Limit shapes of position routes

Mikołaj Marciniak

1 Introduction

Dan Romik and Piotr Śniady in [1] were considering limit shape of bumping routes obtained from insertion tableau, when applying an RSK insertion step with a fixed input $z \in [0, 1]$ to an existing insertion tableau P_n, where P_n is the result of n previous insertion steps applying to n i.i.d. $U(0, 1)$ random inputs $X_1, \dots, X_n$.

In [2] they were considering limit shape of jeu de taquin path obtained from insertion tableau made from n i.i.d. $U(0, 1)$ random inputs $X_1, \dots, X_n$ prepending a fixed number $z \in [0, 1]$.

They were looking into in the first case to $P((X_1, \dots, X_n, z))$ and in the second case to $P((z, X_1, \dots, X_n))$. We can merge this cases and consider $P((X_1, \dots, X_n, z, X_{n+1}, \dots, X_m))$, where m is approximately A times greater than n for any $A \in (1, \infty]$.

After applying an RSK insertion steps consecutive for numbers $X_1, \dots, X_n, z$ in insertion tableau will appear box with number z. During applying an RSK insertion steps consecutive for numbers $X_{n+1}, \dots, X_m$ box with number z can be sliding by the bumping roots. We would like to prove, that exist macroscopic limit shape of path describing the position of box with number z.

2 Position path

Let's take a fixed number $z \in [0,1]$. Let $\{X_j\}_{j=1}^{\infty}$ will be sequence of i.i.d. $U(0,1)$. Let $A \in (1,\infty)$, $n \in \mathbb{N}$ and let function $Pos_n : \{n+1, n+2, \ldots\} \to \mathbb{N}^2$ describe position of box with number z:

$$Pos_n(j) = (a_j^{(n)}, b_j^{(n)}) = box_z(P(X_1, \ldots, X_n, z, X_{n+1}, \ldots, X_j))$$

for $j \in \{n+1, n+2, \ldots\}$. $\exists_{G:[1,\infty)\to\mathbb{R}_+^2} \forall_{\epsilon>0}$

$$lim_{n\to\infty}\mathbb{P}(sup_{1\leq A}||G(A) - \tfrac{Pos_n([An])}{\sqrt{A}\sqrt{n}}|| > \epsilon) = 0$$

Literatura

[1] Dan Romik, Piotr Śniady *Limit shapes of bumping routes in the Robinson-Schensted correspondence.*

[2] Dan Romik, Piotr Śniady *Jeu de taquin dynamics on infinite Young tableaux and second class particles.* The Annals of Probability 2015, Vol. 43, No. 2: 719-724.