Report 3
Modeling and Identification

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1 Regression Estimation

The problem of regression estimation is shown on Fig 1. We have the input signal U_k, the output signal of unknown regression function Y_k, and the noise Z_k which is the part of the output. Our goal is to estimate function $\mu()$ by measuring the input and the output values. To make it consistent we nee to add some assumptions:

- $\mu() \rightarrow$ is continuous function,
- $EZ_k = 0$,
- $varZ_k < \infty$,
- U_k and Z_k are independent.

To accomplish this goal we use two methods: Kernel Regression Estimator and Orthogonal Expansion Method. Description of each method is presented below.

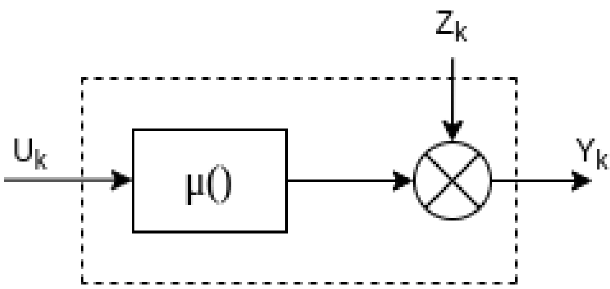

Figure 1: Diagram of Regression Estimation

1.1 Kernel Regression Estimator

Formula for estimating $\mu()$ function by Kernel Regression Estimator looks like this:

$$\hat{\mu}(u) = \frac{\sum_{k=1}^{N} y_k K(\frac{u_k - u}{h})}{\sum_{k=1}^{N} K(\frac{u_k - u}{h})}$$

where:

$$K(u) = \begin{cases} 1 & : \ |u| < h \\ 0 & : \ elsewhere \end{cases}$$

h - size of kernel window.

1.2 Orthogonal Expansion Method

Formula for estimating $\mu()$ function by Orthogonal Expansion Method looks like this:

$$\hat{\mu}(u) = \frac{\sum_{i=1}^{S} \hat{b}_i \varphi_i(u)}{\sum_{i=1}^{S} \hat{a}_i \varphi_i(u)}$$

where:

$$\hat{a}_i = \frac{1}{N} \sum_{k=1}^{N} \varphi_i(u_k)$$

$$\hat{b}_i = \frac{1}{N} \sum_{k=1}^{N} y_k \varphi_i(u_k)$$

$$\varphi_i(x, i) = \begin{cases} \frac{1}{\sqrt{2}} & if\ i = 1 \\ sin(\frac{i}{2}\pi x) & if\ i\ is\ even \\ ccos(\frac{i-1}{2}\pi x) & if\ i\ is\ odd \end{cases}$$

2 Experiments

The function that was chosen as μ is:

$$\mu(u) = \lfloor u * 5 \rfloor$$

Then the 100 random arguments were chosen from interval [-1,1], values for them was calculated form μ function and the noise (2*rand()-1) was added. Nest step was to estimate function μ using samples with noise.

2.1 Kernel Regression Estimator

Plot of the estimated function μ by Kernel Regression Estimator with h=0.1 is shown on Fig 2. Relationship between value of h and corresponding MSE is shown on Fig 3.

2.2 Orthogonal Expansion Method

Plot of the estimated function μ by Orthogonal Expansion Method with S=5 is shown on Fig 4. Relationship between value of h and corresponding MSE is shown on Fig 5.

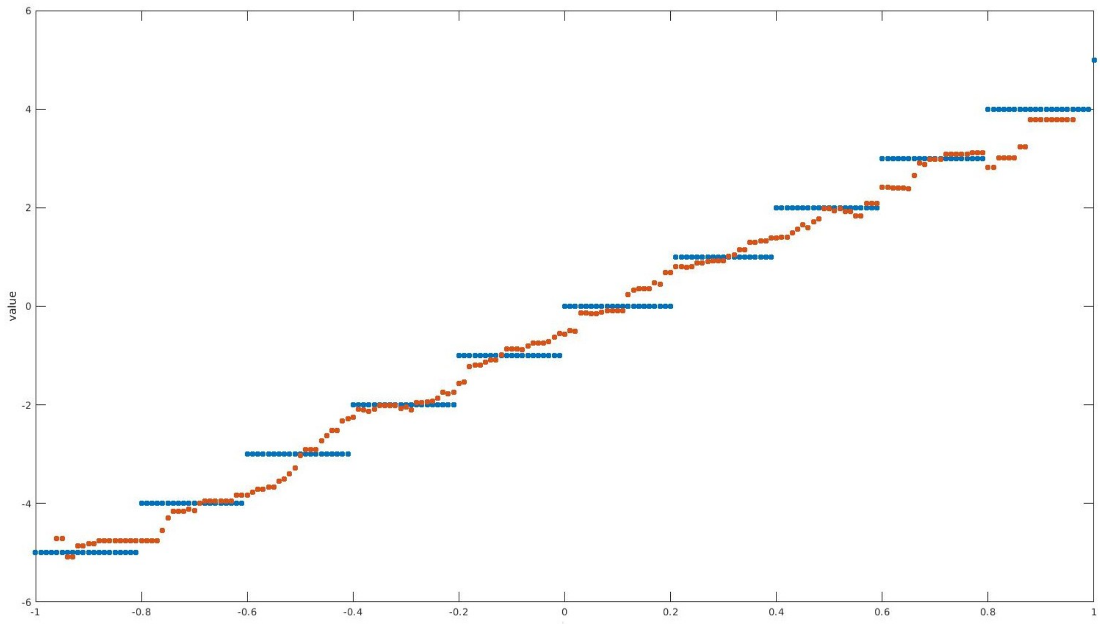

Figure 2: Estimated μ function for h=0.1.

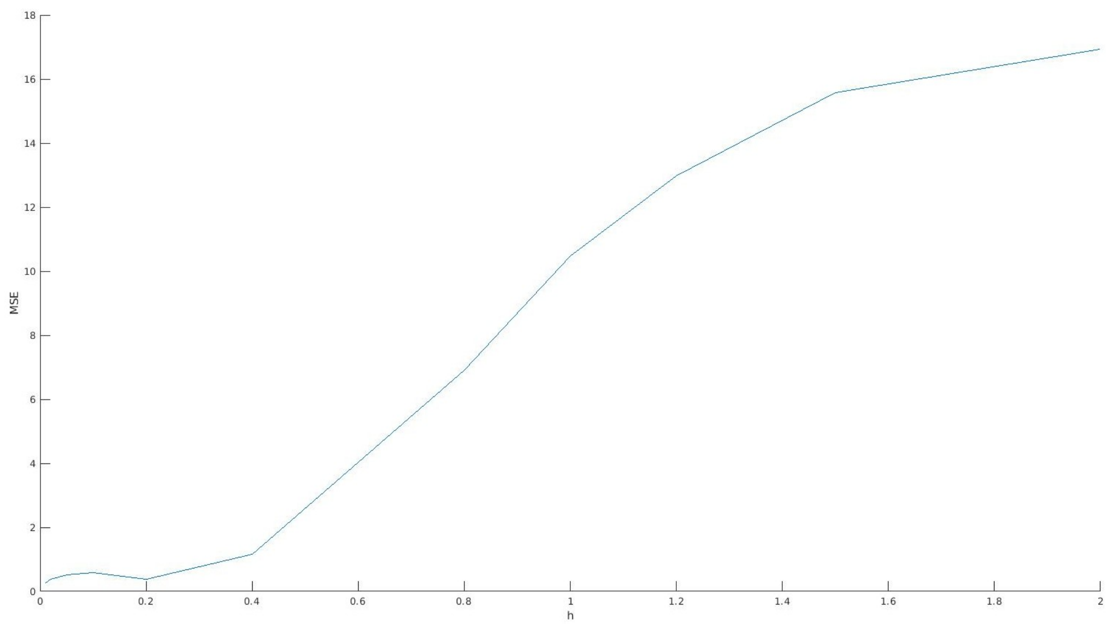

Figure 3: Relationship between MSE and h value for Kernel Regression Estimator.

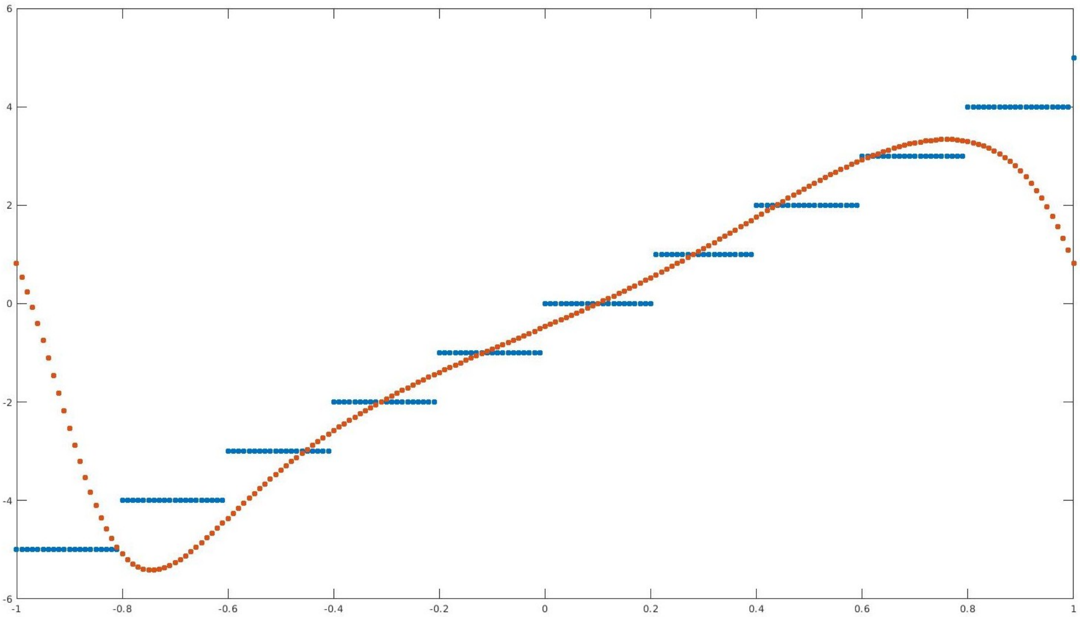

Figure 4: Estimated μ function for S=5.

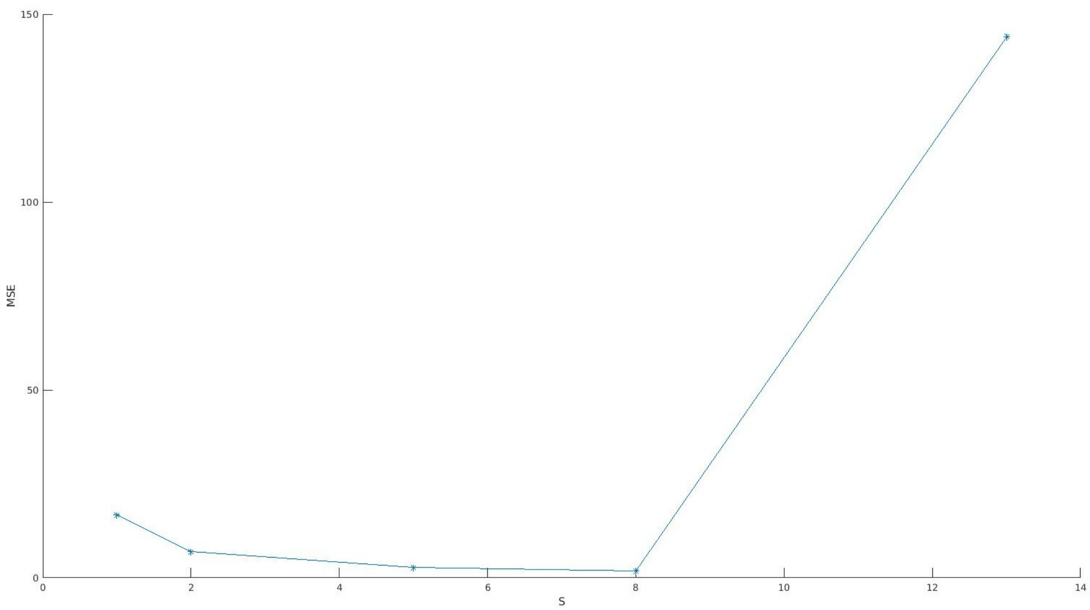

Figure 5: Relationship between MSE and h value for Orthogonal Expansion Method.