

# Task 1 - Linearization around $y = x_2 = 15 \text{ cm}$

➤ given system

$$\dot{x}_1 = -k_1 \sqrt{x_1} + z_{p,1}$$

$$\dot{x}_2 = -k_2 \sqrt{x_2} + k_1 \sqrt{x_1} + z_{p,2} \rightarrow 0$$

➤ equilibrium:  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$$0 = -k_1 \sqrt{x_{1,e}} + z_{p,1,e} \Leftrightarrow k_1 \sqrt{x_{1,e}} = z_{p,1,e}$$

$$0 = -k_2 \sqrt{x_{2,e}} + k_1 \sqrt{x_{1,e}}$$

$$k_2 \sqrt{x_{2,e}} = k_1 \sqrt{x_{1,e}} = z_{p,1,e} = \alpha_1 + \sqrt{\beta_1 + \gamma_1 u_{1,e}}$$

$$k_2 \sqrt{x_{2,e}} - \alpha_1 = \sqrt{\beta_1 + \gamma_1 u_{1,e}}$$

$$\frac{1}{\gamma_1} \left[ \left( k_2 \sqrt{x_{2,e}} - \alpha_1 \right)^2 - \beta_1 \right] = u_{1,e}$$

$$k_1 \sqrt{x_{1,e}} = z_{p,1,e}$$

$$x_{1,e} = \frac{1}{k_1^2} z_{p,1,e}^2$$

$$\Rightarrow \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -k_1/2 \cdot x_1^{-1/2} & 0 \\ k_1/2 \cdot x_1^{-1/2} & -k_2/2 \cdot x_2^{-1/2} \end{bmatrix}$$

$$\frac{\partial f}{\partial u_1} = \begin{bmatrix} \frac{1}{2} (\beta_1 + \gamma_1 u_1)^{-1/2} \cdot \gamma_1 \\ 0 \end{bmatrix}$$

$$\frac{\partial f}{\partial u_2} = \begin{bmatrix} 0 \\ \frac{1}{2} (\beta_2 + \gamma_2 u_2)^{-1/2} \gamma_2 \end{bmatrix}$$