Exercises for Applied Analysis; Part 2

Assignment 1; for 6th of December

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1. Compute the Fourier series of $f(t) = t$ on the interval $[-\frac{1}{2}, \frac{1}{2}]$.

2. Describe the truncation error, if only 3 (9) coefficients are used in the previous example, i.e. S_3 oder S_9. How many coefficients are needed, such that $\|E_N\|_2^2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} |f(t) - S_N(t)|^2 dt$ is less than 1% of the function's energy $\|f\|_2^2$?

3. (a) Please recall (or google) the most important properties of orthonormal bases, their relation to projections and minimal distance in Eucledian spaces. Formulate what you found for the special case of orthonormal basis of complex exponentials $\mathcal{T}_d$.
   (b) Prove the orthonormality of $\mathcal{T}_d$.

4. Prove Proposition 2.2.4.