# Report 5
# Modeling and Identification

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## 1 Identification of Multi-Dimensional Systems

General multi-dimensional system is shown on Figure 1. During experiments we considered system show on Figure 2. Estimate of values A and B of each sub-system we can compute as fallow:

$$\begin{aligned} Y_{iN} &= [y_i^{(1)}, y_i^{(2)}, \ldots, y_i^{(N)}] \\ W_{iN} &= [w_i^{(1)}, w_i^{(2)}, \ldots, w_i^{(N)}] \\ w_i &= (x_i, u_i)^T \end{aligned}$$

$$Y_{iN} = (A_i, B_i) W_{iN} + \xi_i$$

$$\widetilde{w}_i = (\widetilde{x}_i, u_i)^T, \qquad \widetilde{x}_i = H_i y = x_i - \delta_i$$

$$\widetilde{W}_{iN} = [\widetilde{w}_i^{(1)}, \widetilde{w}_i^{(2)}, \ldots, \widetilde{w}_i^{(N)}]$$

$$(\widehat{A}_i^{l.s.}, \widehat{B}_i^{l.s.}) = Y_{iN} \widetilde{W}_{iN}^T \left( \widetilde{W}_{iN} \widetilde{W}_{iN}^T \right)^{-1}$$

Where:
$Y_{iN}-$ matrix of successive outputs
$u_i$ - vector of i-th input
$x_i$ - vector of i-th internal state

### 1.1 Experiments

In system shown on Figure 2 we have three sub-systems. We collected N samples from system. Sample output was generated and disturbances Z which has distribution $U(-0.05, 0.05)$ were added. Then values of A and B was estimated by Least Squared Method using equations shown above. Results of MSE for each

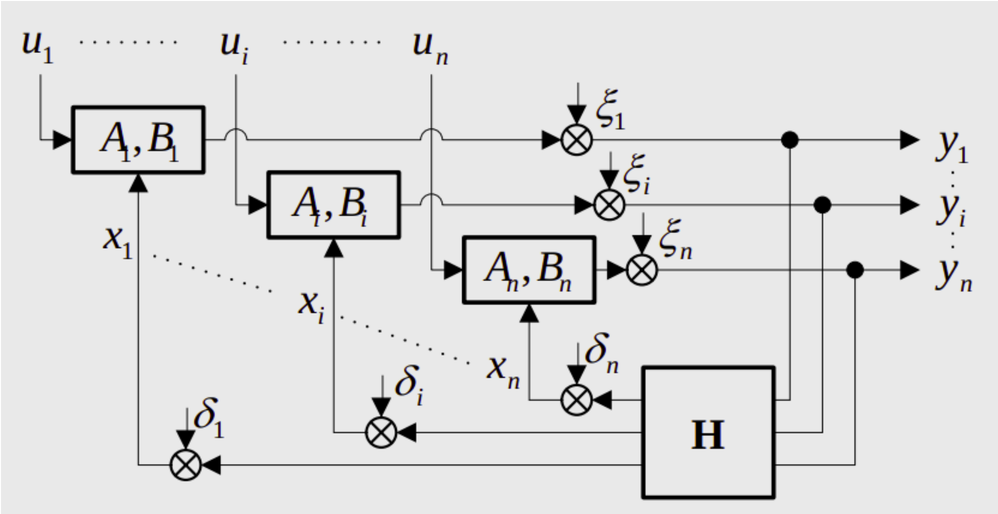


Figure 1: General Multi-Dimensional system diagram.

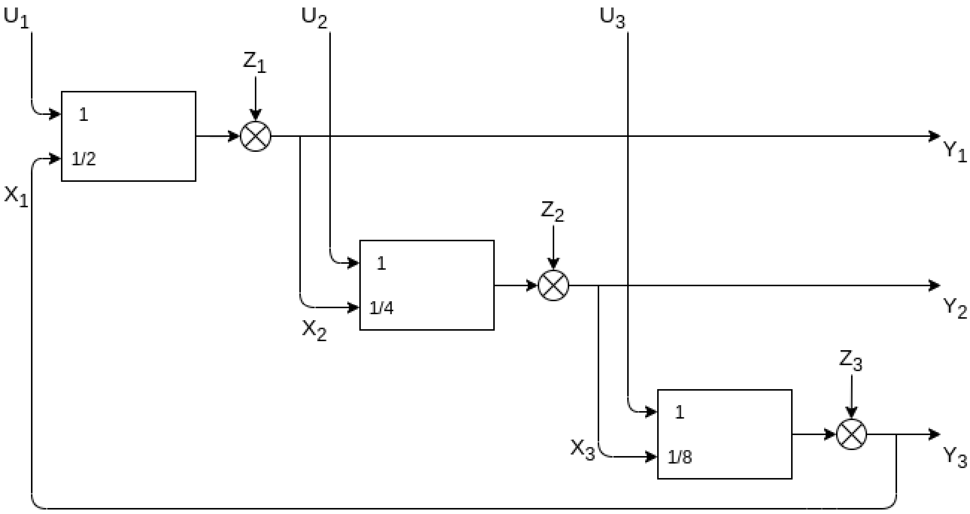


Figure 2: Considered system.

parameter together for all sub-systems with respect to numbers of samples are shown on Figure 3.

## 2 Regression Estimation with Parametric Approach

The problem of regression estimation is shown on Figure 4. We have the input signal $U_k$, the output signal of unknown regression function $Y_k$, and the noise $Z_k$ which is the part of the output.

Our goal is to estimate function $\mu()$ by measuring the input and the output values. To make it consistent we nee to add some assumptions:

- $\mu() \rightarrow$ is continuous function,
- $EZ_k = 0$,

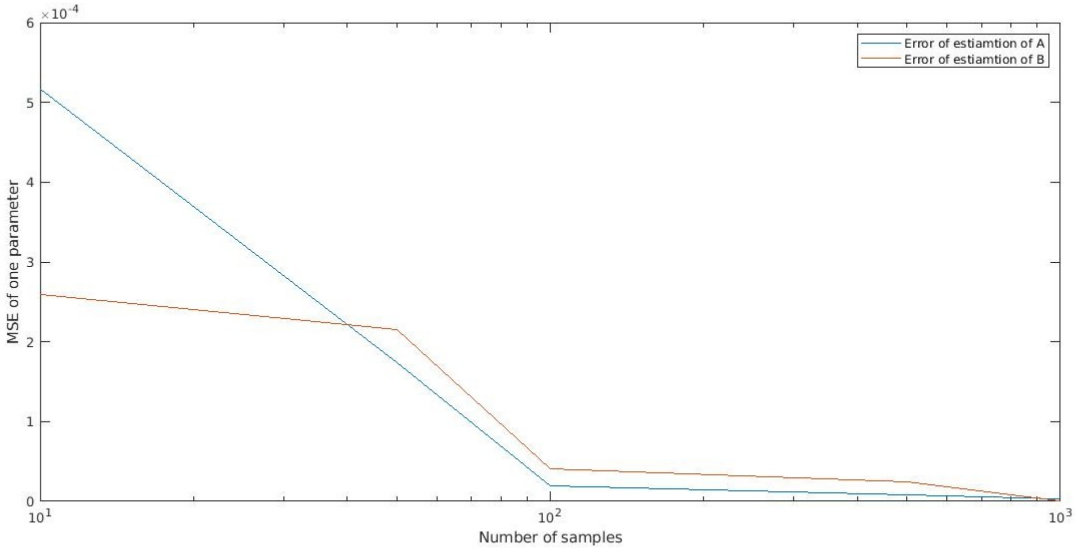


Figure 3: MSE for each parameter for multi-dimensional system.

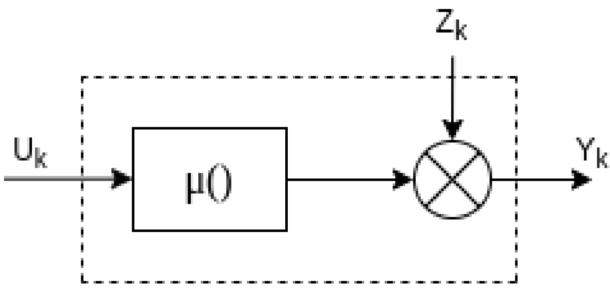


Figure 4: Diagram of Regression Estimation.

- $varZ_k < \infty$,
- $U_k$ and $Z_k$ are independent.

To accomplish this goal we use parametric method. In this case we have knowledge about equation which is as follow:

$$y_k = c_1^* f_1(u) + c_2^* f_2(u) + ... + c_p^* f_p(u)$$

In our case:

$$y_k = 1^* u + 1 * sin(u)$$

We can define:

$$\varphi(u) = \begin{bmatrix} f_1(u) \\ f_2(u) \\ ... \\ f_p(u) \end{bmatrix}$$

$$\Phi_N = (\varphi^T(u_1), \varphi^T(u_2), ..., \varphi^T(u_N))^T$$

To obtain the estimations of C vector we can use LSM:

$$\hat{c} = (\Phi_N^T \Phi_N)^{-1} \Phi_N^T Y_N$$

During experiments disturbances were added to the output, 100 samples were gathered for tests. Results of estimation are shown on Figure 5, and estimated C values are equal to: $[0.9949; 0.9981]$

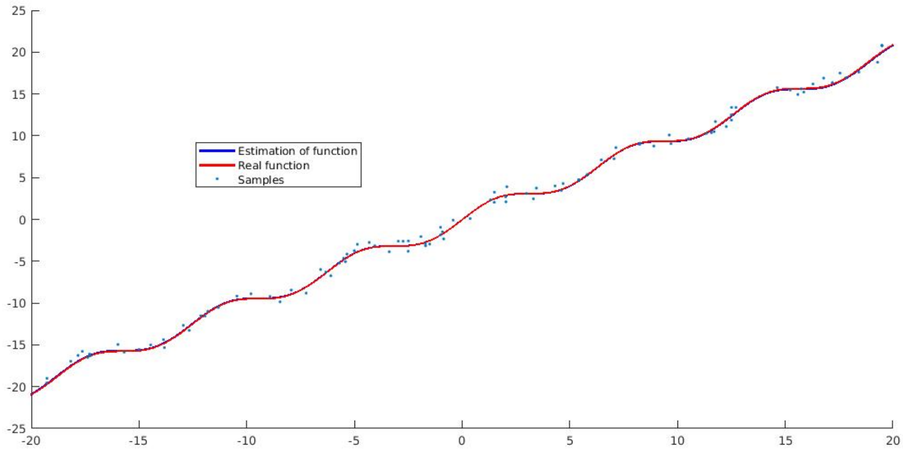


Figure 5: Results of Regression Estimation.

# 3 Conclusions

For the experiment with multi-dimensional system we can clearly see that error converges to 0 as number of samples grows.

The second experiment shows that even for small number of samples estimation seems quite good.