Optimization and Control, Laboratory

Parameter Identification for the “Inverted Pendulum”

Winter term 2021/22

1 Laboratory Experiment

Consider the inverted pendulum setup as depicted in Figure 1. This setup consists of a cart (with mass m_W) which is actuated by a force F. A pendulum is connected via a rotating joint to this cart. The goal of this laboratory class is to perform parameter identification

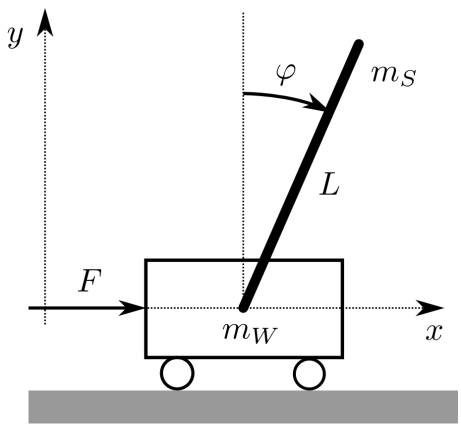

Figure 1: Laboratory experiment "inverted pendulum"

based on measurements from laboratory experiments. Additionally, a controller should be developed which stabilizes the upper (unstable) equilibrium.

2 Mathematical Model

The states consist of the cart position x_W, its velocity v_W, the angle of the pendulum φ and its corresponding angular velocity ω. Applying the Newton's laws and considering the connection between cart and pendulum, results in the mathematical model

$$
\begin{aligned}
\frac{dx_W}{dt} &= v_W \\
\frac{d\varphi}{dt} &= \omega \\
\frac{dv_W}{dt} &= \frac{1}{m_W + m_S(1-\cos^2\varphi)}\Big[F + m_S l\omega^2 \sin\varphi - m_S g \sin\varphi\cos\varphi\Big] \\
\frac{d\omega}{dt} &= \frac{1}{l\left(1 - \frac{m_S}{m_W+m_S}\cos^2\varphi\right)}\Big[g\sin\varphi - \frac{\cos\varphi}{m_W+m_S}F - \frac{m_S}{m_W+m_S} l\omega^2 \sin\varphi\cos\varphi\Big]
\end{aligned} \tag{1}
$$

where g denotes the gravitational acceleration and l half the length of the pendulum.

The force F is generated by a DC-motor. Two different models of the inverted pendulum will be used in the laboratory class. The first model (large pendulum) takes the current i_A

as input to the system. The force F consists then of a force which is directly proportional to this current and a second force which represents viscous friction, i.e.

$$F = V\, i_A - k_1 v_W. \tag{2}$$

In the second model (small pendulum), the input is represented by the motor voltage u. The resulting force is proportional to this input voltage and also to the velocity v_w, i.e.

$$F = \frac{k_M k_G}{r R_A} u - \frac{k_M^2 k_G^2}{r^2 R_A} v_W \tag{3}$$

where k_M denotes the engine constant, k_G the transmission ratio, R_A the rotor resistance und r the equivalent radius which is used to translate rotational to transitional movements.

The state vector of the system is chosen as $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} x_W & \varphi & v_W & \omega \end{bmatrix}^T$.

3 Tasks

As already stated above, we want to identify the unknown parameters of the inverted pendulum followed by designing a controller which stabilizes the pendulum in the upper equilibrium. Unless mentioned otherwise, the tasks below refer to the large pendulum where the unknown parameters are given by l, m_S, m_W, V and k_1.

Task 1 First, we want to identify the parameter l. In order to do so, an experiment was recorded where the position of the cart was fixed and the current i_A was zero.

a) A differential equation describing the dynamics of ω when the cart is fixed can be obtained by inserting $x_W = v_W = \frac{dv_W}{dt} = i_A = 0$ in the mathematical model of the inverted pendulum (1). Write down the resulting differential equation in the form

$$\dot{\omega}(t) = p_1 f_1(t) = p_1 f_1(\varphi(t), \omega(t), \dot{\omega}(t)) \tag{4}$$

where p_1 is constant but unknown and the function $f_1(t)$ does not depend on any unknown parameters.

b) The evolution of φ, ω and $\dot{\omega}$ were recorded during this experiment and are available in the file `measurements_1.mat`. Identify the unknown parameter p_1 in (4) via L_1, L_2 and L_∞ optimization using Yalmip. Calculate l from the identified parameter p_1 resulting from all three identification methods.

c) Implement the model of the pendulum with fixed cart

$$\dot{\varphi} = \omega \qquad\qquad \dot{\omega} = p_1 f_1(\varphi, \omega)$$

in Matlab/Simulink; determine the initial values of the two state variables $\varphi(t)$ and $\omega(t)$ from the recorded data in `measurements_1.mat`. Run the simulation with the parameter p_1 obtained via L_1 optimization and store the resulting evolution of the angle $\varphi_{L_1}(t)$. Repeat the simulation with p_1 identified via L_2 and L_∞ optimization to obtain $\varphi_{L_2}(t)$ and $\varphi_{L_\infty}(t)$. Plot $\varphi_{L_1}(t)$, $\varphi_{L_2}(t)$, $\varphi_{L_\infty}(t)$ and the evolution of the angle recorded in `measurements_1.mat` $\varphi_{meas.}(t)$ together in one Matlab-plot.

d) In order to evaluate how well the mathematical model with the identified parameters represents the real system, simulation results and measurements are compared for a different experiment (an experiment that was not used for parameter identification). Repeat the simulations from the previous task for the experiment recorded in `measurements_2.mat` and plot simulation results and measured angle as before.

Task 2 Now that we have identified the parameter l, we want to identify additional parameters from another recorded experiment where the position of the cart is no longer fixed, i.e. the system dynamics are given by (1).

a) Write down the dynamics of ω from (1) in the form

$$\dot{\omega}(t)l - g\sin(\varphi(t)) = \sum_{i=1}^{N} p_i f_i(t) = \sum_{i=1}^{N} p_i f_i(i_A(t), x_W(t), v_W(t), \varphi(t), \omega(t), \dot{\omega}(t))$$

(where $N \in \mathbb{N}$ is some positive constant) similar to Task 1a. Note that l is assumed to be known since it has already been identified.

b) The evolution of i_A, x_W, φ, v_w and ω was recorded in the file `measurements_3.mat` (the first column of the recorded values is the evolution of i_A; the remaining columns are x_W, φ, v_w, ω and $\dot{\omega}$). Identify the unknown parameters p_i using L_1, L_2 and L_∞ optimization.

c) Can you calculate the values of the unknown parameters m_S, m_W, V and k_1 from the identified parameters p_i? If you can, do so. Otherwise, use $m_S = 0.5kg$ and calculate the remaining parameters. Compare the values of m_S, m_W, V and k_1 resulting from the three identification methods.

d) What will change with respect to the parameter identification when the small pendulum is used?

e) Create a Simulink plan for the nonlinear model. Determine the initial state vector from the experiment recorded in `measurements_3.mat` and use i_A recorded during this experiment as the input of the plant. Run the simulation with the parameters l, m_S, m_W, V and k_1 obtained via L_1 optimization, with the parameters obtained via L_2 optimization and with the parameters obtained via L_∞ optimization. Compare the evolution of φ and x_W from all three simulations to the evolution of φ and x_W recorded in `measurements_3.mat` like in Task 1c.

f) Repeat the simulations from the previous task for the experiment recorded in `measurements_4.mat` and plot simulation results and measured angle as before.

Task 3 Linearize the nonlinear model around the unstable (upper) equilibrium point. Design a linear state controller $u = -\mathbf{k}^T\mathbf{x}$ for the linearized model using the Matlab command `lqr()`; choose reasonable values for the weighting matrices.