

# Signal Output of a Linear Time-Variant System

Mikael Henriksson *and* Danyo Danev (2022)

We define the output of a Linear Time-Variant (LT) system, illustrated in Fig 1, by the following integral equation

$$y(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} h(\tau; t) x(t - \tau) d\tau. \quad (1)$$

Further, we define the Fourier Transform ( $\mathcal{F}$ ) of a possibly multivariate function, with respect to one of its input variable, as

$$\mathcal{F}_{\tau \rightarrow f} \{ h(\tau; t) \} \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} h(\tau; t) e^{-j2\pi f\tau} d\tau = H(f; t), \quad (2)$$

and the Inverse Fourier Transform ( $\mathcal{F}^{-1}$ ) of a possibly multivariate function, with respect to one of its input variables, as

$$\mathcal{F}_{f \rightarrow \tau}^{-1} \{ H(f; t) \} \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} H(f; t) e^{j2\pi f\tau} df = h(\tau; t). \quad (3)$$

With these three definitions, we are able to prove the following relationship between the Inverse Fourier Transform of the product  $X(f) \cdot H(f; t)$ , and the time-domain output from the LT system  $y(t)$ :

$$\begin{aligned} \int_{-\infty}^{\infty} H(f; t) X(f) e^{j2\pi ft} df &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(w; t) e^{-j2\pi fw} dw X(f) e^{j2\pi ft} df = \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(w; t) e^{-j2\pi fw} X(f) e^{2\pi ft} df dw = \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(w; t) e^{-j2\pi fw} X(f) e^{2\pi ft} df dw = \\ &= \int_{-\infty}^{\infty} h(w; t) \int_{-\infty}^{\infty} X(f) e^{-j2\pi fw} e^{2\pi ft} df dw = \\ &= \int_{-\infty}^{\infty} h(w; t) \int_{-\infty}^{\infty} X(f) e^{j2\pi f(t-w)} df dw = \\ &= \int_{-\infty}^{\infty} h(w; t) x(t - w) dw \stackrel{\text{def}}{=} y(t). \end{aligned} \quad (4)$$

We conclude the following important relation for LT systems

$$\underline{\underline{y(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} h(\tau; t) x(t - \tau) d\tau = \int_{-\infty}^{\infty} H(f; t) X(f) e^{j2\pi ft} df.}} \quad (5)$$

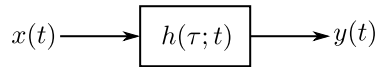


Figure 1: A general Linear and Time-Variant (LT) system.