

# Radial FDM README

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## 1 Introduction

**Radial Pressure Diffusion Solver with Skin Effect**

## 2 Governing Equation

$$\frac{\partial P}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( D(r) \cdot r \cdot \frac{\partial P}{\partial r} \right)$$

where:

- $P(r, t)$ : pressure [Pa],
- $D(r) = \frac{k(r)}{\phi \mu C_t}$ : diffusivity [ $\text{m}^2/\text{s}$ ], potentially varying due to skin effect,
- $r$ : radial distance [m],
- $t$ : time [s].

## 3 Skin Effect Modeling

We use Hawkins' model to adjust permeability near the wellbore:

$$k_{skin} = \left( \frac{s}{k \ln(r_s/r_w)} + 1 \right)^{-1} \cdot k$$

This results in a modified diffusivity:

$$D_{skin} = \frac{k_{skin}}{\phi \mu C_t}$$

## 4 Discretization

We discretize the domain using finite differences on a non-uniform grid:

- $N_r$ : number of radial nodes,
- $N_t$ : number of time steps,
- $\Delta r = \frac{r_e - r_w}{N_r - 1}$ ,
- $\Delta t = \frac{t_{\max}}{N_t}$ .

Let  $P_i^n$  be the pressure at radius index  $i$  and time index  $n$ .

We use the explicit form of the FDM for the radial Laplacian:

$$\alpha_1 = \frac{D_i \Delta t}{2 \Delta r^2} - \frac{D_i \Delta t}{4 r_i \Delta r} \alpha_2 = 1 + \frac{D_i \Delta t}{\Delta r^2} \alpha_3 = \frac{D_i \Delta t}{2 \Delta r^2} + \frac{D_i \Delta t}{4 r_i \Delta r}$$

These coefficients are used to build a tridiagonal matrix  $A$ :

$$A_i = \begin{matrix} \ddots & \ddots & \ddots & -\alpha_1 \alpha_2 - \alpha_3 & \ddots & \ddots & \ddots \end{matrix}$$

## 5 Boundary Conditions

- Dirichlet at the well:  $P_1^n = P_{wf}$ ,
- Neumann (no flow) at the outer boundary:  $\frac{\partial P}{\partial r} \Big|_{r_e} = 0 \Rightarrow P_N = P_{N-1}$ .

## 6 Stability Criterion (Explicit Scheme)

The stability criterion is:

$$\Delta t < \frac{\Delta r^2}{2 D_{\max}}$$

## 7 Oil Production Estimate

Assuming a constant total compressibility and applying material balance:

$$N_p = \frac{\phi V (P_i - P_{avg})}{C_t B_o (P_{avg}) P_i}$$

Linear change of Bo in terms of pressure:

$$B_o(P) \approx B_{oi} (1 + c_o (P_i - P_{avg}))$$



