CHAPTER 1
Functions

SECTION 1.1

1.1.1 If $y = x^2 + x - 30$, the values for which $y = 0$ are

1.1.2 If $y = x^2 + 7x - 8$, for what values of x is $y \geq 0$?

1.1.3 If $y = x^2 + 5x - 9$, for what values of x is $y > 5$?

1.1.4 If $y = 5x^2 - 5$, for what values of x is $y \geq 0$?

1.1.5 If $y = 6x^3 - 3x^2 - 2x + 1$, for what values of x is $y > 0$?

1.1.6 If $y = 4x^3 + x^2 - 4x + 1$, for what values of x is $y \geq 0$?

1.1.7 Use the equation $y = 1 + \sqrt{x}$. For what values of x is $y = 3$?

1.1.8 A ship is sailing at a speed that varies according to the power supplied by the engines. If the speed of the ship is plotted against time on a graph, will the curve be continuous (unbroken)?

1.1.9 If 200 feet of fencing is used to enclose a rectangular plot, what dimensions should the plot have if the area enclosed is to be maximized?

1.1.10 A steel beam is subjected to heat. As it heats it expands. Is the graph of the length of the steel beam over time continuous if the temperature changes during the time period graphed?

1.1.11 $y = 4x^2 - 8x + 9$. Determine at what x the graph of this equation has a minimum.

1.1.12 Use the table to answer the questions.
(a) For what values of x is $y = 1$?
(b) For what values of y is $x = 2$?

x	-2	-1	0	1	2	3
y	4	1	2	3	1	4

1.1.13 Use the equation $y = x^2 - 1$. For what values of x is $y = 0$?

1.1.14 Use the equation $y = x^2 - 4$. For what values of x is $y \leq 0$?

1.1.15 Use the equation $y = x^2 + 5$. For what value of y is $x = 2$?

1.1.16 Does $y = 1 + 3\sqrt{x}$ have a maximum value?

SOLUTIONS

SECTION 1.1

1.1.1 $0 = x^2 + x - 30$

$0 = (x-5)(x+6)$

$x - 5 = 0 \qquad x + 6 = 0$

$x = 5 \qquad x = -6$

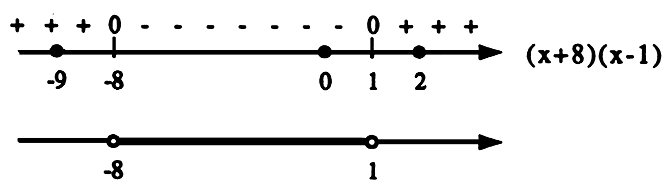
1.1.2 $x^2 + 7x - 8 < 0$

$(x+8)(x-1) < 0$

$S = (-8, 1)$

Choose -9, 0, and 2 as test points within the intervals $(-\infty, -8)$, $(-8, 1)$, and $(1, +\infty)$ respectively.

Interval	Test Point	Sign of $(x+8)(x-1)$ at the Test Point
$(-\infty, -8)$	-9	$(-)(-) = +$
$(-8, 1)$	0	$(+)(-) = -$
$(1, +\infty)$	2	$(+)(+) = +$



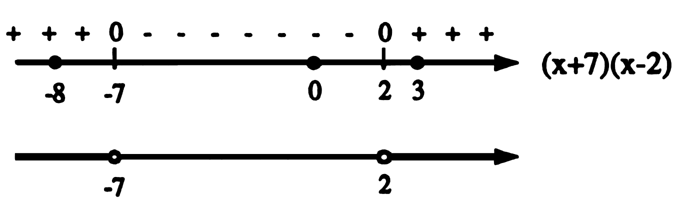
S = (−8,1)

1.1.3 $x^2 + 5x - 14 > 0$

$(x+7)(x-2) > 0$

Choose -8, 0, and 3 as test points within the intervals $(-\infty, -7)$, $(-7, 2)$, and $(2, +\infty)$ respectively.

Interval	Test Point	Sign of $(x+7)(x-2)$ at the Test Point
$(-\infty, -7)$	-8	$(-)(-) = +$
$(-7, 2)$	0	$(+)(-) = -$
$(2, +\infty)$	3	$(+)(+) = +$



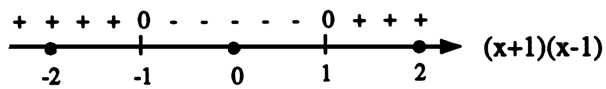
S=(−∞,−7)∪(2,+∞)

1.1.4 $5x^2 - 5 \geq 0$

$(x+1)(x-1) \geq 0$

Choose -2, 0, and 2 as test points within the intervals $(-\infty, -1)$, $(-1, 1)$, and $(1, +\infty)$ respectively.

Interval	Test Point	Sign of $(x+1)(x-1)$ at the Test Point
$(-\infty, -1)$	-2	$(-)(-) = +$
$(-1, 1)$	0	$(+)(-) = -$
$(1, +\infty)$	2	$(+)(+) = +$



$S = (-\infty, -1] \cup [1, +\infty)$

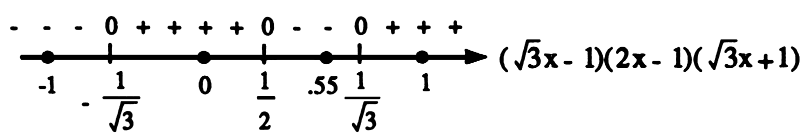
1.1.5 $6x^3 - 3x^2 - 2x + 1 > 0$

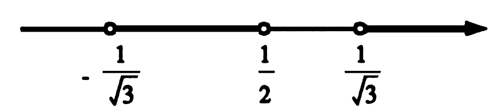
$3x^2(2x-1) - (2x-1) > 0$

$(\sqrt{3}x - 1)(2x-1)(\sqrt{3}x + 1) > 0$

Choose -1, 0, 0.55, and 1 as test poinst within the interval $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$, $\left(-\frac{1}{\sqrt{3}}, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{1}{\sqrt{3}}\right)$, and $\left(\frac{1}{\sqrt{3}}, +\infty\right)$ respectively

Interval	Test Point	Sign of $(2x-1)(\sqrt{3}x-1)(\sqrt{3}x+1)$ at the Test Point
$\left(-\infty, -\frac{1}{\sqrt{3}}\right)$	-1	$(-)(-)(-) = -$
$\left(-\frac{1}{\sqrt{3}}, \frac{1}{2}\right)$	0	$(-)(-)(+) = +$
$\left(\frac{1}{2}, \frac{1}{\sqrt{3}}\right)$	0.55	$(+)(-)(+) = -$
$\left(\frac{1}{\sqrt{3}}, +\infty\right)$	1	$(+)(+)(+) = +$





$S = \left(-\frac{1}{\sqrt{3}}, \frac{1}{2}\right) \cup \left(\frac{1}{\sqrt{3}}, +\infty\right)$

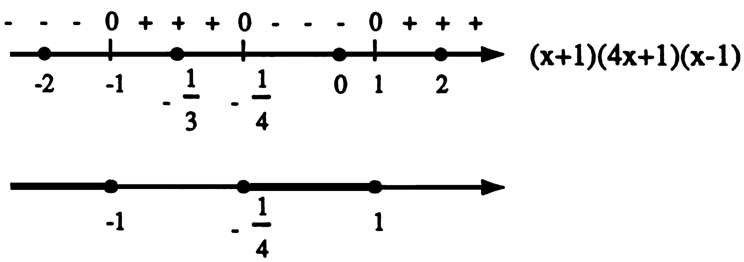
1.1.6 $4x^3 + x^2 - 4x - 1 \le 0$

$x^2(4x+1) - (4x+1) \le 0$

$(x+1)(4x+1)(x-1) \le 0$

Choose -2, $-1/3$, 0, and 2 as test points within the intervals $(-\infty,-1)(-1,-1/4)$, $(-1/4,1)$, and $(1,+\infty)$ respectively.

Interval	Test Point	Sign of $(x+1)(4x+1)(x-1)$ at the Test Point
$(-\infty,-1)$	-2	$(-)(-)(-) = -$
$(-1,-1/4)$	$-1/3$	$(+)(-)(-) = +$
$(-1/4,1)$	0	$(+)(+)(-) = -$
$(1,+\infty)$	2	$(+)(+)(+) = +$



$$S = (-\infty,-1] \cup [-\tfrac{1}{4},1]$$

1.1.7 $3 = 1 + \sqrt{x}$

$2 = \sqrt{x}$

$4 = x$

Check: $3 = 1 + \sqrt{4}$

$= 1 + 2$

$= 3$

1.1.8 Yes, because even if the power suddenly changes the ship will smoothly adjust its speed.

1.1.9 $A = LW$

$P = 200 \text{ feet } = 2L + 2W$

$100 \text{ feet} = L + W$

Let $L = 50 + x$, then $W = 50 - x$

$(50+x)(50-x) = LW = A$

$2,500 - x^2 = A$

This is a parabola that opens downward with a vertex at $(0,0)$, so $x = 0$ maximizes A. 50 ft by 50 ft enclose a maximum area.

1.1.10 It is a continuous curve since the beam responds to temperature slowly.

1.1.11 $x = \dfrac{-(-8)}{2(4)} = 1$

1.1.12 **(a)** -1 and 2 **(b)** 1

1.1.13 $x^2 - 1 = 0$

$$\begin{aligned} x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

1.1.14 Graph the equation with a calculator and observe where the graph is on or below the x-axis.

$-2 \le x \le 2$

1.1.15 $y = 2^2 + 5 = 9$

1.1.16 Use a calculator and observe the graph always increases.

No.

SECTION 1.2

1.2.1 If $h(x) = 3x^2 - 2$, find

(a) $h(0)$ **(b)** $h(2a)$ **(c)** $h(a-4)$.

1.2.2 If $g(x) = \dfrac{x+1}{x}$, find

(a) $g(1)$ **(b)** $g(0)$ **(c)** $g(-1)$ **(d)** $g(x-1)$.

1.2.3 If $f(\theta) = 2\sin\theta + \cos 2\theta$, find

(a) $f(0)$ **(b)** $f(\pi/6)$ **(c)** $f(-\pi/3)$.

1.2.4 If $\phi(x) = 2\sin 2x \cos 3x$, find

(a) $\phi(\pi/6)$ **(b)** $\phi(-\pi/4)$ **(c)** $\phi(\pi/2)$.

1.2.5 Given that

$$s(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

find $s(-1)$, $s(5)$, $s(0)$.

1.2.6 Given that

$$\phi(x) = \begin{cases} 1, & -1 \le x < 2 \\ 4 - x, & 2 < x < 9 \\ x^2 - 4, & x \ge 9 \end{cases}$$

find

(a) $\phi(0)$ **(b)** $\phi(2)$ **(c)** $\phi(4)$.

1.2.7 Express $f(x) = |x+4| - 6$ in piecewise form without using absolute values.

1.2.8 Express $g(y) = 2 - |5y - 10|$ in piecewise form without using absolute values.

1.2.9 Express $h(x) = |x-2| + |x-5|$ in piecewise form without using absolute values.

1.2.10 Find the natural domain and range for $f(x) = \sqrt{3x+4}$.

1.2.11 Find the natural domain and range for $f(x) = \dfrac{2x-5}{3x+2}$.

1.2.12 Find the natural domain for $f(x) = \dfrac{1}{\sqrt{x} - 1}$.

1.2.13 Find the natural domain for $f(x) = \sqrt{\dfrac{x+5}{x-1}}$.

1.2.14 Find the natural domain and range for $g(x) = \dfrac{3x+5}{2x+3}$.

1.2.15 Find the natural domain for $h(x) = \sqrt{4 - 3x - x^2}$.

1.2.16 Find the natural domain for $f(x) = \sqrt{\dfrac{x}{x+2}}$.

1.2.17 Find the x-coordinate of any hole(s) in the graph of $f(x) = \dfrac{x^2 - 144}{x + 12}$.

1.2.18 Find $f(3)$ if $f(x) = x^2 - 2x + 4$.

1.2.19 Use a graphing utility to determine the natural domain of $f(x) = \sqrt{x} + 4$.

1.2.20 $f(x) = \dfrac{1}{(x+3)^2} - 6$. What is the natural domain of this function?

1.2.21 If $f(x) = 4x^3 - 3x^2$, $f(2) =$

SOLUTIONS

SECTION 1.2

1.2.1 **(a)** -2
(b) $12a^2 - 2$
(c) $3(a-4)^2 - 2 = 3a^2 - 24a + 46$

1.2.2 **(a)** 2 **(b)** not defined **(c)** 0 **(d)** $\dfrac{x}{x-1}$

1.2.3 **(a)** 1 **(b)** $3/2$ **(c)** $-\sqrt{3} - 1/2$

1.2.4 **(a)** 0 **(b)** $\sqrt{2}$ **(c)** 0

1.2.5 **(a)** -1 **(b)** 1 **(c)** 0

1.2.6 **(a)** 1 **(b)** not defined **(c)** 0

1.2.7 $f(x) = \begin{cases} x-2, & x \geq -4 \\ -x-10, & x < -4 \end{cases}$

1.2.8 $g(y) = \begin{cases} -5y+12, & y \geq 2 \\ 5y-8, & y < 2 \end{cases}$

1.2.9 $f(x) = \begin{cases} -2x+7, & x < 2 \\ 3, & 2 \leq x < 5 \\ 2x-7, & x \geq 5 \end{cases}$

1.2.10 $3x + 4 \geq 0$ if $x \geq -4/3$, so the domain is $[-4/3, +\infty)$ and the range is $[0, +\infty)$.

1.2.11 $3x + 2 \neq 0$ so the domain is $(-\infty, -2/3) \cup (-2/3, +\infty)$. To get the range, let $y = \dfrac{2x-5}{3x+2}$ and solve for x, thus, $x = \dfrac{5+2y}{2-3y}$ so the range is $(-\infty, 2/3) \cup (2/3, +\infty)$.

1.2.12 $x \geq 0$ and $\sqrt{x} - 1 \neq 0$ so the domain is $[0, 1) \cup (1, +\infty)$.

1.2.13 $\dfrac{x+5}{x-1} \geq 0$ and $x - 1 \neq 0$ if $x \leq -5$ or $x > 1$ so the domain is $(-\infty, -5] \cup (1, +\infty)$.

1.2.14 $2x + 3 \neq 0$ so the domain is $(-\infty, -3/2) \cup (-3/2, +\infty)$. To get the range, let $y = \dfrac{3x+5}{2x+3}$ and solve for x, thus, $x = \dfrac{5-3y}{2y-3}$ so the range is $(-\infty, 3/2) \cup (3/2, +\infty)$.

1.2.15 $4 - 3x - x^2 \geq 0$ if $-4 \leq x \leq 1$ so the domain is $[-4, 1]$.

1.2.16 $\dfrac{x}{x+2} \geq 0$ and $x \neq -2$ if $x < -2$ or $x \geq 0$ so the domain is $(-\infty, -2) \cup [0, +\infty)$.

1.2.17 $\dfrac{x^2 - 144}{x+12} = \dfrac{(x-12)(x+12)}{x+12} = x - 12$
The hole is at $x = -12$.

1.2.18 $3^2 - 2(3) + 4 = 7$

1.2.19 $x \geq 0$

1.2.20 All real numbers except -3.

1.2.21 $4(2)^3 - 3(2)^2 = 20$

SECTION 1.3

1.3.1 Use a graphing utility to determine the number of localized maxima of $f(x) = x^3 + x^2 - 5x + 3$ that are observable in a window set with $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

1.3.2 Use a graphing utility to determine the number of localized maxima of $f(x) = x^5 - x^3 + 2x$ that are observable in a window set with $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

1.3.3 Using a graphing utility, determine how many times the graph of $f(x) = x^4 - 3x^3 - x + 2$ crosses the x-axis if $-10 \leq x \leq 10$.

1.3.4 Using a graphing utility, determine the natural domain of $f(x) = \sqrt{81 - x^2}$.

1.3.5 Using a graphing utility, determine the natural domain of $f(x) = \sqrt{144 - x^2}$.

1.3.6 If the width of the window is twice its height for a certain graphing utility, what would make the graphs displayed on it not appear distorted?

1.3.7 Using a graphing utility, at how many x-coordinates would a graph of $y = \dfrac{x}{x(x-1)}$ have a false line segment?

1.3.8 Using a graphing utility, at how many x-coordinates would a graph of $y = \dfrac{1}{x^3 - 4}$ have a false line segment?

1.3.9 What should the settings be on a graphing utility to show a 20 by 20 window centered at the origin with marks every 5 units on each axis?

1.3.10 If xScl is set at 4, how many equal segments would an x-axis be divided into if xMax $= 20$ and xMin $= -20$?

1.3.11 A student believes a graph crosses the y-axis between 5 and 20. What settings would minimize the y range and still guarantee the window would show the y-intercept if the student's assumption is correct?

1.3.12 What is the smallest domain that is needed to show the entire graph of $f(x) = \sqrt{144 - x^2} + 3$?

1.3.13 What two functions are needed to graph the ellipse $6x^2 + 3y^2 = 15$?

1.3.14 Use a graphing utility to determine where the false line segments of $y = \dfrac{3x}{x^2 - 4}$ are.

1.3.15 Use a graphing utility to determine where the false line segments of $y = \dfrac{3x - 9}{x^2 - 9}$ are.

SOLUTIONS

SECTION 1.3

1.3.1 1

1.3.2 0

1.3.3 2

1.3.4 $-9 \le x \le 9$

1.3.5 $-12 \le x \le 12$

1.3.6 Set the y range at half the x range.

1.3.7 1

1.3.8 1

1.3.9 xMin $= -10$
xMax $= 10$
xScl $= 5$
yMin $= -10$
yMax $= 10$
yXcl $= 5$

1.3.10 The range of x values is 40, separated into segments 4 units long each, so there would be 10 equal segments.

1.3.11 yMin $= 5$
yMax $= 20$

1.3.12 $-12 \le x \le 12$

1.3.13 $6x^2 + 3y^2 = 15$
$3y^2 = 15 - 6x^2$
$y^2 = 5 - 2x^2$
$y = \pm\sqrt{5 - 2x^2}$

1.3.14 $x = \pm 2$

1.3.15 $x = -3$ (Note $x = 3$ is a hole.)

SECTION 1.4

1.4.1 If $f(x) = \dfrac{x}{x+2}$ and $g(x) = f\left(\dfrac{x+2}{2}\right)$, write an expression for $g(x)$ and find its range and domain.

1.4.2 If $f(x) = \sqrt{1-x^2}$ and $g(x) = f(2x)$, write an expression for $g(x)$ and find its range and domain.

1.4.3 If $h(x) = \dfrac{1}{|2-x|+4}$ and $g(x) = h(-2x)$, write an expression for $g(x)$ and find its domain and range.

1.4.4 Let $f(x) = x^2 - x + 1$, find $\dfrac{f(2+h)-f(2)}{h}$.

1.4.5 Let $f(x) = \dfrac{x^2-x-6}{x}$ and $g(x) = x-3$, find

(a) $(f+g)(x)$ **(b)** $(f-g)(x)$ **(c)** $\left(\dfrac{f}{g}\right)(x)$.

1.4.6 Let $f(x) = \dfrac{x}{1+x^2}$ and $g(x) = \sqrt{x}$, find

(a) $f(-2)$ **(b)** $g(x^2)$ **(c)** $f \circ g(x)$.

1.4.7 Let $g(x) = 13x + 3$ and $h(x) = 2x - 1$, find

(a) $g(t+2)$ **(b)** $g \circ h(x)$

(c) $g \circ h(t+2)$ **(d)** the domain of $g \circ h(x)$.

1.4.8 Let $f(x) = \sqrt{x-5}$ and $g(x) = \sqrt{x}$, find

(a) $f \circ g(x)$ **(b)** the domain of $f \circ g(x)$

(c) $g \circ f(x)$ **(d)** the domain of $g \circ f(x)$.

1.4.9 Let $f(x) = x^3$ and $g(x) = \dfrac{2}{\sqrt[3]{x}}$, find

(a) $f \circ g(t+2)$ **(b)** $g \circ f(-x)$.

1.4.10 Let $f(x) = \dfrac{1}{x} + 1$ and $g(x) = 3x^2$, find

(a) $f \circ g(x)$ **(b)** the domain of $f \circ g(x)$

(c) $g \circ f(-x+1)$ **(d)** the domain of $g \circ f(x)$.

1.4.11 Let $f(x) = \sqrt{4-x^2}$ and $g(x) = \dfrac{2}{x}$, find

(a) the domain of $f(x)$ **(b)** $g \circ f(x)$

(c) $f \circ g(x)$ **(d)** the domain of $g \circ f(x)$.

1.4.12 Let $f(x) = |x|$ and $g(x) = x^3 + 1$, find

(a) $f \circ g(x)$ **(b)** $g \circ f(x)$

(c) the domain of $f \circ g(x)$ **(d)** the domain of $g \circ f(x)$.

1.4.13 Let $f(x) = 2x - 1$ and $g(x) = \sqrt{x}$, find the domain for $f \circ g(x)$ and $g \circ f(x)$.

1.4.14 Express $h(x) = \sqrt{x^2 - 4}$ as the composition of two functions such that $h(x) = f \circ g(x)$.

1.4.15 Express $h(x) = |x^3 - 1|$ as the composition of two functions such that $h(x) = f \circ g(x)$.

1.4.16 Express $h(x) = \dfrac{3}{x-4}$ as the composition of two functions such that $h(x) = f \circ g(x)$.

1.4.17 For what values of x does $f(x) = f(x+1)$ and for what values of x does $f(x+1) = f(x) + 1$ if $f(x) = x^2 - 2x + 1$?

1.4.18 For what values of x does $f(x) = f(x+3)$ and for what values of x does $f(x+3) = f(x) + f(3)$ if $f(x) = x^2 - 6x + 9$?

1.4.19 For what values of x does $f(x) = f(x+1)$ if $f(x) = x^3 - x^2 - x + 1$?

1.4.20 Express $h(x) = \sin(x^2)$ as the composition of two functions such that $h(x) = f \circ g(x)$.

1.4.21 Express $h(x) = \cos(2x + \pi/3)$ as the composition of two functions such that $h(x) = f \circ g(x)$.

1.4.22 Let $f(x) = x^2 + 1$ and let h be any nonzero real number. Find $\dfrac{f(x+h) - f(x)}{h}$.

1.4.23 Let $f(x) = 3x - 1$ and let h be any nonzero real number. Find $\dfrac{f(x+h) - f(x)}{h}$.

1.4.24 Sketch the graph of $f(x) = 3 - 4x$, $[0, 2]$.

1.4.25 Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of $f(x) = 1 + \sqrt{-x}$.

1.4.26 Sketch the graph of $f(x) = \sqrt{5 - 4x - x^2}$ by completing the square.

1.4.27 Sketch the graph of $g(x) = -\sqrt{6x - x^2}$.

1.4.28 Sketch the graph of $\phi(x) = \sin(-x/2)$.

1.4.29 Sketch the graph of $g(x) = 2 + \sin x$.

1.4.30 Sketch the graph of $f(x) = 2\sin x + \sin 2x$.

1.4.31 Express $f(x) = |x+2| + 1$ in piecewise form without using absolute values and sketch its graph.

1.4.32 Express $g(x) = 7 - |2x - 4|$ in piecewise form without using absolute values and sketch its graph.

1.4.33 Sketch the graph of $\phi(x) = \begin{cases} x - 2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$.

1.4.34 $f(x) = x^2 - 2$ and $g(x) = x^3 + 1$. Find $f(g(x))$.

1.4.35 Use the graph of $f(x) = |x|$ to sketch the graph of $f(x) = 2 - |2 - x|$.

1.4.36 Is $f(x) = (x+3)^2$ an even function. An odd function, or neither?

1.4.37 The graph of $y = 7 + (x-2)^2$ is obtained from the graph of $y = x^2$ by what translations?

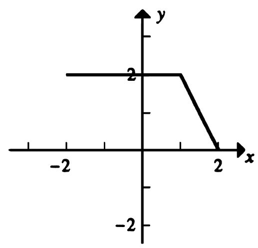
1.4.38 Sketch the graph of $h(x) = (x-2)^3 - 1$.

1.4.39 Sketch the graph of $f(x) = \dfrac{x^2 - 2x - 3}{x - 3}$.

1.4.40 Sketch the graph of $x^2 + 2x - y - 3 = 0$.

1.4.41 Use the graph of $x = y^2$ to sketch the graph of $y^2 - 3y + \dfrac{5}{4} + x = 0$.

1.4.42 A function f with domain $[-2, 2]$ has the graph shown



Use this graph to obtain the graphs of the equations

(a) $y - f(x) + 1$ **(b)** $y = f(x + 1)$ **(c)** $y = f(-x)$ **(d)** $y = -f(x)$.

1.4.43 Determine whether the graph $y = 4x^2 - 2$ is symmetric about the x-axis, the y-axis, or the origin.

1.4.44 Determine whether the graph $y = 4x^3 + x$ is symmetric about the x-axis, the y-axis, or the origin.

1.4.45 Find all intercepts of $x^3 = 2y^3 - y$ and determine symmetry about the x-axis, the y-axis, or the origin.

1.4.46 Find all intercepts of $2x^2 - y^2 = 3$ and determine symmetry about the x-axis, the y-axis, or the origin.

1.4.47 Find all intercepts of $y = \dfrac{1}{3x + x^3}$ and determine symmetry about the x-axis, the y-axis, or the origin.

1.4.48 Find all intercepts of $x = y^4 - 3y^2$ and determine symmetry about the x-axis, the y-axis, or the origin.

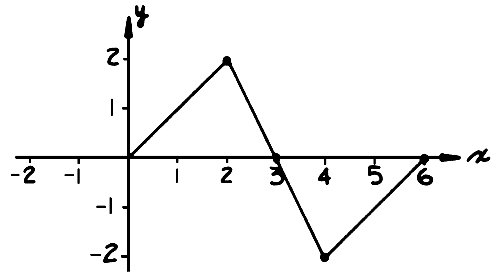
1.4.49 Find all intercepts of $y^4 = |x| + 3$ and determine symmetry about the x-axis, the y-axis, or the origin.

1.4.50 Find all intercepts of $y^3 = |x| - 5$ and determine symmetry about the x-axis, the y-axis, or the origin.

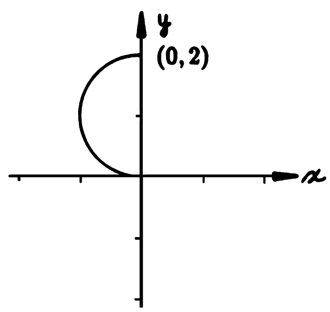
1.4.51 Sketch $y = x^4 - x^2$ in the first quadrant and use symmetry to complete the rest of the graph.

1.4.52 Sketch $y = x^3 - x$ in the first quadrant and use symmetry to complete the rest of the graph.

1.4.53 Extend the graph of the figure given below so that it is symmetric about (a) the origin, (b) the x-axis, and (c) the y-axis.



1.4.54 Extend the graph of the figure given below so that it is symmetric about (a) the origin, (b) the x-axis, and (c) the y-axis.



1.4.55 Show that $y = |x|$ is symmetric about the y-axis and sketch its graph.

1.4.56 Show that $y^2 = 4x + 4$ is symmetric about the x-axis and sketch its graph.

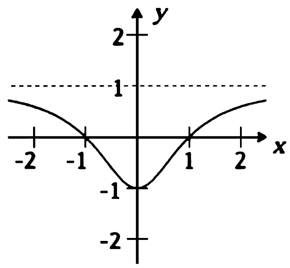
1.4.57 Show that $y = x^3$ is symmetric about the origin and sketch its graph.

1.4.58 Show that $xy = 4$ is symmetric about the origin and sketch its graph.

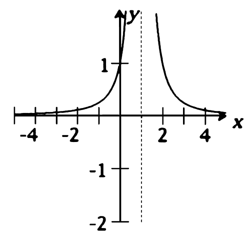
1.4.59 Match the given equations with its graph. [Equations are labeled (a)–(d), graphs are labeled (A)–(D).]

FUNCTION	GRAPH
(a) $y = \dfrac{1}{x^2+1}$	______
(b) $y = \dfrac{x^2-1}{x^2+1}$	______
(c) $y = \dfrac{1}{(x-1)^2}$	______
(d) $y = \dfrac{x}{x^2+1}$	______

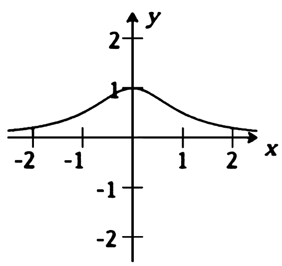
(A)



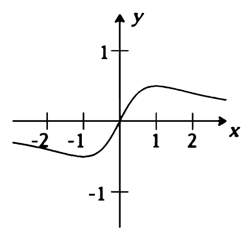
(B)



(C)



(D)



1.4.60 State which of the following statements are true and which are false.

(a) __ A graph which is symmetric about the x-axis and the y-axis must be symmetric about the origin.

(b) __ A graph which is symmetric about the origin must be symmetric about the x-axis and the y-axis.

(c) __ A graph which is symmetric about the origin and about the y-axis must be symmetric about the x-axis.

(d) __ A graph which is not symmetric about the origin is not symmetric about the x-axis and the y-axis.

1.4.61 $f(x) = x^3 + 4x$ is symmetric about what?

SOLUTIONS

SECTION 1.4

1.4.1 $f(x) = \dfrac{x}{x+2}$, $g(x) = \dfrac{\dfrac{x+2}{2}}{\dfrac{x+2}{2}+2} = \dfrac{x+2}{x+6}$. $x+6 \neq 0$ so the domain is $(-\infty,-6)\cup(-6,+\infty)$. To get the range, let $y = \dfrac{x+2}{x+6}$ and solve for x, thus, $x = \dfrac{2-6y}{y-1}$ so the range is $(-\infty,1)\cup(1,+\infty)$.

1.4.2 $g(x) = \sqrt{1-(2x)^2} = \sqrt{1-4x^2}$, $1-4x^2 \geq 0$ if $-1/2 \leq x \leq 1/2$ so the domain is $[-1/2, 1/2]$ and the range is $[0,1]$.

1.4.3 $g(x) = \dfrac{1}{|2-(-2x)|+4} = \dfrac{1}{|2+2x|+4}$, $|2+2x|+4 \neq 0$ so the domain is $(-\infty,+\infty)$ and its range is $(0,1/6]$.

1.4.4
$$\frac{f(2+h)-f(2)}{h} = \frac{[(2+h)^2-(2+h)+1]-[(2)^2-(2)+1]}{h}$$
$$= \frac{3h+h^2}{h} = \frac{h(3+h)}{h} = 3+h, h \neq 0$$

1.4.5 **(a)** $\dfrac{x^2-x-6}{x} + x - 3 = \dfrac{2x^2-4x-6}{x}$

(b) $\dfrac{x^2-x-6}{x} - (x-3) = \dfrac{2x-6}{x}$

(c) $\dfrac{\dfrac{x^2-x-6}{x}}{x-3} = \dfrac{x^2-x-6}{x(x-3)} = \dfrac{(x-3)(x+2)}{x(x-3)} = \dfrac{x+2}{x}, x \neq 3$

1.4.6 **(a)** $-2/5$ **(b)** $\sqrt{x^2} = |x|$ **(c)** $\dfrac{\sqrt{x}}{1+x}$

1.4.7 **(a)** $13t+29$ **(b)** $26x-10$
(c) $26t+42$ **(d)** $(-\infty,+\infty)$

1.4.8 **(a)** $\sqrt{\sqrt{x}-5}$ **(b)** $[25,+\infty)$
(c) $\sqrt{\sqrt{x-5}} = \sqrt[4]{x-5}$ **(d)** $[5,+\infty)$

1.4.9 **(a)** $f \circ g(x) = \dfrac{8}{x}$ so $f \circ g(t+2) = \dfrac{8}{t+2}$

(b) $g \circ f(x) = \dfrac{2}{x}$ so $g \circ f(-x) = \dfrac{2}{-x}$ or $-\dfrac{2}{x}$

1.4.10 **(a)** $f \circ g = \dfrac{1}{3x^2} + 1 = \dfrac{1+3x^2}{3x^2}$ **(b)** $(-\infty,0)\cup(0,+\infty)$

(c) $3\left(\dfrac{1}{-1+x}+1\right)^2 = 3\left(\dfrac{2-x}{1-x}\right)^2$ **(d)** $(-\infty,0)\cup(0,+\infty)$

1.4.11 **(a)** $[-2,2]$ **(b)** $\dfrac{2}{\sqrt{4-x^2}}$

(c) $\sqrt{4-\dfrac{4}{x^2}}$ **(d)** $(-2,2)$

1.4.12 **(a)** $|x^3+1|$ **(b)** $|x|^3+1$
(c) $(-\infty, +\infty)$ **(d)** $(-\infty, +\infty)$

1.4.13 $f \circ g(x)$ is $2\sqrt{x}-1$ so the domain of $f \circ g(x)$ is $[0, +\infty)$, $g \circ f(x)$ is $\sqrt{2x-1}$ so the domain of $g \circ f(x)$ is $[1/2, +\infty)$.

1.4.14 $g(x) = x^2 - 4,\ f(x) = \sqrt{x}$

1.4.15 $g(x) = x^3 - 1,\ f(x) = |x|$

1.4.16 $g(x) = x - 4,\ f(x) = \dfrac{3}{x}$

1.4.17 $x^2-2x+1 = (x+1)^2-2(x+1)+1$ is true if $x = 1/2$ and $(x+1)^2-2(x+1)+1 = (x^2-2x+1)+1$ is true if $x = 1$.

1.4.18 $x^2-6x+9 = (x+3)^2-6(x+3)+9$ is true only if $x = 3/2$ and $(x+3)^2-6(x+3)+9 = (x^2-6x+9)+[(3)^2-6(3)+9]$ is true only if $x = 3/2$.

1.4.19 $x^3 - x^2 - x + 1 = (x+1)^3 - (x+1)^2 - (x+1) + 1$

$$3x^2 + x - 1 = 0$$

solve using the quadratic formula, thus the values of x are $\dfrac{-1-\sqrt{13}}{6}$ and $\dfrac{-1+\sqrt{13}}{6}$.

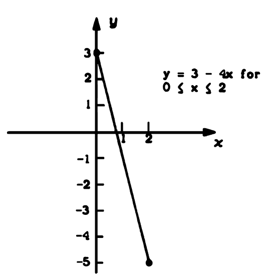
1.4.20 $g(x) = x^2,\ f(x) = \sin x$

1.4.21 $g(x) = 2x + \pi/3,\ f(x) = \cos x$

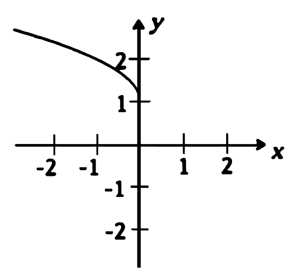
1.4.22 $\dfrac{(x+h)^2+1-(x^2+1)}{h} = \dfrac{2xh+h^2}{h} = 2x+h$

1.4.23 $\dfrac{3(x+h)-1-(3x-1)}{h} = \dfrac{3h}{h} = 3$

1.4.24

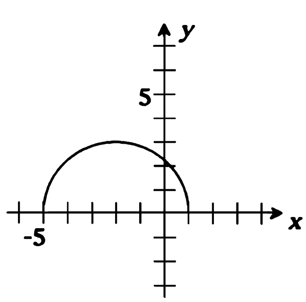


1.4.25

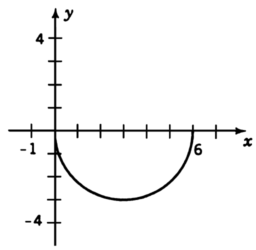


1.4.26 $f(x) = \sqrt{5-4x-x^2} = \sqrt{5+4-(x^2+4x+4)}$

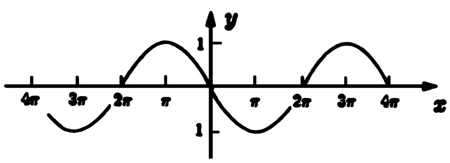
$$= \sqrt{9-(x+2)^2}$$



1.4.27
$$
\begin{aligned}
y &= -\sqrt{6x - x^2} \\
y^2 &= 6x - x^2 \\
(x^2 - 6x) + y^2 &= 0 \\
(x^2 - 6x + 9) + y^2 &= 9 \\
(x - 3)^2 + y^2 &= 9
\end{aligned}
$$

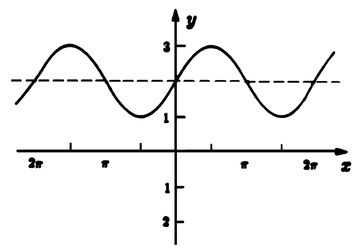


1.4.28



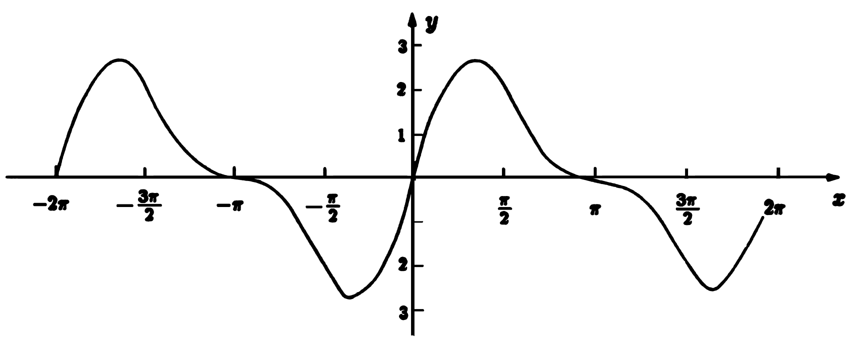
$y = \sin\left(-\frac{x}{2}\right)$

1.4.29

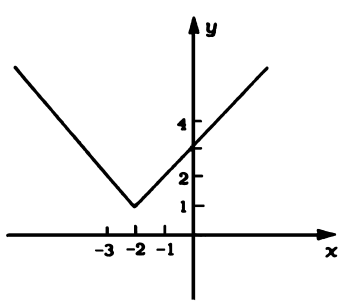


$y = 2 + \sin x$

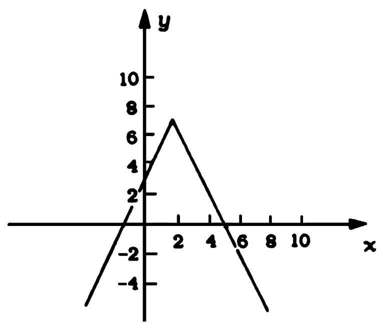
1.4.30



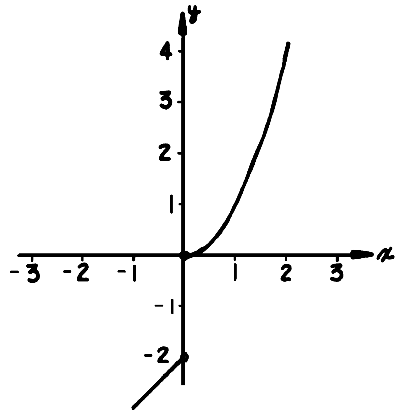
1.4.31 $f(x) = \begin{cases} x + 3, & x \geq -2 \\ -x - 1, & x < -2 \end{cases}$



1.4.32 $g(x) = \begin{cases} 11 - 2x, & x \geq 2 \\ 3 + 2x, & x < 2 \end{cases}$

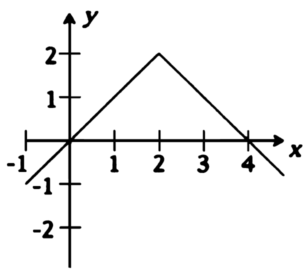


1.4.33 $y = \begin{cases} x - 2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$



1.4.34 $f(g(x)) = (x^3 + 1)^2 - 2 = x^6 + 2x^3 + 1 - 2 = x^6 + 2x^3 - 1$

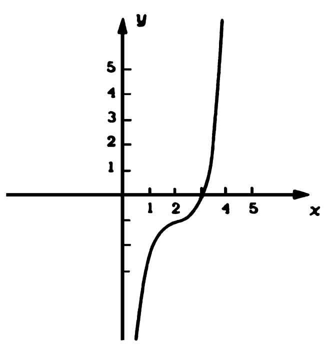
1.4.35



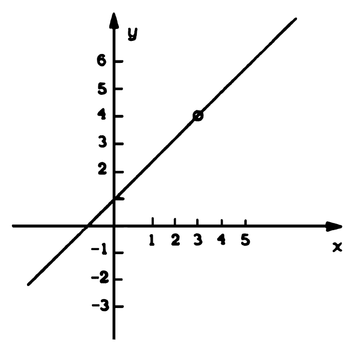
1.4.36 $f(x) = (x + 3)^2 = x^2 + 6x + 9$
$f(-x) = (-x)^2 - 6x + 9 = x^2 - 6x + 9$
neither

1.4.37 First, translate the graph horizontally 2 units to the right.
Second, translate the graph vertically 7 units upward.

1.4.38

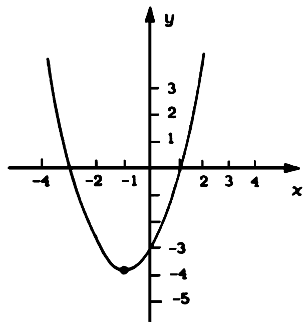


1.4.39

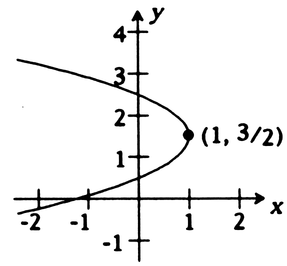


1.4.40 $x^2+2x-y-3=0$

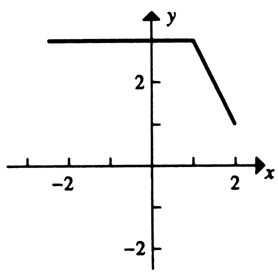
$$\begin{aligned} y &= x^2+2x-3 \\ &= (x^2+2x+1)-4 \\ &= (x+1)^2-4 \end{aligned}$$



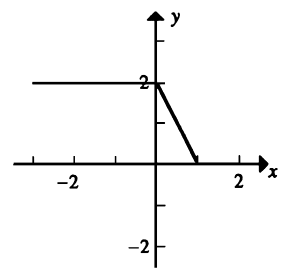
1.4.41



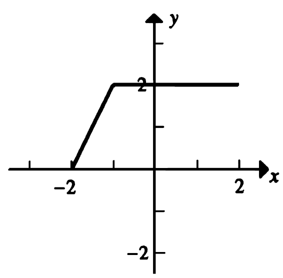
1.4.42 (a)



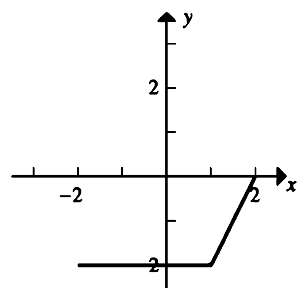
(b)



(c)



(d)



1.4.43 $y=4x^2-2$
Replace x with $(-x)$: $y=4(-x)^2-2=4x^2-2$ thus the graph is symmetric about the y-axis.

1.4.44 $y=4x^3+x$
Replace x with $(-x)$ and y with $(-y)$:

$$\begin{aligned} -y &= 4(-x)^3+(-x) \\ -y &= -(4x^3+x) \\ y &= 4x^3+x \end{aligned}$$

thus the graph is symmetric about the origin.

1.4.45 $x^3 = 2y^3 - y$
Set $y = 0$: $x^3 = 0$, (x-intercept) $x = 0$
Set $x = 0$: $0 = 2y^3 - y = y(2y^2 - 1)$ (y intercepts) $y = 0$ or $y = \pm\sqrt{2}/2$
Replace x with $(-x)$ and y with $(-y)$:

$$\begin{aligned}(-x)^3 &= 2(-y)^3 - (-y)\\ -(x^3) &= -(2y^3 - y)\\ x^3 &= 2y^3 - y\end{aligned}$$

thus the graph is symmetric about the origin.

1.4.46 $2x^2 - y^2 = 3$
Set $y = 0$: $2x^2 = 3$, $x = \pm\dfrac{\sqrt{6}}{2}$ (x-intercepts)
Set $x = 0$: $-y^2 = 3$ has no real solution so no y-intercept.
Replace x with $(-x)$

$$\begin{aligned}2(-x)^2 - y^2 &= 3\\ 2x^2 - y^2 &= 3\end{aligned}$$

thus the graph is symmetric about the y-axis. Replace y with $(-y)$

$$\begin{aligned}2x^2 - (-y)^2 &= 3\\ 2x^2 - y^2 &= 3\end{aligned}$$

thus the graph is symmetric about the y-axis. Since the graph is symmetric about both the x-axis and y-axis, it is symmetric about the origin.

1.4.47 $y = \dfrac{1}{3x + x^3}$
No x or y intercepts
Replace x with $(-x)$ and y with $(-y)$:

$$\begin{aligned}-y &= \frac{1}{3(-x) + (-x)^3} = -\frac{1}{3x + x^3}\\ y &= \frac{1}{3x + x^3}\end{aligned}$$

thus the graph is symmetric about the origin.

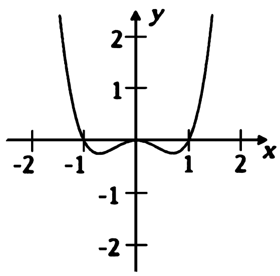
1.4.48 $x = y^4 - 3y^2$
Set $x = 0$: $0 = y^4 - 3y^2 = y^2(y^2 - 3)$ (y-intercepts) $y = 0$ and $y = \pm\sqrt{3}$
Set $y = 0$: $x = 0$ (x-intercept)
Replace y with $(-y)$:
$x = (-y)^4 - 3(-y)^2 = y^4 - 3y^2$ thus the graph is symmetric about the x-axis.

1.4.49 $y^4 = |x| + 3$
Set $y = 0$: $0 = |x| + 3$, $|x| = -3$ no y-intercept
Set $x = 0$: $y^4 = 3$, $y = \pm\sqrt[4]{3}$ (y-intercepts)
Replace y with $(-y)$: $(-y)^4 = y^4 = |x| + 3$
Graph is symmetric about the x-axis
Replace x with $(-x)$: $y^4 = |-x| + 3 = |x| + 3$
Graph is symmetric about the y-axis
Since the graph is symmetric about both the x and y-axes the graph is symmetric about the origin.

1.4.50 $y^3 = |x| - 5$
Set $y = 0$: $0 = |x| - 5$ then $x = \pm 5$ (x-intercepts)
Set $x = 0$: $y^3 = -5$ then $y = \sqrt[3]{-5}$ (y-intercept)
Replace x with $(-x)$: $y^3 = |-x| - 5 = |x| - 5$
The graph is symmetric about the y-axis

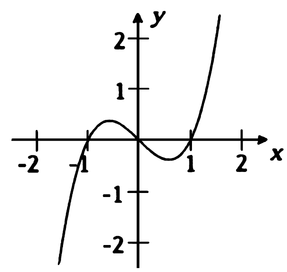
1.4.51 $y = x^4 - x^2$
Set $y = 0$: $0 = x^2(x+1)(x-1)$, $x = 0$ and $x = \pm 1$ are x-intercepts
Set $x = 0$: $y = 0$ is a y-intercept
Replace x by $(-x)$: $y = (-x)^4 - (-x)^2 = x^4 - x^2$ thus the graph is symmetric about the y-axis

x	.5	1.5
y	-0.1875	2.8125



1.4.52 $y = x^3 - x$
Let $x = 0$: $y = 0$ is the y-intercept
Let $y = 0$: $0 = x^3 - x = x(x+1)(x-1)$,
$x = 0$, $x = \pm 1$ are the x-intercepts
Replace x by $(-x)$ and y by $(-y)$:

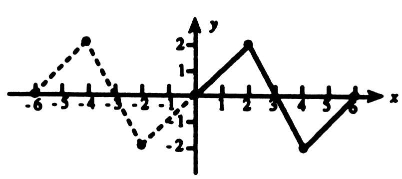
$$-y = (-x)^3 - (-x)$$
$$-y = -(x^3 - x)$$
$$y = x^3 - x.$$



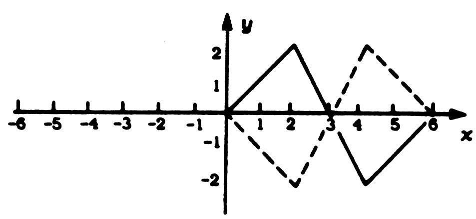
The graph is symmetric about the y-axis

x	.5	1.5
y	-0.375	1.875

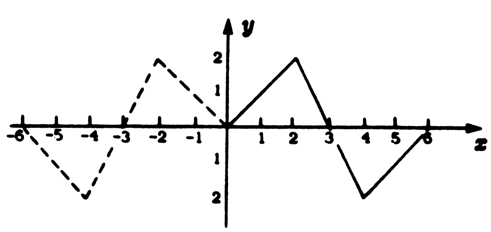
1.4.53 (a)



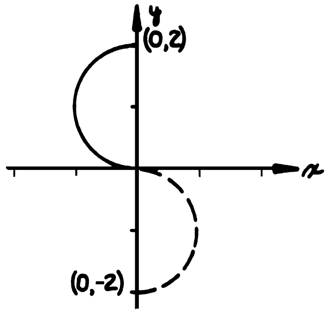
(b)



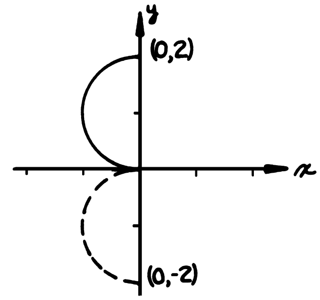
(c)



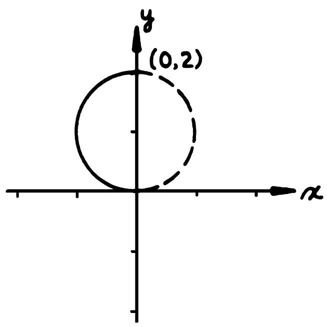
1.4.54 (a)



(b)



(c)

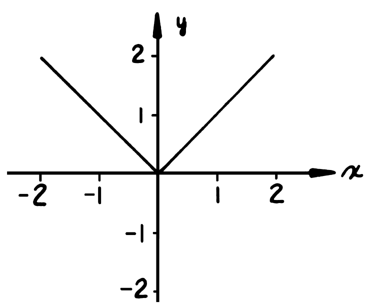


1.4.55 $y = |x|$

Replace x by $-x$:

$$y = |-x| = |x|$$

Thus, the graph is symmetric about the y-axis.

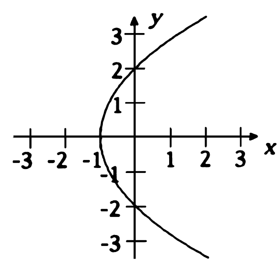


1.4.56 $y^2 = 4x + 4$

Replace y with $(-y)$:

$$(-y)^2 = 4x + 4$$
$$y^2 = 4x + 4$$

Thus, the graph is symmetric about the x-axis.

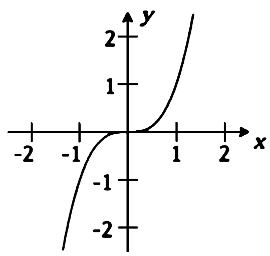


1.4.57 $y = x^3$

Replace x with $(-x)$ and y with $(-y)$:

$$(-y) = (-x)^3$$
$$-y = -x^3$$
$$y = x^3$$

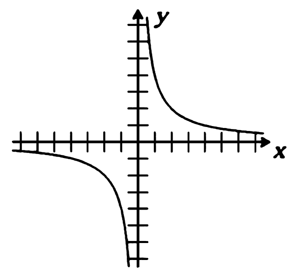
thus, the graph is symmetric about the origin.



1.4.58 Replace x by $(-x)$ and (y) by $(-y)$:

$$(-x)(-y) = 4$$
$$xy = 4$$

Thus, the graph is symmetric about the origin.



1.4.59 **(a)** (C) **(b)** (A) **(c)** (B) **(d)** (D)

1.4.60 **(a)** T **(b)** F **(c)** T **(d)** T

1.4.61 $f(x) = x^3 + 4x$

$f(-x) = (-x)^3 - 4x = -x^3 - 4x$

$f(x)$ is an odd function, so it is symmetric about the origin.

SECTION 1.5

1.5.1 Find the slope of a line drawn perpendicular to the line through $(-2,-4)$ and $(3,5)$.

1.5.2 Find the slope of a line drawn perpendicular to the line through $(3,5)$ and $(6,-3)$.

1.5.3 Show that the line through $(-2,14)$ and $(1,8)$ is
(a) parallel to the line through $(1,-2)$ and $(2,-4)$;
(b) perpendicular to the line through $(1,1)$ and $(3,2)$.

1.5.4 Show that the line through $(3,-4)$ and $(7,5)$ is
(a) parallel to the line through $(1,-11)$ and $(5,-2)$;
(b) perpendicular to the line through $(0,0)$ and $(9,-4)$.

1.5.5 Use slopes to show that $(-2,4)$, $(2,0)$, and $(6,4)$ are vertices of a right triangle.

1.5.6 Use slopes to show that $(9,-6)$, $(-3,0)$, and $(0,6)$ are vertices of a right triangle.

1.5.7 Use slopes to show that $(-1,-8)$, $(5,0)$, and $(6,-7)$ are vertices of a right triangle.

1.5.8 Use slopes to show that $(-1,1)$, $(4,2)$, $(3,-2)$, and $(-2,-3)$ are vertices of a parallelogram.

1.5.9 Use slopes to show that $(-1,-3)$, $(8,3)$, $(3,4)$, and $(0,2)$ are vertices of a trapezoid.

1.5.10 Show that $(-1,3)$, $(6,6)$, $(8,2)$, and $(1,-1)$ are vertices of a parallelogram but not a rectangle.

1.5.11 Use slopes to show that $(3,-5)$, $(7,-2)$, $(2,-2)$ and $(-2,-5)$ are vertices of a rhombus.

1.5.12 Use slopes to show that $(-6,-1)$, $(-2,5)$, $(1,3)$ and $(-3,-3)$ are vertices of a rectangle.

1.5.13 Find the equation of the line through $(-1,3)$ with slope $m=-2$.

1.5.14 Find the equation of the line through $(-3,-7)$ with slope $m=3$.

1.5.15 Find the equation of the line through $(1,3)$ and $(-2,1)$.

1.5.16 Find the equation of the line through $(-2,-3)$ and $(5,-6)$.

1.5.17 Find the equation of the line through $(2,-1)$ and parallel to $3y+5x-6=0$.

1.5.18 Find the equation of the line through $(3,4)$ and parallel to $4x+3y+7=0$.

1.5.19 Find the equation of the line through $(1,-1)$ and perpendicular to $2x-3y-8=0$.

1.5.20 Find the equation of the line through $(5,2)$ and perpendicular to $4x-7y-10=0$.

1.5.21 Find the equation of the line through $(2,2)$ and parallel to the line through $(3,4)$ and $(6,2)$.

1.5.22 Find the equation of the line that has an angle of inclination of $\phi = \dfrac{1}{4}\pi$ and passes through the point $(3,-2)$.

1.5.23 Find the equation of the line that passes through the point $(7, 3)$ and has an angle of inclination $\phi = \frac{1}{3}\pi$.

1.5.24 A person driver 50 miles at 50 mi/hr and 120 miles at 60 mi/hr. Find the average speed the person drives to the nearest mi/hr.

1.5.25 A spring is stretched from 4.00 m, its natural length, to 4.02 m when a 5 kg object is suspended from it. If an additional 15 kg is added to the suspended mass, what would the new length of the spring be?

1.5.26 A particle moves with a velocity given in cm/s according to the equation $v = 4t - 2$. What is the velocity when $t = 0$?

1.5.27 Find the angle of inclination of the line $-2x + y = 7$.

1.5.28 A spring has a natural length of 3 m. If a 10-kg weight is hung from the spring, the spring stretches to 3.02 m. How long will it be if a 15-kg weight is hung from the spring?

1.5.29 A particle moves with a velocity in cm/s given by $v = t^2 - 3t$. How fast is the particle going at $t = 4$ seconds?

1.5.30 Find the slope of all lines parallel to $y = 7x + 9$.

SOLUTIONS

SECTION 1.5

1.5.1 $m = \dfrac{5+4}{3+2} = \dfrac{9}{5}$, so any line with slope $-5/9$ will be perpendicular to the line through $(-2,-4)$ and $(3,5)$.

1.5.2 $m = \dfrac{-3-5}{6-3} = -\dfrac{8}{3}$, so any line with slope $3/8$ will be perpendicular to the line through $(3,5)$ and $(6,-3)$.

1.5.3 Let m_1 be the slope of the line through $(-2,14)$ and $(1,8)$ and let m_2 and m_3 be the slope of the lines in parts (a) and (b), then

$$m_1 = \frac{8-14}{1+2} = -2$$

(a) $m_2 = \dfrac{-4+2}{2-1} = -2$, thus, $m_1 = m_2$ and the lines are parallel;

(b) $m_3 = \dfrac{2-1}{3-1} = \dfrac{1}{2}$, thus, $m_1 m_3 = -1$ and the lines are perpendicular.

1.5.4 Let m_1 be the slope of the line through $(3,-4)$ and $(7,5)$ and let m_2 and m_3 be the slopes of the lines in parts (a) and (b), then

$$m_1 = \frac{5+4}{7-3} = \frac{9}{4}$$

(a) $m_2 = \dfrac{-2+11}{5-1} = \dfrac{9}{4}$, thus, $m_1 = m_2$ and the lines are parallel;

(b) $m_3 = \dfrac{-4-0}{9-0} = -\dfrac{4}{9}$, thus, $m_1 m_3 = -1$ and the lines are perpendicular.

1.5.5 Let $A(-2,4)$, $B(2,0)$, and $C(6,4)$ be the given vertices and let a, b, and c be the sides opposite the vertices, then

$$m_a = \frac{4-0}{6-2} = 1 \qquad m_b = \frac{4-4}{6+2} = 0 \quad \text{and} \quad m_c = \frac{0-4}{2+2} = -1$$

Since $m_a m_c = -1$, sides a and c are perpendicular thus ABC is a right triangle.

1.5.6 Let $A(9,-6)$, $B(-3,0)$, and $C(0,6)$ be the given vertices and let a, b, and c be the sides opposite the vertices, then

$$m_a = \frac{6-0}{0+3} = 2 \qquad m_b = \frac{-6-6}{9-0} = -\frac{4}{3} \quad \text{and} \quad m_c = \frac{-6-0}{9+3} = -\frac{1}{2}$$

Since $m_a m_c = -1$, sides a and c are perpendicular thus ABC is a right triangle.

1.5.7 Let $A(-1,-8)$, $B(5,0)$, and $C(6,-7)$ be the given vertices and let a, b, and c be the sides opposite the vertices, then

$$m_a = \frac{-7-0}{6-5} = -7 \qquad m_b = \frac{-7+8}{6+1} = \frac{1}{7} \quad \text{and} \quad m_c = \frac{0+8}{5+1} = \frac{4}{3}$$

Since $m_a m_b = -1$, sides a and b are perpendicular thus, ABC is a right triangle.

1.5.8 The line through $(-1,1)$ and $(4,2)$ has slope $m_1 = 1/5$, the line through $(4,2)$ and $(3,-2)$ has slope $m_2 = 4$, the line through $(3,-2)$ and $(-2,-3)$ has slope $m_3 = 1/5$, the line through $(-2,-3)$ and $(-1,1)$ has slope $m_4 = 4$; since $m_1 = m_3$ and $m_2 = m_4$, opposite sides are parallel so the figure is a parallelogram.

1.5.9 The line through $(-1,-3)$ and $(8,3)$ has slope $m_1 = 2/3$, the line through $(8,3)$ and $(3,4)$ has slope $m_2 = -1/5$, the line through $(3,4)$ and $(0,2)$ has slope $m_3 = 2/3$, the line through $(0,2)$ and $(-1,-3)$ has slope $m_4 = 5$. So $m_1 = m_3$, the figure is a trapezoid since it has two parallel sides.

1.5.10 The line through $(-1,3)$ and $(6,6)$ has slope $m_1 = 3/7$, the line through $(6,6)$ and $(8,2)$ has slope $m_2 = -2$, the line through $(8,2)$ and $(1,-1)$ has slope $m_3 = 3/7$, the line through $(1,-1)$ and $(-1,3)$ has slope $m_4 = -2$; since $m_1 = m_3$ and $m_2 = m_4$, opposite sides are parallel so the figure is a parallelogram; since $m_1 m_2 \neq -1$, adjacent sides are not perpendicular and thus, the parallelogram is not a rectangle.

1.5.11 The line through $(3,-5)$ and $(7,-2)$ has slope $m_1 = \dfrac{-2+5}{7-3} = \dfrac{3}{4}$ the line through $(7,-2)$ and $(2,-2)$ has slope $m_2 = \dfrac{-2+2}{2-7} = 0$, the line through $(2,-2)$ and $(-2,-5)$ has slope $m_3 = \dfrac{-5+2}{-2-2} = \dfrac{3}{4}$ and the line through $(-2,-5)$ and $(3,-5)$ has slope $m_4 = \dfrac{-5+5}{3+2} = 0$. Since $m_1 = m_3$ and $m_2 = m_4$, opposite sides of the quadrilateral are parallel. The diagonal from $(3,-5)$ to $(2,-2)$ has slope $m_a = \dfrac{-2+5}{2-3} = -3$ and the diagonal from $(7,-2)$ to $(-2,-5)$ has slope $m_b = \dfrac{-5+2}{-2+7} = \dfrac{1}{3}$. Since $m_a m_b = -1$, the diagonals are perpendicular so the quadrilateral is a rhombus.

1.5.12 The line through $(-6,-1)$ and $(-2,5)$ has slope $m_1 = \dfrac{5+1}{-2+6} = \dfrac{3}{2}$, the line through $(-2,5)$ and $(1,3)$ has slope $m_2 = \dfrac{3-5}{1+2} = -\dfrac{2}{3}$, the line through $(1,3)$ and $(-3,-3)$ has slope $m_3 = \dfrac{-3-3}{-3-1} = \dfrac{3}{2}$, and the line through $(-3,-3)$ and $(-6,-1)$ has slope $m_4 = \dfrac{-1+3}{-6+3} = -\dfrac{2}{3}$. Since $m_1 = m_3$ and $m_2 = m_4$ opposite sides of the quadrilateral are parallel and since $m_1 m_2 = -1$, adjacent sides are perpendicular so the quadrilateral is a rectangle.

1.5.13
$$\begin{aligned} y - 3 &= (-2)(x+1) \\ y &= -2x + 1 \end{aligned}$$

1.5.14
$$\begin{aligned} y - (-7) &= 3[x - (-3)] \\ y &= 3x + 2 \end{aligned}$$

1.5.15 The slope of the line through $(1,3)$ and $(-2,1)$ is $m = \dfrac{1-3}{-2-1} = \dfrac{2}{3}$; the required equation is $y - 3 = \dfrac{2}{3}(x-1)$ or $y = \dfrac{2}{3}x + \dfrac{7}{3}$.

1.5.16 The slope of the line through $(-2,-3)$ and $(5,-6)$ is $m = \dfrac{-6-(-3)}{5-(-2)} = -\dfrac{3}{7}$; the required equation is $y - (-3) = -\dfrac{3}{7}[x - (-2)]$ or $y = -\dfrac{3}{7}x - \dfrac{27}{7}$.

1.5.17 Place $3y+5x-6=0$ into slope-intercept form to yield $y=-\dfrac{5}{3}x+2$. The lines will be parallel if the slope of the line $m=-5/3$, thus

$$\begin{aligned} y-(-1) &= -\frac{5}{3}(x-2) \\ y &= -\frac{5}{3}x+\frac{7}{3} \end{aligned}$$

1.5.18 Place $4x+3y+7=0$ into slope-intercept form to yield $y=-\dfrac{4}{3}x-\dfrac{7}{3}$. The lines will be parallel if the slope of the line $m=4/3$, thus,

$$\begin{aligned} y-4 &= -\frac{4}{3}(x-3) \\ y &= -\frac{4}{3}x+8 \end{aligned}$$

1.5.19 Place $2x-3y-8=0$ into slope-intercept form to yield $y=\dfrac{2}{3}x-\dfrac{8}{3}$. The lines will be perpendicular if the slope of the line $m=-3/2$, thus,

$$\begin{aligned} y-(-1) &= -\frac{3}{2}(x-1) \\ y &= -\frac{3}{2}x+\frac{1}{2} \end{aligned}$$

1.5.20 Place $4x-7y-10=0$ into slope-intercept form to yield $y=\dfrac{4}{7}x-\dfrac{10}{7}$. The lines will be perpendicular if the slope of the line $m=-7/4$, thus,

$$\begin{aligned} y-2 &= -\frac{7}{4}(x-5) \\ y &= -\frac{7}{4}x+\frac{43}{4} \end{aligned}$$

1.5.21 The slope of the line through $(3,4)$ and $(6,2)$ is

$$\frac{2-4}{6-3}=-\frac{2}{3}$$

The lines will be parallel if the slope of the line $m=-2/3$, thus,

$$\begin{aligned} y-2 &= -\frac{2}{3}(x-2) \\ y &= -\frac{2}{3}x+\frac{10}{3} \end{aligned}$$

1.5.22 $m=\tan\dfrac{\pi}{4}=1$ so

$$\begin{aligned} y-(-2) &= x-3 \\ y+2 &= x-3 \\ y &= x-5 \end{aligned}$$

1.5.23 $m = \tan\frac{\pi}{3} = \sqrt{3}$

$$\begin{aligned} y - 3 &= \sqrt{3}(x-7) \\ y &= \sqrt{3}x - 7\sqrt{3} + 3 \end{aligned}$$

1.5.24
$$\begin{aligned} d &= rt \\ 50\text{ mi} &= (50\text{ mi/hr})t_1 \\ t_1 &= \frac{50}{50}\text{hr} = 1\text{ hr} \\ 120\text{ mi} &= (60\text{ mi/hr})t_2 \\ t_2 &= \frac{120}{60}\text{ hr} = 2\text{ hr} \\ t_{\text{total}} &= t_1 + t_2 = 1\text{ hr} + 2\text{ hr} = 3\text{ hr} \end{aligned}$$

$$\frac{170}{3}\text{mi/hr} = 57\text{ mi/hr (rounded)}$$

1.5.25 $F = kx$, so $k = \frac{F}{x}$
Since the acceleration is g, $k \propto \frac{m}{x}$

$$\begin{aligned} \frac{m_1}{x_1} &= \frac{m_2}{x_2} \\ x_2 &= \frac{m_2}{m_1}x_1 = \frac{20\text{ kg}}{5\text{ kg}}(0.02\text{ m}) = 4(0.02\text{ m}) = 0.08\text{ m} \end{aligned}$$

1.5.26
$$\begin{aligned} v(t) &= 4t - 2 \\ v(0) &= 4(0) - 2 \\ &= -2\text{ cm/s} \end{aligned}$$

1.5.27 Put the equation into slope-intercept form, $y = 2x + 7$.
The slope is 2.
Use the $\tan^{-1}$ key on a calculator that is set in degrees mode.
$\tan^{-1} 2 = 63.4°$

1.5.28 $10\text{ kg }(9.8\text{ m/s}^2) = 0.02\text{ m }(k)$
$k = 500(9.8\text{ m/s}^2)\text{ kg/m}$
$F = kx$
$15\text{ kg}(9.8\text{ m/s}^2) = 500(9.8\text{ m/s}^2)\text{ kg/m } x$
$x = 0.03\text{ m}$
The spring will stretch to 3.03 m.

1.5.29 $v = 4^2 - 3(4) = 4\text{ cm/s}$

1.5.30 $m = 7$

SECTION 1.6

1.6.1 What do all members of the family of lines of the form $y = 2x + b$ have in common?

1.6.2 What do all members of the family of lines of the form $y = ax + 8$ have in common?

1.6.3 What do all members of the family of lines of the form $ax = by$ have in common?

1.6.4 Use a graphing utility to graph on the same window $y_1 = \sqrt{x}$, $y_2 = \sqrt[4]{x}$, and $y_3 = \sqrt[6]{x}$.

1.6.5 What points do all curves of the form $y = \sqrt[n]{x^2}$, where n is an add integer, have in common?

1.6.6 Determine the vertical asymptote(s) of $y = \dfrac{x-1}{x^2+5x-14}$.

1.6.7 Determine the vertical asymptote(s) of $y = \dfrac{x}{x^2(x-1)}$.

1.6.8 Use a calculating utility to approximate $\sin\left(\dfrac{\pi}{8}\right)$ to four decimal places.

1.6.9 Use a calculating utility to approximate $\tan\left(\dfrac{\pi}{7}\right)$ to four decimal places.

1.6.10 A sphere whose radius is 0.5 m rolls through an angle of $60°$. How far does it roll?

1.6.11 The amplitude of $3\cos(8\pi x + 2)$ is

1.6.12 The amplitude of $4\sin(\pi x + 6)$ is

1.6.13 The amplitude of $8\cos(x) - 12$ is

1.6.14 What is the phase shift of $\tan\left(x - \dfrac{\pi}{6}\right)$?

1.6.15 What is the period of $\sin(3x + 4)$?

1.6.16 A point source of energy in space radiates energy that spreads so its magnitude is inversely proportional to r^3. If $E = 5$ w when $r = 1$ m, what is E when $r = 2$ m?

1.6.17 What is the amplitude of $2\cos\left(3x + \dfrac{\pi}{2}\right) - 7$?

1.6.18 What is the phase shift of $2\cos\left(3x + \dfrac{\pi}{2}\right) - 7$?

1.6.19 What is the period of $2\cos\left(3x + \dfrac{\pi}{2}\right) - 7$?

1.6.20 Determine the vertical asymptotes of $y = \dfrac{7}{x^2 - 7x}$.

SOLUTIONS

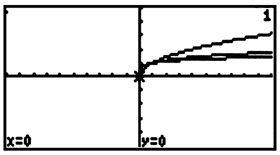
SECTION 1.6

1.6.1 They all have a slope of 2.

1.6.2 They all have a y-intercept of 8.

1.6.3 They all go through the origin.

1.6.4



1.6.5 They all go through $(-1, 1)$, $(0, 0)$, and $(1, 1)$.

1.6.6 $y = \dfrac{x-1}{x^2+5x-14} = \dfrac{x+1}{(x+7)(x-2)}$

Setting the denominator equal to 0 yields

$$\begin{gathered}(x+7)(x-2) = 0 \\ x+7=0 \qquad x-2=0 \\ x=-7 \qquad\quad x=2\end{gathered}$$

1.6.7 $y = \dfrac{x}{x^2(x-1)} = \dfrac{1}{x(x-1)}$

Setting the denominator equal to 0 yields

$$\begin{gathered}x(x-1) = 0 \\ x-1=0 \\ x=0 \qquad x=1\end{gathered}$$

1.6.8 0.3827

1.6.9 0.4816

1.6.10 The circumference of the sphere is $2\pi r = 2(0.5\text{ m})\pi = \pi$ m

$60°$ is $\dfrac{60}{360} = \dfrac{1}{6}$ of one full rotation.

$\dfrac{1}{6}$ of a circumference is $\dfrac{\pi}{6}$ m.

1.6.11 3

1.6.12 4

1.6.13 The amplitude of $8\cos(x) - 12$ is 8. The shift of 12 downward moves the curve, but does not alter the amplitude.

1.6.14 $\dfrac{\pi}{6}$. The phase shift has a sign opposite the one that appears in the expression.

1.6.15 If $3x = 2\pi$, then $x = \dfrac{2\pi}{3}$.

1.6.16 $E = \dfrac{k}{r^3}$ or $k = Er^3$

$k = 1(5) = 5$

$5 = E(2)^3$

$E = \dfrac{5}{8}$ w

1.6.17 2

1.6.18 $2\cos\left(3x + \dfrac{\pi}{2}\right) = 2\cos\left(3\left(x + \dfrac{\pi}{6}\right)\right) - 7$
$\dfrac{\pi}{6}$ units to the left.

1.6.19 $3x = 2\pi$
The period is $\dfrac{2\pi}{3}$.

1.6.20 $x^2 - 7x = 0$
$x(x - 7) = 0$
The vertical asymptotes are $x = 0$ and $x = 7$.

SECTION 1.7

1.7.1 The accompanying table gives the number of centimeters that a spring is stretched by various weights.

(a) Use linear regression to express the stretch as a function of the amount of weight placed on the spring.

(b) Is the linear regression a good model for the data?

Weight (N)	Stretch (cm)
0.0	0.000
0.5	0.123
1.0	0.243
1.5	0.369
2.0	0.482
2.5	0.604

1.7.2 The accompanying table gives the number of centimeters that a spring is stretched by various weights.

(a) Use linear regression to express the stretch as a function of the amount of weight placed on the spring.

(b) Is the linear regression a good model for the data?

Weight (N)	Stretch (cm)
0.0	0.000
0.5	0.156
1.0	0.310
1.5	0.471
2.0	0.631
2.5	0.784

1.7.3 The accompanying table gives the number of centimeters that a spring is stretched by various weights.

(a) Use linear regression to express the stretch as a function of the amount of weight placed on the spring.

(b) Is the linear regression a good model for the data?

Weight (N)	Stretch (cm)
0.0	0.000
0.5	1.34
1.0	2.65
1.5	3.99
2.0	5.37
2.5	6.71

1.7.4 The table below provides the heights of a particle thrown upward at various times after it was thrown. Use a quadratic regression feature of a graphing utility to determine the height as a function of time.

time (s)	Height (m)
0.0	200.00
0.1	205.84
0.2	211.36
0.3	216.56
0.4	221.44
0.5	226.00
0.6	230.24

1.7.5 The table below provides the heights of a particle thrown upward at various times after it was thrown. Use a quadratic regression feature of a graphing utility to determine the height as a function of time.

time (s)	Height (m)
0.0	20.00
0.1	22.84
0.2	25.36
0.3	27.56
0.4	29.44
0.5	31.00
0.6	32.24

1.7.6 The table below provides the heights of a particle thrown upward at various times after it was thrown. Use a quadratic regression feature of a graphing utility to determine the height as a function of time.

time (s)	Height (m)
0.0	3.20
0.1	4.52
0.2	5.52
0.3	6.20
0.4	6.56
0.5	6.60
0.6	6.32

1.7.7 The table below provides the heights of a particle thrown upward at various times after it was thrown. Use a quadratic regression feature of a graphing utility to determine the height as a function of time.

time (s)	Height (m)
0.0	19.40
0.1	21.11
0.2	22.50
0.3	23.57
0.4	24.32
0.5	24.75
0.6	24.86

1.7.8 A particle in an experiment has its position measured at various times, as given in the table. Find the least squares line for the data points.

time (s)	Position (cm)
0	0.0
1	1.4
2	2.9
3	3.1
4	4.6
5	6.1
6	7.5

1.7.9 A particle in an experiment has its position measured at various times, as given in the table. Find the least squares line for the data points.

time (s)	Position (cm)
0	0.00
1	8.21
2	16.85
3	25.01
4	33.45
5	42.02
6	49.98

1.7.10 A particle in an experiment has its position measured at various times, as given in the table. Find the least squares line for the data points.

time (s)	Position (cm)
0	20.2
1	30.4
2	40.3
3	50.1
4	60.9
5	71.2
6	81.3

1.7.11 A particle in an experiment has its position measured at various times, as given in the table. Find the least squares line for the data points.

time (s)	Position (cm)
0	16.8
1	18.9
2	20.1
3	22.5
4	24.6
5	26.8
6	28.4

1.7.12 A particle in an experiment has its position measured at various times, as given in the table. Find the least squares line for the data points.

time (s)	Position (cm)
0	20.1
1	18.0
2	16.3
3	14.2
4	12.0
5	10.1
6	7.9

1.7.13 A particle in an experiment has its position measured at various times, as given in the table. Find the exponential function for the data points.

time (s)	Position (cm)
0	0.1
1	1.2
2	4.1
3	16.3
4	32.8
5	64.1
6	128.3

1.7.14 A particle in an experiment has its position measured at various times, as given in the table. Find the quadratic function for the data points.

time (s)	Position (cm)
0	0.1
1	1.2
2	4.1
3	16.3
4	32.8
5	64.1
6	128.3

1.7.15 A particle in an experiment has its position measured at various times, as given in the table. Find the exponential function for the data points.

time (s)	Position (cm)
0	0.2
1	2.3
2	8.4
3	32.9
4	65.1
5	130.1
6	265.8

1.7.16 A particle in an experiment has its position measured at various times, as given in he table. Find the quadratic function for the data points.

time (s)	Position (cm)
0	0.2
1	2.3
2	8.4
3	32.9
4	65.1
5	130.1
6	265.8

SOLUTIONS

SECTION 1.7

1.7.1 **(a)** $S = 0.241Wt + 0.00186$ **(b)** Yes, $r = 0.9999$

1.7.2 **(a)** $S = 0.315Wt - 0.00129$ **(b)** Yes, $r = 0.9999$

1.7.3 **(a)** $S = 2.685Wt - 0.0124$ **(b)** Yes, $r = 0.9999$

1.7.4 $h(t) = -16t^2 + 60t + 200$

1.7.5 $h(t) = -16t^2 + 30t + 20$

1.7.6 $h(t) = -16t^2 + 14.8t + 3.2$

1.7.7 $h(t) = -16t^2 + 18.7t + 19.4$

1.7.8 $y = 1.2t + 0.57$

1.7.9 $y = 8.363t - 0.0143$

1.7.10 $y = 10.196t + 20.039$

1.7.11 $y = 1.968t + 16.682$

1.7.12 $y = -2.025t + 20.161$

1.7.13 $y = 0.275(3.081)^t$

1.7.14 $y = 5.549t^2 - 14.039t + 5.255$

1.7.15 $y = 0.541(3.102)^t$

1.7.16 $y = 11.642t^2 - 30.239t + 11.490$

SECTION 1.8

1.8.1 A particle moves according to $x = 4t$, $y = t^2$. Find the position of the particle at $t = 2$.

1.8.2 A particle moves according to $x = \cos \pi t$, $y = t^2$. Find the position of the particle at $t = 2$.

1.8.3 Given $x = t + 2$, $y = 8t - 1$, eliminate the parameter t and write the equation in terms of x and y.

1.8.4 Given $x = t^2$, $y = t^3$, eliminate the parameter t and write the equation in terms of x and y.

1.8.5 Describe the graph of $x = 2 + \sin t$, $y = 3 + \cos t$, $0 \le t \le 2\pi$.

1.8.6 Describe the graph of $x = 5 \sin t$, $y = 2 \cos t$, $0 \le t \le 2\pi$.

1.8.7 Describe the graph of $x = 2 + 5 \cos t$, $y = 4 + 5 \sin t$, $0 \le t \le 2\pi$.

1.8.8 Describe the graph of $x = 4$, $y = t$.

1.8.9 Where is the ellipse $x = 4 + 3 \cos t$, $t = 2 + 8 \sin t$, $0 \le t \le 2\pi$ centered?

1.8.10 Where is the circle $x = 6 + 2 \cos t$, $y = 4 + 2 \sin t$, $0 \le t \le 2\pi$ centered?

1.8.11 Describe the graph of $x = 5 \sin t$, $y = \cos t$, $0 \le t \le \pi$.

1.8.12 A particle moves according to $x = t$, $y = t^2$. The shape of the trace of the particle, assuming t can be either positive or negative, is a parabola that opens in what direction?

1.8.13 The graph of $x = t + 2$, $y = 3 - t$, $2 \le t \le 5$ is

1.8.14 Represent $x = 2 + 3 \cos t$, $y = 4 + 3 \sin t$, $0 \le t \le 2\pi$ in rectilinear coordinates.

1.8.15 Where is the circle defined by $x = 8 + 9 \cos t$, $y = 2 + 9 \sin t$, $0 \le t \le 2\pi$ centered?

1.8.16 What is the radius of the circle defined by $x = 8 + 9 \cos t$, $y = 2 + 9 \sin t$, $0 \le t \le 2\pi$?

SOLUTIONS

SECTION 1.8

1.8.1 At $t = 2$, $x = 4(2) = 8$ and $y = 2^2 = 4$.
The particle will be at $(8, 4)$.

1.8.2 $x = \cos(2\pi) = 1$
$y = 2^2 = 4$
The particle will be at $(1, 4)$.

1.8.3 $x = t + 2$
$t = x - 2$
Substituting: $y = 8(x - 2) - 1$
$y = 8x - 16 - 1$
$y = 8x - 17$

1.8.4 $y = t^3$
$t = \sqrt[3]{y}$
Substituting: $x = (\sqrt[3]{y})^2 = y^{2/3}$
$x = y^{2/3}$

1.8.5 The graph is a circle with a radius of 1 and centered at $(2, 3)$.

1.8.6 The graph is an ellipse centered at $(0, 0)$ with x-intercepts $(-5, 0)$ and $(5, 0)$ and y-intercepts $(0, -2)$ and $(0, 2)$.

1.8.7 The graph is a circle centered at $(2, 4)$ with radius 5.

1.8.8 This is a vertical line at $x = 4$.

1.8.9 $(4, 2)$

1.8.10 $(6, 4)$

1.8.11 This is the right semi-circle of a circle centered at $(0, 0)$ with radius 5.

1.8.12 upward

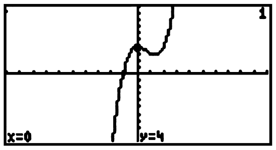
1.8.13 This is a line segment from P $(4, 1)$ to Q $(7, -2)$.

1.8.14 This is a circle of radius 3 centered at $(3, 4)$.
$(x - 2)^2 + (y - 4)^2 = 9$

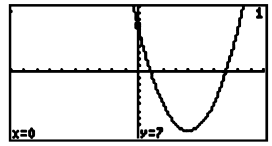
1.8.15 $(8, 2)$

1.8.16 $\sqrt{9} = 3$

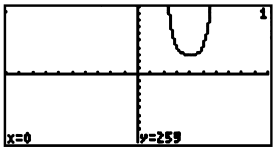
SUPPLEMENTARY EXERCISES, CHAPTER 1

1. 

For what value(s) of x is $y = 4$?

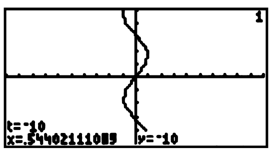
2. 

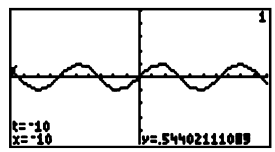
For what values of x is $y \leq 0$?

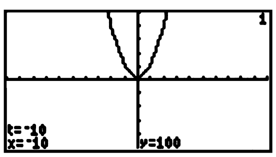
3. 

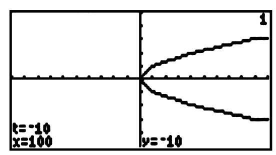
For what value of x does the graph have a minimum?

4. Find the natural domain for $f(x) = \sqrt{x^2 - 64}$.

5. A. 

B. 

C. 

D. 

In the accompanying figures, which show y as a function of x?

6. For a given temperature T and wind speed, v, the windchill index (WCI) is the equivalent temperature that exposed skin would feel with a wind speed of 4 mi/h. An empirical formula for the WCI (based on experience and observation) is

$$\text{WCI} = \begin{cases} T & 0 \leq v \leq 4 \\ 91.4 + (91.4 - 7)(0.0203v - 0.304\sqrt{v} - 0.474 & 4 < v < 45 \\ 1.6T - 55 & v \geq 45 \end{cases}$$

Find the actual temperature to the nearest degree if the WCI is reported as 30°F and the wind speed is 50 mi/h.

7. What is the smallest viewing window that shows the entire graph of $f(x) = -\sqrt{x^2 - 9}$?

In Exercises 8–12, find the natural domain of f and then evaluate f (if defined) at the given values of x.

8. $f(x) = \sqrt{4 - x^2}$; $x = -\sqrt{2}, 0, \sqrt{3}$

9. $f(x) = 1/\sqrt{(x-1)^3}$, $x = 0, 1, 2$

10. $f(x) = (x-1)/(x^2 + x - 2)$; $x = 0, 1, 2$

11. $f(x) = \sqrt{|x| - 2}$; $x = -3, 0, 2$

12. $f(x) = \begin{cases} x^2 - 1, & x \le 2 \\ \sqrt{x-1}, & x > 2 \end{cases}$; $x = 0, 2, 4$

In Exercises 13 and 14, find

(a) $f(x^2) - (f(x))^2$ **(b)** $f(x+3) - [f(x) + f(3)]$
(c) $f(1/x) - 1/f(x)$ **(d)** $(f \circ f)(x)$.

13. $f(x) = \sqrt{3-x}$

14. $f(x) = \dfrac{3-x}{x}$

In Exercises 15–22, sketch the graph of f and find its domain and range.

15. $f(x) = (x-2)^2$

16. $f(x) = -\pi$

17. $f(x) = |2 - 4x|$

18. $f(x) = \dfrac{x^2 - 4}{2x + 4}$

19. $f(x) = \sqrt{-2x}$

20. $f(x) = -\sqrt{3x+1}$

21. $f(x) = 2 - |x|$

22. $f(x) = \dfrac{2x - 4}{x^2 - 4}$

23. In each part, complete the square, and then find the range of f.
(a) $f(x) = x^2 - 5x + 6$ **(b)** $f(x) = -3x^2 + 12x - 7$

24. Express $f(x)$ as a composite function $(g \circ h)(x)$ in two different ways.
(a) $f(x) = x^6 + 3$ **(b)** $f(x) = \sqrt{x^2+1}$ **(c)** $f(x) = \sin(3x + 2)$

In Exercises 25–28, sketch the graph of the given equation.

25. $xy + 4 = 0$

26. $y = |x - 2|$

27. $y = \sqrt{4 - x^2}$

28. $y = x(x-2)$

29. Show that the point $(8, 1)$ is not on the line through the points $(-3, -2)$ and $(1, -1)$.

30. Where does the circle of radius 5 centered at the origin intersect the line of slope $-3/4$ through the origin?

31. Find the slope of the line whose angle of inclination is

(a) $30°$ **(b)** $120°$ **(c)** $90°$

In Exercises 32–34, find the slope-intercept form of the line satisfying the stated conditions.

32. The line through $(2, -3)$ and $(4, -3)$.

33. The line with x-intercept -2 and angle of inclination $\phi = 45^\circ$.

34. The line parallel to $x + 2y = 3$ that passes through the origin.

35. Find an equation of the perpendicular bisector of the line segment joining $A(-2, -3)$ and $B(1, 1)$.

36. Consider the triangle with vertices $A(5, 2)$, $B(1, -3)$, and $C(-3, 4)$. Find the point-slope form of the line containing

(a) the median from C to AB **(b)** the altitude from C to AB.

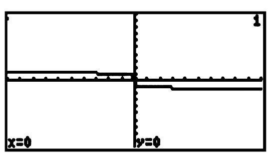
37. Use slopes to show that the points $(5, 6)$, $(-4, 3)$, $(-3, -2)$, and $(6, 1)$ are vertices of a parallelogram. Is it a rectangle?

38. For what value of k (if any) will the line $2x + ky = 3k$ satisfy the stated condition?

(a) Have slope 3 **(b)** Have y-intercept 3
(c) Be parallel to the x-axis **(d)** Pass through $(1, 2)$

39. Find an equation of a family of lines of lines with a slope of 4.

40.



This is the graph of $x^{-1/9}$, $x^{1/9}$, $-x^9$, or x^9?

41. The graph of $y = \dfrac{1}{x^2 + 2x + 1}$ is found by making appropriate transformations to the graph of what basic power function?

42. A particle moves according to $y = 3x^2 + 2x + 1$. Find the position y when the time x is 2.

43. A spring is stretched 2 mm by a 10-kg mass. How much will it be stretched by a 40-kg mass?

44. Sketch the graph of $x = t$, $y = t + 5$.

45. Find the radius and center of the circle $x = 4 + 2\sin t$, $y = -3 + 2\cos t$, $0 \le t \le 2\pi$.

46. Identify the curve that the parametric equation $x = 5 + 2\cos t$, $y = 2 + 3\sin t$ defines.

SOLUTIONS

SUPPLEMENTARY EXERCISES, CHAPTER 1

1. Reading from the graph: $x = 0$, $x = 2$.

2. Reading from the graph: $(1, 7)$.

3. Reading from the graph: 4.

4. $x^2 - 64 \geq 0$
$(x - 8)(x + 8) \geq 0$
Partitioning the x-axis at -8 and 8, and using test points, the answer is $(-\infty, -8] \cup [8, \infty)$. The endpoints are included.

5. A and D fail the vertical line test. The answer is B and C.

6. $v = 50$ mi/h implies WCI $= 1.6T - 55$.
$30 = 1.6T - 55$
$80 = 1.6T$
$T = 53°F$

7. $-3 \leq x \leq 3$ and $-3 \leq y \leq 0$

8. $\sqrt{4 - x^2}$ is real if and only if $4 - x^2 \geq 0$, thus $4 \geq x^2$, so the domain is $|x| \leq 2$; $f(-\sqrt{2}) = \sqrt{2}$, $f(0) = 2$, $f(\sqrt{3}) = 1$.

9. domain: $x > 1$; $f(0)$ and $f(1)$ are not defined, $f(2) = 1$.

10. $f(x) = \dfrac{(x-1)}{(x+2)(x-1)}$, domain: all x except -2 and 1; $f(0) = 1/2$, $f(1)$ is not defined, $f(2) = 1/4$.

11. domain: $|x| \geq 2$; $f(-3) = 1$, $f(0)$ is not a real number, $f(2) = 0$.

12. domain: all x; $f(0) = -1$, $f(2) = 3$, $f(4) = \sqrt{3}$.

13. **(a)** $f(x^2) - (f(x))^2 = \sqrt{3 - x^2} - (3 - x)$

(b) $f(x+3) - [f(x) + f(3)] = \sqrt{3 - (x+3)} - [\sqrt{3 - x} + \sqrt{3 - 3}] = \sqrt{-x} - \sqrt{3 - x}$

(c) $f(1/x) - 1/f(x) = \sqrt{3 - 1/x} - 1/\sqrt{3 - x}$

(d) $f(f(x)) = \sqrt{3 - \sqrt{3 - x}}$

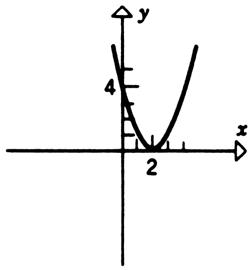
14. **(a)** $f(x^2) - (f(x))^2 = \dfrac{3 - x^2}{x^2} - \left(\dfrac{3 - x}{x}\right)^2 = \dfrac{3 - x^2}{x^2} - \dfrac{9 - 6x + x^2}{x^2} = \dfrac{-2x^2 + 6x - 6}{x^2}$

(b) $f(x+3) - [f(x) + f(x)] = \dfrac{3 - (x+3)}{x+3} - \left[\dfrac{3 - x}{x} + \dfrac{3 - 3}{3}\right] = -\dfrac{9}{x(x+3)}$

(c) $f(1/x) - 1/f(x) = \dfrac{3 - 1/x}{1/x} - \dfrac{x}{3 - x} = 3x - 1 - \dfrac{x}{3 - x} = \dfrac{3x^2 - 9x + 3}{x - 3}$

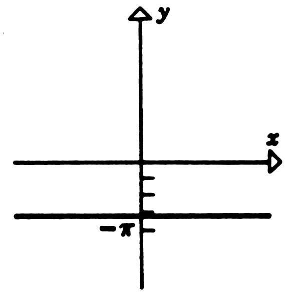
(d) $f(f(x)) = f\left(\dfrac{3 - x}{x}\right) = \dfrac{3 - \dfrac{3 - x}{x}}{\dfrac{3 - x}{x}} = \dfrac{4x - 3}{3 - x}$

15.



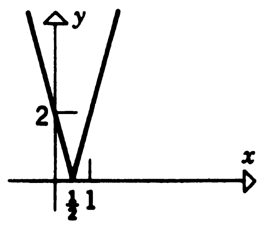
domain: all x
range: $y \geq 0$

16.



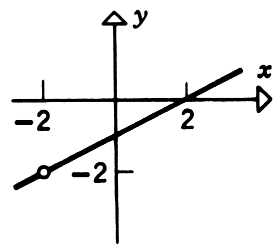
domain: all x
range: $y = -\pi$

17.



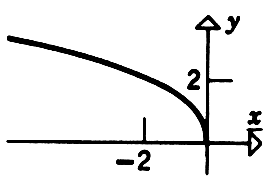
domain: all x
range: $y \geq 0$

18.



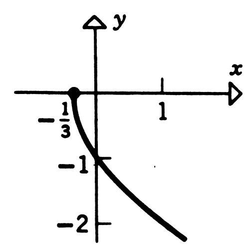
$$f(x) = \frac{x^2-4}{2x+4} = \frac{1}{2}(x-2)$$
$x \neq -2$
domain: all x except -2
range: all y except -2

19.



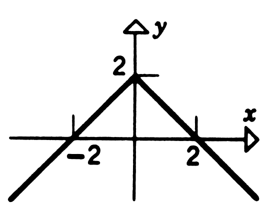
domain: $x \leq 0$
range: $y \geq 0$

20.



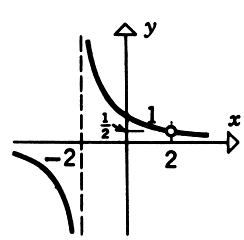
domain: $x \geq -1/3$
range: $y \leq 0$

21.



domain: all x
range: $y \geq 2$

22.



$$f(x) = \frac{2x-4}{x^2-4} = \frac{2}{x+2},\ x \neq 2$$
domain: all x except $-2, 2$
range: all y except 0, $1/2$

23. **(a)** $y = f(x) = \left(x^2 - 5x + \frac{25}{4}\right) + 6 - \frac{25}{4} = \left(x - \frac{5}{2}\right)^2 - \frac{1}{4}$; range: $y \geq -\frac{1}{4}$

(b) $y = f(x) = -3(x^2 - 4x + 4) - 7 + 12 = -3(x-2)^2 + 5$; range: $y \leq 5$

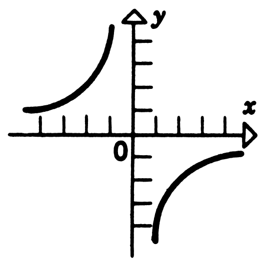
24. Some possible answers are:

(a) $h(x) = x^3$, $g(x) = x^2 + 3$; $h(x) = x^6$, $g(x) = x + 3$

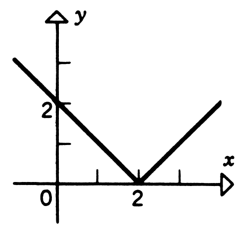
(b) $h(x) = x^2 + 1$, $g(x) = \sqrt{x}$; $h(x) = x^2$, $g(x) = \sqrt{x+1}$

(c) $h(x) = 3x + 2$, $g(x) = \sin x$; $h(x) = 3x$, $g(x) = \sin(x + 2)$

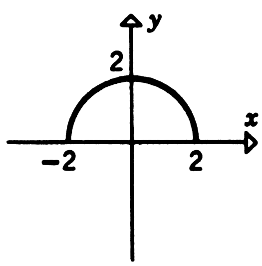
25.



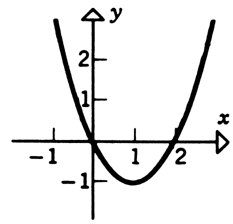
26. If $x \geq 2$, then $y = x - 2$;
if $x < 2$, then $y = -x + 2$.



27.



28.



29. $x - 4y = 5$ is an equation of the line through $(-3, -2)$ and $(1, -1)$, but $(8, 1)$ does not satisfy it.

30. Equation of circle is $x^2 + y^2 = 25$, equation of line is $y = -\frac{3}{4}x$.
Eliminate $y : x^2 + \left(-\frac{3}{4}x\right)^2 = 25$, $x^2 + \frac{9}{16}x^2 = 25$, $\frac{25}{16}x^2 = 25$, $x^2 = 16$, so $x = \pm 4$.
The points of intersection are $(-4, 3)$ and $(4, -3)$.

31. **(a)** $\tan 30° = 1/\sqrt{3}$ **(b)** $\tan 120° = -\sqrt{3}$ **(c)** $\tan 90°$ is not defined

32. $m = \dfrac{-3+3}{4-2} = 0$, so $y = -3$

33. $m = \tan 45° = 1$, and $(-2, 0)$ is on the line, so $y - 0 = (1)(x + 2)$, $y = x + 2$.

34. For $x + 2y = 3$, $m = -\frac{1}{2}$. A parallel line through the origin is $y - 0 = -\frac{1}{2}(x - 0)$, $y = -\frac{1}{2}x$.

35. The line segment joining $A(-2,-3)$ and $B(1,1)$ has slope $m = \dfrac{4}{3}$ and midpoint $M\left(-\dfrac{1}{2}, -1\right)$. The perpendicular bisector has slope $-\dfrac{3}{4}$ and goes through M, so $y + 1 = -\dfrac{3}{4}\left(x + \dfrac{1}{2}\right)$, $y = -\dfrac{3}{4}x - \dfrac{11}{8}$.

36. **(a)** The median from C to AB is the line segment joining C and the midpoint of AB. The midpoint of AB is $M(3, -1/2)$, thus the slope of the line through C and M is $-3/4$, so $y - 4 = (-3/4)(x+3)$.

(b) The altitude to AB is perpendicular to AB. The slope of AB is $5/4$, thus the slope of the line perpendicular to AB is $-4/5$, so $y - 4 = (-4/5)(x+3)$.

37. Label the points as $A(5,6)$, $B(-4,3)$, $C(-3,-2)$, and $D(6,1)$. Then $m_{AB} = 1/3$, $m_{BC} = -5$, $m_{CD} = -1/3$, and $m_{DA} = -5$, so $ABCD$ is a parallelogram because opposite sides are parallel $(m_{AB} = m_{CD}, m_{BC} = m_{DA})$. It is not a rectangle because sides AB and BC do not form a right angle $(m_{AB} \neq -1/m_{BC})$.

38. **(a)** $y = -2x/k + 3$, if $k \neq 0$; $m = -2/k = 3$ if $k = -2/3$

(b) $k \neq 0$ (if $k = 0$, then the line coincides with the y-axis and does not have a unique y-intercept).

(c) $-2/k = 0$ is impossible for any real value of k.

(d) $(1,2)$ must satisfy $2x + ky = 3k$, so $2(1) + k(2) = 3k$ which gives $k = 2$.

39. $y = 4x + b$

40. $yu \ - x^{1/9}$

41. $y = \dfrac{1}{(x+1)^2}$

So, $y = x^{-2}$

42. $y = 3(2)^2 + 2(2) + 1 = 3(4) + 4 + 1 = 17$

43. The mass is directly proportional to the amount the spring stretches.

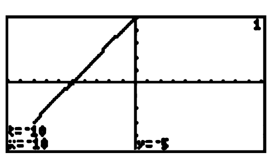
$10 = 2k$

$k = 5$

$40 = 5x$

$x = 8$ mm

44.



45. Center: $(4, -3)$; Radius 2

46. This is an ellipse centered at $(5,2)$ with a vertical major axis that extends 3 units above and below the center and a horizontal minor axis that extends 2 units left and right of the center.