Report 6
Modeling and Identification

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1 Non-Parametric Identification

Considered system is shown on Figure 1. Equation that describe system look like:

$$y = 0,9 * y_{n-1} + u_n + z_n$$

We simulate run of the system by generating N samples: $\left\{(u_k, y_k)\right\}_{k=1}^{N}$. During identification we calculate succeeding coefficients $(\hat{\gamma}_0, \hat{\gamma}_1, ..., \hat{\gamma}_S)$ where S is the scale of the model:

$$\hat{\gamma}_\tau = \frac{1}{N-\tau} \sum_{N-\tau}^{n=1} u_n y_{n+\tau}$$

After calculating coefficients we can calculate estimation:

$$\hat{y_k} = \sum_{S}^{i=1} \hat{\gamma}_i u_{k-1}$$

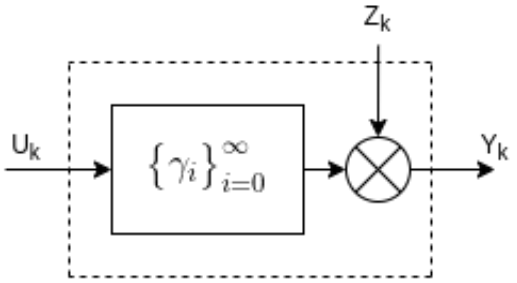

Figure 1: Considered System.

1.1 Experiments

For the testing new data was generated to validate the model. Error measure used in experiments:

$$\Delta_S = \frac{1}{N}\sum_{n=1}^{N}(y_n - \hat{y_n})$$

Plot of error (delta) for given S in shown on Figure 2.

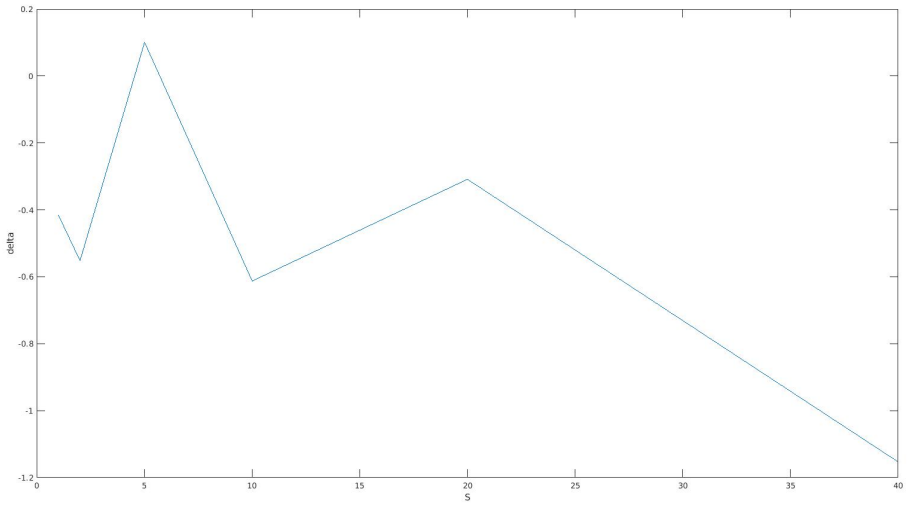

Figure 2: Plot of delta for given S.

2 Generalized Least Squares Method

In this task we perform estimation of the system using Generalized Least Squares Method. Considered system is shown on Figure 3. In this case we add disturbances it the form of:

$$Z_n = \epsilon_n + \epsilon_{n-1}$$

Where $\epsilon \sim N(0,1)$. Covariance matrix of disturbances looks as shown:

$$C_{NxN} = \begin{bmatrix} 2 & 1 & 0 & ... & 0 \\ 1 & 2 & 1 & ... & ... \\ 0 & 1 & ... & ... & 0 \\ ... & ... & ... & 2 & 1 \\ 0 & ... & 0 & 1 & 2 \end{bmatrix}$$

The equation for GLS estimation:

$$\hat{\theta}_{GLS} = (\Phi_N^T C^{-1} \Phi_N)^{-1} \Phi_N^T C^{-1} Y_N$$

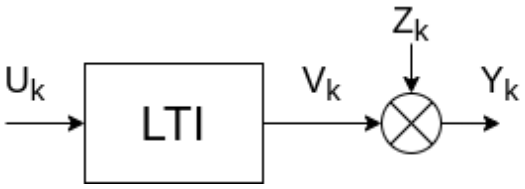

Figure 3: Considered system.

2.1 Experiments

In experiments we compare GLS with known covariance matrix with LS. Figure 4 shown error of estimation for GLS and LS method relative to number of samples.

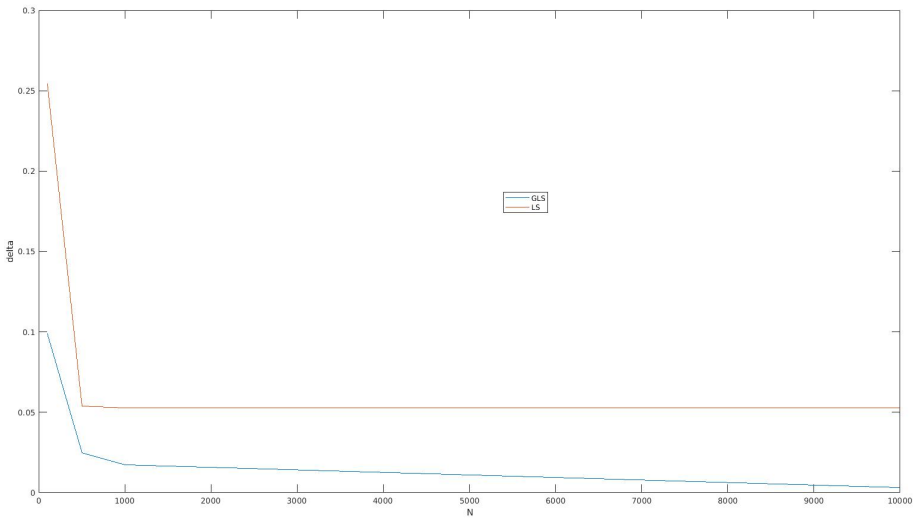

Figure 4: Estimation Error Comparison.

3 Conclusion

We can clearly see form Figure 4 that GLS method works much better in comparison with LS.