Exercises for Applied Analysis; Part 2

Assignment 2; for 13th of December

Monika Dörfler

1. Prove linearity of the Fourier transform on $L^1(\mathbb{R}^d)$.

2. Prove the properties stated in Examples 3.1.9 and 3.1.10; that is:

   (a) For any real number x_0, if $g(x) = T_{x_0} f(x) := f(x - x_0)$, then $\hat{g}(\omega) = e^{-2\pi i x_0 \omega} \hat{f}(\omega)$.

   (b) For any real number ω_0, if $g(x) = M_{\omega_0} := e^{2\pi i x \omega_0} f(x)$, then $\hat{g}(\omega) = \hat{f}(\omega - \omega_0)$.

   (c) Let $a \neq 0 \in \mathbb{R}$. Set $g(x) = D_{\frac{1}{a}} f(x) := f(ax)$. Let $\hat{f}$ be the Fourier transform of f. Then, the Fourier transform of g is given by

$$\hat{g}(\omega) = \frac{1}{a} \hat{f}(\frac{\omega}{a}).$$

3. In Example 3.1.8, the Fourier transform of the Box-function was computed. Use the previous exercises to compute the Fourier transform of the following function:

$$\Pi(x) := \begin{cases} 1 & \text{for } -\frac{3}{2} < x < \frac{1}{2} \\ 0 & \text{else} \end{cases}$$

4. This is voluntary, and should be considered playful... :
   Find or record a sound, read it into your preferred Software (Matlab, Octave, Python). Compute convolutions with various dilated, shifted, modulated Sinc-functions. Can it be done in various ways? Listen to and interpret the results.