Specification of NBBG model for propagule pressure simulation experiment

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Introduction

We introduced 20 *Tribolium* flour beetles to novel environments that were either stable or randomly fluctuating through time, varying the number of introduction events used to distribute them (1, 2, 4, or 5 events). We want to simulate the exercise using reasonable choices for different kinds of stochasticity that may affect the population (demographic stochasticity, environmental stochasticity, and demographic heterogeneity including sex ratio). The goal is to see whether we see similar results with simulations as we do with the manipulative experiment, and to determine what factors may drive any divergence in observations (*e.g.* how big does environmental stochasticity need to be in order to change the outcome of the simulated experiment?). The first step is to estimate the parameters corresponding to stochasticity using a Negative binomial-binomial-gamma stochastic model built on a Ricker skeleton of density dependent population growth.

The Data Model

$$N_{(t+1)i} \sim NegBinom(\mu_i, k_D F_{ti})$$

Where $N_{(t+1)i}$ is the population size in the next generation for the i^{th} population, μ_i is the expected per capita population growth rate (births minus density independent deaths) for the i^{th} population in the E environment, α is the adult search rate for eggs, N_{ti} is the population size in the current generation for the i^{th} population, k_D is the shape parameter of the gamma distribution for demographic stochasticity, and F_{ti} is the expected number of females in the i^{th} population.

The Process Model

$$\mu_i = \frac{1}{p} F_i R_i e^{-\alpha N_{ti}}$$

Where p is the probability of an individual being female ($p = 0.5$), F_i is the number of adult females in the i^{th} population, and R_E is the per-capita population growth rate for the E environment assuming an even sex ratio.

The Parameter Model

Parameters

$$F_i \sim Binom(N_{ti}, p)$$
$$R_E \sim Gamma(k_E, \frac{R_0}{k_E})$$

Where k_E is the shape parameter of the gamma distribution for environmental stochasticity and R_0 is the expectation of per-capita population growth rate in the average environment assuming an even sex ratio.

Priors

$$\begin{aligned}
R_0 &\sim Gamma(2.6, 1) \\
k_E &\sim Gamma(17.6, 1) \\
k_D &\sim Gamma(1.07, 1) \\
\alpha &\sim Gamma(0.0037, 1)
\end{aligned}$$

All Together

$$\begin{aligned}
N_{(t+1)i} &\sim NegBinom(\mu_i, k_D F_{ti}) \\
\mu_i &= \frac{1}{p} F_i R_i e^{-\alpha N_{ti}} \\
F_i &\sim Binom(N_{ti}, p) \\
R_E &\sim Gamma(k_E, \frac{R_0}{k_E}) \\
R_0 &\sim Gamma(2.6, 1) \\
k_E &\sim Gamma(17.6, 1) \\
k_D &\sim Gamma(1.07, 1) \\
\alpha &\sim Gamma(0.0037, 1) \\
p &= 0.5
\end{aligned}$$

The Posterior Distribution

$$[R_0, k_E, k_D, \alpha, \boldsymbol{R_E}, \boldsymbol{F} | \boldsymbol{N_t}, \boldsymbol{N_{t+1}}] \propto \prod_{i=1}^{n} \left([N_{(t+1)i} | \left(\frac{1}{p} F_i R_E \right) e^{-\alpha N_{ti}}, k_D F_{ti}][F_i | N_{ti}, p] \right) [R_0][k_E][k_D][\alpha]$$

Full Conditional Distributions

$$
\begin{aligned}
[R_0|\cdot] &\propto \prod_{i=1}^{n} \left(\left[N_{(t+1),i} | \left(\frac{1}{p} F_{mated_{t,i}} R_{E_i} \right) e^{-\alpha N_{t,i}}, k_D F_{mated_{t,i}} \right] \right) [R_0] \\
[k_E|\cdot] &\propto \prod_{i=1}^{n} \left(\left[N_{(t+1),i} | \left(\frac{1}{p} F_{mated_{t,i}} R_{E_i} \right) e^{-\alpha N_{t,i}}, k_D F_{mated_{t,i}} \right] \right) [k_E] \\
[k_D|\cdot] &\propto \prod_{i=1}^{n} \left(\left[N_{(t+1),i} | \left(\frac{1}{p} F_{mated_{t,i}} R_{E_i} \right) e^{-\alpha N_{t,i}}, k_D F_{mated_{t,i}} \right] \right) [k_D] \\
[\alpha|\cdot] &\propto \prod_{i=1}^{n} \left(\left[N_{(t+1),i} | \left(\frac{1}{p} F_{mated_{t,i}} R_{E_i} \right) e^{-\alpha N_{t,i}}, k_D F_{mated_{t,i}} \right] \right) [\alpha] \\
[R_{E_i}|\cdot] &\propto \left[N_{(t+1),i} | \left(\frac{1}{p} F_{mated_{t,i}} R_{E_i} \right) e^{-\alpha N_{t,i}}, k_D F_{mated_{t,i}} \right] [R_{E_i}] \\
[F_{migrants_{t,i}}|\cdot] &\propto \left[N_{(t+1),i} | \left(\frac{1}{p} F_{mated_{t,i}} R_{E_i} \right) e^{-\alpha N_{t,i}}, k_D F_{mated_{t,i}} \right] \left[F_{migrants_{t,i}} | N_{migrants_{(t,i)}}, p \right] \\
[F_{residents_{t,i}}|\cdot] &\propto \left[N_{(t+1),i} | \left(\frac{1}{p} F_{mated_{t,i}} R_{E_i} \right) e^{-\alpha N_{t,i}}, k_D F_{mated_{t,i}} \right] \left[F_{residents_{t,i}} | N_{residents_{(t,i)}}, p \right] \\
F_{mated_{t,i}} &= \begin{cases} F_{migrants_{t,i}} & \text{when } F_{migrants_{t,i}} + F_{residents_{t,i}} = N_{t,i} \\ F_{migrants_{t,i}} + F_{residents_{t,i}} & \text{when } F_{migrants_{t,i}} + F_{residents_{t,i}} < N_{t,i} \end{cases}
\end{aligned}
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