Suitably impressive thesis title

Mikkel Bjørn

St. Anne's College

University of Oxford

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Acknowledgements

I would very much like to thank myself, for doing such a huge and impressive amount of work.

Abstract

Oh my have I done a lot of things. I have made the best measurement of γ evah! I'll describe that in more detail at an appropriate time.

Contents

Preface

The work presented in this thesis has been resulted in two papers, either under review or published in the Journal of High Energy Physics. These are

[1] *Measurement of the CKM angle γ using $B^{\pm} \to [K^0_S h^+ h^-]_D h^{\pm}$ decays*, submitted to JHEP.
This paper describes a measurement of the CKM angle γ using pp collision data taken with the LHCb experiment during the Run 1 of the LHC, in 2011 and 2012, and during the full Run 2, in 2015–2018. The measurement uses the decay channels $B^{\pm} \to Dh^{\pm}$ where $D \to K^0_S h'^+ h'^-$, in which h and h' denotes pions or kaons. It obtains a value of $\gamma = (?\pm?)^{\circ}$, which constitutes the world's best single-measurement determination of γ. The work is the main focus of this thesis and described in detail in Chapter 5.

[2] *CP violation and material interaction of neutral kaons in measurements of the CKM angle γ using $B^{\pm} \to DK^{\pm}$ decays where $D \to K^0_S \pi^+ \pi^-$*, JHEP 19 (2020) 106.
This paper describes a phenomenological study of the impact of neutral kaon *CP* violation and material interaction on measurements of γ. With the increased measurement precision to come in the near future, an understanding of these effects is crucial, especially in the context of $B \to D\pi$ decays; however no detailed study had been published at the start of this thesis. The study is the subject of Chapter 4.

All of the work described in this thesis is my own, except where clearly referenced to others. Furthermore, I contributed significantly to an analysis of $B^{\pm} \to DK^{\pm}$ decays with LHCb data taken in 2015 and 2016, now published in

[3] *Measurement of the CKM angle γ using $B^{\pm} \to DK^{\pm}$ with $D \to K^0_S \pi^+ \pi^-$ $K^0_S K^+ K^-$ decays*, JHEP 08 (2018) 176.

I was responsible for the analysis of the signal channel, whereas the control channel was analysed by Nathan Jurik. The measurement is superseded by that of Ref. [1] and is not described in detail in the thesis.

Le roi est mort, vive le roi!

— Traditional French proclamation at the death of one monarch and the ascension of a new

1 Introduction

"The King is dead, long live the King!". Idea of the quote: the Standard Model is dead but, unlike French monarchs, sadly lives on forever. What a conundrum.

Some savoury flavour text on why precision flavour physics is important, and where it fits into the grand scheme of things. Can this be made sexy? Good question.

1.1 Structure of the thesis

2
Theoretical background

Some things will need to be said, about *CP* violation and such.

2.1 The C and P symmetries, and their violation

- What does C and P do, fundamentally?
- How does that translate to QFTs?
- *CP* violation historically: kaons, B physics, now even D physics!
- Three sorts of CP violation, and some early equations on general CP violation (leading on the K^0_S chapter)

2.2 CP violation in the Standard Model

Then we turn to the SM, where *CP* violation is fundamentally included.

2.2.1 The CKM matrix and the Unitarity Triangle

- Define the CKM matrix and count parameters; Wolfenstein parameterisation
- How does that give rise to CP violation? Interference is necessary: two sorts. Relate to general discussion earlier
- Unitarity constraints and the triangles, pretty pictures..

2.2.2 Measuring the CKM angle γ in tree level decays

The amplitude for the decay $B^- \to D(\to K^0_S(\to \pi^+\pi^-)\pi^+\pi^-)K^-$, denoted $\mathcal{A}^-_S$, can be written as a superposition of the amplitude for a $B^- \to D^0K^-$ decay and the suppressed $B^- \to \overline{D}^0K^-$ decay

$$\mathcal{A}^-_S(s_-, s_+) = A_B A_{K^0_S} \left(A^D_S(s_-, s_+) + r_B \exp[i(\delta_B - \gamma)] A^{\overline{D}}_S(s_-, s_+)\right), \tag{2.1}$$

where r_B is the ratio of the magnitudes of the $B^- \to \overline{D}^0K^-$ and $B^- \to D^0K^-$ amplitudes, δ_B is the strong phase between them, A_B is the amplitude of the $B^- \to D^0K^-$ decay.

2.3 The GGSZ method: measuring γ in multi-body D decays

$A_{K^0_S}$ is the amplitude of the $K^0_S \to \pi^+\pi^-$ decay, s_- and s_+ are the squared invariant masses of the $K^0_S\pi^-$ and $K^0_S\pi^+$ particle combinations, respectively, and the D decay amplitudes are defined as

$$A^{\overset{(-)}{D}}_{S(L)}(s_-, s_+) = A(\overset{(-)}{D}{}^0 \to K^0_{S(L)}\pi^+\pi^-). \tag{2.2}$$

Biases from the effects of $D^0 - \overline{D}^0$ mixing are potentially significant, but can be confined to 0.1° with an appropriate measurement strategy. These effects are described in Ref. [?] and are not discussed further in this study. Direct *CP* violation in the D decay is assumed to be negligible, as the effect is expected to be very small for $D \to K^0_S\pi^+\pi^-$ decays in the SM, and has been analysed in Ref. [?]. Under the further assumption that K^0_S is a *CP* eigenstate, the D decay amplitudes satisfy

$$A^{\overline{D}}_S(s_{-+}) = A^D_S(s_{+-}), \tag{2.3}$$

where the short-hand notation $(s_{-+}) = (s_-, s_+)$ and $(s_{+-}) = (s_+, s_-)$ is employed to simplify equations. The differential $B^- \to D(\to K^0_S\pi^+\pi^-)K^-$ decay rate to a given point in the D decay phase space is

$$\begin{aligned} \mathrm{d}\Gamma^-(\mathrm{s}_{-+}) \propto |\mathcal{A}^-_S|^2 = |A_B|^2|A_{K^0_S}|^2 & \\ \times \Big[|A^D_S(s_{-+})|^2 + r_B^2|A^D_S(s_{+-})|^2 + 2r_B|A^D_S(s_{-+})||A^D_S(s_{+-})| & \\ \times (\cos[-\Delta\delta_D(s_{-+})]\cos[\delta_B - \gamma] - \sin[-\Delta\delta_D(s_{-+})]\sin[\delta_B - \gamma])\Big]\,. & \end{aligned} \tag{2.4}$$

Here, $\Delta\delta_D(s_{-+}) = \delta_D(s_{-+}) - \delta_D(s_{+-})$, where $\delta_D(s_{-+})$ denotes the phase of $A^D_S(s_{-+})$. The equivalent expressions to Eq. (2.1) and Eq. (2.4) for $B^+ \to D(\to K^0_S\pi^+\pi^-)K^+$ decays are obtained via the substitutions $\gamma \to -\gamma$ and $A^D_S(s_{-+}) \leftrightarrow A^{\overline{D}}_S(s_{-+})$, where the latter substitution is equivalent to $A^D_S(s_{-+}) \leftrightarrow A^D_S(s_{+-})$.

- Bring in a high-level discussion of why the phase-space dependence gives a great handle on γ
- Include plots of the phase as a function of phase-space etcetera. Pretty plots can be made using the amplitude model
- Do I want to discuss the properties of amplitude models here? It might not actually be a bad idea, because I will need it in a following chapter
- Also be explicit about where and why I need the amplitude phase-space symmetry

2.3.1 A model-independent approach

Based on Eq. (2.4), it is possible to measure γ (and the nuisance parameters r_B and δ_B) from the phase-space distribution of $B^\pm \to D(\to K^0_S\pi^+\pi^-)K^\pm$ decays, given knowledge of $\delta_D(s_{-+})$ or $\Delta\delta_D(s_{-+})$. A series of γ measurements have used amplitude models of the D decay to describe $\Delta\delta_D(s_{-+})$ [?, ?, ?, ?, ?, ?, 4, 5]. More recently, a model-independent approach, which is critical when looking for evidence of beyond-the-SM effects [?, 3, 6, 7], has been favoured. The D decay phase space is split into $2 \times N$ regions, or bins, numbered $i = -N$ to N (omitting zero) that are defined to be symmetric around $s_- = s_+$, with $i > 0$ for $s_- > s_+$. The binning scheme used in current experimental measurements of $B^\pm \to DK^\pm$ decays [?, 3, 6] has $N = 8$ and is shown in Fig. ??. Then, the average of $\cos\Delta\delta_D(s_{-+})$ over bin i of the D decay phase space is

$$c_i = \frac{\int_i \mathrm{d}s^2 |A^D_S(s_{-+})||A^D_S(s_{+-})| \cos[\Delta\delta_D(s_{-+})]}{\sqrt{\int_i \mathrm{d}s^2 |A^D_S(s_{-+})|^2}\sqrt{\int_i \mathrm{d}s^2 |A^D_S(s_{+-})|^2}}, \tag{2.5}$$

where $\int_i \mathrm{d}s^2$ denotes integration over bin i of the D decay phase space, and with an analogous definition of s_i, the average of $\sin\Delta\delta_D(s_{-+})$. Integrating Eq. (2.4), and the corresponding expression for $B^+ \to DK^+$ decays, the yields of $B^\pm \to D(\to K^0_S\pi^+\pi^-)K^\pm$ in bin i are

$$\begin{aligned} N_i^- &= h_B^- \left(K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}\,(c_i x_- + s_i y_-)\right), \\ N_i^+ &= h_B^+ \left(K_{-i} + r_B^2 K_i + 2\sqrt{K_i K_{-i}}\,(c_i x_+ - s_i y_+)\right), \end{aligned} \tag{2.6}$$

in terms of the integrals

$$K_i = \frac{1}{N_K}\int_i \mathrm{d}s^2 |A^D_S(s_{-+})|^2, \qquad N_K = \int \mathrm{d}s^2 |A^D_S(s_{-+})|^2, \tag{2.7}$$

the normalisation constants $h_B^{\pm}$, and the *CP* violation observables $x_{\pm} = r_B \cos(\delta_B \pm \gamma)$ and $y_{\pm} = r_B \sin(\delta_B \pm \gamma)$.

In model-independent experimental measurements of γ, the signal decay yields in each phase-space bin are expressed via Eq. (2.6) and $x_{\pm}$ and $y_{\pm}$ are determined via a maximum likelihood fit to the data. As Eq. (2.6) leads to 32 observable yields but 36 unknown parameters, it is necessary to use some external information. It is possible to measure c_i and s_i using quantum correlated $D^0\overline{D}^0$ pairs produced at the $\psi(3770)$ resonance [?,?]. Such measurements have been made by the CLEO collaboration [?], and these have been employed in a range of γ measurements [?,3,6,7]. More precise measurements of c_i and s_i are expected from the BESIII collaboration, and further measurements could also be made from an analysis of charm mixing [?], or from reconstructed decays of the $\psi(3770)$ meson at LHCb [?]. The K_i parameters can also be measured in external samples. It is advantageous to determine the K_i parameters from a sample that has as similar kinematics to the $B^{\pm} \to DK^{\pm}$ sample as possible, in order to automatically include corrections from experimental effects, such as reconstruction efficiency and resolution. A sample of flavour-tagged D decays is commonly used. Given the current yields of B decays, both the strong phase and K_i parameters are taken from external input in order to maximise the sensitivity to $x_{\pm}$ and $y_{\pm}$.

The interference in $B^{\pm} \to D(\to K_S^0\pi^+\pi^-)K^{\pm}$ decays presents itself primarily in different distributions over the D-decay phase space between signal decays originating from B^+ and B^- mesons. From an experimental point of view this is highly desirable, as production and detection asymmetries that affect the phase-space-integrated yields can be ignored. In addition to the asymmetry in the distribution over the phase space there is a further *CP* asymmetry in the phase-space-integrated yields, expected to be around 1 %. This *CP* asymmetry is very challenging to measure with useful precision, due to the limited sample sizes currently available and possible biases of a similar magnitude from production and detection asymmetries. Hence, in most studies it is ignored and makes no contribution to the overall determination of γ.

2.3.2 Measuring strong-phase inputs at charm factories

- The thesis should certainly feature a reasonably detailed discussion of how c_i and s_i are actually measured

- If I have time, I would like to calculate the impacts of K_S^0*CP* violation and material interaction for these parameters also

2.3.3 Extending the method to multiple decay channels

- Just a quick definition of the ξ parameters and a reference to Jordi's paper (maybe also Matt's just to show that I know about it)
- Important to mention that this is the first time it has been used, what it brings to the table, and that we were the first to carefully show that it works ...

3
The LHCb experiment

We have a detector? I thought ntuples were made of magic.

3.1 Subdetectors

3.1.1 The VELO

3.1.2 Magnet and tracking stations

3.1.3 The RICH

3.1.4 Calorimeters

3.1.5 Muon detectors

3.2 The LHCb triggerring system

3.2.1 The level-0 hardware trigger

3.2.2 High-level triggers

3.2.3 Offline data filtering: the LHCb stripping

3.3 Track reconstruction

3.4 Simulation

4

Neutral kaon CP violation and material interaction in GGSZ measurements

Let's see how much I can get away with simply copying from the paper. Large amounts of the initial theory can be moved to the theory chapter of course.

4.1 CP violation and material interaction of neutral kaons

The derivation in the preceding section assumed that K^0_{S} is a *CP* eigenstate, however this is known to be an approximation. Including *CP* violation in the neutral kaons, the mass eigenstates K^0_{S} and K^0_{L} are given by

$$|K^0_{\mathrm{S}}\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left[|K_1\rangle + \epsilon|K_2\rangle\right], \qquad |K^0_{\mathrm{L}}\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left[|K_2\rangle + \epsilon|K_1\rangle\right], \tag{4.1}$$

in terms of the *CP* even (odd) eigenstates $K_{1(2)} = (K^0 + (-)\overline{K}^0)/\sqrt{2}$ and the *CP* violation parameter $\epsilon = A(K^0_{\mathrm{L}} \to \pi^+\pi^-)/A(K^0_{\mathrm{S}} \to \pi^+\pi^-)$, measured to be [?]

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}, \qquad \arg\epsilon = (43.52 \pm 0.05)^\circ. \tag{4.2}$$

The phase-convention $\hat{C}\hat{P}|K^0\rangle = |\overline{K}^0\rangle$ is used and direct *CP* violation in the kaon decay is ignored, as the effect is three orders of magnitude smaller than indirect *CP* violation [?]. The non-zero amplitude for the $K^0_{\mathrm{L}} \to \pi^+\pi^-$ decay introduces a direct dependence on the $A_{\mathrm{L}}^{\overset{(-)}{D}}$ amplitudes. Whereas Eq. (2.4) assumed that $\mathrm{d}\Gamma^-(t, s_{-+}) \propto \left|\psi^-_{\mathrm{S}}(t, s_{-+})\right|^2$, the actual decay rate satisfies

$$\mathrm{d}\Gamma^-(t, s_{-+}) \propto \left|\psi^-_{\mathrm{S}}(t, s_{-+}) + \epsilon\psi^-_{\mathrm{L}}(t, s_{-+})\right|^2, \tag{4.3}$$

where $\psi_{\rm S}^-$ and $\psi_{\rm L}^-$ are the $K^0_{\rm S}$ and $K^0_{\rm L}$ components of the neutral kaon state.[1] The $A_{\rm L}^{\overset{(-)}{D}}$ amplitudes do not satisfy Eq. (2.3), but instead $A_{\rm L}^{D}(s_{-+}) \simeq -A_{\rm L}^{\bar{D}}(s_{+-})$, and therefore the presence of the $K^0_{\rm L}$ term leads to corrections to the yield expressions in Eq. (2.6). In an experimental setting, the dependence on the $A_{\rm L}^{\overset{(-)}{D}}$ amplitudes is further enhanced by material interactions of the neutral kaon, because different nuclear interaction strengths of the K^0 and $\overline{K}^0$ mesons introduce a non-zero $K^0_{\rm S} \leftrightarrow K^0_{\rm L}$ transition amplitude for neutral kaons traversing a detector segment. This effect was predicted early in the history of kaon physics [?] and is commonly denoted *kaon regeneration*. The general expression for the time dependent neutral kaon state components is [?, ?]

$$\begin{aligned}
\psi_{\rm S}(t, s_{-+}) &= e^{-i\Sigma t}\left(\psi_{\rm S}^0(s_{-+})\cos\Omega t + \frac{i}{2\Omega}\left(\Delta\lambda\psi_{\rm S}^0(s_{-+}) - \Delta\chi\psi_{\rm L}^0(s_{-+})\right)\sin\Omega t\right),\\
\psi_{\rm L}(t, s_{-+}) &= e^{-i\Sigma t}\left(\psi_{\rm L}^0(s_{-+})\cos\Omega t - \frac{i}{2\Omega}\left(\Delta\lambda\psi_{\rm L}^0(s_{-+}) + \Delta\chi\psi_{\rm S}^0(s_{-+})\right)\sin\Omega t\right),
\end{aligned} \tag{4.4}$$

in terms of the parameters

$$\begin{aligned}
\Delta\chi &= \chi - \bar{\chi},\\
\Delta\lambda &= \lambda_{\rm L} - \lambda_{\rm S} = (m_{\rm L} - m_{\rm S}) - \frac{i}{2}(\Gamma_{\rm L} - \Gamma_{\rm S}),\\
\Sigma &= \frac{1}{2}\left(\lambda_{\rm S} + \lambda_{\rm L} + \chi + \bar{\chi}\right),\\
\Omega &= \frac{1}{2}\sqrt{\Delta\lambda^2 + \Delta\chi^2},
\end{aligned} \tag{4.5}$$

where $m_{\rm S(L)}$ and $\Gamma_{\rm S(L)}$ are the mass and decay width of the $K^0_{\rm S}$ ($K^0_{\rm L}$) mass eigenstates, and the parameters χ and $\bar{\chi}$ describe the material interaction of the K^0 and $\overline{K}^0$ flavour eigenstates. The χ ($\bar{\chi}$) parameter is proportional to the forward scattering amplitude of a K^0 ($\overline{K}^0$) meson in a traversed material. In Eq. (4.4), $\psi_{\rm S}^0$ and $\psi_{\rm L}^0$ are the initial $K^0_{\rm S}$ and $K^0_{\rm L}$ components of the neutral kaon state, which depend on the phase-space coordinates of the D decay: $\psi_{\rm S/L}^0 \propto \mathcal{A}_{\rm S/L}(s_{-+})$. Thus, for $\Delta\chi \neq 0$, $\psi_{\rm S}(t)$ depends on $\mathcal{A}_{\rm L}(s_{+-})$ irrespective of the $K^0_{\rm L} \to \pi^+\pi^-$ decay, due to kaon regeneration.

In addition, the relations $A_{\rm S}^{\bar{D}}(s_{-+}) = A_{\rm S}^{D}(s_{+-})$ and $A_{\rm L}^{\bar{D}}(s_{-+}) = -A_{\rm L}^{D}(s_{+-})$ are not exact for $\epsilon \neq 0$, as $K^0_{\rm S}$ and $K^0_{\rm L}$ are not exact *CP* eigenstates. This leads to further corrections to the yield expressions in Eq. (2.6). It it beneficial to express $A_{\rm S(L)}^{D}$ in terms of the amplitudes $A_{1(2)}^{D}$, defined analogously to Eq. (2.2)

[1]The time dependence of Eq. (4.3) is kept implicit in Eq. (2.4), as it contributes a factor that is constant over phase-space in the $\epsilon = 0$ approximation.

but for the CP even (odd) eigenstates K_1 (K_2). After the decay of a D^0 meson to a neutral kaon, the kaon state is

$$\begin{aligned}\psi^0 &= A_1^D|K_1\rangle + A_2^D|K_2\rangle \\ &= N\left[(A_1^D - \epsilon A_2^D)|K_{\rm S}^0\rangle + (A_2^D - \epsilon A_1^D)|K_{\rm L}^0\rangle\right],\end{aligned} \tag{4.6}$$

with the normalisation constant $N = \sqrt{1+|\epsilon|^2}/(1-\epsilon^2)$. Thus it can be seen that

$$\begin{aligned}A_{\rm S}^D(s_{+-}) &= N\left[(A_1^D(s_{+-}) - \epsilon A_2^D(s_{+-}))\right], \\ A_{\rm L}^D(s_{+-}) &= N\left[(A_2^D(s_{+-}) - \epsilon A_1^D(s_{+-}))\right],\end{aligned} \tag{4.7}$$

with an analogous expression for the $\overline{D}^0$ decay amplitudes. Therefore, the generalised relations between the D^0 and $\overline{D}^0$ amplitudes are

$$\begin{aligned}A_{\rm S}^{\overline{D}}(s_{+-}) =&\ N[A_1^{\overline{D}}(s_{+-}) - \epsilon A_2^{\overline{D}}(s_{+-})] \\ =&\ N[A_1^D(s_{-+}) + \epsilon A_2^D(s_{-+})] = \ A_{\rm S}^D(s_{-+}) + 2N\epsilon A_2^D(s_{-+}), \\ A_{\rm L}^{\overline{D}}(s_{+-}) =&\ N[A_2^{\overline{D}}(s_{+-}) - \epsilon A_1^{\overline{D}}(s_{+-})] \\ =& -N[A_2^D(s_{-+}) + \epsilon A_1^D(s_{-+})] = -A_{\rm L}^D(s_{-+}) - 2N\epsilon A_1^D(s_{-+}).\end{aligned} \tag{4.8}$$

In order to calculate the full corrections to the yield expressions in Eq. (2.6), models of $A_{\rm S}^D$ and $A_{\rm L}^D$ (or A_1^D and A_2^D) are needed. While there are several amplitude models available to describe the decay amplitude $A(D^0 \to K_{\rm S}^0\pi^+\pi^-)$ [?, ?, ?, ?, ?], no models have been published for $D^0 \to K_{\rm L}^0\pi^+\pi^-$ decays. However, following the assumptions laid out in Ref. [?], the amplitudes $A_1^D(s_{+-})$ and $A_2^D(s_{+-})$ can be related, allowing both qualitative and quantitative estimates of the bias effects to be made with existing $D^0 \to K_{\rm S}^0\pi^+\pi^-$ models. In the isobar formalism, the decay amplitude $A(D^0 \to K_1\pi^+\pi^-)$ is expressed as a non-resonant constant amplitude plus a sum of resonances

$$A(D^0 \to K_1\pi^+\pi^-) = k_{NR} + \sum_{CF} k_i R^i(s_{K\pi^-}) + \sum_{DCS} k_j R^j(s_{K\pi^+}) + \sum_{R_{\pi\pi}} k_k R^k(s_{\pi^+\pi^-}). \tag{4.9}$$

The resonances are split into Cabibbo-favoured (CF) K^{*-} resonances, doubly Cabibbo-suppressed (DCS) K^{*+} resonances and $\pi\pi$ resonances. The R functions are taken to describe all kinematical dependence and are well described in eg. Refs. [?, ?] and references therein. In modern models, the $\pi\pi$ and $K\pi$ S-wave components are modelled via the K-matrix formalism and LASS parametrisations, respectively, instead of sums of individual resonances [?]. This does not alter the arguments below, as the R functions of Eq. (4.9) can equally well represent such terms. The CF resonances couple to the $\overline{K}^0$ component of $K_1(\propto K^0 + \overline{K}^0)$, and therefore the corresponding k_i in the $K_2(\propto K^0 - \overline{K}^0)$ amplitude will have a relative minus sign. The DCS resonances couple to the K^0 component of K_1, and so the corresponding k_j in the K_2 amplitude will have a relative plus sign. For the h^+h^- resonances, there will be a coupling to both the K^0 and $\overline{K}^0$ components, however the coupling to the K^0 component is expected to be suppressed with a Cabibbo suppression factor $r_k e^{i\delta_k}$, where $r_k \simeq \tan^2\theta_C \simeq 0.05$ is determined by the Cabibbo angle θ_C and δ_k can take any value. Therefore, the k_k for these resonances have a relative $-(1-2r_k e^{i\delta_k})$ factor in the K_2 amplitude. The same effect leads to the differences in decay rates between $D^0 \to K^0_{\mathrm{S}}\pi^0$ and $D^0 \to K^0_{\mathrm{L}}\pi^0$ decays [?, ?]. An important consequence of these substitution rules is that

$$A_2^D(s_{+-}) = -A_1^D(s_{+-}) + r_A \Delta A(s_{+-}), \tag{4.10}$$

where $r_A \simeq \tan^2\theta_C$ and $\Delta A(s_{+-}) \sim A_1^D(s_{+-})$ are of the same order of magnitude (at least when averaged over the bins used in γ measurements). This relation is sufficient to make the qualitative arguments of Section **??**, while the full set of substitution rules above are used in the quantitative studies of Section **??**.

4.1.1 Impact on γ measurements: principles

With suitable models to calculate $A^{\overset{(-)}{D}}_{\mathrm{S(L)}}$ (or $A^{\overset{(-)}{D}}_{1/2}$) and knowledge of $\Delta\chi$ for the materials relevant to an experimental setting, Eqs. (4.3), (4.4), and (4.8) can be integrated to calculate the expected phase-space bin yields, $N_i^{\pm}$, including the effects of kaon *CP* violation and material interaction. Preliminary to doing this in Section **??**, it is useful to look at the lowest order corrections to Eq. (2.6) in ϵ and $r_\chi = \frac{1}{2}\frac{\Delta\chi}{\Delta\lambda}$, the dimensionless parameter governing material interactions. For LHCb and Belle II the average $|r_\chi| \simeq 10^{-3}$, as detailed in the Section **??**. The studies of this section are made with the assumption of a flat phase-space efficiency and uniform acceptance over all decay times. Time-acceptance effects will be treated in Section **??**. To first order in r_χ, the expression in Eq. (4.4) simplifies to [?]

$$\begin{aligned}
\psi_{\mathrm{S}}(t, s_{+-}) &= e^{-\frac{i}{2}(\chi+\bar{\chi})t} e^{-i\lambda_{\mathrm{S}} t}\left(\psi^0_{\mathrm{S}}(s_{+-}) - r_\chi\left(1 - e^{-i\Delta\lambda t}\right)\psi^0_{\mathrm{L}}(s_{+-})\right),\\
\psi_{\mathrm{L}}(t, s_{+-}) &= e^{-\frac{i}{2}(\chi+\bar{\chi})t} e^{-i\lambda_{\mathrm{L}} t}\left(\psi^0_{\mathrm{L}}(s_{+-}) + r_\chi\left(1 - e^{+i\Delta\lambda t}\right)\psi^0_{\mathrm{S}}(s_{+-})\right).
\end{aligned} \tag{4.11}$$

In model-independent measurements, the K_i are obtained in a data-driven way using flavour-tagged D samples, and by averaging over D^0 and $\overline{D}^0$ decays. Therefore it proves beneficial to introduce the parameters

$$\hat{K}_i = \frac{1}{1 + |\epsilon + r_\chi|^2 \frac{\Gamma_{\mathrm{S}}}{\Gamma_{\mathrm{L}}}}\left(K_i^{(1)} + |\epsilon + r_\chi|^2 \frac{\Gamma_{\mathrm{S}}}{\Gamma_{\mathrm{L}}} K_i^{(2)}\right), \tag{4.12}$$

in which the $K_i^{(1/2)}$ parameters are phase-space integrals, defined as in Eq. (2.7) but for $A^D_{1/2}$. The $\hat{K}_i$ correspond to the expected measured value of $K_i^{\text{meas}} = (N_i^D + N_{-i}^{\bar{D}})/(\sum_j N_j^D + N_{-j}^{\bar{D}})$ to lowest order in ϵ and r_χ, where N_i^D ($N_i^{\bar{D}}$) is the expected yield of flavour tagged D^0 ($\overline{D}^0$) mesons into bin i of the D decay phase-space. In fits of amplitude models where both flavour tagged D^0 and $\overline{D}^0$ decays are used to fit the $D \to K^0_S\pi^+\pi^-$ amplitude, related via Eq. (2.3), one will effectively fit an amplitude describing $N_i^D + N_{-i}^{\bar{D}}$, and therefore the arguments below, based on $\hat{K}_i$, will also hold for model-dependent measurements. Employing Eq. (4.10) in Eq. (4.12), the expected yields can be written

$$\begin{aligned} N_i^- &= h_B^{-'}\left(\hat{K}_{+i} + r_B^2\hat{K}_{-i} + 2\sqrt{\hat{K}_{+i}\hat{K}_{-i}}(x_-\hat{c}_i + y_-\hat{s}_i) + O(r\epsilon)\right), \\ N_i^+ &= h_B^{+'}\left(\hat{K}_{-i} + r_B^2\hat{K}_{+i} + 2\sqrt{\hat{K}_{+i}\hat{K}_{-i}}(x_+\hat{c}_i - y_+\hat{s}_i) + O(r\epsilon)\right), \end{aligned} \tag{4.13}$$

where $O(r\epsilon)$ denotes terms of $O(r_A\epsilon)$, $O(r_B\epsilon)$, $O(r_A r_\chi)$, and $O(r_B r_\chi)$. Since $r_B \sim r_A \sim 10^{-1}$ (in $B^\pm \to DK^\pm$ decays) and $r_\chi \sim \epsilon \sim 10^{-3}$, these terms are all of the same order of magnitude. The new normalisation constants $h_B^{\pm'} = h_B^\pm(1 + |\epsilon + r_\chi|^2\frac{\Gamma_S}{\Gamma_L} \mp \Delta h)$ are defined in terms of

$$\Delta h = 2\text{Re}[\epsilon + r_\chi] - 4\frac{\Gamma_S}{\Gamma_L + \Gamma_S}\frac{\text{Re}[\epsilon + r_\chi] + \mu\text{Im}[\epsilon + r_\chi]}{1 + \mu^2}, \quad \mu = 2\frac{m_L - m_S}{\Gamma_L + \Gamma_S}. \tag{4.14}$$

The parameters $(\hat{c}_i, \hat{s}_i)$ have been introduced to denote the *measured* average strong-phases, which are expected to differ from (c_i, s_i) at $O(\epsilon)$, since neutral kaon *CP* violation is not taken into account in the measurements by CLEO. The corrections are thus in the neglected $O(r_B\epsilon)$ terms.

Two observations can be made from the expression in (4.13). The first is that the phase-space *distribution* is only changed at $O(r\epsilon)$ compared to the expression in Eq. (2.6), if the measured $\hat{K}_i$ are used in the experimental analysis. As the $D^0 - \overline{D}^0$ interference term that provides sensitivity to γ enters at order $O(r_B)$, the impact on γ measurements can be expected to be $\Delta\gamma/\gamma \sim O(r\epsilon/r_B)$. For $B \to DK$ analyses, where $r_B \simeq 0.1$, this is at the permille level, so the induced $\Delta\gamma$ bias can be expected to be smaller than $1°$. This holds true, unless the integrated material interaction, and thereby effective r_χ, varies significantly across the D-decay phase-space due to experimental effects. However, this is unlikely to be the case in practice, since no significant correlation between the phase-space coordinates and the travel direction of the kaon is expected.

The second observation relates to potential future measurements of γ, which may also include sensitivity from the total, phase-space-integrated yield asymmetry

$$A_{\text{total}} = \frac{N^- - N^+}{N^- + N^+} = \frac{2\sum_i c_i \sqrt{\hat{K}_i \hat{K}_{-i}} r_B \sin\delta_B \sin\gamma + \Delta h}{1 + r_B^2 + 2\sum_i c_i \sqrt{\hat{K}_i \hat{K}_{-i}} r_B \cos\delta_B \cos\gamma} + O(r\epsilon), \tag{4.15}$$

which was considered in Ref. [?]. In the limit $r_B \to 0$ the expression agrees with the result for the analogous asymmetry in $D^\pm \to \pi^\pm K^0_S$ decays in Ref. [?], evaluated to $O(\epsilon)$ for an infinite and uniform time-acceptance. The asymmetry due to *CP* violation in the neutral kaon sector, governed by Δh, is of approximately the same order of magnitude as the asymmetry due to γ being non-zero. This is illustrated in Fig. ??, where the expression in Eq. (4.15) is plotted in the default case where $\Delta h = 0$, using the model in Ref. [?] to calculate K_i and c_i, as well as including neutral kaon *CP* violation and material interaction effects, calculated using $r_\chi = \epsilon$, with ϵ taking the value in Eq. (4.2). The asymmetry changes significantly when including the latter effects. Therefore, measurements based only on the global asymmetry will suffer relative biases of tens of degrees, not a few degrees, if neutral kaon *CP* violation and material interaction is not taken into account. The contribution to A_{total} due to *CP* violation in the B decay is an order of magnitude smaller than the $O(r_B)$ expectation described in Ref. [?] because $\sum_i c_i \sqrt{K_i K_{-i}} \simeq 0.1 \ll 1$. The reason is that $K^0_S\pi^+\pi^-$ is not a *CP* eigenstate and the strong-phase $\Delta\delta_D(s_{-+})$ has a non-trivial phase-space dependence. This results in the CF and DCS interference term, which governs the *CP* asymmetry, changing sign over phase-space and therefore giving a small contribution to the phase-space-integrated yields.

4.2 Detector descriptions for LHCb and Belle II

In order to estimate the effects of *CP* violation and material interaction on γ measurements that are based on the phase-space distribution of signal decays, the equations of Section ?? need to be evaluated to at least $O(r\epsilon)$. Therefore a set of numerical studies are carried out, in which calculations are made to all orders in ϵ, r_χ, r_A and r_B. Furthermore, the bias effects depend on the specific detector material budget and time acceptance of a given experiment. The bias is calculated considering the conditions at the two main flavour physics experiments where measurements of γ will be performed in the next decade: LHCb and Belle II.

4.2.1 LHCb material budget in simulation

- I will need to make a deep dive on LHCb, at the same time motivating a comparison to simpler estimates, in order to show that they are probably ok to use for Belle II

4.2.2 Simplified description of Belle II

Experiment specific biases are obtained for LHCb and Belle II, by assuming time acceptances, momentum distributions, and detector geometries typical of the experiments. The LHCb experiment is a forward arm spectrometer where the B mesons are produced in proton-proton collisions at 13 TeV. Subsequent decays of the K_S^0 are highly boosted and can occur within different detector subsystems, which leads to two distinct categories of candidates, with different mean lifetimes and material traversed. Therefore two scenarios are considered for LHCb: one in which the decay products of the K_S^0 leave reconstructed tracks in both the silicon vertex detector and downstream tracking detectors (denoted *long-long* or LL), and one in which the decay products of the K_S^0 only leave tracks in the downstream tracking detectors (denoted *down-down* or DD). At Belle II, B mesons are produced from decays of $\Upsilon(4S)$ mesons, produced in asymmetric electron-positron collisions. This leads to substantially different decay kinematics in comparison to those found at LHCb. A single scenario is considered for Belle II, because nearly all the K_S^0 mesons produced in signal decays in Belle II decay within the tracking volume, with more than 90 % decaying in the vertex detector according to the studies described below. Thus, three scenarios are considered in total: LL LHCb, DD LHCb, and Belle II.

In order to model the experimental time acceptance, the time-dependent integral in Eq. (4.3) is only carried out over a finite time interval (τ_1, τ_2). The intervals are defined for each of the three experimental categories, by requiring that a neutral kaon, if produced at $x = y = z = 0$ with momentum $p = (p_T, p_z)$, decays within the relevant part of the corresponding detector. The time acceptance has a significant impact for the LHCb categories, where some 20 % of the kaons escape the tracking stations completely before decaying, whereas the resulting cut-off, τ_2, is large enough in Belle II to have negligible significance. A discussion on the exact requirements placed, and corresponding decay lengths, is found in appendix **??**.

The neutral kaon momentum distribution in LHCb is obtained using `RapidSim` [?], which can generate decays of B mesons with the kinematic distribution found in LHCb collisions, and falling in the LHCb acceptance. The momentum distribution in Belle II is estimated by decaying B mesons with a momentum of 1.50 GeV/c

along the z-axis using `RapidSim`, corresponding to the $\gamma\beta = 0.28$ boost of the centre-of-mass system in Belle II when operated at the $\Upsilon(4S)$ resonance [?]. A perfect 4π angular acceptance is assumed. The generated $D \to K^0_S\pi^+\pi^-$ decays are uniformly distributed in phase space. The `RapidSim` samples for LHCb are reweighted to take the relevant time acceptance into account. This is not necessary for Belle II, as all produced K^0_S mesons decay in the tracking volume. The resulting momentum distributions for the three types of sample are shown in Fig. ??.

The parameter $\Delta\chi$ describes the matter-interaction effect, as detailed in Section ??. It depends on kaon momentum and varies along a given kaon path, as the kaon intersects detector components made of different materials. In these studies, the calculations are simplified by using a constant set of average material parameters for each experimental scenario. The average material parameters can be estimated for a given experimental scenario by considering the type and length of material traversed by a kaon in the relevant sub-detector(s). A detailed description of the calculation is given in appendix ??. The average value of the dimensionless parameter $r_\chi = \frac{1}{2}\frac{\Delta\chi}{\Delta\lambda}$, which governs the size of the matter regeneration effect, can be calculated for the three considered experimental scenarios, and the averages are found to satisfy $|r_\chi^{\mathrm{LL}}| = 2.7 \times 10^{-3}$, $|r_\chi^{\mathrm{DD}}| = 2.2 \times 10^{-3}$, and $|r_\chi^{\text{Belle II}}| = 1.0 \times 10^{-3}$.

The LHCb detector is undergoing a significant upgrade prior to the start of the LHC Run 3. However, the material budget and geometry of the relevant sub-detectors will be similar to the sub-detectors used during Run 1 and 2 [?, ?]. Hence the results of this study will be valid for measurements during the upgrade phases of LHCb, even though the detector parameters presented in this section relate to the original LHCb detector.

4.3 Impact on GGSZ measurements of γ: the full study

In the numerical bias studies studies, the amplitude model for $D^0 \to K^0_S\pi^+\pi^-$ decays in Ref. [?] is taken to represent the $A_1(s_{+-})$ amplitude. Then $A_2(s_{+-})$ is obtained as described in Section ??. In terms of A_1 and A_2, the amplitudes $A^{\overset{(-)}{D}}_{\mathrm{S(L)}}(s_{+-})$ can be expressed and related via Eqs. (4.7) and (4.8), and the full $\mathcal{A}^{\pm}_{\mathrm{S/L}}(s_{+-})$ amplitudes calculated for a given set of input parameters $(\gamma^0, r^0_B, \delta^0_B)$. Then Eq. (4.4) gives the kaon state as a function of time, phase-space coordinates, and the material parameter $\Delta\chi$. The neutral kaon state components, $\psi_{\mathrm{S}}(t)$ and $\psi_{\mathrm{S}}(t)$, are inserted into Eq. (4.3), which is integrated numerically over time and the phase-space bins of Fig. ?? to obtain the expected yields in each bin. These integrals use the experimental time acceptance that was described in Section **??**. The signal yields depend on the momentum via the time-acceptance parameters τ_1 and τ_2, and because the material interaction parameter $\Delta\chi$ is momentum dependent. Therefore, the yields are averaged over the K^0_S momentum distributions of LHCb and Belle II. The neutral kaon momentum in the lab frame is correlated with $m^2(\pi^+\pi^-)$, and in order to take this correlation into account in the averaging, the kaon p, p_z, and p_T distributions are extracted for a number of different $m^2(\pi^+\pi^-)$ values, using the `RapidSim` samples described in Section **??**. In order to keep the calculations manageable, the distributions of kaon p, p_z, and p_T for each phase-space point are divided into 5 quantiles and the 5 medians of these quantiles are used to represent the overall distribution.

The parameters $x_\pm$ and $y_\pm$ are determined by a maximum likelihood fit to the calculated yields, using the default yield expression in Eq. (2.6), which ignores the presence of *CP* violation and material interaction in the neutral kaon sector. The fit result and covariance matrix are interpreted in terms of the physics parameters (γ, r_B, δ_B) using another maximum likelihood fit [**?**], to allow for the extraction of the bias $\Delta\gamma = \gamma - \gamma^0$. In the fits, the K_i are obtained using the definition $K_i = K_i^{\text{meas}} = (N_i^D + N_{-i}^{\bar{D}})/(\sum_j N_j^D + N_{-j}^{\bar{D}})$, in terms of the expected yields N_i^D $(N_i^{\bar{D}})$ of a flavour-tagged D^0 $(\bar{D}^0)$ decays in bin i of the D decay phase space, calculated as described above for $r_B^0 = 0$. This corresponds to experimentally measuring the K_i in a control channel, and takes the effect of neutral kaon *CP* violation and material interaction on K_i measurements into account, as well the experimental time acceptance. The (c_i, s_i) are calculated using $A_1(s_{+-})$ and the experimental time acceptance is taken into account in this calculation as well. While a model-independent method is specifically used here to determine biases, it is expected that traditional and new unbinned methods such as those in Refs [**?**, **?**, **?**, **?**, **?**, **?**, 4] and Ref. [**?**], respectively, will be similarly biased if the kaon *CP*-violation and regeneration are not accounted for.

4.3.1 Results

The obtained bias $\Delta\gamma$ is shown as a function of input γ^0 for the various experimental conditions in Fig. **??**. The calculations are made using $(r_B^0, \delta_B^0) = (0.1, 130°)$, approximately equal to the physics parameters relevant for $B^\pm \to DK^\pm$ decays [**?**, **?**]. The bias does not vary significantly with γ^0 in the plotted range, which includes the world average value of direct γ measurements as well as the values obtained in full unitarity-triangle fits [**?**, **?**, **?**], and for all cases, the bias is found to be below $0.5°$, corresponding to relative biases of about half a percent. Thus the biases are of $O(r\epsilon/r_B)$ as expected, given the arguments of Section **??**. The contributions from the individual $K^0_{\rm S}$ CPV and material interaction effects are also shown. It is seen that the neutral kaon *CP* violation and material interaction effects leads to approximately equal biases in all three cases.

Given the decay-time acceptance and momentum distribution for each experimental category, the mean life time, $\langle\tau\rangle$, of the reconstructed kaons can be calculated. In terms of the $K^0_{\rm S}$ lifetime $\tau_{K^0_{\rm S}} = (0.895 \pm 0.004) \times 10^{-11}\,{\rm s}$ [**?**], $\langle\tau_{\rm LL}\rangle \simeq 0.1\tau_{K^0_{\rm S}}$ for the LHCb LL category, $\langle\tau_{\rm DD}\rangle \simeq 0.8\tau_{K^0_{\rm S}}$ for the LHCb DD category, and at Belle II $\langle\tau_{\text{Belle II}}\rangle \simeq \tau_{K^0_{\rm S}}$. The difference in average kaon lifetime is reflected in the observed biases, which are found to be larger in the samples with longer lived kaons. The very small effect in the LL category is to be expected because the *CP*-violation effect due to $K^0_{\rm S}$ not being *CP*-even is approximately cancelled by the *CP*-violation effect arising from $K^0_{\rm S} - K^0_{\rm L}$ interference for kaons with decay times much smaller than $\tau_{K^0_{\rm S}}$ [**?**]. The time dependence of the bias effect means that it can potentially be beneficial to restrict a measurement to using short-lived $K^0_{\rm S}$ mesons in a future scenario, where the impact of $K^0_{\rm S}$ *CP* violation is comparable to the statistical precision of the measurement. For example, the bias can reduced by 40 % in the Belle II scenario if only $K^0_{\rm S}$ mesons decaying within 8 cm of the beam axis are included in the measurement. This requirement only removes 20 % of the signal yield, and hence only increases the statistical uncertainty of the measurement by 10 %.

The uncertainty bands in Fig. **??** are calculated by repeating the study while varying some of the inputs. The model dependence of the predicted biases is probed by repeating the study using two other amplitude models as input for $A_1(s_{+-})$ and $A_2(s_{+-})$: the model published in Ref. [**?**] and the model included in EvtGen [**?**]. The use of different models change the predicted biases by up to 0.05°. When defining $A_2(s_{+-})$ in terms of $A_1(s_{+-})$, there is an uncertainty due to the unknown (r_k, δ_k) parameters used to describe the $\pi\pi$ resonance terms. This uncertainty is assessed by making the study with 50 different random realisations of the parameter set. The phases δ_k are sampled uniformly in the interval $[0, 2\pi]$ while the r_k are sampled from a normal distribution with $\mu = \tan^2\theta_C$ and $\sigma = \mu/2$. The uncertainty is about 0.05° across the three experiments considered. The studies are repeated while varying the time acceptances and material densities with ±10 %. The largest deviations in biases are found to be below 0.05°. The dependence on the handling of the momentum distribution is estimated by repeating the study using 10 and 20 quantiles to describe the momentum distributions at each point in phase space, instead of 5. The variation in the results is taken as the systematic uncertainty, and found to be below 0.01° for all experiments. There is an additional uncertainty due to the use of simulation samples generated with `RapidSim` to describe the kaon momentum distribution, in lieu of full detector simulations. The uncertainty has not been considered here. Full detector simulation should be used if specific experimental measurements are to be corrected for the biases described in this study.

There is also an uncertainty from the use of (c_i, s_i) as calculated using $A_1(s_{+-})$. It is to be expected that the measured values $(\hat{c}_i, \hat{s}_i)$ from the CLEO collaboration differ by those calculated using $A_1^D(s_-, s_+)$ by terms of $O(\epsilon)$ due to neutral kaon *CP* violation, which is not taken into account in the measurement [**?**]. These corrections can be calculated via a procedure analogous to the one used to estimate the corrections on measurements of γ in this paper. However, as these corrections are much smaller than the experimental uncertainties in the measurement, they have not been studied further.

It is interesting to evaluate the bias obtained if the K_i are calculated from $A_1(s_{+-})$, without any corrections due to neutral kaon *CP* violation and material interaction. If this is done, while the full experimental time acceptance is taken into account, the biases only change by up to 0.01°, across the experiments. This is because the $O(\epsilon)$ corrections in Eq. (4.12), where the expected measured K_i is given to lowest order in ϵ and r_χ, only affect the overall normalisation. If the time acceptance *is not* taken into account, biases of several degrees can occur, irrespective of the presence of neutral kaon *CP* violation or material interaction effects.

While the $B^\pm \to DK^\pm$ decay mode provides the best sensitivity to γ, it is also possible to measure γ in other B decay channels, such as $B^\pm \to D^*K^\pm$, $B^\pm \to DK^{*\pm}$, $B^0 \to DK^{*0}$, and $B^\pm \to D\pi^\pm$. For the purpose of the study presented here, the main difference between the decay channels is that they have different values of r_B and δ_B. Figure **??** shows $\Delta\gamma$ as a function of input δ_B^0, for $\gamma^0 = 75°$ and three different values of r_B^0. Aside from $r_B^0 = 0.1$, the results are shown for $r_B^0 = 0.005$, which corresponds to the expectation in $B^\pm \to D\pi^\pm$ decays [**?**] and $r_B^0 = 0.25$, which corresponds to $B^0 \to DK^{*0}$ decays [**?**, **?**]. Three features are notable, namely that the biases depend on δ_B^0, that the biases are large for the small $r_B^0 = 0.005$ case, and that the oscillation period of the δ_B dependence is different between the $r_B^0 = 0.005$ case and the $r_B^0 \in \{0.1, 0.25\}$ cases. It is to be expected that $\Delta\gamma$ oscillates as a function of δ_B^0, because δ_B^0 enters the yield equations via $\cos(\delta_B^0 \pm \gamma)$ and $\sin(\delta_B^0 \pm \gamma)$ terms. The r_B^0 dependent behaviour is governed by the relative importance of different $O(r\epsilon)$ correction terms to the phase-space distribution. There are terms of both $O(r_A\epsilon)$ and $O(r_B\epsilon)$[2], which lead to expected

[2] There are similar terms of $O(r_A r_\chi)$ and $O(r_B r_\chi)$, but as ϵ and r_χ are of the same order of magnitude, these terms can be treated completely analogously to the $O(r_A\epsilon)$ and $O(r_B\epsilon)$ terms, and have been left out of the discussion for brevity.

biases of size $O(r_A\epsilon/r_B)$ and $O(r_B\epsilon/r_B) = O(\epsilon)$, respectively, cf. the discussion of Section **??**. The $O(r_A\epsilon)$ terms are independent of δ_B^0, whereas the $O(r_B\epsilon)$ terms have factors of $\cos(\delta_B^0 \pm \gamma)$ and $\sin(\delta_B^0 \pm \gamma)$. Therefore the $O(r_A\epsilon)$ and $O(r_B\epsilon)$ terms introduce biases with different dependence on δ_B^0. In the $B^\pm \to D\pi^\pm$ case, the $O(r_A\epsilon)$ correction terms dominate because $r_A/r_B \simeq (0.05/0.005) = 10$. This explains the relatively large bias, as $|r_A\epsilon/r_B^{D\pi}| \simeq 4\%$, and the simple dependence on δ_B^0. The bias is seen to be up to $\pm 1.5°$, but only about $+0.2°$ with the expected value of $\delta_B^{D\pi} \simeq 300°$ [**?**, **?**]. In the $r_B^0 = 0.1$ and $r_B^0 = 0.25$ cases the $O(r_B\epsilon)$ correction terms dominate, and the biases are of $O(\epsilon)$, independent of the r_B^0 value. Therefore both cases have biases of similar size and with similar δ_B^0 dependence. While the input value of $\gamma^0 = 75°$ was chosen for these studies, there is minimal variation in the results if another value of γ^0 in the range $[65°, 85°]$ is used.

The γ measurements treated in this paper can be made using other D-decay final states, such as $D \to K_S^0 K^+ K^-$ and $D \to K_S^0 \pi^+\pi^-\pi^0$. The biases from neutral kaon *CP* violation and material interaction on measurements of γ based the D decay phase-space distributions should be of similar size in these decay channels, as those presented for $D \to K_S^0\pi^+\pi^-$ in this paper. The impact on γ measurements based on the phase-space-integrated yield asymmetry can be expected to be tens of degrees for the $D \to K_S^0 K^+ K^-$ channel, where the yield asymmetry is expected to be around 2 %, for the reasons explained in Section **??**. The $D \to K_S^0\pi^+\pi^-\pi^0$ decay, however, is dominantly *CP*-odd [**?**], and the bias in measurements based on the total asymmetry is therefore expected to be $O(r_B\epsilon)$, ie. a few degrees [**?**]. More precise calculations of the biases would require a repeat of the study included here, with relevant amplitude models and binning schemes in place.

The studies presented here can be used to assign systematic uncertainties to measurements while the statistical uncertainties continue to dominate. As the statistical uncertainty becomes comparable with the bias effects described in this paper, the systematic uncertainty should be assigned by repeating the studies with a detailed detector simulation. This would incorporate a more accurate description of the K_S^0 decay-time acceptance, of the full selection criteria, and the traversed material. The detailed calculations can also be used to apply a bias correction if desired.

5 A GGSZ measurement with $B^{\pm} \to Dh^{\pm}$ decays

First I will return to describing the overall strategy a bit, then one can proceed with the data analysis section

5.1 Candidate selection

5.2 Signal and background components

5.3 Measurement of the CP-violation observables

5.4 Systematic uncertainties

5.5 Obtained constraints on γ

6
Conclusions

Say something clever

Bibliography

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