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ESTIMATING FREQUENCY RESPONSE FUNCTIONS BY POLE-RESIDUE OPERATIONS

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ABSTRACT

This article presents a new approach for estimating *frequency response functions* (FRFs) of linear multiple-degree-of-freedom (MDOF) systems by pole-residue operations. In this new approach, the poles and residues associated with the input and output signals are extracted by using the multi-signal Prony-ss method, which is an extension and improvement of the classical Prony's method. In turn, the system poles are simply the output poles minus the input poles. The remaining work is to compute each corresponding system residue, which is obtained by dividing the output residue at the system pole location with a quantity computed from the input function at the system pole location in the Laplace domain. The numerical examples in this paper estimate the electrical impedance function—which is a FRF between voltage and current—of a piezoelectric tube transducer. Both finite element model simulations and lab experiments are conducted in this paper to verify the effectiveness of the newly developed pole-residue method.

Keywords: Frequency response function, Piezoelectric transducer, pole/residue, Laplace domain

1. INTRODUCTION

Traditionally, there are two kinds of methods for estimating frequency response functions: (i) stepped harmonic analysis (i.e. single frequency) method, and (ii) Fourier-based analysis method using transient signals. The stepped harmonic analysis is to repeatedly apply a harmonic input signal to get the corresponding steady state response of the given frequency, and is generally considered to be an accurate way to obtain FRFs. The Fourier method computes the frequency response function through the usage of the fast Fourier transform (FFT) of both input and output signals, and is considered to be an efficient way on estimating frequency response functions. Both methods have drawbacks though. The stepped harmonic analysis method is costly in computational time; and the Fourier-based analysis method always suffers from the problems related to the periodic assumption of both input and output signals, such as frequency leakage and resolution [1].

A recent article [2] developed an efficient pole-residue method for computing the dynamic response of the MDOF system, which showed that the poles and residues of dynamic response could be easily obtained from those of the input and system functions. Following the same theoretical principle, a new approach for estimating frequency response functions of linear MDOF systems by pole-residue operations is presented in this article to overcome the drawbacks of the traditional methods.

2. POLE-RESIDUE METHOD

The displacement response $x_{jk}(t)$ at coordinate j to the load p_k at coordinate k in the time domain can be expressed as

$$x_{jk}(t) = \int_0^t h_{jk}(t-\tau)p_k(\tau)d\tau \tag{1}$$

where $h_{jk}(t)$ is the *unit impulse response function* associated with coordinates j and k. In the Laplace domain (s-domain), one has

$$\tilde{x}_{jk}(s) = T_{jk}(s)\tilde{p}_k(s) \tag{2}$$

Throughout this paper, functions depending on time t are in lower cases, and their Laplace transforms on s denoted by the same letters with a tilde. As shown in [2], the poles and residues of $\tilde{x}_{jk}(s)$ can be obtained from those of $\tilde{p}_k(s)$ and the *transfer function* $T_{jk}(s)$. When $\tilde{p}_k(s)$ and $T_{jk}(s)$ are written in their pole-residue forms

$$\tilde{p}_k(s) = \sum_{\ell=1}^{L} \frac{\alpha_\ell}{s-\lambda_\ell} \tag{3}$$

and

$$T_{jk}(s) = \sum_{n=1}^{2N} \frac{\beta_n}{s-\mu_n} \tag{4}$$

respectively, substituting Eq. 3 and Eq. 4 into Eq. 2 leads to

$$\tilde{x}_{jk}(s) = \left(\sum_{n=1}^{2N} \frac{\beta_n}{s-\mu_n}\right)\left(\sum_{\ell=1}^{L} \frac{\alpha_\ell}{s-\lambda_\ell}\right) = \sum_{m=1}^{2N+L} \frac{\gamma_m}{s-\nu_m} \tag{5}$$

It is evident that the poles ν_m of the response $\tilde{x}_{jk}(s)$ consist of L excitation poles λ_ℓ and $2N$ system poles μ_n. Let the first L poles of $\tilde{x}_{jk}(s)$ be the excitation poles λ_ℓ, namely, $\nu_m = \lambda_m$ for $m = 1, \cdots, L$, and the last $2N$ poles of $\tilde{x}_{jk}(s)$ be the system poles μ_n, namely, $\nu_{m+L} = \mu_m$ for $m = 1, \cdots, 2N$. It has been shown in [2] that the first L residues γ_m (for $m = 1, \cdots, L$) of $\tilde{x}_{jk}(s)$ at the excitation poles λ_ℓ are

$$\gamma_m = \lim_{s\to\nu_m} (s-\nu_m)\left(\sum_{\ell=1}^{L} \frac{\alpha_\ell}{s-\lambda_\ell}\right) T_{jk}(s) = \alpha_m T_{jk}(\lambda_m) \tag{6}$$

and the last $2N$ residues γ_{m+L} (for $m = 1, \cdots, 2N$) of $\tilde{x}_{jk}(s)$ at the system poles μ_n are

$$\gamma_{m+L} = \lim_{s\to\nu_{m+L}} (s-\nu_{m+L})\tilde{p}_k(s)\left(\sum_{n=1}^{2N} \frac{\beta_n}{s-\mu_n}\right) = \beta_m \tilde{p}_k(\mu_m) \tag{7}$$

Eqs. 6 and 7 show that all residues of $\tilde{x}_{jk}(s)$ can be computed from simple operations of the poles and residues of the excitation $\tilde{p}_k(s)$ and the system transfer function $T_{jk}(s)$.

In the proposed system identification method, the poles and residues associated with the input signal $p_k(t)$ and output signal $x_{jk}(t)$ are first extracted by using the multi-signal Prony-SS method [4]. By using this multi-signal method to process the input and output signals simultaneously, it numerically guarantees that the poles of the input signals will also appear at those of the output signals. The outcomes of this multi-signal method include both the poles and residues associated with the input and output signals. The way to figure out input poles is by observing the fact that the value of the residue corresponding to every input pole would not be negligibly small. After discerning the input poles from the output poles, the system poles μ_n (see Eq. 4) are simply the output poles ν_m (see Eq. 5) minus the input poles λ_ℓ (see Eq. 3). As the transfer function $T_{jk}(s)$ is desired, the remaining work is to compute the residue β_n corresponding to each system pole μ_n in Eq. 4. In accordance with Eq. 7, the computed β_m (for $m = 1, \cdots, 2N$) is equal to the output residue γ_{m+L} (at the system pole μ_m) divided by a quantity obtained from the input function calculated at the system pole location $\tilde{p}_k(\mu_m)$, that is,

$$\beta_m = \frac{\gamma_{m+L}}{\sum\limits_{\ell=1}^{L} \frac{\alpha_\ell}{\mu_m - \lambda_\ell}}, \quad m = 1, \cdots, 2N \tag{8}$$

Note that the input poles λ_ℓ and residues α_ℓ, together with the output residues γ_{m+L} and the system poles μ_m in Eq. 8, have already been obtained through the multi-singal Prony-SS method. Once the system poles μ_n and the residues β_n associated with $T_{jk}(s)$ are available, one can also numerically compute the corresponding complex frequency response function (FRF)

$$H_{jk}(\omega) = \sum_{n=1}^{2N} \frac{\beta_n}{i\omega - \mu_n} \tag{9}$$

3. NUMERICAL EXAMPLES

The numerical examples in this paper estimate the electrical impedance function—an FRF between voltage and current—of a radially polarized piezoelectric tube transducer. Shown in Fig. 1 is a piezoceramic transducer model, with the inner diameter $D_i = 16 \times 10^{-3}$ m, outer diameter $D_o = 19 \times 10^{-3}$ m and length $\ell = 20 \times 10^{-3}$ m. Fig. 2 compares the estimated impedance functions from the proposed method, the Fourier-based method, and the stepped harmonic method. One can easily conclude that the impedance function estimated from the proposed method agrees well with that from the stepped single frequency method.

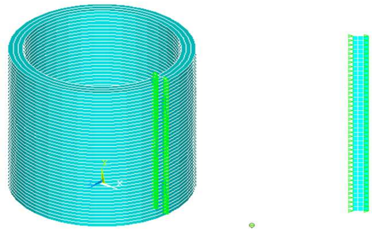

Figure 1: Finite element model of a tube transducer: (a) 3D model, and (b) cross section

Ultimately, the effectiveness of the proposed method must be tested for a real transducer. In this experiment, the transducer tube was mounted on a "soft" base (a sponge pad) to simulate a free-free mechanical boundary condition. Both the stepped harmonic signals and the transient signal have been utilized as the input voltage siganl. The harmonic input signals were generated repeatedly at uniformly spaced frequencies with intervals of 0.24 KHz. The transient signal was an exponential decay signal with a maximum of 1V and a decay ratio of -3×10^3. Using the transient signals, the impedance functions estimated by the direct FFT and proposed methods are shown in Fig. 3. Also included is the impedance function estimated by the stepped harmonic method. It is clear that the proposed method significantly outperforms the traditional Fourier-based method in a noisy experimental environment.

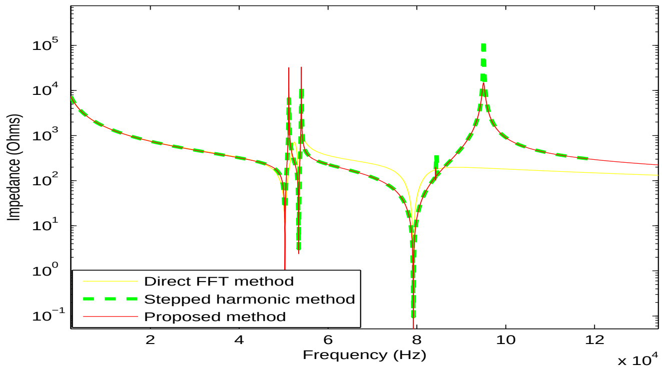

Figure 2: The comparison of the estimated impedance functions using a finite element model

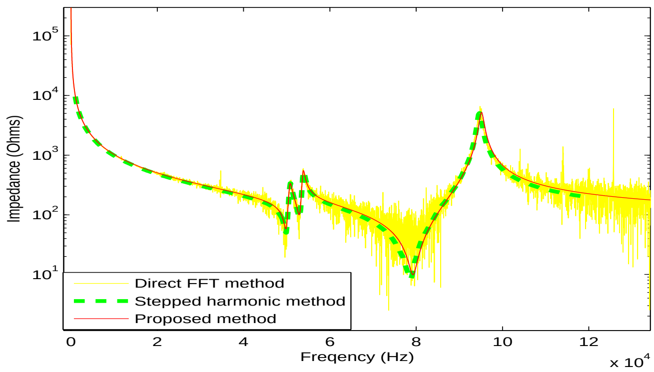

Figure 3: The comparison of the impedance functions using experimental data

4. CONCLUSION

An efficient pole-residue method for estimating frequency response functions of linear systems has been developed and examined in this paper. After having both the input and output signals in their pole-residue forms, the corresponding frequency response function could be easily obtained. In comparison to traditional Fourier-based methods, the proposed method did not suffer any leakage or resolution problems. Another advantage of the proposed method is that it requires only very short input and output signals. The accuracy and efficiency of the proposed method has been demonstrated by evaluating the impedance function of a piezoelectric transducer, using data from both a finite element model and lab experiments.

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