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ECGR 2254
2/27/2022

Project 1 Report

Problem 1:

a.

To find the current I_s after the switch closed, first had to convert the inductance to impedance.

$L = 4.5mH$

$\omega = 2\pi 60\, rad/s$

$Z_L = j\omega L = j1.69\Omega$

After switch closes:

$Z_{eq} = Z_L + Z_R = 0.1 + j1.69\Omega = 1.69e^{j86.6}\Omega$

Apply Euler's identity:

$V_s(t) = 480\sqrt{2}\cos(2\pi 60t - \Phi)$ V $= 480\sqrt{2}e^{j2\pi 60}e^{-j\Phi}$

$I_s(0\text{-}) = \frac{V_s(0^-)}{Z_{eq}} = 400e^{j\Phi}$A

$\rightarrow$A = 400

b.

L = 4.5 mH

R = 0.1Ω

$V_S = 480\sqrt{2}\cos(2\pi 60t - \Phi)$

Apply KCL:

$V_S - V_R - V_L = 0$

$V_R = I * R$

$V_L = L\frac{di}{dt}$

$\rightarrow L\frac{di}{dt} + IR = V_S$ divide both sides by L

$\frac{di}{dt} + \frac{R}{L}I = \frac{V_S}{L}$

c.

Transient:

$i(t) = ce^{st}, s = -22.22$

$\tau = \frac{-1}{s} = \frac{1}{22.2}$

Steady State:

$\omega = 2\pi 60$

$T = \frac{2\pi}{\omega} = \frac{1}{60}$

$\Delta t << \frac{T}{100}$ and $\Delta t << \frac{5\tau}{100}$

$\frac{T}{100} < \frac{5\tau}{100}$

$\rightarrow \Delta t = \frac{T}{100} = \frac{1}{6000}$

d.

```
clear all
y_0 = 0;
tau = 0.045;
T = 1/60;
delta_t = T/100;
A = 6788; %starting voltage divided by resistance of 0.1
omega = 2 * pi * 60;
phi = 87.5 * pi/180;
a = tau/delta_t;

t = [0:delta_t:1];
y = zeros(size(t));
x = A* cos(omega * t - phi);
%x = A * exp(j*omega*t)*exp(-j*phi);
for n = 1:1:length(t)-1
   if n == 1
      y(1) = y_0;
      y(n+1) = ((x(n))-y(n)+a*y(n))/a;
   else
      y(n+1) = ((x(n))-y(n)+a*y(n))/a;
   end
end
figure(1);
plot(t,y);
```

To find the value of Φ that gave the largest value after t = 0, I started by using generic values such as 0, 45, 90, 135, and 180. Using these values, I found that Φ = 90 gave the largest value, so I tried to break it down and use numbers close to 90 to see if I could find the Φ value that gave the absolute largest value after t = 0. Doing so, I found that Φ = 87.5 gave the value of 733.737, which was the largest value after t = 0.

e.

L = 4.5 mH

$R_s = 0.1\Omega$

$R_s = 23.04\Omega$

$V_s = 480\sqrt{2}e^{j2\pi 60}e^{j\Phi}$V

$Z_L = j\omega L = j1.69$

$Z \;=\; 23.14 + j1.69\Omega = 23.2e^{j4.18}$

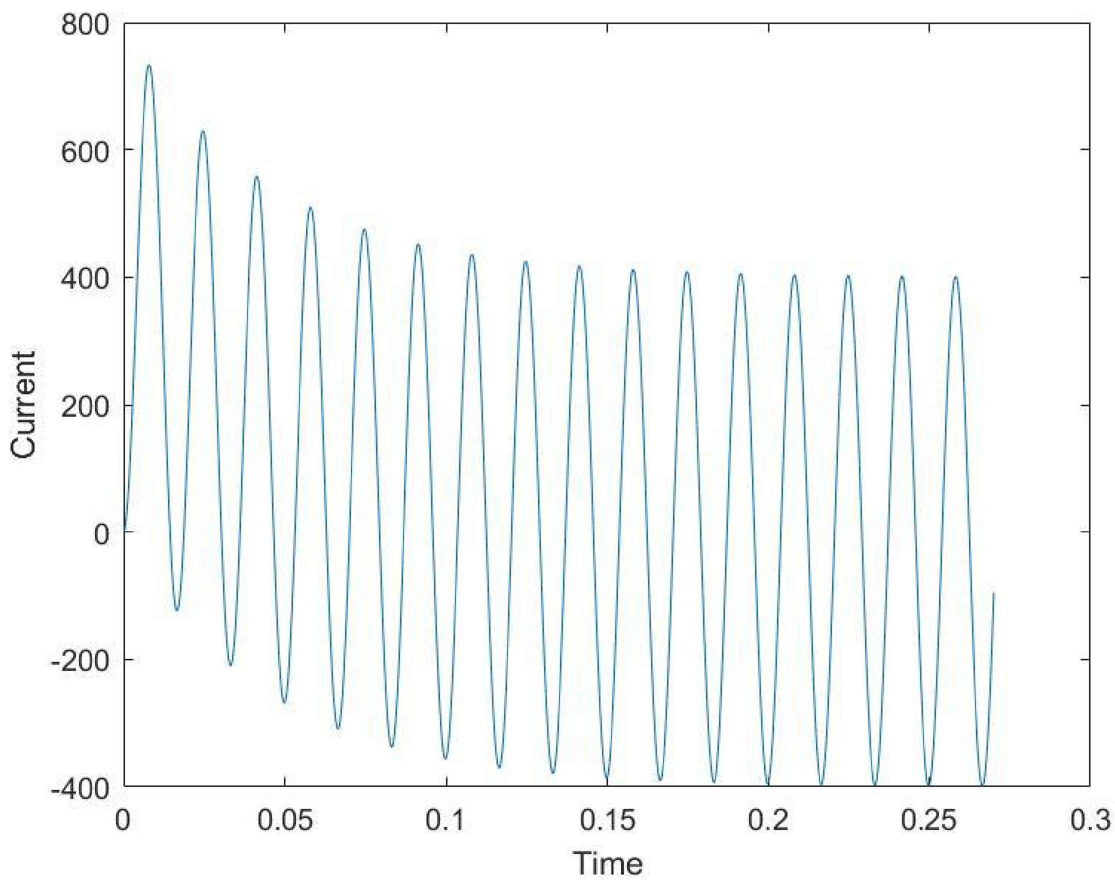
$I_s(0\text{-}) = \frac{V_s}{Z} \;=\; 29.26e^{j\Phi} = 29.29\cos(2\pi 60t - \Phi)$ A

Plug in t = 0 and Φ = 87.5

29.26cos(- 87.5) = 1.276 A

f.

After plugging in y_0 = 1.276A, the current peaked at 734.805A.



Problem 2:

a.

To find the current Is after the switch opened, first had to convert the inductance to impedance.

$L = 2.65mH$

$\omega = 2\pi 60\ rad/s$

$Z_L = j\omega L = j0.999\Omega$

$R_{eq} = 144\Omega||7.2\Omega = 6.86\Omega = Z_R$

$Z_{eq} = Z_L + Z_R = 6.86 + j0.999\Omega = 6.93e^{j8.28°}\Omega$

Apply Euler's identity to $V_s(t)$:

$V_s(t) = 120\sqrt{2}\cos(2\pi 60t) = 120\sqrt{2}e^{j2\pi 60t}$

$i_s(t) = \frac{V_s(t)}{Z_{eq}} = 24.48e^{j2\pi 60t}e^{-j8.28°}\text{A} = 24.48\cos(2\pi 60\text{t}-8.28°)\text{A}$

$\rightarrow i_s(0^-)=24.48cos(2\pi 60(0) - 8.28°)A = 24.22A$

b.

L = 2.65 mH

R = 144Ω

$V_S = 120\sqrt{2}\cos(2\pi 60t)$

Apply KCL:

$V_S - V_R - V_L = 0$

$V_R = I * R$

$V_L = L\frac{di}{dt}$

$\rightarrow L\frac{di}{dt} + IR = V_S$ divide both sides by L

$\frac{di}{dt} + \frac{R}{L}I = \frac{V_S}{L}$

C.

Transient:

$i(t) = ce^{\frac{-t}{\tau}}$

C = 24.22

$\tau = \frac{L}{R} = 1.84e - 5$

Steady State:

$\omega = 2\pi 60$

$T = \frac{2\pi}{\omega} = \frac{1}{60}$

$\Delta t << T\ and\ \Delta t << \tau$

Check $\frac{T}{100}$ and $\frac{5\tau}{100}$

$\frac{T}{100} > \frac{5\tau}{100}$

$\rightarrow \Delta t = \frac{5\tau}{100} = 9.2 * 10^{-7}$

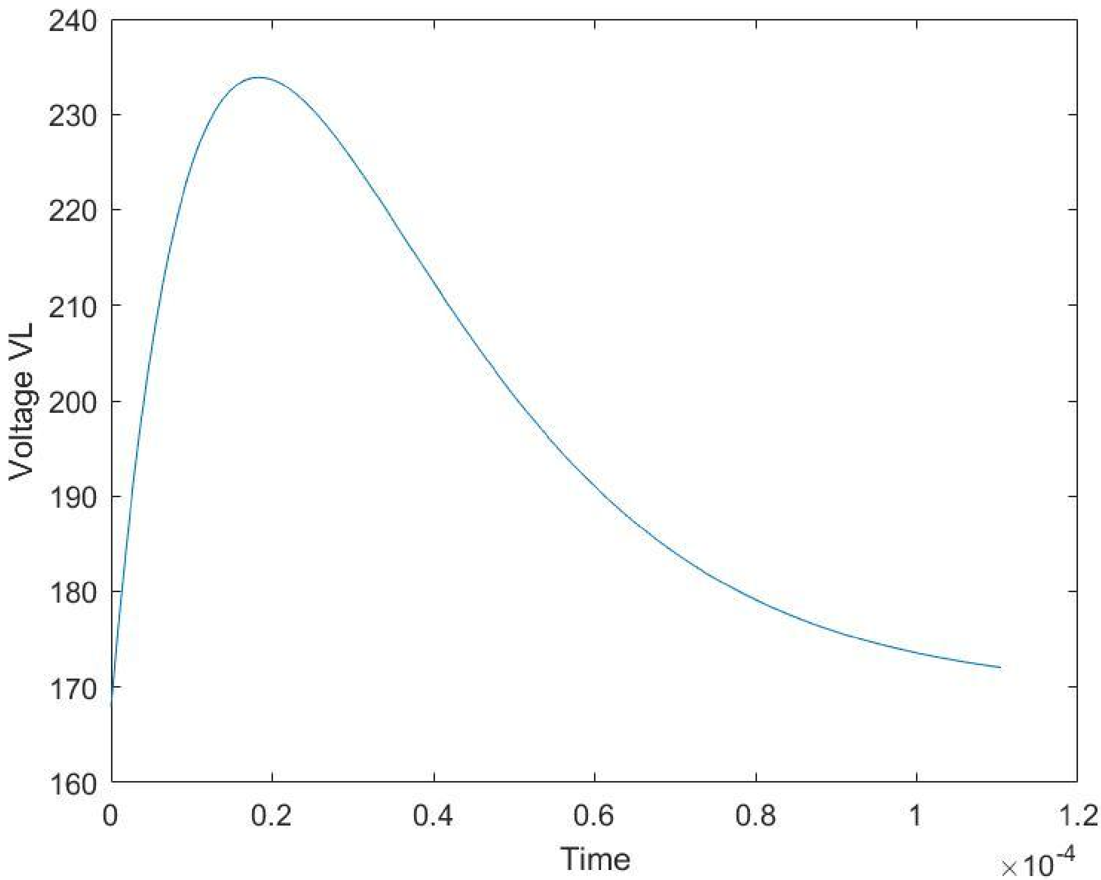
d.

```
clear all
y_0 = 167.9;%VL(0-)
abs(y_0);
angle(y_0)*180/pi;
R = 144;%ohms
L = 0.00265;%mH
tau = L/R;
T = 1/60;
delta_t = 5*tau/100;% < T/100
phi = 0;
A = 1.1785;%mag of i for t>0
omega = 2 * pi * 60;
a = tau/delta_t;%used to discretize diff eq
Z = 144 + j*0.999;%impedence after switch opens

%particular sol
t = [0:delta_t:6*tau];
yp = zeros(size(t));
x = R*  (A* cos(omega * t - phi*pi/180)+1.1785*exp(-t/tau));%multiply by R to get VL
for n = 1:1:length(t)-1
   if n == 1
      yp(1) = y_0;
      yp(n+1) = ((x(n))-yp(n)+a*yp(n))/a;
   else
      yp(n+1) = ((x(n))-yp(n)+a*yp(n))/a;
   end
end
figure(1);
plot(t,yp);
xlabel('Time');
ylabel('Voltage VL');
```

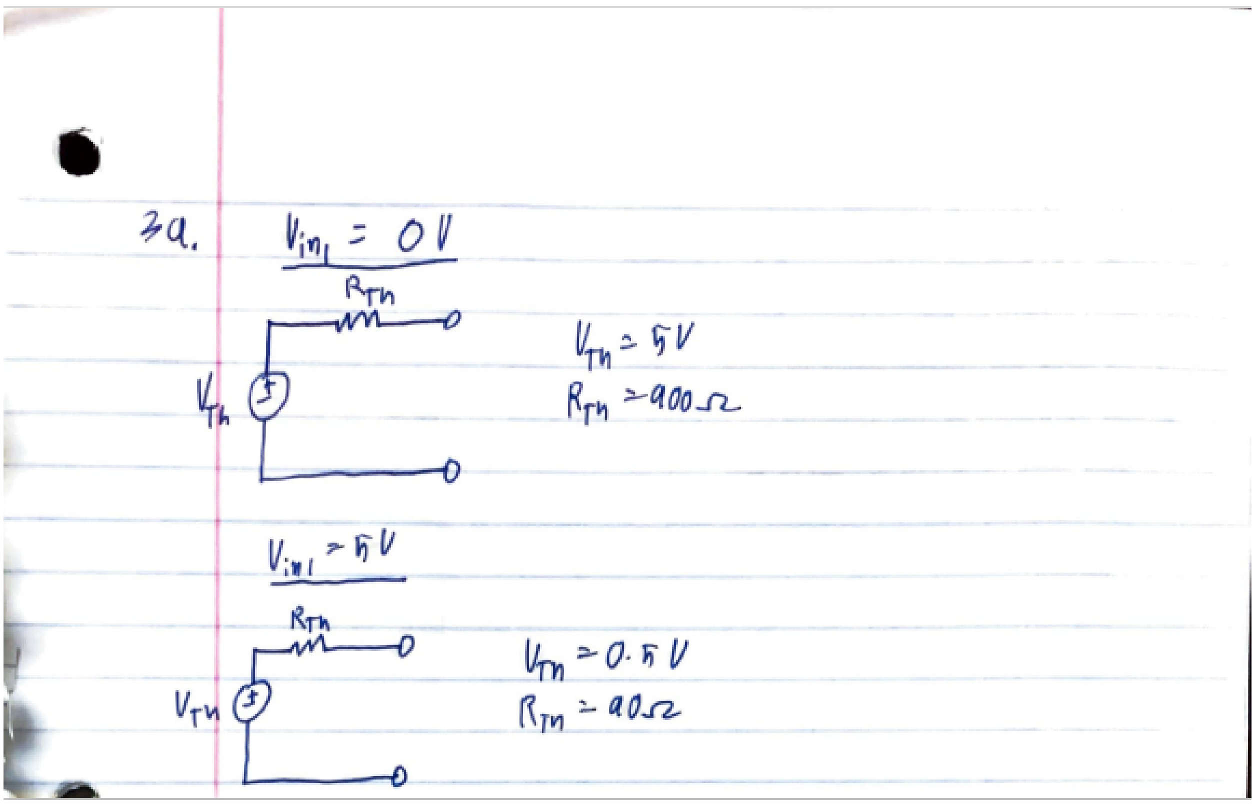
e.

The maximum value of the voltage was approximately 233.8 volts.



Problem 3:

A.



b.

- V_{th} = 5V.

  Since the circuit is an open circuit, there is no current flowing which mean the voltage on the node for V_{th} would remain 5.

- $R_{th} = 900\Omega$

Since there is no current, the two resistors are not connected in series or parallel so R_{th} would be the value of the load resistor.

- $V_{out1} \cong 5V$

  Assuming the circuit has been in its state for a long time, most of the voltage would be absorbed in the capacitor and the inductor, which is what V_{out1} consists of.

- $V_{in2} \cong 5V$

  The same value as V_{out1}. It is safe to assume most of the voltage was absorbed in the capacitor which gives the voltage for V_{in2}.

- $I_L \cong 0A$

  If the circuit has been in the same state for a long time, most of the voltage would have been absorbed in the capacitor which would slow down the flow of the current.

c.

$L = 100nH$

$C_{GS2} = 0.1pF$

$R_L = 900\Omega$

$R_{on} = 100\Omega$

Apply KCL:

$V_{th} - V_R - V_L - V_c = 0$

$\rightarrow V_R + V_L + V_C = V_{th}$

$V_R = i(t) * R$

$V_L = L * \frac{di(t)}{dt}$

$V_c = \frac{1}{C}\int i(t)dt$

$\rightarrow i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = V_{th}$

$Sub\ i(t) = C\frac{dv_c(t)}{dt}$

$C\frac{dv_c(t)}{dt}R + L\frac{dC\frac{dv_c(t)}{dt}}{dt} + \frac{1}{C}\int C\frac{dv_c(t)}{dt}dt = V_{th}$

$\rightarrow LC\frac{d^2v_c(t)}{d^2t} + RC\frac{dv_c(t)}{dt} + V_c(t) = V_{th}$

Divide by LC

$\rightarrow \frac{d^2v_c(t)}{d^2t} + \frac{R}{L}\frac{dv_c(t)}{dt} + \frac{1}{LC}V_c(t) = \frac{V_{th}}{LC}$

- $V_{in2}(0) \cong 0V$

  The value of V_{in1} jumping to 5V makes V_{in2} and V_{out1} approximately equal to 0 .

- $\frac{dv_{in}(0)}{dt} \cong \infty$
  The value of V_{in1} instantaneously jumped to 5V at t = 0, which means the rate of change was nearly infinite.
- V_{th} = 0.5V
  Since the switch closed, current would be flowing and 4.5v would be absorbed by the 900 Ω resistor, leaving 0.5V.
- $R_{th} = 90\Omega$
  The switch closing creates a parallel relationship between the two capacitors, which when combined yield an equivalent resistance of 90Ω.

d.

Transient

Assume v(t) = ce^{st}

$v'(t) = sce^{st}$

$v''(t) = s^2ce^{st}$

$s^2ce^{st} + \frac{R}{L}sce^{st} + \frac{1}{LC}ce^{st} = 0$

$s = (-4.5 \pm j * 8.93) * 10^9$

$\tau = \frac{1}{4.5*10^9} = 2.2 * 10^{-10}$

$\omega_d = 8.93 * 10^9$

$T = \frac{2\pi}{\omega_d} = 7.03 * 10^{-10}$

$\Delta t << T \; and \; \Delta t << 5\tau$

$\frac{T}{100} < \frac{5\tau}{100}$

Therefore, $\Delta t = \frac{T}{100} = 7.04 * 10^{-12}$

e.

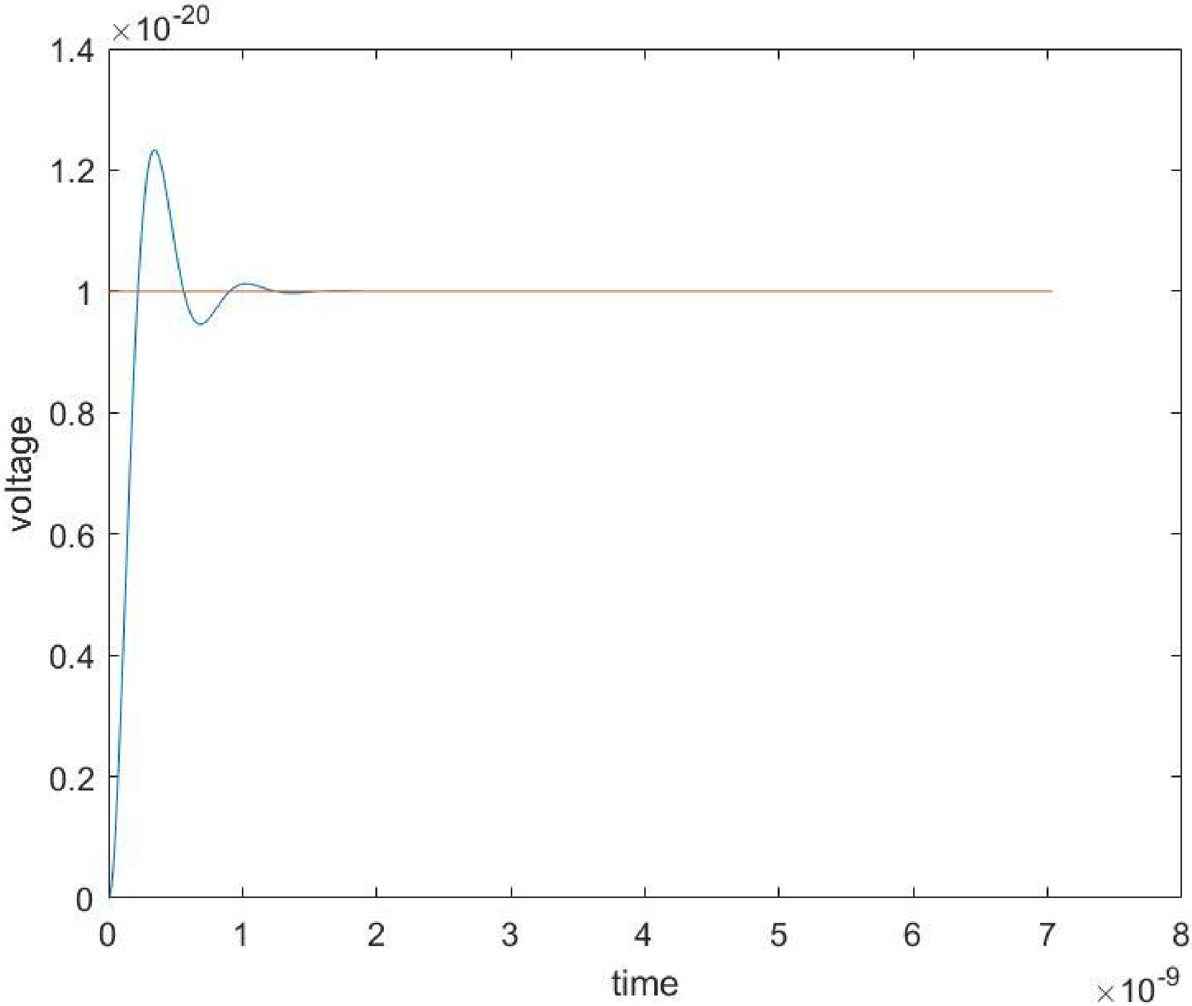
```
clear all
%values of circuit elements
R = 900;
C = 0.1 * 10^(-12);
L = 100 * 10^(-9);
%initial voltage
y_0 = 0;
y_prime_0 = 0;
%variables for diff eq
b = R/L;
c = 1/(L*C);
s = roots([1 b c]);
tau = -1/real(s(1))%time constant
omega_d = imag(s(1))%damping coeff
T_d = 2*pi/omega_d;%period
delta_t = T_d/100;%delta t to discretize

t = [0:delta_t:10*T_d];
y1 = zeros(size(t));
y2 = zeros(size(t));
x = ones(size(t));

for n = 1:1:length(t)-1
    if n == 1
        y1(1) = y_0;
        y2(1) = y_prime_0;
    end
    y1(n+1) = y1(n) + delta_t*y2(n);
    y2(n+1) = delta_t*(x(n) - b*y2(n) -c*y1(n))+y2(n);
end
figure(1);
plot(t,y1,t,(1/c)*ones(size(t)));
xlabel('time')
ylabel('voltage')
```

f.

It took v(t) until time t≅$2.6*10^{-9}$s to become reliably less than 1V.



Problem 4:

a.

$x_{out}(t) \ = \ x(t)cos(2\boldsymbol{\pi}ft)$

$x(t) \ = \ x_{audio}(t)cos(2\boldsymbol{\pi}ft)$

Sub x(t) into $x_{out}(t)$

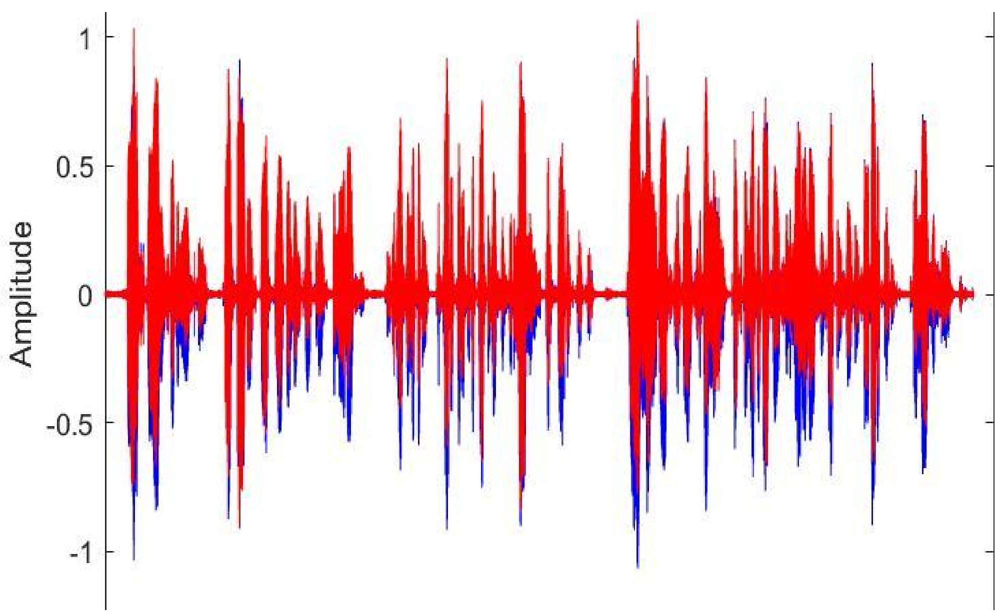
$x_{out}(t) \ = x_{audio}(t)cos^2(2\boldsymbol{\pi}ft)$

Use trig identity: $cos^2(x) \ = \ \frac{1}{2}(1 \ + \ cos(2x)$

$\rightarrow x_{out}(t) \ = \frac{1}{2}(x_{audio}(t)(1 \ + \ cos(4\boldsymbol{\pi}ft)))$

b.

Zooming in on the graph, it was clear that x_out had a higher frequency. This makes sense, as in part a, x_out was in terms of $cos(4\boldsymbol{\pi}ft)$, which would effectively double the frequency compared to x, which was in terms of $cos(2\boldsymbol{\pi}ft)$**.**



x_out-red
x-blue

c.

Start by using KVL on left hand loop

$x_{out}(t) \; = \; V_L + V_C$

Sub $V_L \; = \; L\frac{di_L(t)}{dt}$

Apply KCL at node

$i_c = i_L - i_R$

Sub $i_R(t) \; = \; \frac{V_C(t)}{R}$

Sub $i_C \; = \; C\frac{dV_C(t)}{dt}$

Solve for i_L

$i_L \; = C\frac{dV_C(t)}{dt} + \frac{V_C(t)}{R}$

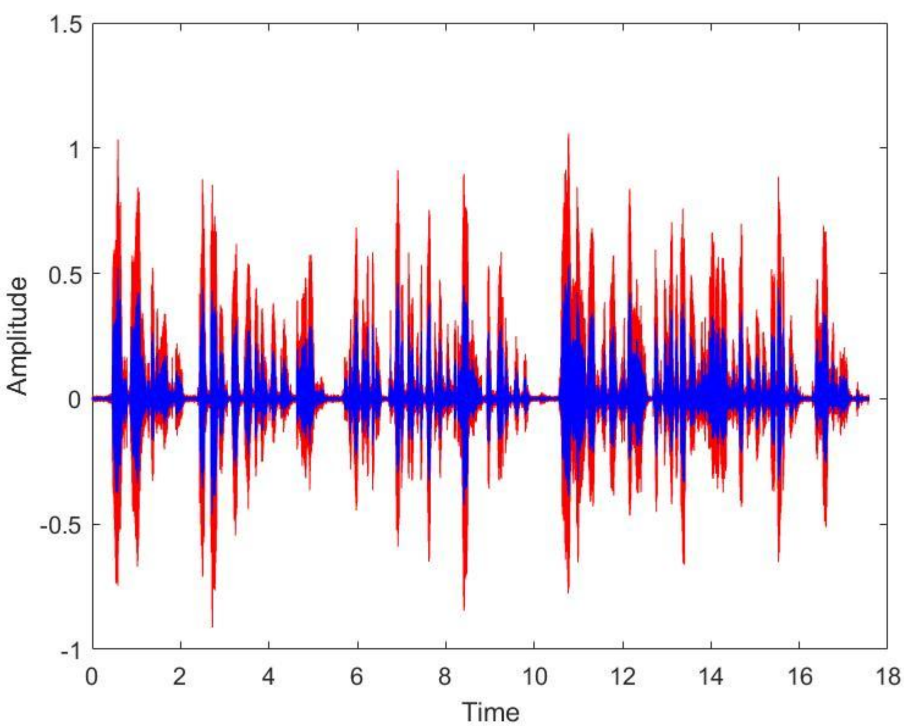
Sub i_L *into KVL equation*

$x_{out}(t) \; = L\frac{d}{dt}(C\frac{dV_C(t)}{dt} + \frac{V_C(t)}{R}) \; + V_C$

$x_{out}(t) \; = LC\frac{d^2V_C(t)}{d^2t} + \frac{L}{R}\frac{dV_C(t)}{dt} + V_C$

Divide by LC

$\frac{x_{out}(t)}{LC} \; = \frac{d^2V_C(t)}{d^2t} + \frac{1}{RC}\frac{dV_C(t)}{dt} + \frac{1}{LC}V_C(t)$



X_out-red
y-blue

d.

```
clear all

%define variables
fileID = fopen('problem4.bin','r');
x = fread(fileID, 'single');
fclose(fileID);
f = 50*10^3;%frequency
f_samp = 2.205 * 10^(6);%frequency of sample
delta_t = 1/f_samp;
endpt = (size(x)-1) * delta_t;%end of time vector

%create time vector and x_out
t = [0:delta_t:endpt];
t = transpose(t);%t same dim as x and demodulator
demod = cos(2*pi*f*t);
x_out = x .* demod;%V source for low pass filter

%plots for x and x_out
plot(t,x,'b')
hold on
plot(t,x_out,'r')
hold off

%values for low pass filter
L = 253.3*10^(-6);%henry
C = 1*10^(-6);%farad
R = 11.254;%ohms
b = 1/(R*C);
c = 1/(L*C);
```

```
y_0 = 0;
y_prime_0 = 0;
y1 = zeros(size(t));
y2 = zeros(size(t));
%x = ones(size(t));

%discretized diff eq for low pass filter
for n = 1:1:length(t)-1
    if n == 1
        y1(1) = y_0;
        y2(1) = y_prime_0;
    end
    y1(n+1) = y1(n) + delta_t*y2(n);
    y2(n+1) = delta_t*(c*x_out(n) - b*y2(n) -c*y1(n))+y2(n);
end

%plot comparing y(t) to x_out
figure(1);
plot(t,x_out,'r')
hold on
plot(t,y1, 'b');
hold off

%used to play sample
x_audio = downsample(y1,100);
%sound(x_audio,22.05e3);
%sound(downsample(x_out,100),22.05e3);
```

The downsampled version of x_out sounded a lot less clear and had somewhat of a high pitched echo present. This is because x_out did not go through the low pass filter, which allowed the higher frequencies to be played which made it sound less clear and more high pitched. When tampering with the frequency by even the slightest bit, it made the audio sound completely distorted and unlegible.

Problem 5:

a.

We needed an $x_3(t)$ that when multiplied by $x(t)$, left $x_{aud}(t)$ alone.

$x(t) = x_{aud}(t)e^{j2\boldsymbol{\pi}ft}$

Need $x_{out}(t) = x_{aud}(t)$

$x_{out}(t) = x_3(t)x(t) = x_3(t)x_{aud}(t)e^{j2\boldsymbol{\pi}ft}$

$x_{aud}(t) = x_3(t)x_{aud}(t)e^{j2\boldsymbol{\pi}ft}$

$x_3(t) = e^{-j2\boldsymbol{\pi}ft}$

b.

```
clear all

%define variables
fileID = fopen('problem5.bin','r');
x_in = fread(fileID, 'single');
fclose(fileID);
f = 50*10^3;%frequency
f_samp = 2.205 * 10^(6);%frequency of sample
delta_t = 1/f_samp;


%index variables
index = 0;

%create x vector
for n = 1:1:length(x_in)/2
   x(n) = x_in(n+index)+j*x_in(n+index+1);
   index = index + 1;
end
x = transpose(x);
%%end of part b
```

c.

```
clear all

%define variables
fileID = fopen('problem5.bin','r');
x_in = fread(fileID, 'single');
fclose(fileID);
f = 50*10^3;%frequency
f_samp = 2.205 * 10^(6);%frequency of sample
delta_t = 1/f_samp;


%index variables
index = 0;

%create x vector
for n = 1:1:length(x_in)/2
   x(n) = x_in(n+index)+j*x_in(n+index+1);
   index = index + 1;
end
x = transpose(x);
%%end of part b

%create time vector
endpt = (length(x)-1) * delta_t;%end of time vector
t = [0:delta_t:endpt];
t = transpose(t);

x_3 = exp(-j*2*pi*f*t);%x3 vector

%x_out vectors
x_out = x .* x_3;
x_real = real(x_out);
x_imag = imag(x_out);

x_audio = downsample(x_real,100);
x_audio2 = downsample(x_imag,100);
%sound(x_audio,22.05e3);
sound(x_audio2,22.05e3);
```