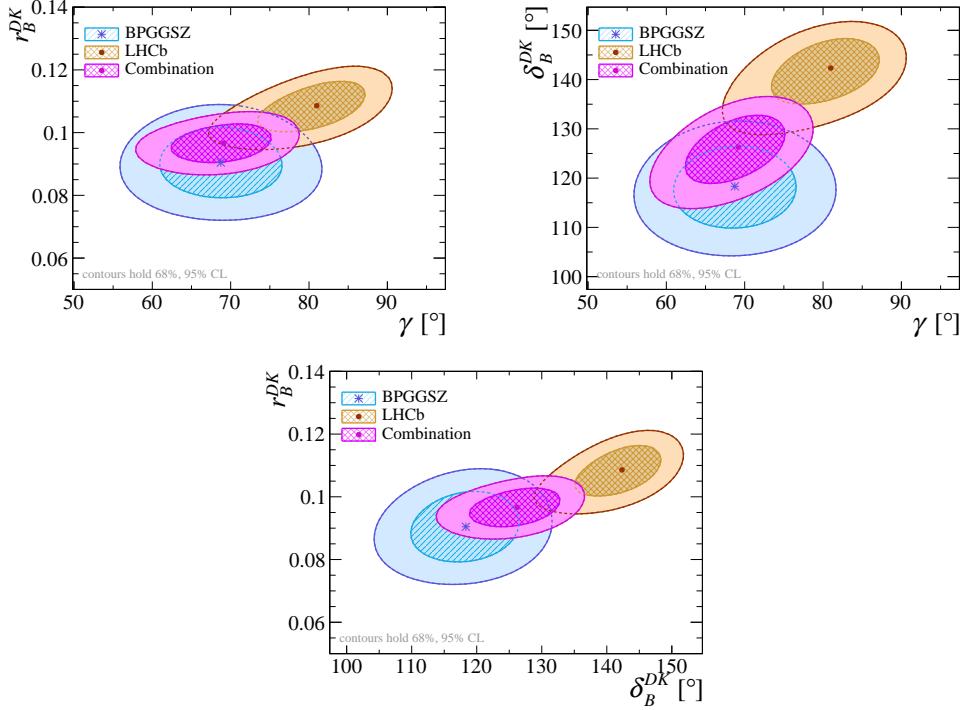


Figure 5.60: The 68 % and 95 % confidence regions for $(\gamma, r_B^{DK\pm})$, $(\gamma, \delta_B^{DK\pm})$, and $(\delta_B^{DK\pm}, r_B^{DK\pm})$ for this measurement, the 2018 LHCb combination [50] where the BPGGSZ results have been excluded, and the combination thereof, calculated via the PROB method of `gammacombo`.

set. The most important update is that of the two-body ADS/GLW measurement in $B^\pm \rightarrow DK^\pm$ decays because that measurement, and the BPGGSZ measurement presented in this thesis, have the largest impact in the combination.

The latest LHCb combination in which $B^\pm \rightarrow D\pi^\pm$ parameters were determined is from 2016 [49]. Two solutions existed for $(r_B^{D\pi^\pm}, \delta_B^{D\pi^\pm})$ which made the interpretation problematic. As can be seen in Fig. 5.61, the measurement presented in this thesis picks out one of these solutions, with which it is in excellent agreement. This solution agrees with the theoretically expected value of $r_B^{D\pi^\pm} \sim 0.005$ [51]. Thus, the inclusion of the results presented here are expected to lead to a much less problematic inclusion of the $B^\pm \rightarrow D\pi^\pm$ channel in future LHCb combinations.

Table 5.22: List of the LHCb measurements used in the combination that the results obtained in the present thesis is compared to. These correspond to the 2018 LHCb combination [50], except that the earlier BPGGSZ results made with $B^\pm \rightarrow DK^\pm$ decays have not been included in the comparison. In the method column, TD stands for "time-dependent", the ADS/GLW/BPGGSZ acronyms were defined in Chapter 2, and references are provided for the remaining methods.

B decay	D decay	Method	Ref.	Data set
$B^+ \rightarrow DK^+$	$D \rightarrow h^+h^-$	GLW	[154]	2011-16
$B^+ \rightarrow DK^+$	$D \rightarrow h^+h^-$	ADS	[151]	2011-12
$B^+ \rightarrow DK^+$	$D \rightarrow h^+\pi^-\pi^+\pi^-$	GLW/ADS	[151]	2011-12
$B^+ \rightarrow DK^+$	$D \rightarrow h^+h^-\pi^0$	GLW/ADS	[166]	2011-12
$B^+ \rightarrow DK^+$	$D \rightarrow K_S^0 K^+ \pi^-$	GLS [167]	[168]	2011-12
$B^+ \rightarrow D^*K^+$	$D \rightarrow h^+h^-$	GLW	[154]	2011-16
$B^+ \rightarrow DK^{*+}$	$D \rightarrow h^+h^-$	GLW/ADS	[169]	2011-16
$B^+ \rightarrow DK^{*+}$	$D \rightarrow h^+\pi^-\pi^+\pi^-$	GLW/ADS	[169]	2011-16
$B^+ \rightarrow DK^+\pi^+\pi^-$	$D \rightarrow h^+h^-$	GLW/ADS	[170]	2011-12
$B^0 \rightarrow DK^{*0}$	$D \rightarrow K^+\pi^-$	ADS	[171]	2011-12
$B^0 \rightarrow DK^+\pi^-$	$D \rightarrow h^+h^-$	GLW-Dalitz [172]	[153]	2011-12
$B^0 \rightarrow DK^{*0}$	$D \rightarrow K_S^0 \pi^+\pi^-$	BPGGSZ	[67]	2011-12
$B_s^0 \rightarrow D_s^\mp K^\pm$	$D_s^\pm \rightarrow h^+h^-\pi^+$	TD	[173]	2011-12
$B^0 \rightarrow D^\mp\pi^\pm$	$D^\pm \rightarrow K^+\pi^-\pi^+$	TD	[174]	2011-12
Measurements included in Ref. [50] but not in the present comparison				
$B^+ \rightarrow DK^+$	$D \rightarrow K_S^0 h^+h^-$	BPGGSZ	[82]	2011-12
$B^+ \rightarrow DK^+$	$D \rightarrow K_S^0 h^+h^-$	BPGGSZ	[3]	2015-16

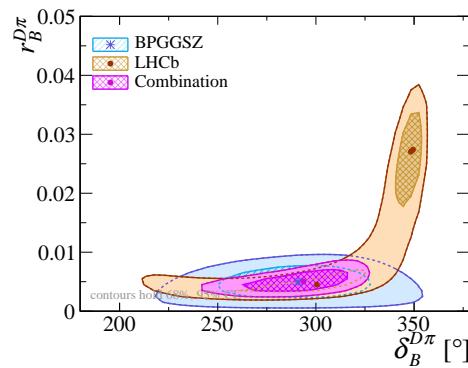


Figure 5.61: The 68 % and 95 % confidence regions for $(\delta_B^{D\pi^\pm}, r_B^{D\pi^\pm})$ obtained from the results of this measurement, in the 2016 LHCb combination [49], and the combination thereof, calculated via the PROB method of `gammacombo`.

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Summary and outlook

3337 Say something clever

Appendices

A

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Projections of the main fit to data



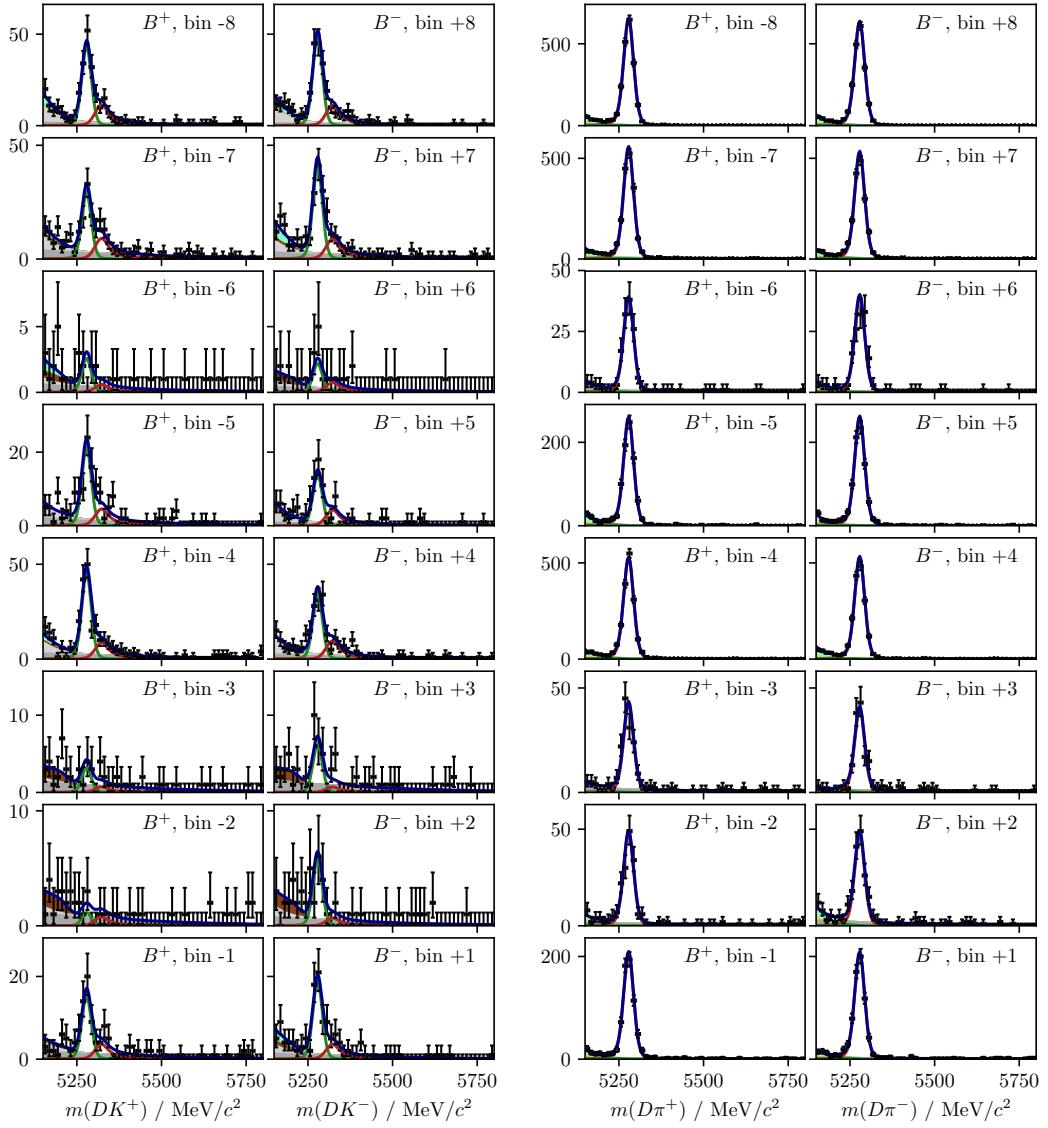


Figure A.1: Projections of the main fit to data described in Section 5.5. The two columns on the left show $B^\pm \rightarrow DK^\pm$ candidates, split by charge, while the two columns on the right show $B^\pm \rightarrow D\pi^\pm$ candidates. These projections are for the $D \rightarrow K_S^0 \pi^+ \pi^-$ mode where the K_S^0 meson is in the LL category.

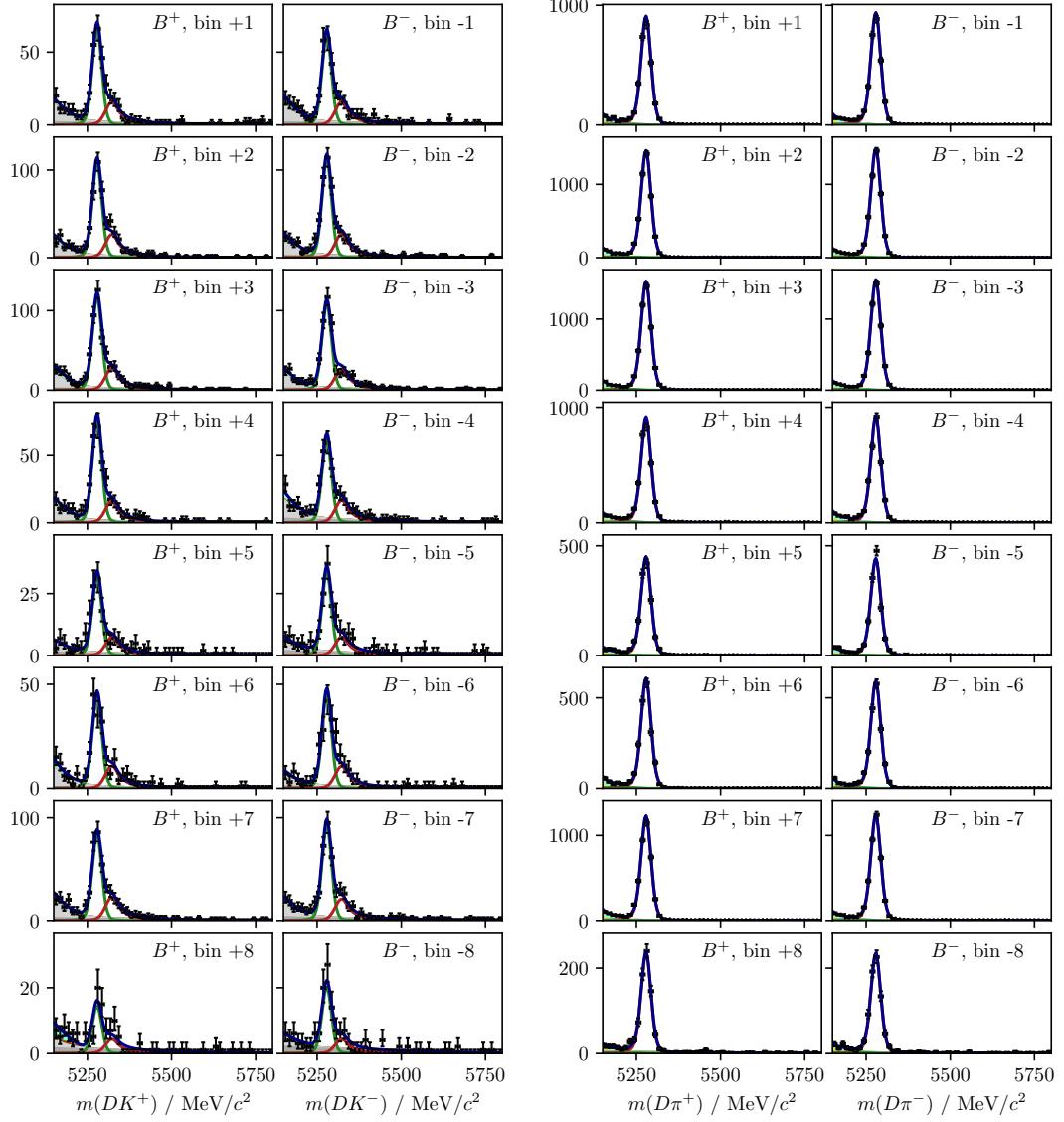


Figure A.2: Projections of the main fit to data described in Section 5.5. The two columns on the left show $B^\pm \rightarrow DK^\pm$ candidates, split by charge, while the two columns on the right show $B^\pm \rightarrow D\pi^\pm$ candidates. These projections are for the $D \rightarrow K_S^0 \pi^+ \pi^-$ mode where the K_S^0 meson is in the LL category.

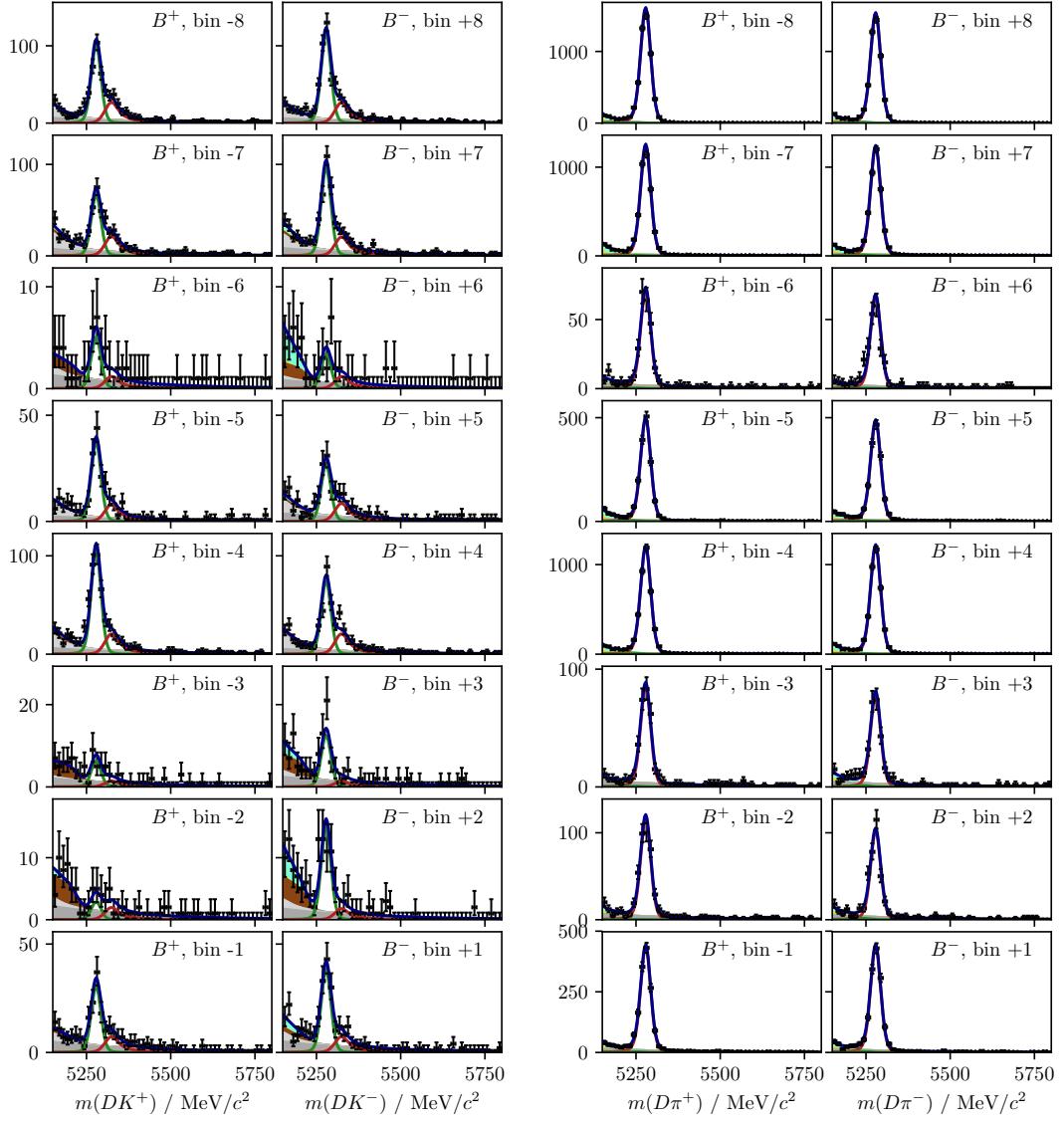


Figure A.3: Projections of the main fit to data described in Section 5.5. The two columns on the left show $B^\pm \rightarrow DK^\pm$ candidates, split by charge, while the two columns on the right show $B^\pm \rightarrow D\pi^\pm$ candidates. These projections are for the $D \rightarrow K_S^0 \pi^+ \pi^-$ mode where the K_S^0 meson is in the DD category.

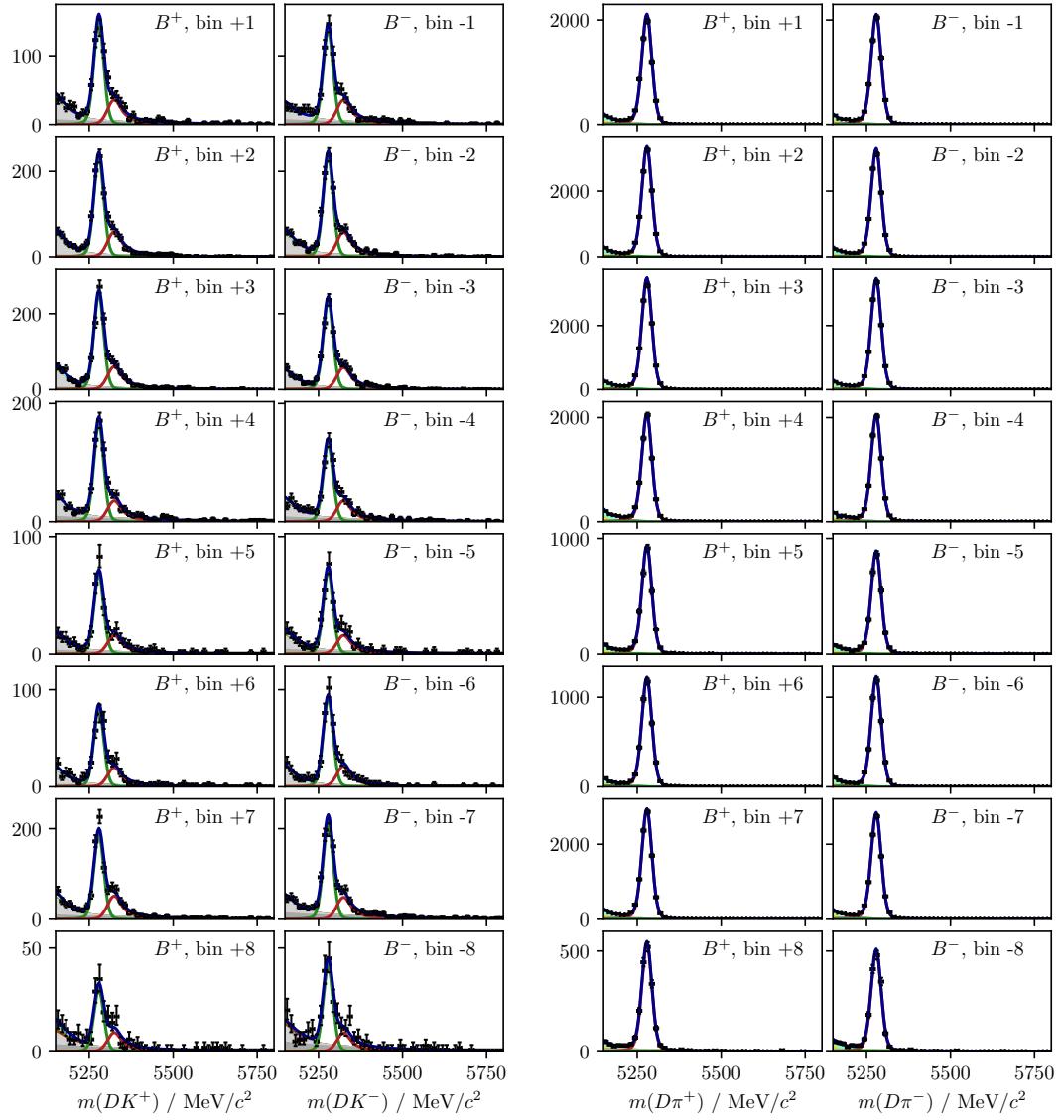


Figure A.4: Projections of the main fit to data described in Section 5.5. The two columns on the left show $B^\pm \rightarrow DK^\pm$ candidates, split by charge, while the two columns on the right show $B^\pm \rightarrow D\pi^\pm$ candidates. These projections are for the $D \rightarrow K_S^0\pi^+\pi^-$ mode where the K_S^0 meson is in the DD category.

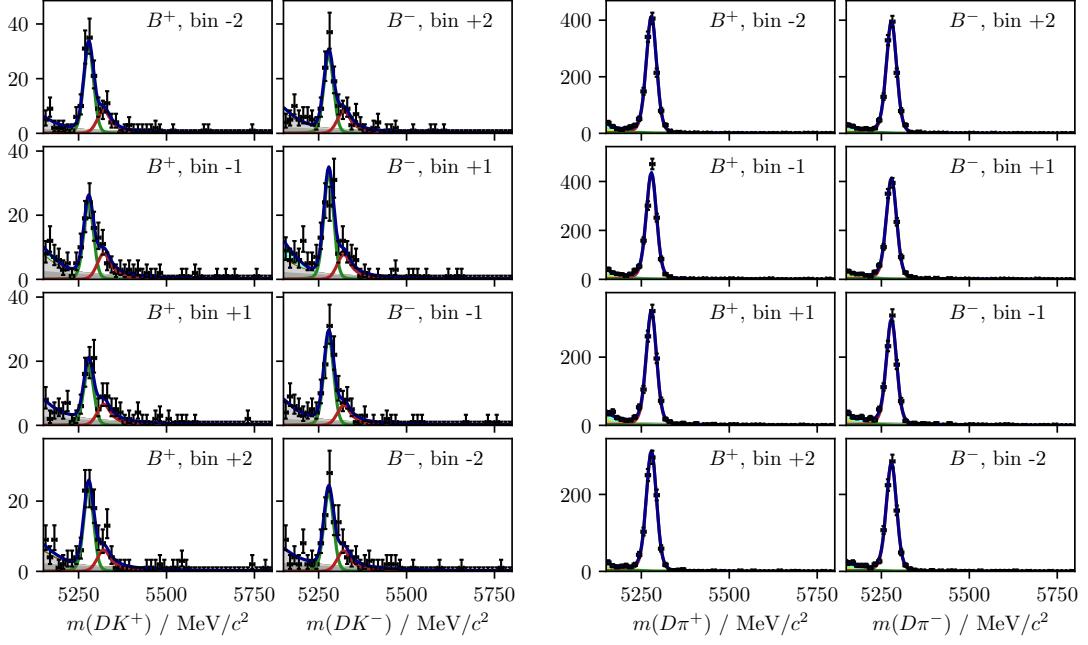


Figure A.5: Projections of the main fit to data described in Section 5.5. The two columns on the left show $B^{\pm} \rightarrow DK^{\pm}$ candidates, split by charge, while the two columns on the right show $B^{\pm} \rightarrow D\pi^{\pm}$ candidates. These projections are for the $D \rightarrow K_S^0 K^+ K^-$ mode where the K_S^0 meson is in the LL category.

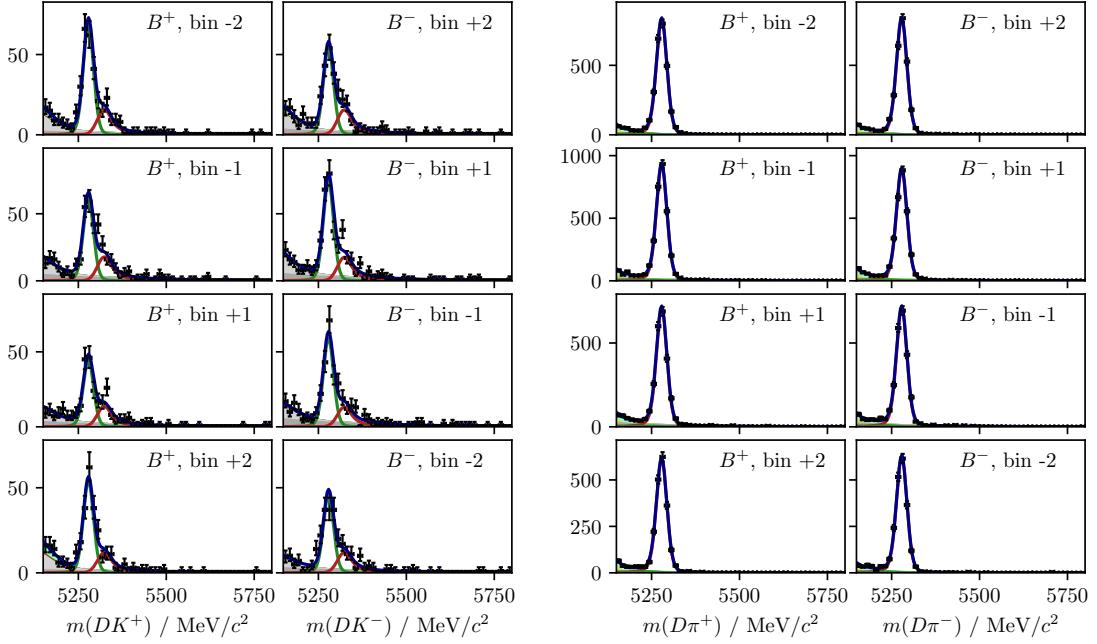


Figure A.6: Projections of the main fit to data described in Section 5.5. The two columns on the left show $B^{\pm} \rightarrow DK^{\pm}$ candidates, split by charge, while the two columns on the right show $B^{\pm} \rightarrow D\pi^{\pm}$ candidates. These projections are for the $D \rightarrow K_S^0 K^+ K^-$ mode where the K_S^0 meson is in the DD category.

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