# EECS 240
# Analog Integrated Circuits

# Topic 11: Feedback

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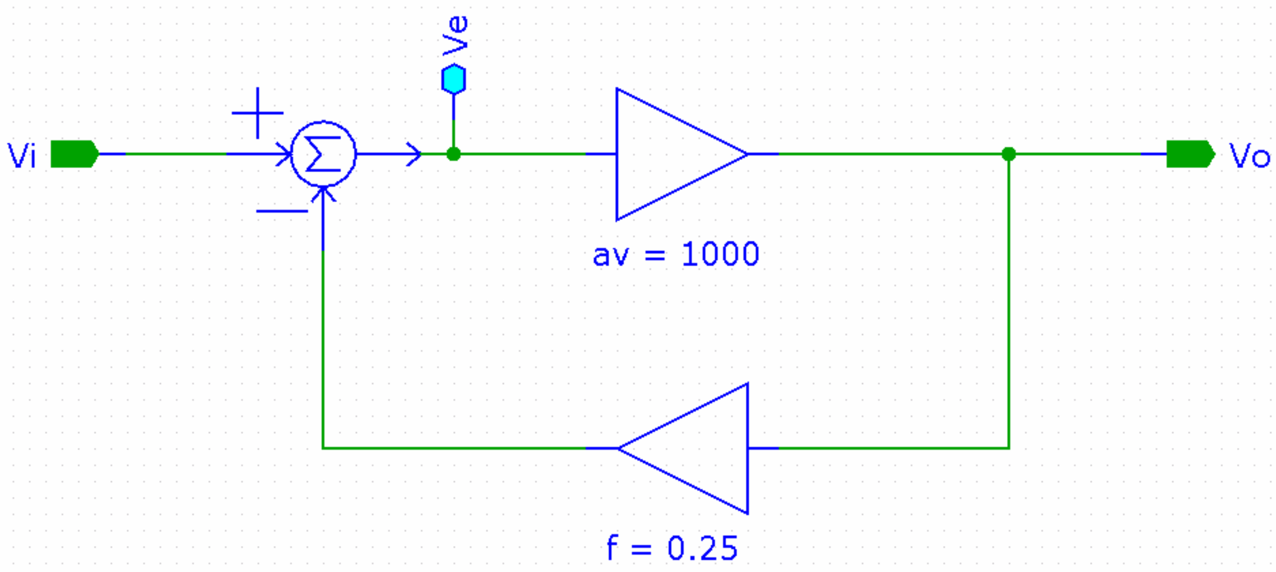
# Feedback

- Benefits
  - Reduced sensitivity to
    - Gain variations
    - Nonlinearity
  - Increased bandwidth

- Caveat: potential instability

- Stability test
  - Bounded input, bounded output: no general test available
  - Linear system:
    - Poles in LHP (“left half-plane”)
    - Nyquist criterion
    - Bode criterion
  - Hand-analysis
  - SPICE

# Generic Feedback Circuit




$$\text{open-loop gain}: \quad a_v$$

$$\text{feedback factor}: \quad f$$

$$\text{loop gain}: \quad T = a_v f$$

$$\text{closed-loop gain}:$$

$$A = \frac{V_o}{V_i} = \frac{a}{1+T} = \frac{1}{f}\frac{1}{1+\frac{1}{T}} \approx \frac{1}{f} \qquad \text{for} \qquad T >> 1$$

# Electronic Feedback Circuit

open - loop gain : $a_v$

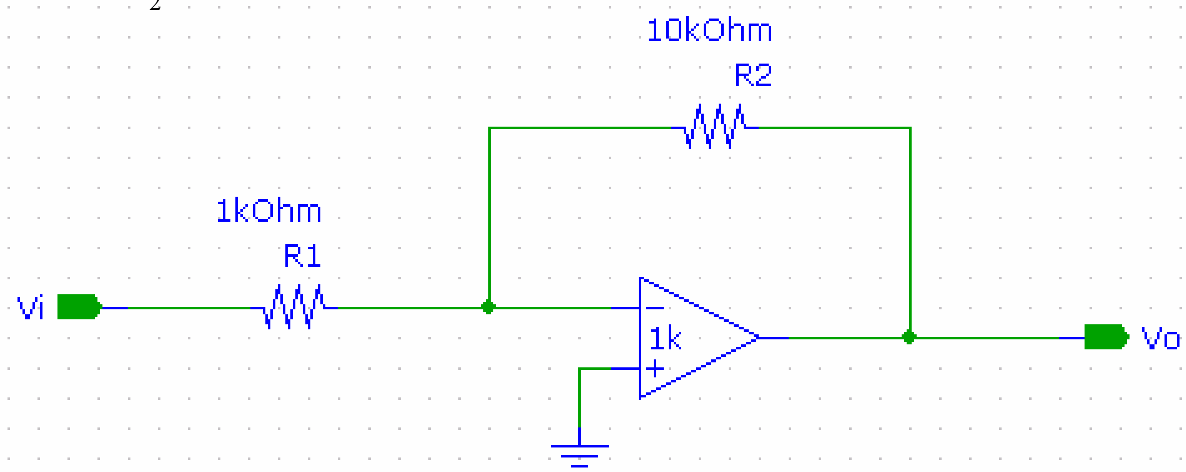
feedback factor : $f = \dfrac{R_1}{R_1 + R_2}$ (sometimes difficult to isolate)

loop gain : $T = a_v f$

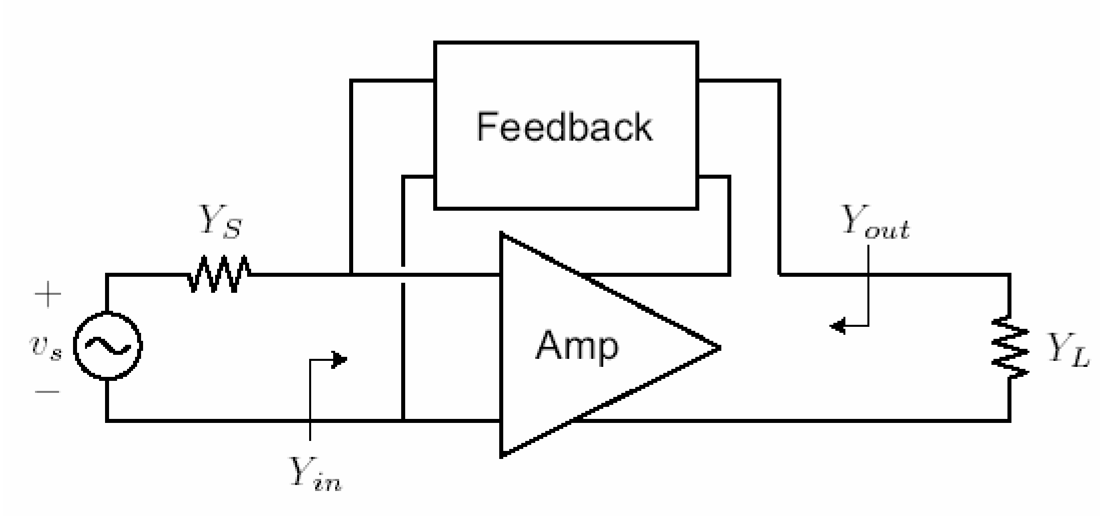
closed - loop gain :

$$A = \frac{V_o}{V_i} = -\frac{R_2}{R_1}\frac{1}{1+\dfrac{R_1}{R_2}\dfrac{1}{T}} \approx -\frac{R_2}{R_1} \qquad \text{for} \qquad \frac{R_2}{R_1}T >> 1$$

$$\underline{\underline{\neq \frac{1}{f}}}$$



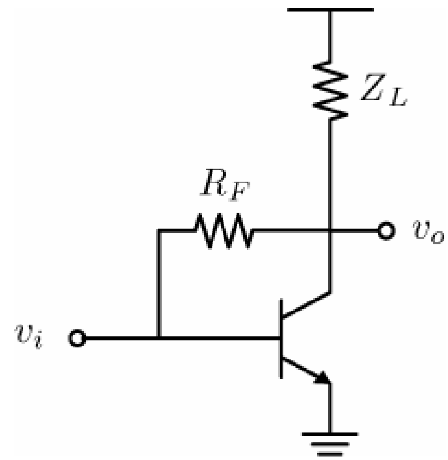
# Two-Port Analysis (Review)




- Amplifiers can often be decomposed into a *unilateral* forward gain and a feedback section.
- Based on the type of feedback (series versus shunt at each port), we should use the simplest two-port equations.

# Examples: Y-Parameters

- If the feedback is shunt at both ports, then the currents at the input and output are summing so *Y* parameters are natural.
- Since the output is connected in shunt, we sample the output voltage. Since the input is in shunt too, we add a feedback current into the input.
- Thus shunt feedback is appropriate for a trans-resistance amplifier (current->voltage)




$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

# Admittance Parameters

- Notice that $y_{11}$ ($y_{22}$) is the short-circuit input (output) admittance

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$$

- The forward trans-conductance is described by $y_{21}$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$

- The reverse trans-conductance is given by $y_{12}$. For a *unilateral* amplifier $y_{12} = 0$

# Voltage Gain

- Since $i_2$=-$v_2$ $Y_L$, we can write

$$(y_{22} + Y_L)v_2 = -y_{21}v_1$$

- The "internal" voltage gain is thus given by

$$A_v = \frac{v_2}{v_1} = \frac{-y_{21}}{y_{22} + Y_L}$$

- The input admittance is now easily given by

$$Y_{in} = \frac{i_1}{v_1} = y_{11} + y_{12}\frac{v_2}{v_1} \qquad \boxed{Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L}}$$

# Output Admittance

- By symmetry we can write down the output admittance by inspection

$$Y_{out} = y_{22} - \frac{y_{12}y_{21}}{y_{11} + Y_S}$$

- Note that for a unilateral amplifier $y_{12} = 0$ implies that

$$Y_{in} = y_{11}$$

$$Y_{out} = y_{22}$$

- The input and output impedance are de-coupled!

# External Voltage Gain

- The gain from the voltage source to the output can be derived by a simple voltage divider equation

$$A_v' = \frac{v_2}{v_s} = \frac{v_2}{v_1}\frac{v_1}{v_s} = A_v \frac{Y_S}{Y_{in} + Y_S} = \frac{-Y_S y_{21}}{(y_{22} + Y_L)(Y_S + Y_{in})}$$

- If we substitute and simplify the above equation we have

$$A_v' = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22}) - y_{12} y_{21}}$$

# Feedback Interpretation

- Note that in an ideal feedback system, the amplifier is unilateral and $\frac{y}{x} = \frac{A}{1+Af}$
- We know that the voltage gain of a general two-port driven with source admittance $Y_S$ is given by

$$A_v' = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22}) - y_{12}y_{21}}$$

- If we unilaterize the two-port by arbitrarily setting $y_{12} = 0$, we have an "open" loop forward gain of

$$A_{vu} = A_v'\big|_{y_{12}=0} = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22})}$$

# Identification of Loop Gain

- Re-writing the gain $A'_v$ by dividing numerator and denominator by the factor $(Y_S + y_{11})(Y_L + y_{22})$ we have

$$A'_v = \frac{\frac{-Y_S y_{21}}{(Y_S+y_{11})(Y_L+y_{22})}}{1 - \frac{y_{12}y_{21}}{(Y_S+y_{11})(Y_L+y_{22})}}$$

- We can now see that the "closed" loop gain with $y_{12}$ is given by

$$A'_v = \frac{A_{vu}}{1+T}$$

- where $T$ is identified as the loop gain

$$\boxed{T = A_{vu} f = \frac{-y_{12}y_{21}}{(Y_S + y_{11})(Y_L + y_{22})}}$$

# Feedback Factor

- Using the last equation also allows us to identify the feedback factor

$$\boxed{f = \frac{Y_{12}}{Y_S}}$$

- If we include the loading by the source $Y_S$, the input admittance of the amplifier is given by

$$Y_{in} = Y_S + y_{11} - \frac{y_{12}y_{21}}{Y_L + y_{22}}$$

- Note that this can be re-written as

$$Y_{in} = (Y_S + y_{11})\left(1 - \frac{y_{12}y_{21}}{(Y_S + y_{11})(Y_L + y_{22})}\right)$$

# Feedback and Terminal Impedance

- The last equation can be re-written as

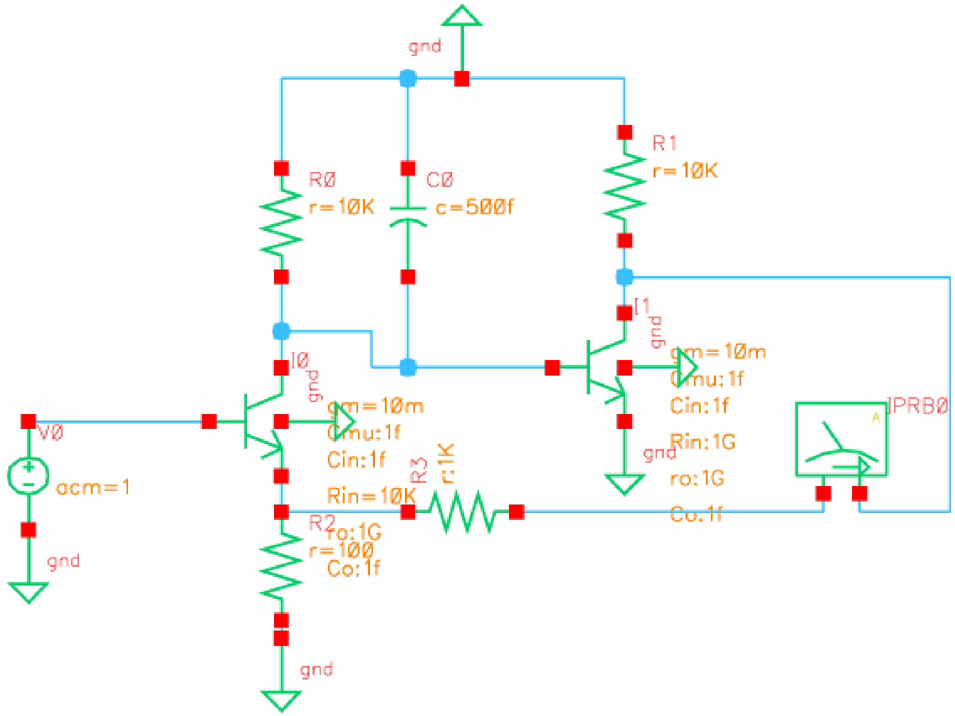
$$\boxed{Y_{in} = (Y_S + y_{11})(1 + T)}$$

- Since $Y_S + y_{11}$ is the input admittance of a unilateral amplifier, we can interpret the action of the feedback as raising the input admittance by a factor of $1 + T$.
- Likewise, the same analysis yields

$$Y_{out} = (Y_L + y_{22})(1 + T)$$

- It's interesting to note that the same equations are valid for series feedback using Z parameters, in which case the action of the feedback is to boost the input and output impedance. This same is true for hybrid feedback.

# Direct Feedback Partition




- We'd like to partition our system into an ideal feedback system. Real feedback circuits load the amplifier.
- What parameters should we use for the above amplifier?

# Real Feedback Two-Port

- Pick the appropriate two-port representation and include loading in calculations … review G&M Ch.8.
- I invite you to analyze the previous circuit …

# Return Ratio Analysis

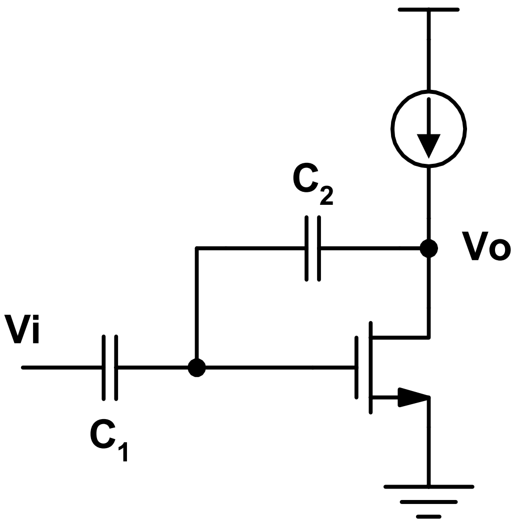
- We really only care about loop gain $T$!!!
- The loop gain can be calculated directly by breaking the feedback loop and injecting a test signal and observing the "return" ratio. The return signal should have negative phase for negative feedback.
- Problem: Loading effects must be taken into account when loop is broken …

# Stability Analysis

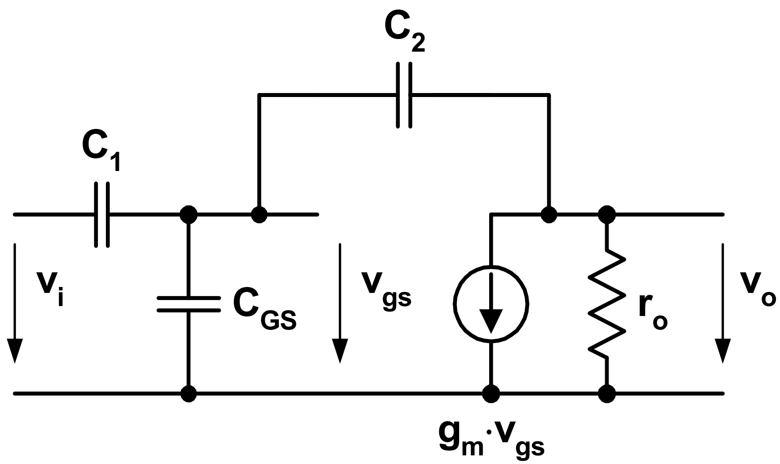
- Depends on $T(s)$
  - NOT $a(s)$

- Finding $T(s)$:
  - Hand analysis:
    - Break loop at controlled source (e.g. $g_m$)
    - $T = - s_r / s_t$
  - SPICE:
    - Controlled sources not accessible
      a) Break loop, model load (approximation), or
      b) Determine T from $T_v$ and $T_i$ (exact)

# Simple Circuit Example




Feedback Amplifier
(Biasing for $V_{gs}$ not shown)



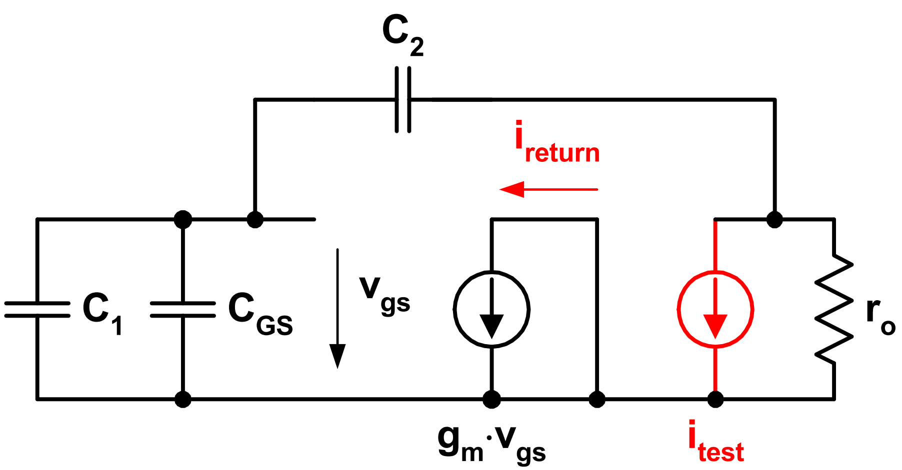

Small Signal Equivalent

Loop Gain = ?

# Return Ratio Analysis [HLGM 01]

1. Set all independent sources to zero ($v_i$=0)
2. Disconnect (ideal) controlled source from circuit
3. Replace with test source
4. Find ratio return signal/test signal = "Return Ratio" = Loop Gain

$$T(s) = F \cdot g_m \cdot r_0 \cdot \frac{1}{1 + \frac{s}{p_1}}$$

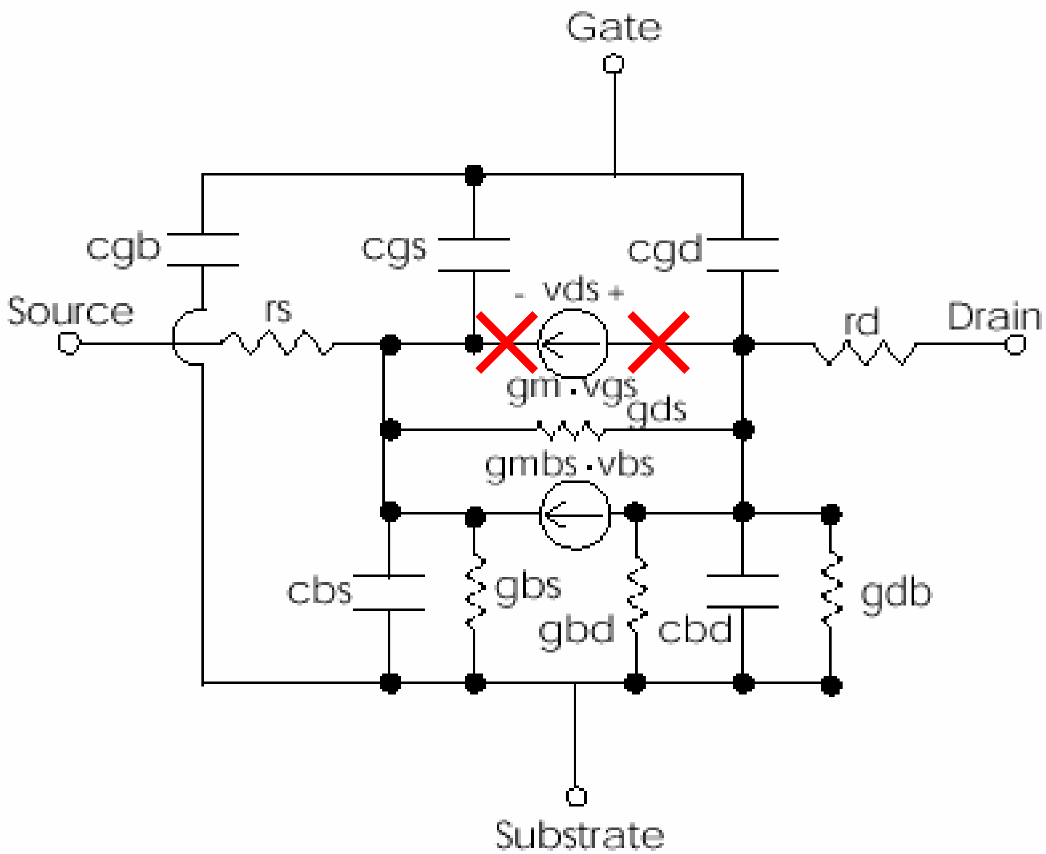



$$F = \frac{C_2}{C_1 + C_2 + C_{GS}}$$

$$p_1 = -\frac{1}{r_o \cdot \frac{C_2(C_1 + C_{GS})}{C_2 + C_1 + C_{GS}}}$$

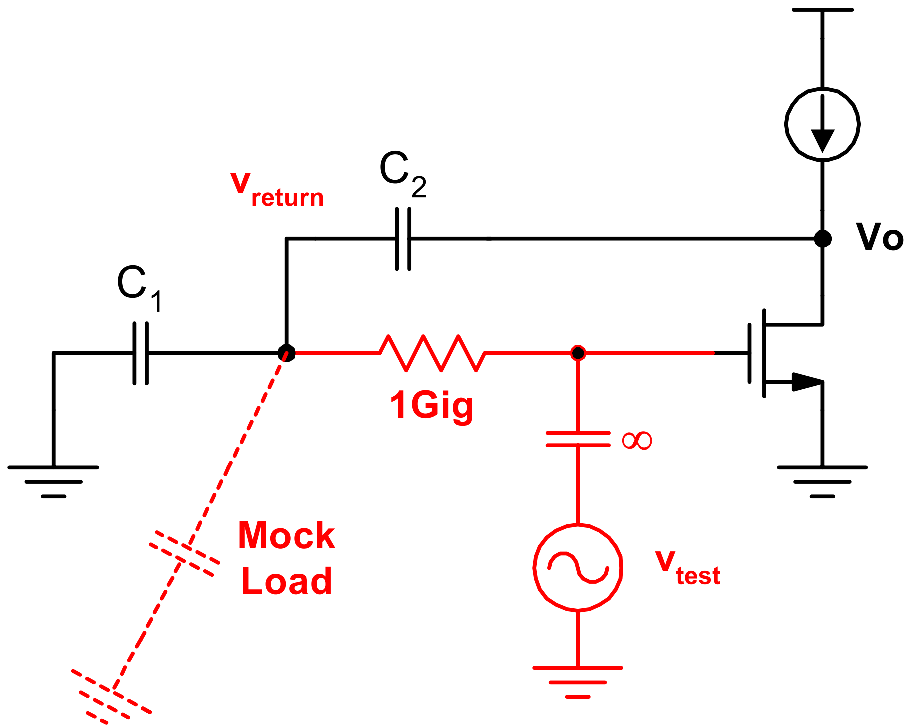
Easy! Why not do the same thing in SPICE?

# MOSFET AC Simulation Model




Small-signal model not accessible in SPICE!

# Popular Simulation Approach




$$T(s) \cong \frac{v_{return}}{v_{test}}$$

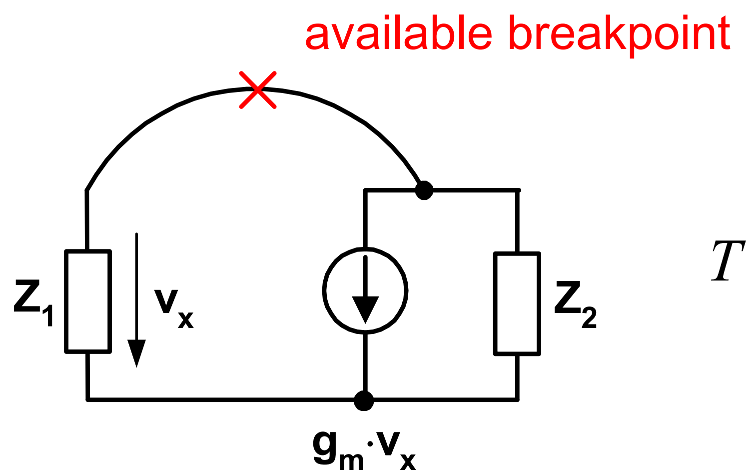
- Inaccurate
- Cumbersome
- Different results for different breakpoints

An ideal loop gain test circuit would:

- not alter node impedances
- not affect the DC bias point

# Problem Generalization

*Any* “single loop“ feedback circuit can be represented as:

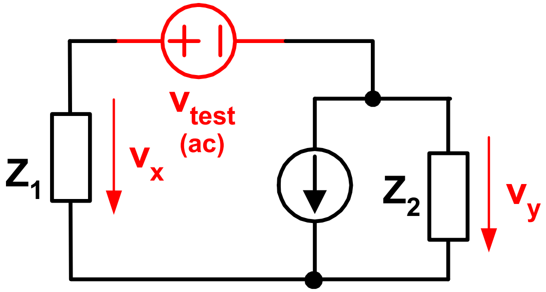



$$T(s) = g_m \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$$

Breakpoint at ideal source is not available.
But there is a breakpoint “between finite impedances“

# Middlebrook Double Injection

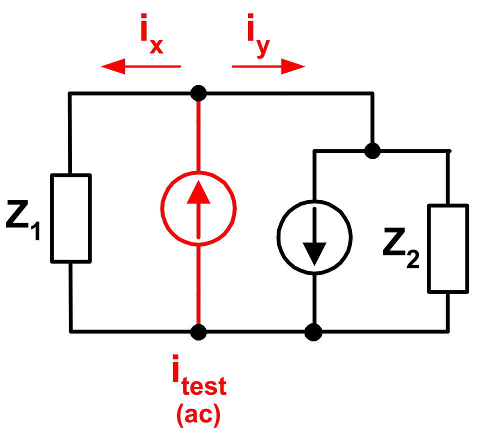
## [Middlebrook 75]




$$\frac{v_y}{v_x} \equiv T_v = g_m \cdot Z_2 + \frac{Z_2}{Z_1}$$

True Loop Gain:

$$T = g_m \cdot \frac{Z_1 Z_2}{Z_1 + Z_2}$$




$$\frac{i_y}{i_x} \equiv T_i = g_m \cdot Z_1 + \frac{Z_1}{Z_2}$$

Solving yields:

$$\boxed{T = \frac{T_v T_i - 1}{T_v + T_i + 2}}$$

- No “DC“ break in the loop, all loading effects covered.
- Measure $T_v$ and $T_i$, then calculate actual T

# Potential Accuracy Problem of Middlebrook Method

$$T = \frac{T_v T_i - 1}{T_v + T_i + 2}$$

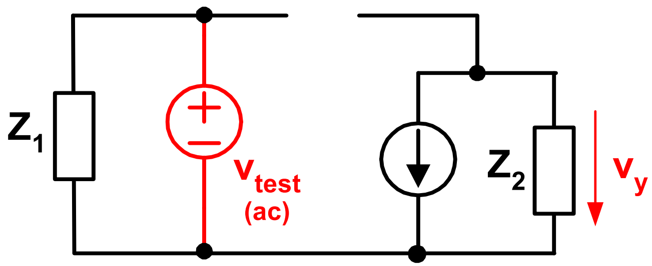
- For small |T|, evaluation of the above expression becomes sensitive to errors in the individual $T_i$ and $T_v$ measurements

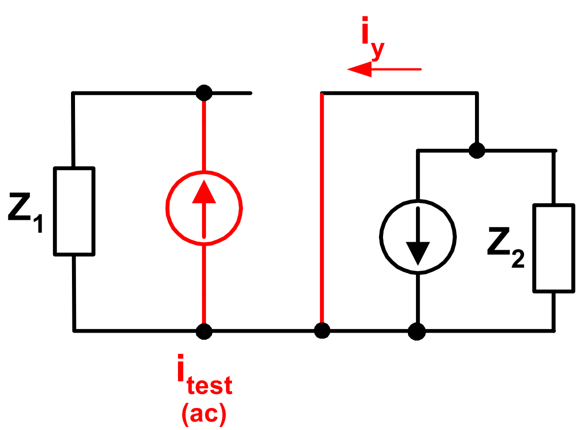
- Sensitivity analysis: $$S_x^y = \frac{\text{fractional change in y}}{\text{fractional change in x}}$$

- For small |T|, it can be shown that: $$S_{T_v,T_i}^T \cong \frac{1}{|T|}$$

- E.g. at |T| =0.01 simulation accuracy decreases by a factor of 100
- Not a problem in a typical circuit simulation/application
- Alternative approach for the “purist“: Rosenstark method [Rosenstark 84,Hurst 94]

# Same Idea: Double Injection




$$\frac{v_y}{v_{test}} \equiv T_v = g_m \cdot Z_2$$

True Loop Gain: $$T = g_m \cdot \frac{Z_1 Z_2}{Z_1 + Z_2}$$

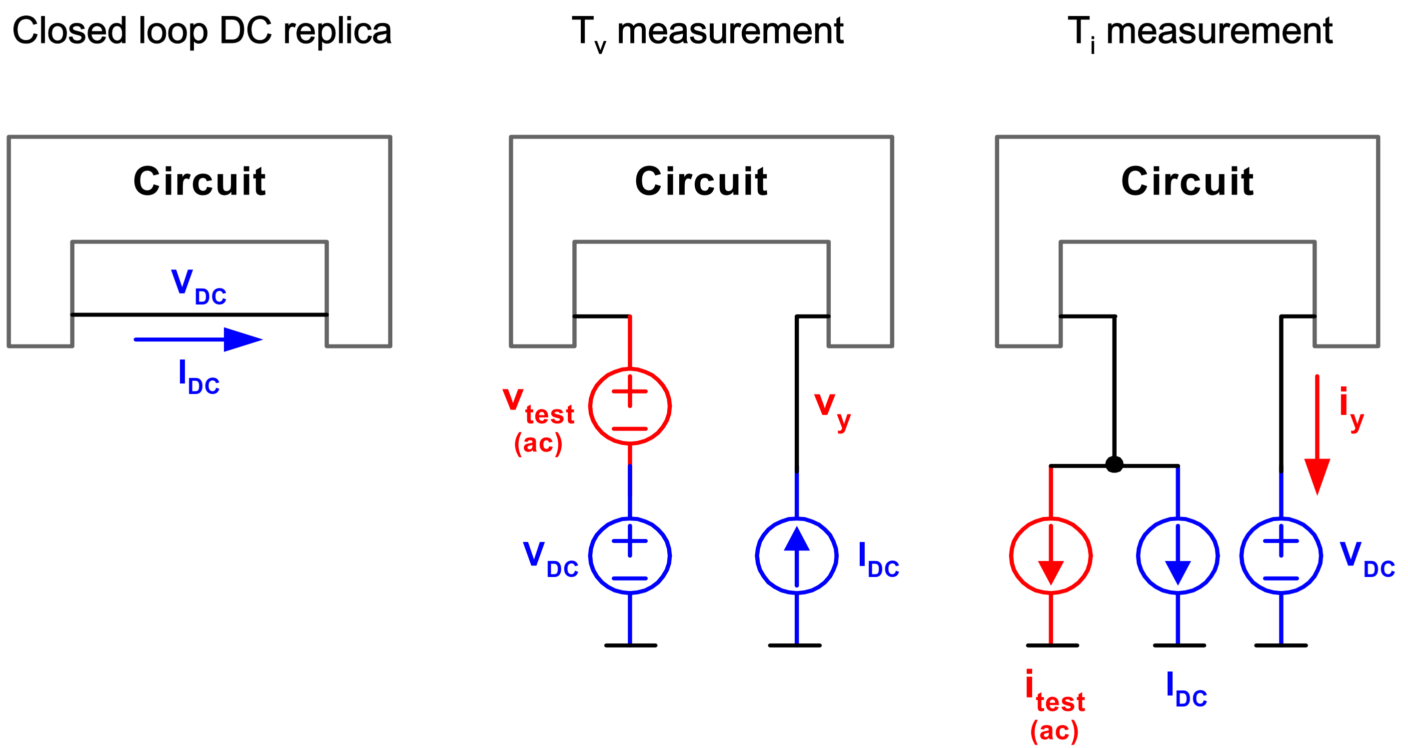
$$\frac{i_y}{i_{test}} \equiv T_i = g_m \cdot Z_1$$

Solving yields:

$$\boxed{T = \frac{T_v T_i}{T_v + T_i}}$$

- Final loop gain calculation has no accuracy problems!
- Problem: Broken loop, no DC path to establish bias point

# Solution to DC problem




- Replicate DC conditions using a closed loop dummy circuit
- Looks complicated, but all sources can be conveniently combined in a subcircuit.

# Multiple Loops ?

- *All* practical feedback circuits have multiple loops:
  - Fully differential circuits have two feedback loops
  - Intrinsic device feedback through $C_{gd}$, $R_{source}$
  - Compensation capacitors
  - ...

- Solutions:
  - Decompose fully differential circuit into common/diff. mode loops
  - If a local feedback loop can be modeled as a combination of a stable controlled source and passive impedances, the multi-loop circuit reduces to a single loop  [Hurst 94].
  - If there is a common breakpoint that breaks all feedback loops simultaneosly, stability can be checked by finding the return ratio at the single breakpoint [Hurst 94].

# Last Resort: General Nyquist Criterion

[Bode 45]:

"If a circuit is stable when all its tubes have their nominal gains, the total number of clockwise and counterclockwise encirclements of the critical point must be equal to each other in the series of Nyquist diagrams for the individual tubes obtained by beginning with all tubes dead and restoring the tubes successively in any order to their nominal gains"

Suggestion: take a controls class if you need this!

# Comments and Observations

- Problem: Simulating the “Nyquist diagrams for the individual tubes“ (return ratios) in principle requires access to the ideal breakpoints of controlled sources
- Single loop case is special in that the return ratio for the active device(s) can be found by breaking the loop *anywhere* in the circuit
- It is not clear how to apply the general Nyquist criterion without having ideal source breakpoints available. Best bet: Break all loops at “near ideal“ breakpoints (voltage/current drive)? Time for a good publication on this topic!
- If there is a “single tube“ that breaks all feedback, this “tube“ can be put back last in the Nyquist plot sequence and therefore establishes stability

# Conclusion

- Presented two methods for loop gain simulation in single loop amplifiers
- Most circuits with (parasitic) multi-loops can be reduced to a single loop problem
- Assessment of stability in a general multi-loop circuit requires Nyquist stability check
- Loop gain simulations would greatly simplify if AC transistor models had a built-in ideal “break and inject“ capability
- Stability analysis (as discussed here) assumes a linear system

# *Always run a* transient analysis *for a true stability check!*

[Bode 45]:

"... thus the circuit may sing when the tubes begin to lose their gain because of age, and it may also sing, instead of behaving as it should, when the gain increases from zero as power is supplied to the circuit ..."

# References

[Bode 45] H.W. Bode, *Network Analysis and Feedback Amplifier Design*, Van Nostrand, New York, 1945.

[Middlebrook 75] R.D. Middlebrook, “Measurement of Loop Gain in Feedback Systems,“ Int. J. Electronics, Vol. 38, No.4, .pp. 485-512, 1975.

[Rosenstark 84] S. Rosenstark, “Loop Gain Measurement in Feedback Amplifiers,“ Int. J. Electronics, Vol. 57, No.3., pp. 415-421, 1984.

[Hurst 91] P.J. Hurst, “Exact Simulation of Feedback Circuit Parameters,“ Trans. on Circuits and Systems, pp.1382-1389, Nov. 1991.

[Hurst 94] P.J. Hurst, S.H. Lewis, “Simulation of Return Ratio in Fully Differential Feedback Circuits,“ Proc. CICC 1994, pp.29-32.

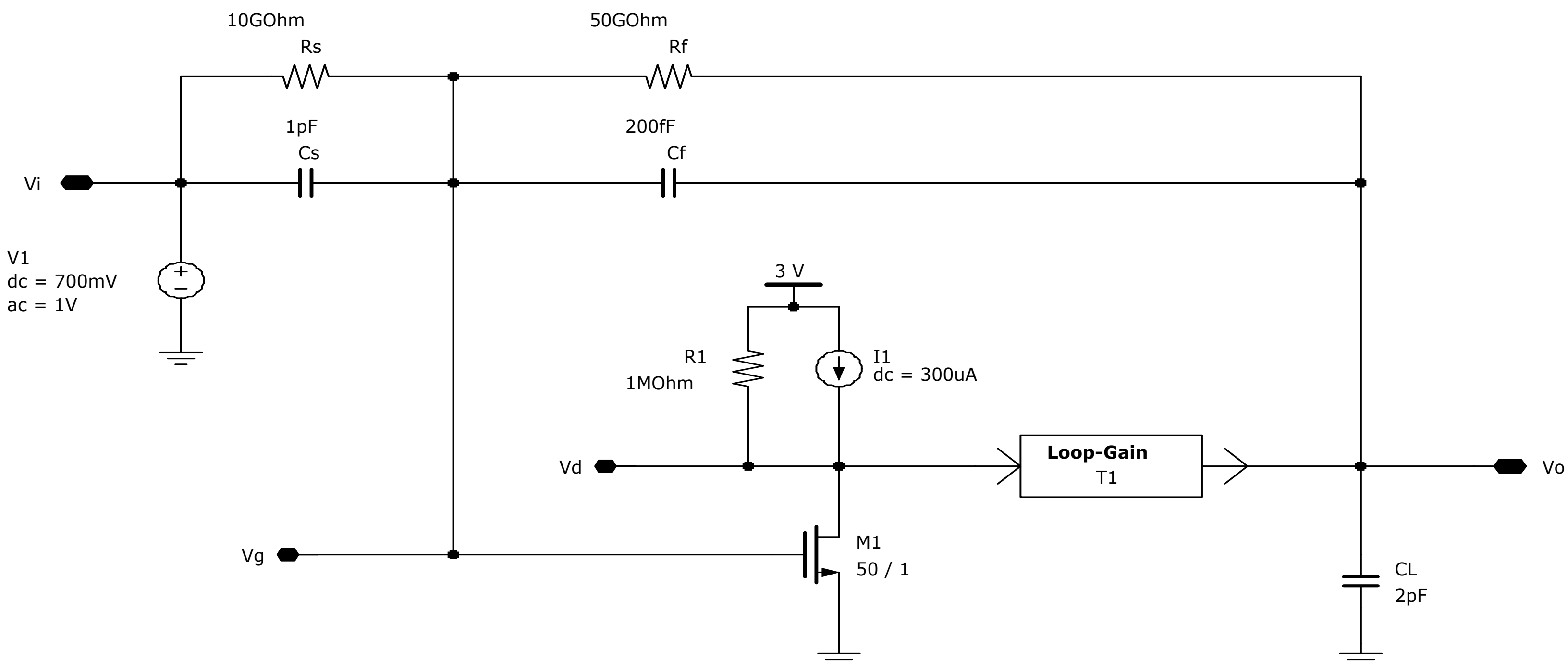
[HLGM 01] P.Gray, P.Hurst, S.Lewis, R. Meyer, *Analysis and Design of Analog Integrated Circuits*, 4th ed., Wiley & Sons, 2001.

# Example

**Loop-Gain Analysis**

**DC Analysis** DC1
Device V1
sweep from 0 to 3 (51 steps)

**AC Analysis** AC1
log sweep from 1k to 10G (101 steps)



# Loop-Gain from SPICE

```
loopgain_example
simulator lang=spectre
output_options options save=all

CL ( Vo 0 ) capacitor c=2p
Cs ( Vi Vg ) capacitor c=1p
Cf ( Vg Vo ) capacitor c=200f
I1 ( p Vd ) isource type=dc dc=300u
VDD ( p 0 ) vsource type=dc dc=3
V1 ( Vi 0 ) vsource type=dc dc=700m mag=1 xfmag=1 pacmag=1
Rs ( Vi Vg ) resistor r=10G
Rf ( Vg Vo ) resistor r=50G
R1 ( p Vd ) resistor r=1M
T1 ( Vd Vo ) tech_misc_loopgain_log start=10k stop=100G points=101
M1 ( Vd Vg 0 0 ) tech_cmos35_nmos w=50u l=1u ad=37.5p pd=51u

DC1 dc start=0 stop=3 lin=51 dev=V1
AC1 ac start=1k stop=10G log=101
```

# SPICE (cont.)

```
subckt tech_misc_loopgain_log (vx vy)
   parameters start=1k stop=10G points=100
   VX (v vx) vsource
   VY (v vy) vsource
   I  (0 v)  isource
   start_ti alter dev=I param=mag value=1
        loopgain_ix xf probe=VX start=start stop=stop log=points
        loopgain_iy xf probe=VY start=start stop=stop log=points
   end_ti alter dev=I param=mag value=0
   start_tv alter dev=VX param=mag value=1
        loopgain_vx (vx 0) xf start=start stop=stop log=points
        loopgain_vy (vy 0) xf start=start stop=stop log=points
   end_tv alter dev=VX param=mag value=0
ends tech_misc_loopgain_log

model tech_cmos35_nmos bsim3v3 type=n …
```

# SPICE (cont.)

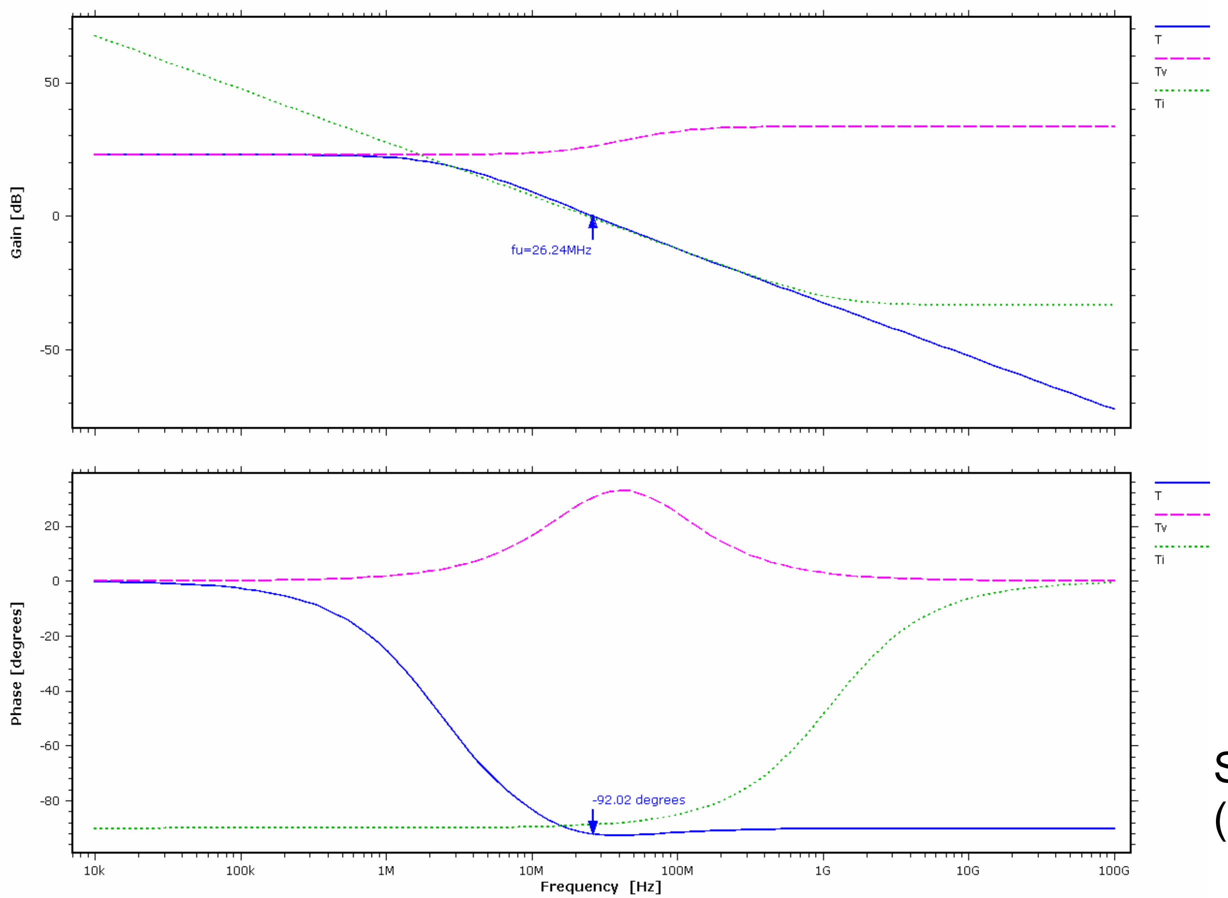
```
Vy = T1.loopgain_vy/T1.VX;                    // analysis / trace
Vx = T1.loopgain_vx/T1.VX;
Iy = T1.loopgain_iy/T1.I;
Ix = T1.loopgain_ix/T1.I;
freq = T1.loopgain_ix/freq;

Tv = Vx / Vy;                                 // compute result
Ti = -Ix / Iy;

T = (Tv * Ti - 1) / (Tv + Ti + 2);

plot(freq, -T, "T");
plot(freq, -Tv, "Tv");
plot(freq, -Ti, "Ti");
```

# SPICE Result

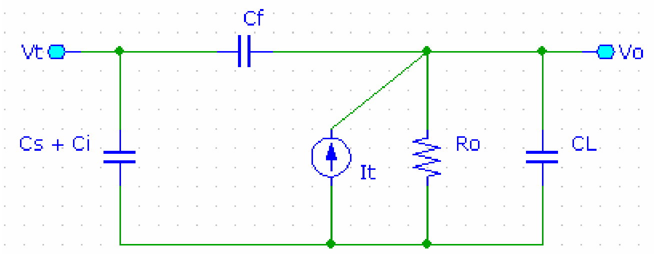



Stable, $\phi_m$ = 88 degrees (Bode criterion)

# Loop-Gain by Hand

$$F = \frac{C_f}{C_f + C_s + C_i}$$

$$\cong \frac{200}{200+1000+170} = \frac{1}{6.85}$$



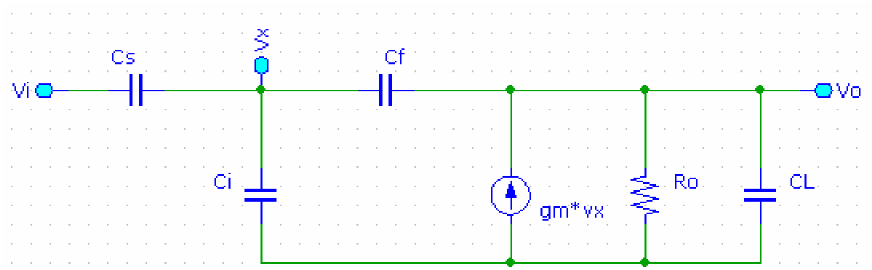

$$T = \frac{1}{s}\frac{g_m F}{C_L + C_f(1-F)} = \frac{1}{s}\frac{g_m F}{C_{Leff}}$$

$$f_u = \frac{1}{2\pi}\frac{g_m F}{C_L + C_f(1-F)}$$

$$\cong \frac{1}{2\pi}\frac{2.2\text{mS}}{2\text{pF}\times 6.85} = \underline{\underline{26\text{MHz}}}$$

Stable, $\phi_m$ = 90 degrees
(single-pole system—trivial example)

# Closed-Loop Gain




$$A = \frac{v_o}{v_i}$$

$$= -\frac{C_s}{C_f} \frac{1 - s\frac{C_f}{g_m}}{1 + s\frac{C_L + C_f(1-F)}{Fg_m}}$$

$$A_{vo} = -c = -\frac{C_s}{C_f}$$

$$\omega_z = +\frac{g_m}{C_f}$$

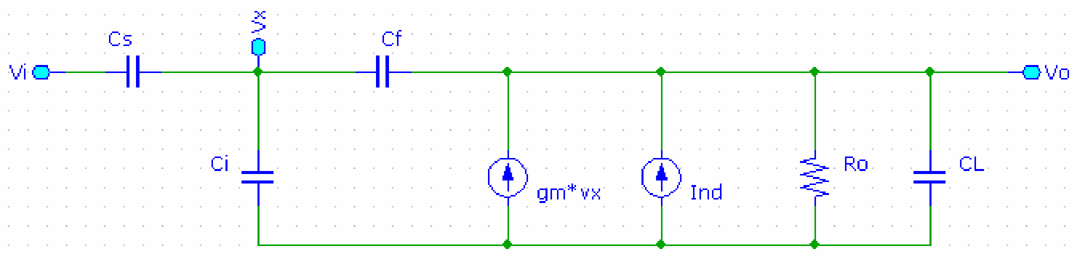
$$\omega_p = -F\frac{g_m}{C_L + C_f(1-F)}$$

$$\approx -F\omega_u \qquad \text{for} \quad 1 - F << 1$$

$$v_x C_T - v_i C_s - v_o C_f = 0$$

$$v_o s\left(C_L + C_f\right) - v_x s C_f + v_x g_m = 0 \qquad \left(R_o \to \infty\right)$$

# Noise




$$v_o = \frac{i_{nd}}{1 + s\dfrac{C_L + C_f(1-F)}{Fg_m}}$$

$$\overline{i_{nd}^2} = 4k_B T g_m \Delta f$$

$$v_x C_T - v_o C_f = 0$$

$$v_o s(C_L + C_f) - v_x s C_f + v_x g_m = i_{nd} \qquad (R_o \to \infty)$$

$$\overline{v_{oT}^2} = \frac{k_B T}{C_L + C_f(1-F)} \frac{1}{F}$$

- Noise gain: 1/F
- Signal gain: c < 1/F = 1 + c + $C_i/C_f$
- $C_i$ has no effect on c but increases 1/F → increases noise gain
- Choose $C_i/C_f$ << 1+c to minimize noise enhancement
- $C_f$ inceases load → lowers noise (depends also on switch resistance)