OAC Laboratory

Task 1.)

a.) fixed cart $\rightarrow x_w = v_w = \frac{dv_w}{dt} = i_A = 0$ $\rightarrow F=0$

I: $\frac{dx_w}{dt} = v_w \Leftrightarrow 0 = 0$

II: $\frac{d\varphi}{dt} = \omega \quad / \int : \varphi = \int \omega + \varphi_0$

III: $\frac{dv_w}{dt} = 0 = \cancel{\frac{1}{m_w + m_s(1-\cos^2\varphi)}} \left[\overset{0}{\cancel{F}} + m_s \ell \omega^2 \sin(\varphi) - m_s g \sin(\varphi) \cos(\varphi) \right]$

$\Leftrightarrow \cancel{m_s} g \cancel{\sin(\varphi)} \cos(\varphi) = \cancel{m_s} \ell \omega^2 \cancel{\sin(\varphi)}$

$g \cos(\varphi) = \ell \omega^2$

IV: $\frac{d\omega}{dt} = \frac{1}{\ell(1 - \frac{m_s}{m_w + m_s}\cos^2(\varphi))} \left[g \sin(\varphi) - \overset{0}{\cancel{\frac{\cos(\varphi)}{m_w + m_s} F}} - \frac{m_s}{m_w + m_s} \ell \omega^2 \sin(\varphi) \cos(\varphi) \right]$

II: $\dot{\varphi} = \omega$

III: $g \cos(\varphi) = \ell \omega^2$

IV: $\dot{\omega} = \frac{1}{\underbrace{\ell(1 - \frac{m_s}{m_w + m_s}\cos^2(\varphi))}_{A}} \cdot \left[g \sin(\varphi) - \frac{m_s}{m_w + m_s} \ell \omega^2 \sin(\varphi) \cos(\varphi) \right]$

goal: $\dot{\omega} = p_1 f_1(t) = p_1 f_1(\varphi, \omega, \dot{\omega})$

f_1 ← all known parameters

p_1: const. but unknown

unknown's: $\ell, m_s, m_w, \underbrace{V, k_1}_{\text{params of } F}$

$$\text{IV:}\quad \dot{\omega} = \frac{1}{\underbrace{1-\frac{m_s}{m_w+m_s}\cdot\underbrace{\frac{\ell^2\omega^4}{g^2}}_{\text{III}:\ =\cos^2(\varphi)}}_{A}}\cdot\left[\frac{g}{\ell}\sin(\varphi)-\frac{m_s}{m_w+m_s}\frac{\ell}{\ell}\omega^2\sin(\varphi)\underbrace{\cos(\varphi)}_{\text{III}:\ =\frac{\ell\omega^2}{g}}\right]$$

$$\dot{\omega} = A\cdot\left[\frac{g}{\ell}\sin(\varphi)-\frac{m_s}{m_w+m_s}\omega^2\sin(\varphi)\frac{\ell\omega^2}{g}\right]$$

$$\dot{\omega} = \frac{(m_w+m_s)g^2}{(m_w+m_s)g^2-m_s\ell^2\omega^4}\cdot\left[\frac{g}{\ell}\sin(\varphi)-\frac{m_s}{m_w+m_s}\frac{\ell\omega^4}{g}\sin(\varphi)\right]$$

$$\dot{\omega} = A\cdot\left[\frac{g^2(m_w+m_s)\sin(\varphi)-m_s\ell^2\omega^4\sin(\varphi)}{\ell(m_w+m_s)g}\right]$$

$$= \frac{\cancel{(m_w+m_s)g^2}}{\cancel{(m_w+m_s)g^2-m_s\ell^2\omega^4}}\cdot\left[\frac{\cancel{\{g^2(m_w+m_s)-m_s\ell^2\omega^4\}}\sin(\varphi)}{\ell\cancel{(m_w+m_s)}\cancel{g}}\right]$$

$$\underline{\underline{\dot{\omega} = \frac{g}{\ell}\sin(\varphi)}}$$

$$\Rightarrow \underline{\underline{p_1 = \frac{g}{\ell}\ ;\ f_1 = \sin(\varphi)}}$$

Optimization & Control L1

Task 2.)

a.)

$$\boxed{\begin{aligned}
\dot{x}_w &= v_w \\
\dot{\varphi} &= \omega \\
\dot{v}_w &= \frac{1}{m_w + m_s(1-\cos^2(\varphi))}\left[F + m_s l \omega^2 \sin(\varphi) - m_s g \sin(\varphi)\cos(\varphi)\right] \\
\dot{\omega} &= \frac{1}{l\left(1 - \frac{m_s}{m_w+m_s}\cos^2(\varphi)\right)} \cdot \left[g \sin(\varphi) - \frac{\cos(\varphi)}{m_w+m_s} F - \frac{m_s}{m_w+m_s} l \omega^2 \sin(\varphi)\cos(\varphi)\right]
\end{aligned}}$$

$$\boxed{F = V i_a - k_1 v_w \quad \text{(large pendulum)}}$$

unknown params: m_s, m_w, V, k_1

$\underline{N}$: $\dot{\omega} l - \dot{\omega} \frac{m_s \cdot l}{m_w+m_s} \cos^2(\varphi) = \dots$

$$\dot{\omega} l - g \sin(\varphi) = \frac{m_s}{m_w+m_s} l \dot{\omega} \cos^2(\varphi) - \frac{m_s}{m_w+m_s} l \omega^2 \sin(\varphi)\cos(\varphi) \underbrace{- \frac{1}{m_w+m_s}\cos(\varphi) F}_{(\alpha)}$$

(*) → Task 2 starts with this line

$$(\alpha): \frac{1}{m_w+m_s}\cos(\varphi)\cdot\left(V i_a - k_1 v_w\right) = \frac{V}{m_w+m_s}\cos(\varphi)\cdot i_a - \frac{k_1}{m_w+m_s}\cdot\cos(\varphi) v_w$$

$$(\alpha) \rightarrow \underline{N}: \quad \dot{\omega} l - g\sin(\varphi) = \frac{m_s}{m_w+m_s} l \dot{\omega} \cos^2(\varphi) - \frac{m_s}{m_w+m_s} l \omega^2 \sin(\varphi)\cos(\varphi) - \frac{V}{m_w+m_s}\cos(\varphi)\cdot i_a + \frac{k_1}{m_w+m_s}\cos(\varphi) v_w$$

$$\dot{\omega} l - g\sin(\varphi) = \underbrace{\frac{m_s}{m_w+m_s}}_{p_1} \cdot \underbrace{\left[l\dot{\omega}\cos^2(\varphi) - l\omega^2 \sin(\varphi)\cos(\varphi) \right]}_{f_1} +$$

$$- \underbrace{\frac{V}{m_w+m_s}}_{p_2} \cdot \underbrace{\left[\cos(\varphi)\cdot i_a \right]}_{f_2} + \underbrace{\frac{k_1}{m_w+m_s}}_{p_3} \underbrace{\left[\cos(\varphi)\, v_w \right]}_{f_3}$$

d.) $\boxed{\text{small pendulum: } F = \frac{k_M k_G}{r R_A} u - \frac{k_M^2 k_G^2}{r^2 R_A} v_w}$

$$(\alpha): \frac{1}{m_w+m_s}\cos(\varphi)\cdot\left[\frac{k_M k_G}{r R_A} u - \frac{k_M^2 k_G^2}{r^2 R_A} v_w\right] =$$

$$= \frac{1}{m_w+m_s}\frac{k_M k_G}{r R_A} u\cos(\varphi) - \frac{1}{m_w+m_s}\frac{k_M^2 k_G^2}{r^2 R_A}\cos(\varphi) v_w$$

$$(\alpha)\to \text{IV}: \dot{\omega} l - g\sin(\varphi) = \frac{m_s}{m_w+m_s} l\dot{\omega}\cos^2(\varphi) -$$

$$\frac{m_s}{m_w+m_s} l\omega^2\cos(\varphi)\sin(\varphi) -$$

$$\frac{1}{m_w+m_s}\frac{k_M k_G}{r R_A} u\cos(\varphi) +$$

$$\frac{1}{m_w+m_s}\frac{k_M^2 k_G^2}{r^2 R_A}\cos(\varphi) v_w =$$

$$\dot{\omega} l - g\sin(\varphi) = \overbrace{\frac{m_s}{m_w+m_s}}^{p_{1,s}} \cdot \left[\underbrace{l\dot{\omega}\cos^2(\varphi) - l\omega^2\cos(\varphi)\sin(\varphi)}_{f_{1,s}}\right] -$$

$$\underbrace{\frac{1}{m_w+m_s}\frac{k_M k_G}{r R_A}}_{p_{2,s}} \underbrace{u\cos(\varphi)}_{f_{2,s}} +$$

$$\underbrace{\frac{1}{m_w+m_s}\frac{k_M^2 k_G^2}{r^2 R_A}}_{p_{3,s}} \underbrace{\cos(\varphi) v_w}_{f_{3,s}}$$

$\Rightarrow$ the unknowns are: $m_s, m_w, k_M, k_G, r, R_A$

$\hookrightarrow$ for the small pendulum, there are 6 unknowns, but only 3 equations to solve. $\Rightarrow$ We have to assume 2 additional parameters to $m_s = 0.5\,kg$.