Report 7
Modeling and Identification

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1 Hammerstein System

Considered system is shown on Figure 1. This is Hammerstein system which is cascade connection of nonlinear static block and linear dynamic block.

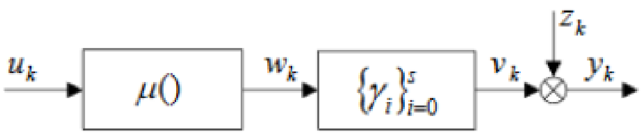

Figure 1: Considered Hammerstein System.

Nonlinear static block is described by:

$$\mu(u) = c_1 * f_1(u) + c_2 * f_2(u)$$

basis function:

$$f_1(u) = c_1 * u^2$$

$$f_2(u) = c_2 * u$$

and parameters:

$$c_1 = 1$$

$$c_2 = 2$$

Linear dynamic block is described by:

$$v_k = \gamma_1 * w_k + \gamma_2 * w_{k-1} + \gamma_3 * w_{k-2}$$

and parameters:

$$\gamma_1 = 3$$

$$\gamma_2 = 2$$
$$\gamma_3 = 1$$

Entire system equation we can write in matrix form as:

$$y_k = \begin{bmatrix} \gamma_1 c_1 \\ \vdots \\ \gamma_1 c_m \\ \vdots \\ \gamma_s c_m \end{bmatrix} * \begin{bmatrix} f_1(u_k) \\ \vdots \\ f_m(u_k) \\ \vdots \\ f_m(u_{k-s}) \end{bmatrix} + z = \begin{bmatrix} 3 \\ 6 \\ 2 \\ 4 \\ 1 \\ 2 \end{bmatrix} * \begin{bmatrix} f_1(u_k) \\ \vdots \\ f_m(u_k) \\ \vdots \\ f_m(u_{k-s}) \end{bmatrix} + z = \theta * F(u) + z$$

1.1 Experiments

For the experiments we use:

$$u = U(-5, 5)$$

and we used following error measure:

$$\Delta = \left\| \theta - \hat{\theta} \right\| = \sqrt{\sum_{i=1}^{6} (\theta_i - \hat{\theta}_i)^2}$$

Plot of error (delta) for given N (number of data) in shown on Figure 1.

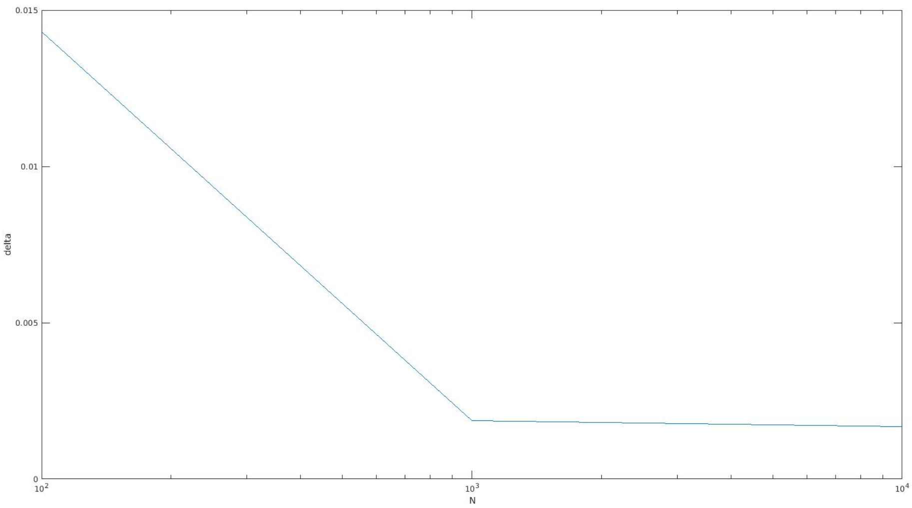

Figure 2: Delta for given number of data.

2 Hammerstein System Nonparametric Approach

System that we consider in this task is shown on Figure 1 form previous section. In this task we use equation for nonlinear static block:

$$\mu(u) = |u|$$

and the same equation for linear dynamics block:

$$v_k = \gamma_1 * w_k + \gamma_2 * w_{k-1} + \gamma_3 * w_{k-2}$$

and parameters:

$$\gamma_1 = 3$$

$$\gamma_2 = 2$$

$$\gamma_3 = 1$$

In this case regression function looks like:

$$R(u) = \gamma_1 \mu(u) + (\gamma_2 + \gamma_3) E w = 3\mu(u) + \frac{3}{2}$$

For estimating μ function we use:

$$\hat{\mu} = \hat{R}(u) - \hat{R}(0)$$

as a regression function we use Kernel Regression. After estimation of μ we use LS for estimating γ parameters.

2.1 Experiments

For the experiments we use:

$$u = U(-1, 1)$$

Figure 3 shows system output and resulted model output. In this case we obtained:

$$\gamma = \begin{bmatrix} 3.4436 \\ 2.0526 \\ 0.9434 \end{bmatrix}$$

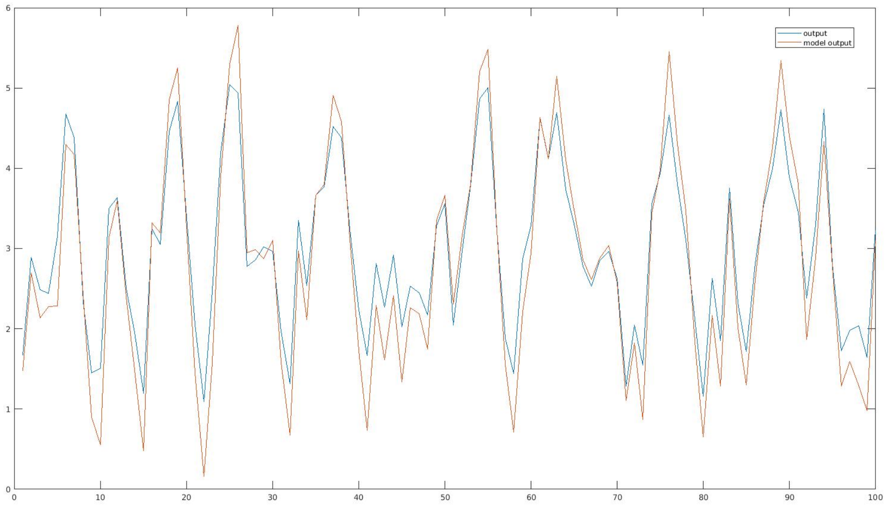

Figure 3: System output vs model output.

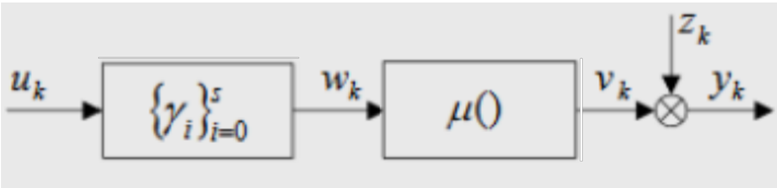

Figure 4: Wiener System.

3 Wiener System

Considered system is shown on Figure 4. This is Wiener system which is cascade connection of linear dynamic block and nonlinear static block.

For identification of linear dynamics we perform with LS but we enrich this estimator with kernel selection.

$$\hat{\theta} = (\sum_{k=1}^{N} \varphi_k \varphi_k^T K(\frac{\|\varphi_k\|}{h}))^{-1} (\sum_{k=1}^{N} \varphi_k y_k K(\frac{\|\varphi_k\|}{h}))$$

We denote:

$$\varphi = \begin{bmatrix} U_k \\ U_{k-1} \end{bmatrix}$$

Then we estimate nonlinear static block by Kernel Regression Estimator.

3.1 Experiments

Parameters of the linear dynamic block: $S = 2$, $\gamma_1 = 2$, $\gamma_2 = 1$.
For the nonlinear static we take: $\mu = sin()$ Other parameters:

$$U_k = U(-1, 1)$$

$$Z_k = U(-0.1, 0.1)$$

As a error measure we take squared error of real output and model output called delta. Results of the experiments are shown on Figure 5. Plot shows delta to corresponding size of window h.

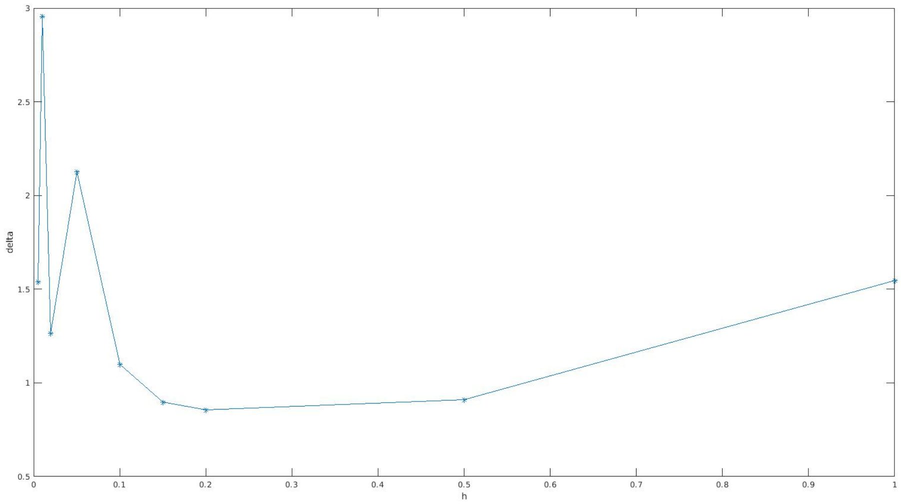

Figure 5: Size of h to delta for Wiener System.