Applied Analysis, WS 2021 – Problem Sheet 2

P. 4 Consider a quadratic equation, with 2 ways to perturb it by a small term with $\varepsilon = 0.001$:

(1) $\quad x^2 + 2\varepsilon x - 1 = 0$

(2) $\quad \varepsilon x^2 + 2x - 1 = 0$

(a) Which of the perturbations is regular, which singular ?

(b) For a regular case, compute an asymptotic expansion up to $O(\varepsilon^2)$.
How many digits are correct in second order, how many in first order ?

P. 5 Classify the following initial value problems into regularly and singularly perturbed problems and, if applicable, calculate asymptotic expansions up to second order:

(a) $\varepsilon y' + y = x, \qquad y(0) = 1$

(b) $\varepsilon y' + y = x, \qquad y(0) = 0$

(c) $\varepsilon y' + y = x, \qquad y(0) = \varepsilon$

(d) $\varepsilon^2 y' + y = x, \qquad y(0) = \varepsilon$

(e) $y' + \varepsilon y = x, \qquad y(0) = 1$

(f) $y' + y = \varepsilon x, \qquad y(0) = 1$

P. 6 Calculate the first three terms of the asymptotic expansion of the solution of

$$\begin{aligned} &y' = -y + \varepsilon y^2, \qquad t > 0, \quad 0 < \varepsilon \ll 1, \\ &y(0) = 1 \end{aligned}$$

in two ways:

(a) as in the lecture (inserting the expansion ansatz into the ODE)

(b) by expanding the exact solution in a power series in ε. (Hint for the exact solution: use the substitution $z = 1/y$.)

P. 7 Take the following initial boundary value of a PDE with a small perturbation ($0 < \varepsilon \ll 1$):

$$\begin{aligned} &\tfrac{\partial}{\partial t} u(x,t) + \tfrac{\partial^2}{\partial x^2} u(x,t) - \varepsilon u(x,t)^2 = 0, && x \in (0,1),\ t > 0, \\ &u(x,0) = u_0(x), && x \in (0,1), \\ &u(0,t) = u(1,t) = 0, && t > 0 \end{aligned}$$

(a) Why to you expect this problem to be ”regular” ?

(b) Find the equations for the first three terms of the formal asymptotic expansion of the solution of this problem.

(c) Calculate the reduced solution for $u_0(x) = \sin(\pi x)$. (Hint: separation)