University of Vienna

Seminar:
Joint RICAM Seminar

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Contents

1 Intro

The following questions are anwsered:

- iterative regularization with NN functions
- application of NNs on inverse problems
- What generalized NNs are best suited for IPs?

1.1 Posing the problem

Consider linear operator equation between Hilbertspaces $\mathbf{X}$ and $\mathbf{Y}$

$$F\mathbf{x} = \mathbf{y}. \tag{1}$$

For the problem modeling we introduce a function, called **Coding** $\Psi : \vec{P} \to \mathbf{X}$ which maps NN parameters to images functions. Our problem can be written as follows

$$N(\vec{p}) = F\Psi(\vec{p}) = \mathbf{y}, \tag{2}$$

where X is the image space, Y the data space and $\vec{P}$ the parameter space. In the case the operator in question F is nonlinear then we would of course have a nonlinear equation, which we are not considering right now.

1.2 Shallow neural network coders

Shallow neural network coders are of the following form

$$\Psi : \mathcal{D}(\Psi) := \mathbb{R}^{n_*} = \mathbb{R}^N \times \mathbb{R}^{n\times N} \times \mathbb{R}^N \to \mathbf{X} := L^2\left([0,1]^n\right), \tag{3}$$

$$\vec{p} = (\vec{\alpha}, \mathbf{w}, \vec{\theta}) \mapsto \left(\vec{x} \to \sum_{j=1}^{N} \alpha_j \sigma\left(\vec{\mathbf{w}}_j^T \vec{x} + \omega_j\right)\right), \tag{4}$$

where σ is an activation function, such as tanh or sigmoid.

2 Solution

The solution ivolves either recostructing the function or the coefficient use Tikhonov regularization(TODO: Tikhonov regularization introduction!) or use newton type methods.