Exercises for Applied Analysis; Part 3

Assignment 3; for 11th of January

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1. If $f(x)$ has Fourier transform $\hat{f}(w)$, what is then the Fourier transform of $\sin(x)f(3x-1)$?

2. Show that the Fourier transform of $f(x) = e^{-|x|}$ is $\hat{f}(w) = \frac{1}{(2\pi w)^2+1}$ and use this result to compute, for some $a \in \mathbb{R}$:
$$\int_{-\infty}^{\infty} \frac{\cos(aw)}{(2\pi w)^2+1} dw \,.$$

3. Consider the vector $s \in \mathbb{C}^N$, whose entries are given by the samples of a sine function of frequency ω_0, that is: $s[n] = \sin(2\pi\omega_0 n/N)$, for some $0 < \omega_0 \leq N$. Compute the (finite, discrete) Fourier transform of s.

4. A circulant matrix is a square matrix in which all row vectors are composed of the same elements and each row vector is rotated one element to the right relative to the preceding row vector. Show that the convolution of a vector $f \in \mathbb{C}^N$ with another, fixed, vector $g \in \mathbb{C}^N$, can be expressed by a circulant matrix applied to f. Determine, for $g = s$, with the vector s from the previous exercise, the matrix entries.
Then, by considering an impulse input f, that is, a vector that is 1 for $n = 1$ and 0 else, show that the impulse response of this linear time-invariant system is given by s.