Exercises for Applied Analysis; Part 4

Assignment 4; for 18th of January

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1. For the finite discrete Fourier transform on $\mathbb{C}^N$, show that $\widehat{\text{III}_m}[k] = \frac{1}{m}\text{III}_{N/m}[k]$.

2. Show that, if $f \in \mathcal{S}(\mathbb{R})$, then $\hat{f} \in \mathcal{S}(\mathbb{R})$.

3. Consider ξ as a tempered distribution, that is, by acting on $\varphi \in \mathcal{S}(\mathbb{R})$ by $\int_\xi \xi \varphi(\xi) d\xi$. Show that $\hat{\xi} = i \cdot \delta'$.

4. Now understand III as a tempered distribution and show that $\widehat{\text{III}} = \text{III}$. If you are courageous, consider the general case of $\text{III}_T = \sum_{n\in\mathbb{Z}} \delta_{nT}$.

5. Check the details of Lemma 4.2.5 in the lecture notes (Fourier transform of $sinc$) and compute the (inverse) Fourier transform of $1_{[-\frac{a}{2},\frac{a}{2}]}$ ($d = 1$).