

Radial FDM README

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1 Introduction

Radial Pressure Diffusion Solver with Skin Effect

2 Governing Equation

$$\frac{\partial P}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(D(r) \cdot r \cdot \frac{\partial P}{\partial r} \right)$$

where:

- $P(r, t)$: pressure [Pa],
- $D(r) = \frac{k(r)}{\phi \mu C_t}$: diffusivity [m²/s], potentially varying due to skin effect,
- r : radial distance [m],
- t : time [s].

3 Skin Effect Modeling

We use Hawkins' model to adjust permeability near the wellbore:

$$k_{skin} = \left(\frac{s}{k \ln(r_s/r_w)} + 1 \right)^{-1} \cdot k$$

This results in a modified diffusivity:

$$D_{skin} = \frac{k_{skin}}{\phi \mu C_t}$$

4 Discretization

We discretize the domain using finite differences on a non-uniform grid:

- N_r : number of radial nodes,
- N_t : number of time steps,
- $\Delta r = \frac{r_e - r_w}{N_r - 1}$,
- $\Delta t = \frac{t_{\max}}{N_t}$.

Let P_i^n be the pressure at radius index i and time index n .

We use the explicit form of the FDM for the radial Laplacian:

$$\alpha_1 = \frac{D_i \Delta t}{2\Delta r^2} - \frac{D_i \Delta t}{4r_i \Delta r} \alpha_2 = 1 + \frac{D_i \Delta t}{\Delta r^2} \alpha_3 = \frac{D_i \Delta t}{2\Delta r^2} + \frac{D_i \Delta t}{4r_i \Delta r}$$

These coefficients are used to build a tridiagonal matrix A :

$$A_i = \begin{smallmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{smallmatrix} - \alpha_1 \alpha_2 - \alpha_3 \begin{smallmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{smallmatrix}.$$

5 Boundary Conditions

- Dirichlet at the well: $P_1^n = P_{wf}$,
- Neumann (no flow) at the outer boundary: $\frac{\partial P}{\partial r}\Big|_{r_e} = 0 \Rightarrow P_N = P_{N-1}$.

6 Stability Criterion (Explicit Scheme)

The stability criterion is:

$$\Delta t < \frac{\Delta r^2}{2D_{\max}}$$

7 Oil Production Estimate

Assuming a constant total compressibility and applying material balance:

$$N_p = \frac{\phi V(P_i - P_{avg})}{C_t B_o(P_{avg}) P_i}$$

Linear change of Bo in terms of pressure:

$$B_o(P) \approx B_{oi} (1 + c_o(P_i - P_{avg}))$$



