Mikołaj Buczak

Kolokwium I

1.

$$|\Omega| = \binom{20}{8} = \frac{20!}{8!(20-8)!} = \cancel{\frac{13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 19\cdot 20}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}} \quad \frac{13\cdot \cancel{14}\cdot 15\cdot \cancel{16}^{2}\cdot 17\cdot \cancel{18}\cdot 19\cdot \cancel{20}}{\cancel{2}\cdot \cancel{3}\cdot \cancel{4}\cdot \cancel{5}\cdot \cancel{6}\cdot \cancel{7}\cdot \cancel{8}}$$

$$= 13\cdot 15\cdot 2\cdot 17\cdot 19$$

A- zdarzenie, że wśród wybranych detali ~~…~~ są 3 standardowe

$$|A| = \binom{6}{3}\binom{14}{5} = \frac{4\cdot 5\cdot \cancel{6}}{\cancel{2}\cdot \cancel{3}} \cdot \frac{\cancel{10}\cdot 11\cdot \cancel{12}\cdot 13\cdot 14}{\cancel{2}\cdot \cancel{3}\cdot \cancel{4}\cdot \cancel{5}} = 4\cdot 5\cdot 11\cdot 13\cdot 14$$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{\cancel{4}^{2}\cdot \cancel{5}^{1}\cdot 11\cdot \cancel{13}\cdot 14}{\cancel{13}\cdot \cancel{15}_{3}\cdot \cancel{2}\cdot 17\cdot 19} = \frac{2\cdot 11\cdot 14}{3\cdot 17\cdot 19} = \frac{308}{969} \approx 0{,}32$$

2.

$$|\Omega| = 12+20+18 = 50$$

A - zdarzenie, że wylosowany detal I-szej jakości został wyprodukowany w zakładzie A

Prawdopodobieństwo, że detal jest I-szej jakości w A - 0,9

$$P(A) = \frac{12}{50}\cdot 0{,}9 = 0{,}216$$

3.

długość boku kwadratu opisującego monetę: $a-2r$

Prawdopodobieństwo, że moneta nie spadnie na żadną krawędź to stosunek powierzchni kwadratu opisującego monetę do powierzchni oryginalnego kwadratu, stąd wzór:

$$P(A) = \frac{(a-2r)^2}{a^2}$$

4.

$$f(x)=\begin{cases} c(2-x) & x\in[0,2] \\ 0 & x\notin[0,2]\end{cases}$$

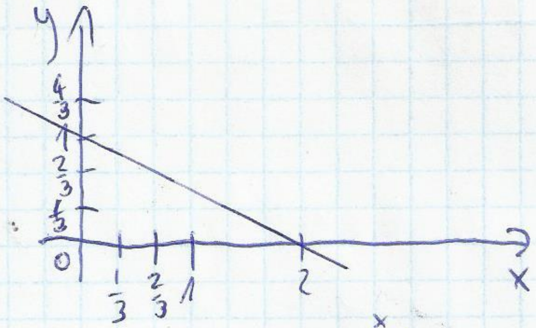
a)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 c(2-x)dx = c\int_0^2 (2-x)dx = c\left(2x\Big|_0^2 - \frac{x^2}{2}\Big|_0^2\right) = c(4-2) = 2c = 1 \Rightarrow$$

$$c=\frac{1}{2}$$

$$f(x)=\frac{1}{2}(2-x) \quad x\in[0,2]$$



Dystrybuanta: $F(x)=\int_{-\infty}^{x} f(t)dt$

- $x<0$ $F(x)=\int_{-\infty}^{x} 0dx = 0$
- $x\in[0,2]$ $F(x)=\int_{-\infty}^{0} 0dx + \int_0^x \frac{1}{2}(2-x)dx = \frac{1}{2}\left(2x\Big|_0^x - \frac{x^2}{2}\Big|_0^x\right) =$

$$\frac{1}{2}\left(2x-\frac{1}{2}x^2\right) = x - \frac{1}{4}x^2$$

- $x>2$ $F(x)=\int_{-\infty}^{0} 0dx + \int_0^2 \frac{1}{2}(2-x)dx + \int_2^x 0dx = \frac{1}{2}\left(2x\Big|_0^2 - \frac{x^2}{2}\Big|_0^2\right) = \frac{1}{2}(4-2) = 1$

$$F(x)=\begin{cases} 0; & x<0 \\ -\frac{1}{4}x^2+x; & x\in[0,2) \\ 1; & x>2\end{cases}$$

b) $E(\xi)=\int_0^2 x f(x)dx$

$E(\xi)=\int_0^2 x\frac{1}{2}(2-x)\,dx=\int_0^2\left(x-\frac{x^2}{2}\right)dx=\frac{x^2}{2}\Big|_0^2-\frac{x^3}{6}\Big|_0^2=2-\frac{4}{3}=\frac{2}{3}$

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$$f(x)=\begin{cases}|x-1| & 0<x\leq 2\\ 0 & x\notin[0,2]\end{cases}$$

$E(\xi)=\int_{-\infty}^{\infty}x f(x)dx=\int_{-\infty}^{0}0\,dx+\int_2^{\infty}0\,dx+\int_0^1(-x+1)dx+\int_1^2(x-1)dx=\left(-\frac{1}{2}x^2+x\right)\Big|_0^1+$
$\left(\frac{1}{2}x^2-x\right)\Big|_1^2=\left(-\frac{1}{2}+1\right)+\left(2-2-\frac{1}{2}+1\right)=\frac{1}{2}+\frac{1}{2}=1$

$V(\xi)=E\xi^2-(E\xi)^2=\int_{-\infty}^{\infty}x^2f(x)\,dx-(E\xi)^2=\int_0^1 x^2(-x+1)dx+\int_1^2 x^2(x-1)dx-(E\xi)^2=$
$\int_0^1(-x^3+x^2)dx+\int_1^2(x^3-x^2)dx-(E\xi)^2=\left(-\frac{1}{4}x^4+\frac{1}{3}x^3\right)\Big|_0^1+\left(\frac{1}{4}x^4-\frac{1}{3}x^3\right)\Big|_1^2-(E\xi)^2=$

$=\left(-\frac{1}{4}+\frac{1}{3}\right)+\left(4-\frac{8}{3}\right)-\left(\frac{1}{4}-\frac{1}{3}\right)-1^2=\frac{1}{12}+4-\frac{8}{3}+\frac{1}{12}-1=\frac{1}{6}+\frac{24}{6}-\frac{16}{6}-\frac{6}{6}=\frac{1}{2}$

6.

$E\xi=1$ $\qquad P(|\xi-E\xi|\geq\varepsilon)\leq\frac{D^2\xi}{\varepsilon^2}$

$E\xi^2=1{,}01$ $\qquad P\{\xi\geq\varepsilon\}\leq\frac{E\xi}{\varepsilon}$

$P\{\xi\notin[0{,}7;1{,}3]\}$

$|\xi-E\xi|=|\xi-1|\geq 0{,}3$

$P\{\xi\notin[0{,}7;1{,}3]\}=P(|\xi-1|\geq 0{,}3)\leq\frac{D^2\xi}{(0{,}3)^2}$

$D^2\xi=E\xi^2-(E\xi)^2=1{,}01-1^2=0{,}01$

$P\{\xi\notin[0{,}7;1{,}3]\}=P(|\xi-1|\geq 0{,}3)\leq\frac{D^2\xi}{(0{,}3)^2}=\frac{0{,}01}{0{,}09}=\frac{1}{9}$

7.

$$f(x)=\begin{cases} -\frac{3}{4}x^2+\frac{9}{2}x-6 & 2\leq x\leq 4 \\ 0 & x\notin[2,4] \end{cases}$$

$$f'(x) = -\frac{3}{2}x+\frac{9}{2}=0 \qquad -\frac{3}{2}x=-\frac{9}{2}$$

$$x=3$$ wartości modalna

$$\int_2^x \left(-\frac{3}{4}x^2+\frac{9}{2}x-6\right)dx=\frac{1}{2}$$

$$\left(-\frac{1}{4}x^3+\frac{9}{4}x^2-6x\Big|_2^x\right)=\frac{1}{2}$$

$$-\frac{1}{4}x^3+\frac{9}{4}x^2-6x-\left(-\frac{1}{4}\cdot 8+\frac{9}{4}\cdot 4-6\cdot 2\right)=\frac{1}{2}$$

$$-\frac{1}{4}x^3+\frac{9}{4}x^2-6x+5=\frac{1}{2}$$

$$-\frac{1}{4}x^3+\frac{9}{4}x^2-6x=-4\frac{1}{2} \iff x=3$$ mediana

8.

$$p=0{,}1$$

$$D^2\xi=0{,}45$$

$$D^2\xi=npq \qquad q=1-p=1-0{,}1=0{,}9$$

$$0{,}45=n\cdot 0{,}1\cdot 0{,}9$$

$$0{,}45=0{,}09n \Rightarrow n=5$$

9.

$$F(x)=\begin{cases} 0 & x<0 \\ \sin 2x & 0<x<\frac{\pi}{4} \\ 1 & x>\frac{\pi}{4} \end{cases}$$

$$P(\xi>a) = F(a)=\frac{1}{3}$$

$$\sin 2a=\frac{1}{3}$$

$$2a=\arcsin\left(\frac{1}{3}\right)$$

$$a=\frac{\arcsin\left(\frac{1}{3}\right)}{2}$$

10. A - zdarzenie, że przy 150 strzałach tarcza będzie trafiona dokładnie 100 razy

$$P(A) = \binom{150}{100} 0{,}7^{100}\ 0{,}3^{50}$$