Report 2
Modeling and Identification

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1 Mean Squared Error Analysis

Mean Squared Error (MSE) - measures the average of the squares of the errors which is, the average squared difference between the estimated values and the actual value. It may be written as the sum of the variance and the squared bias of the estimator:

$$MSE(\hat{\theta}) = E[\hat{\theta} - E\hat{\theta}]^2 + E[E\hat{\theta} - \theta^*]^2 = Var\hat{\theta} + Bias^2\hat{\theta}$$

If Bias equals to 0:

$$MSE(\hat{\theta}) = Var\hat{\theta}$$

1.1 Experiments

In this experiment we wanna show one of the MSE feature which is convergence to 0. We gonna use variance to show that it goes to 0 as number of measurements goes to infinity.

1.2 Results

Results of experiments are shown on Figure 1

2 Non-parametric Estimation Of Probability Density Function

This estimation is based on estimating coefficients from gathered samples and constructing function estimation in the form of:

$$\hat{f}(x) = \sum_{S}^{i=1} \hat{a}_i \varphi_i(x)$$

Estimation of coefficients, for every sample x_k:

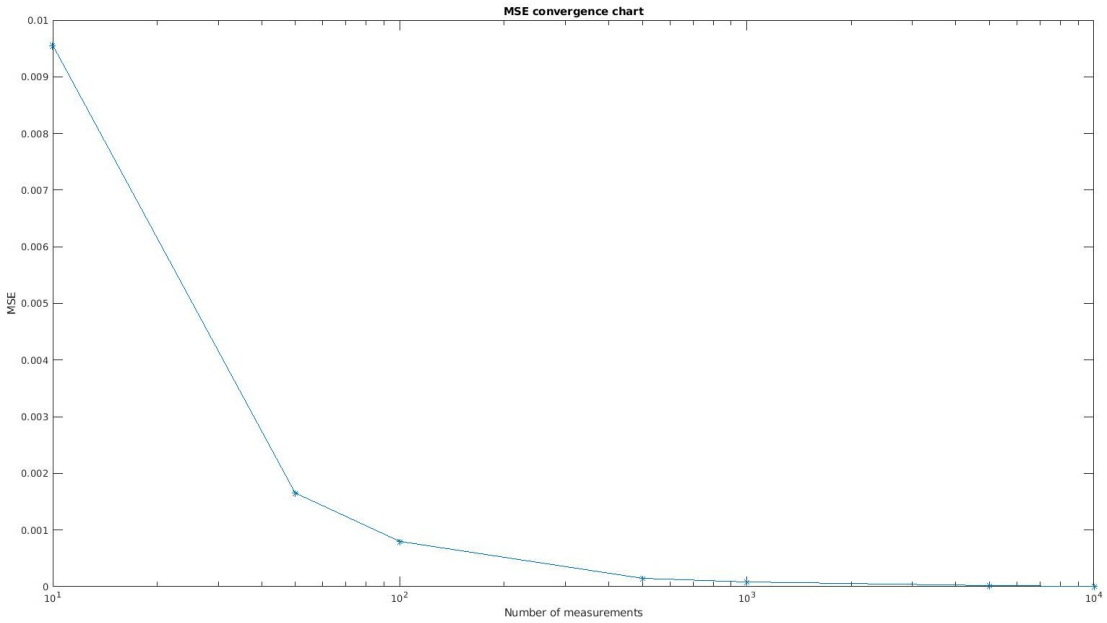

Figure 1: MSE variance convergence to 0.

$$\hat{a}_i = \frac{1}{N} \sum_{k=1}^{N} \varphi_i(x_k)$$

Proposed basis function is in the form of:

$$\varphi_i(x,i) = \begin{cases} \frac{1}{\sqrt{2}} & if\ i = 1 \\ sin(\frac{i}{2}\pi x) & if\ i\ is\ even \\ ccos(\frac{i-1}{2}\pi x) & if\ i\ is\ odd \end{cases}$$

2.1 Experiments

In this experiment we wanna show estimation of PDF according to number of S (coefficients), and plot the graph of MSE to number of S.

2.2 Results

PDF estimation according to number of S is shown on Figure 2.
Comparison of MSE to a number of S is shown on Figure 3.

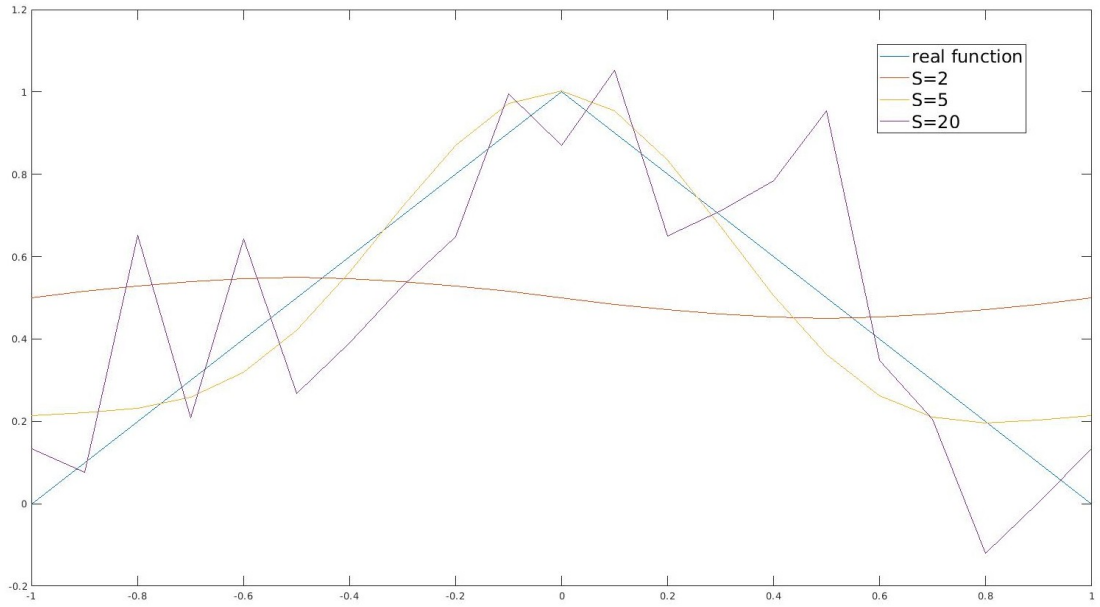

Figure 2: PDF estimation.

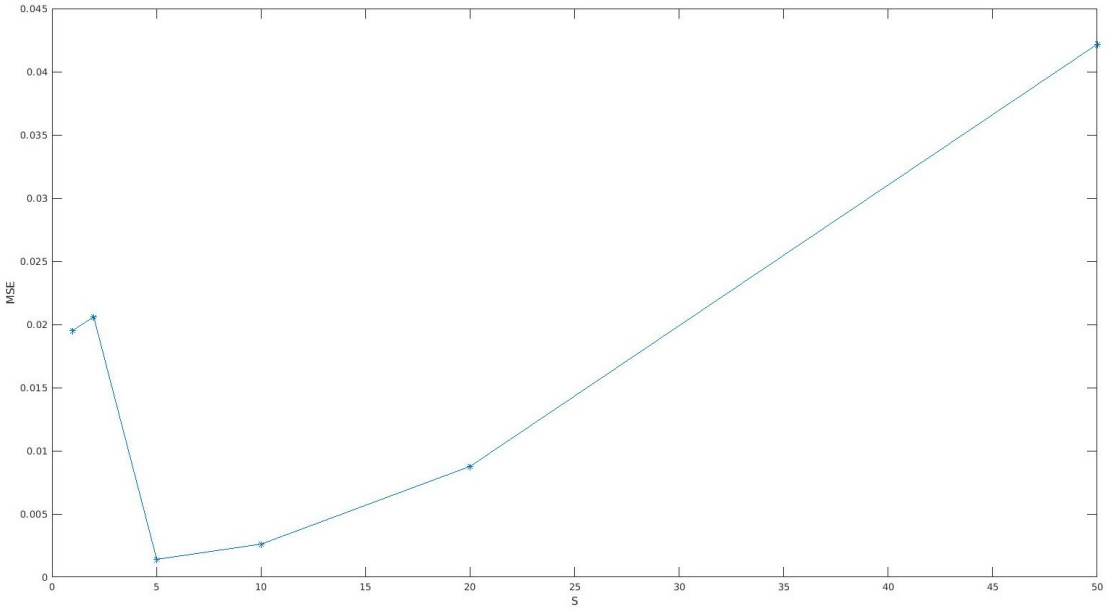

Figure 3: MSE to number of S.