Typicality: A computational account

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Outline of the presentation

1. Introduction
2. Formalization of typicality
3. Experimental evaluation
4. Summary & Questions

Publications

Belohlavek, R. and T. Mikula (2022). “**Typicality: A Formal Concept Analysis Account**”. In: *International Journal of Approximate Reasoning* 142, pp. 349–369.

Belohlavek, R. and T. Mikula (2024). “**Are Human Categories Formal Concepts? A Case Study Using Dutch Data**”. In: *International Journal of General Systems*, pp. 1–50.

Belohlavek, R. and T. Mikula (2024). “**Similarity Metrics vs Human Judgment of Similarity for Binary Data: Which Is Best to Predict Typicality?**” In: *Applied Soft Computing* 153, p. 111270.

Belohlavek, R. and T. Mikula (2024). “**Schemes to Compute Typicality: Revisiting Rosch and Mervis’s Approach**”. *Preprint (to be submitted).*

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Introduction

Concept

- Many definitions across psychological literature.

A concept of x is a body of knowledge about x that is stored in long term memory and that is used by default in the processes underlying most, if not all, higher cognitive competences when these processes result in judgments about x. *(Machery, 2009, p. 12)*

Classic theory

- ***Necessity*** and ***sufficiency***.
    - *The entity is a category member if it possesses all attributes from the definition (necessity).*
    - *At the same time, definitional attributes are jointly sufficient to the entity being a member of a given category (sufficiency).*
- Object is either a member of the given category or not.

Graded membership of concepts

- Ground-breaking publication:

  Rosch, E. and C. B. Mervis (1975). “**Family Resemblances: Studies in the Internal Structure of Categories**”. In: *Cognitive Psychology* 7.4, pp. 573-605.

- Categories have a **graded structure** where membership is related to the **typicality of the object**.

- Fall of Classic theory?


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https://unsplash.com/photos/dAYGcP1fc9E

Typicality

- **Typical members** are good examples of a category.
- **Atypical members** are known to be members of a category but are unusual in some sense.
- Works across multiple levels of concepts.
  - Superordinate, basic level, subordinate.

Strong evidence

- Category judgment inconsistency (Rips, Shoben, and Smith, 1973)
- Category learning (Rosch, Simpson, and Miller, 1976)
- Category inference (Rips, 1975)
- Exemplar generation frequency (Mervis, Catlin, and Rosch, 1976)
- Linguistics (Kelly, Bock, and Keil, 1986)
- fMRI research (Iordan et al., 2016)

Family resemblance

- Determines the graded structure of category:

> *The basic hypothesis was that members of a category come to be viewed as prototypical of the category as a whole in* ***(1) proportion to the extent to which they bear a family resemblance to (have attributes which overlap those of) other members of the category.*** *Conversely, items viewed as most prototypical of one category will be those with* ***(2) least family resemblance to or membership in other categories****. (added parts numbering)*

Rosch, E. and C. B. Mervis (1975). “Family Resemblances: Studies in the Internal Structure of Categories”. In: *Cognitive Psychology* 7.4, pp. 573–605.

Formalization of typicality

Research goals

- Multidisciplinary approach.
  - New insights into data science.
  - New insights into cognitive psychology.

Concept vs. Formal concept vs. Category

- All formalizations are proposed for a set of objects A.
  - For example, an extent of a formal concept, a cluster from the k-means algorithm, etc.
- An object is represented as a set of binary attributes.
- We use FCA notation:

$$\{x\}^{\uparrow} = \{y; x \ \ \mathit{has} \ \ y\} \qquad \{y\}^{\downarrow} = \{x; y \ \ \mathit{has} \ \ x\}$$

Rosch and Mervis' scheme

...each attribute was weighted in accordance with the number of items in the category possessing it. The basic measure of the degree of family resemblance for an item was the ***sum of the weighted scores of each of the attributes*** *that had been listed for that item.*
(Rosch and Mervis, 1975)

$$typ_{\mathrm{RM}}(x, A) = \sum_{y \in Y,\ \langle x,y \rangle \in I} w(y, A) \qquad w(y, A) = |\{x; x \ \mathit{is\ in}\ A \ \mathit{and\ does\ have}\ y\}|$$

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Extending Rosch and Mervis' formula

- Rosch and Mervis’ formula can be extended to account for the absence of the attributes.

$$typ^{a^+,a^-}_{\mathrm{RM}^{\pm}}(x,A) = a^+ \cdot \sum_{y \in Y,\ \langle x,y\rangle \in I} w^+(y,A) + a^- \cdot \sum_{y \in Y,\ \langle x,y\rangle \notin I} w^-(y,A)$$

$$w^+(y,A) = |\{x; x \texttt{ is in } A \texttt{ and does have } y\}|$$

$$w^-(y,A) = |\{x; x \texttt{ is in } A \texttt{ and does not have } y\}|$$

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Similarity scheme

...members of a category come to be viewed as prototypical of the category as a whole in proportion ***to the extent to which they bear a family resemblance*** *to (have attributes which overlap those of) other members of the category.* *(Rosch and Mervis, 1975)*

*...**distance of items** from the origin of the space **is determined by their degree of family resemblance**.* *(Rosch and Mervis, 1975)*

$$typ_{sim}(x, A) = \frac{\sum_{x_1 \in A} sim(x, x_1)}{|A|} \qquad sim : X \times X \to \mathbb{R}$$

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Mutual relationship of the schemes

Theorem (Rosch and Mervis scheme): For the function $sim_{rm}(x_1, x_2) = \dfrac{|\{x_1\}^{\uparrow} \cap \{x_2\}^{\uparrow}|}{|Y|}$ we have:

$$typ_{\mathrm{RM}}(x, A) = |A| \cdot |Y| \cdot typ_{\mathrm{rm}}(x, A)$$

where $typ_{\mathrm{rm}}(x, A)$ is determined by sim_{rm} according to $typ_{sim}(x, A) = \dfrac{\sum_{x_1 \in A} sim(x, x_1)}{|A|}$.

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Mutual relationship of the schemes

- Why is this important?
  - Particular instance of the similarity approach and Rosch and Mervis' scheme are the two sides of the same coin.
  - Psychology based support for Russel-Rao similarity.

Looking outside of the concept

- Let us recall the family resemblance hypothesis:

> *The basic hypothesis was that members of a category come to be viewed as prototypical of the category as a whole in* ***(1)*** *proportion to the extent to which they bear a family resemblance to (have attributes which overlap those of) other members of the category. Conversely, items viewed as most prototypical of one category will be those with* ***(2)*** *least family resemblance to or membership in other categories.* *(Rosch and Mervis, 1975) (added parts numbering)*

Attribute characteristicness as a weight

- New attribute weight: $w(y, A) = \dfrac{|\{y\}^{\downarrow} \cap A|}{|A|} \cdot \dfrac{|\{y\}^{\downarrow} \cap A|}{|\{y\}^{\downarrow}|}$

$\dfrac{|\{y\}^{\downarrow} \cap A|}{|A|}$ extent to which objects from A have the attribute y

$\dfrac{|\{y\}^{\downarrow} \cap A|}{|\{y\}^{\downarrow}|}$ extent to which objects with attribute y belongs to A

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Attribute characteristicness as a weight

- Attribute weight can be then used in the formula:

$$typ_w(x, A) = \sum_{y \in \{x\}^{\uparrow}} w(y, A)$$

Experimental evaluation

Available data

- **Zoo dataset** (101 exemplars, 17 attributes).
  - Small, no typicality rating.
  - We have collected typicality ratings for three categories from the Zoo data: https://github.com/mikulatomas/zoo-typicality.
- **Dutch data** (249 + 166 exemplars, 225–1,295 attributes, **2 domains**).
  - Large, typicality ratings, similarity ratings and many more.
  - Version with fixed errors: https://github.com/mikulatomas/dutch-concepts.

Exemplar vs Category attributes

- **Exemplar**
  - Gathered for exemplar as stimuli (e.g., sparrow).
- **Category**
  - Gathered for category name as stimuli (e.g., bird).

What is the best similarity to predict typicality?

- Experimental evaluation of **69 similarity coefficients:**
  - For each object in the category, 69 typicality ratings were calculated.
  - Kendall tau-b correlation coefficient used to compare rank order to the ground truth.
  - Both category and domain datasets.

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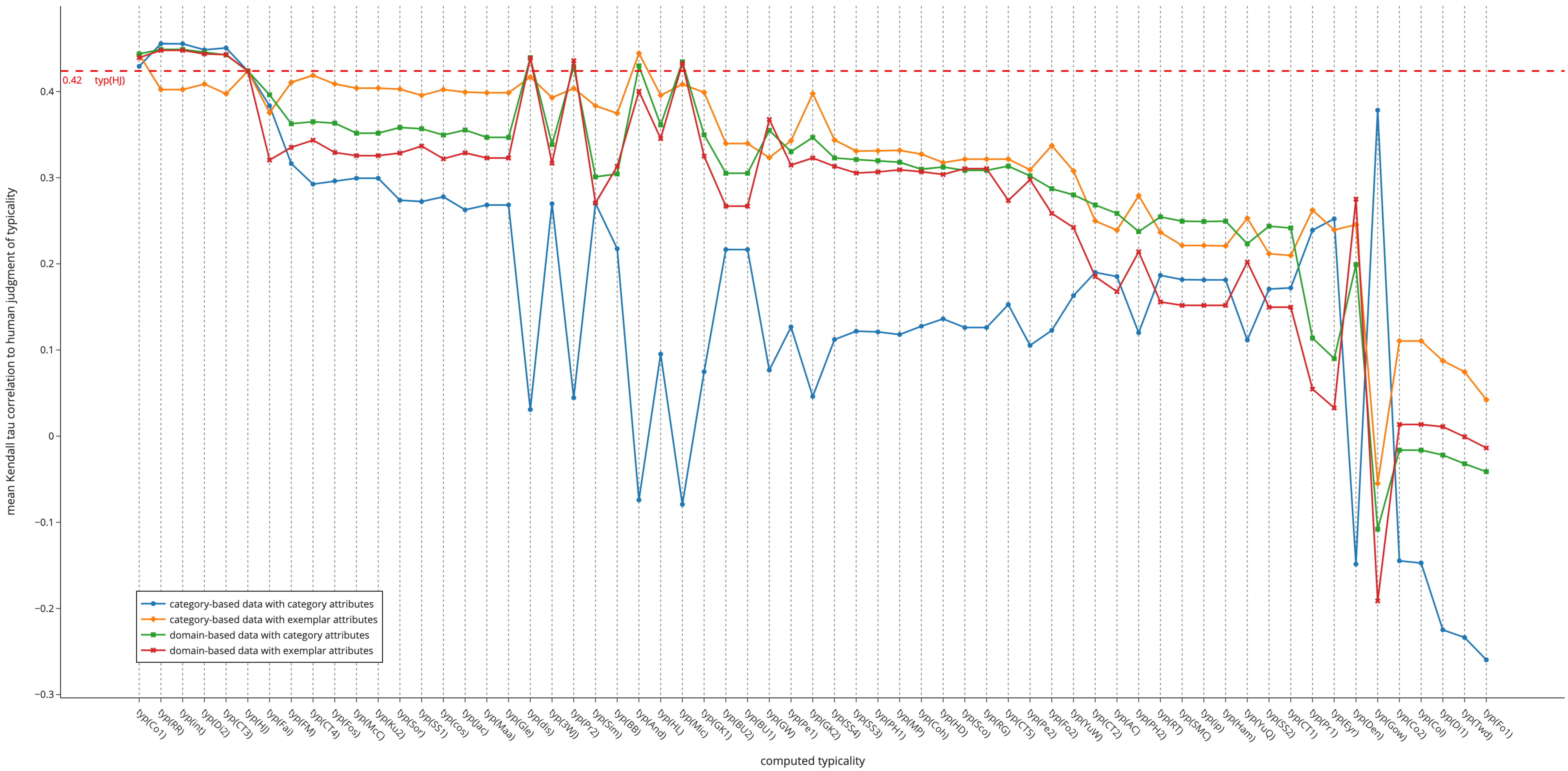
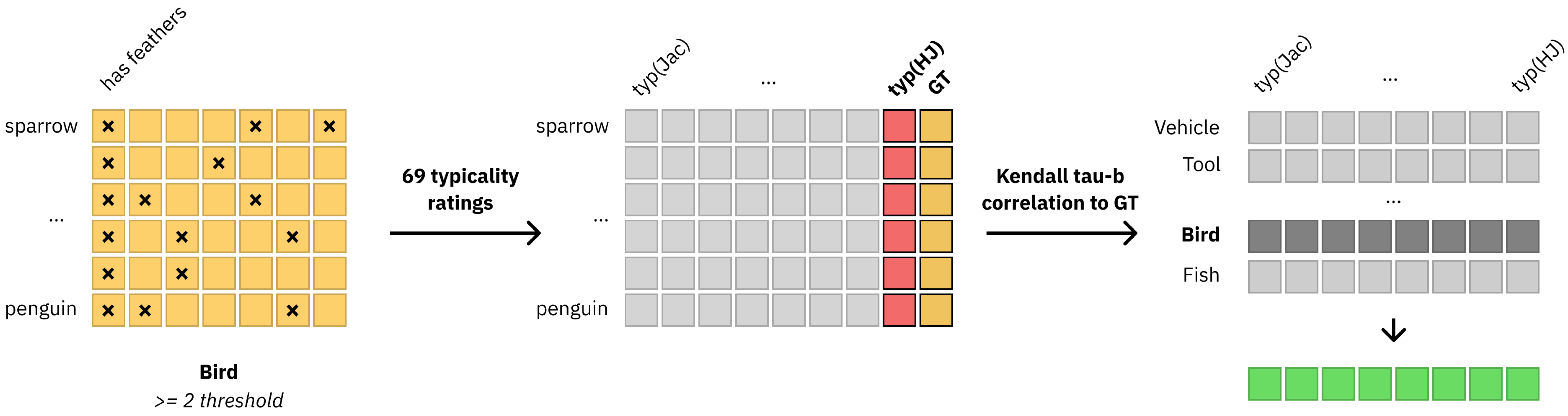
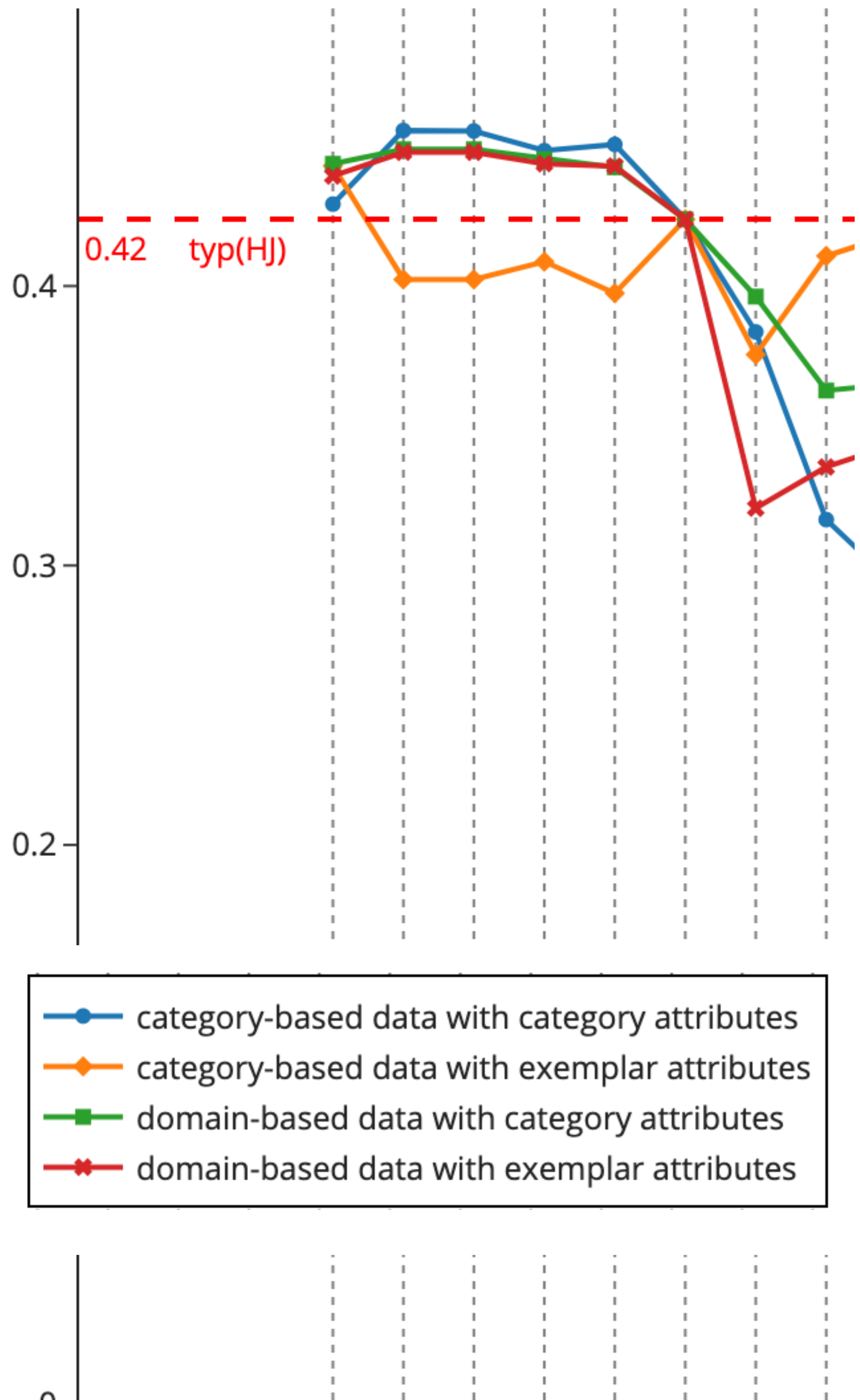


mean Kendall tau correlation to human judgment of typicality
0.42 typ(HJ)
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
category-based data with category attributes
category-based data with exemplar attributes
domain-based data with category attributes
domain-based data with exemplar attributes
typ(Co1) typ(RR) typ(int) typ(Di2) typ(CT3) typ(HJ) typ(Fai) typ(FM) typ(CT4) typ(Fos) typ(McC) typ(Ku2) typ(Sor) typ(SS1) typ(cos) typ(Jac) typ(Maa) typ(Gle) typ(dis) typ(3WJ) typ(Pr2) typ(Sim) typ(BB) typ(And) typ(HL) typ(Mic) typ(GK1) typ(BU2) typ(BU1) typ(GW) typ(Pe1) typ(GK2) typ(SS4) typ(SS3) typ(PH1) typ(MP) typ(Coh) typ(HD) typ(Sco) typ(RG) typ(CT5) typ(Pe2) typ(Fo2) typ(YuW) typ(CT2) typ(AC) typ(PH2) typ(RT) typ(SMC) typ(ip) typ(Ham) typ(YuQ) typ(SS2) typ(CT1) typ(Pr1) typ(Eyr) typ(Den) typ(Gow) typ(Co2) typ(Col) typ(Di1) typ(Twd) typ(Fo1)
computed typicality


has feathers
sparrow
...
penguin
Bird
>= 2 threshold
69 typicality ratings
typ(Jac)
...
typ(HJ)
GT
sparrow
...
penguin
Kendall tau-b correlation to GT
typ(Jac)
...
typ(HJ)
Vehicle
Tool
...
Bird
Fish



1.Co1 (Cole 1)

2.RR (Russel-Rao)

3.int (Intersection)

4.Di2 (Dice 2)

5.CT3 (Consonni-Todeschini 3)

6.HJ (Human judgement)

7.Fai (Faith)

8.FM (Fager-McGowan)

Is there better way to predict typicality?

- Experimental evaluation of new scheme on Dutch domain data.

- The newly proposed scheme was evaluated in the context of previous results.

  - This time, only on domain data since we need to “look behind the category itself”.

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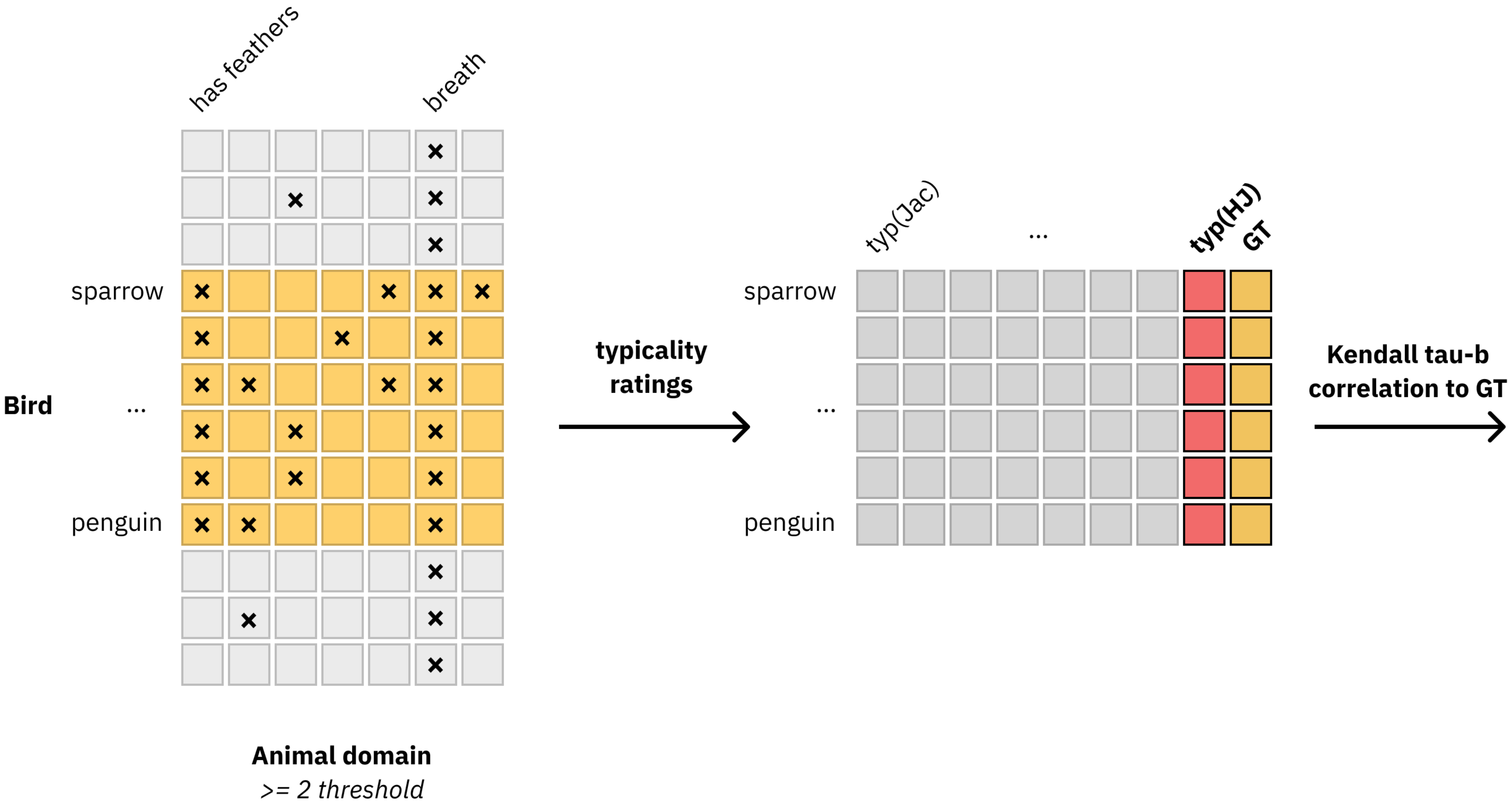
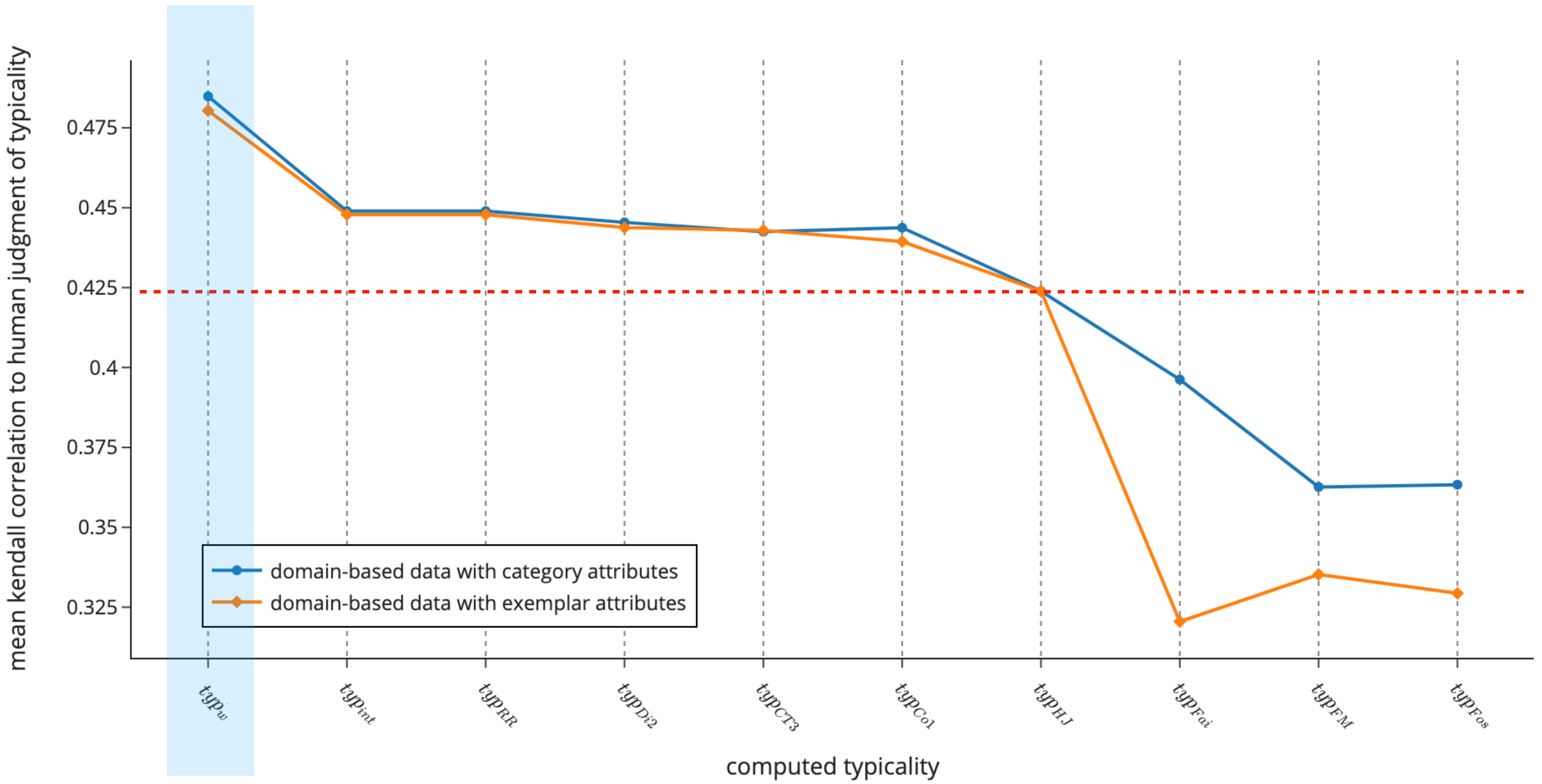

has feathers
breath
sparrow
Bird
...
penguin
Animal domain
>= 2 threshold
typicality
ratings
typ(Jac)
...
typ(HJ)
GT
sparrow
...
penguin
Kendall tau-b
correlation to GT



mean kendall correlation to human judgment of typicality
0.475
0.45
0.425
0.4
0.375
0.35
0.325
domain-based data with category attributes
domain-based data with exemplar attributes
typ_w
typ_{int}
typ_{RR}
typ_{Di2}
typ_{CT3}
typ_{Co1}
typ_{HJ}
typ_{Fai}
typ_{FM}
typ_{Fos}
computed typicality

Comparing similarity vs. human judgement of similarity

- Experimental evaluation of **69 similarity coefficients.**
- Direct comparison:
    - Calculate similarity for all pairs of objects for all 69 similarity coefficients.
    - Kendall tau-b was used to compare rank-order of these pairs to the pairs evaluated by the human respondents.

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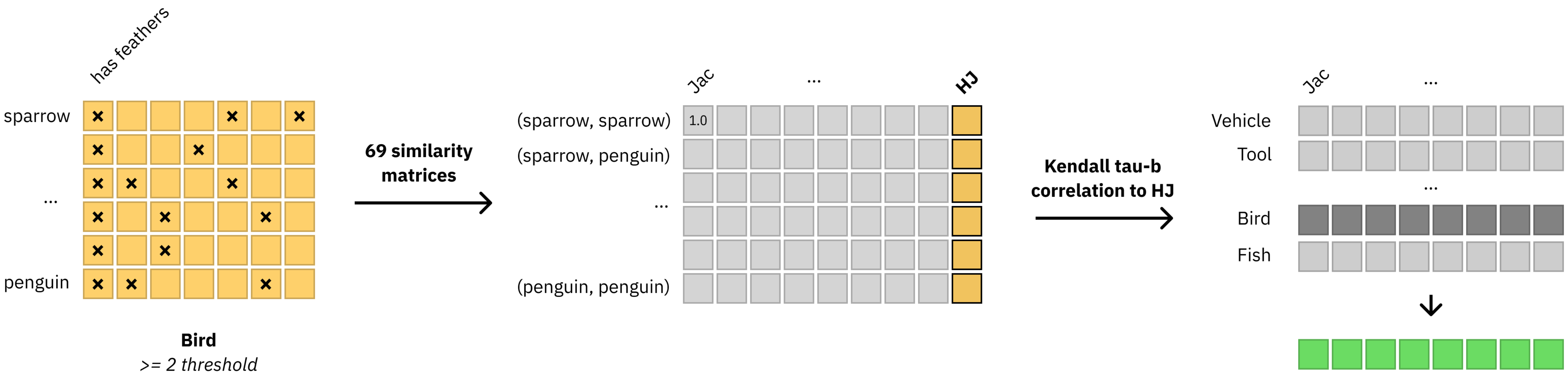
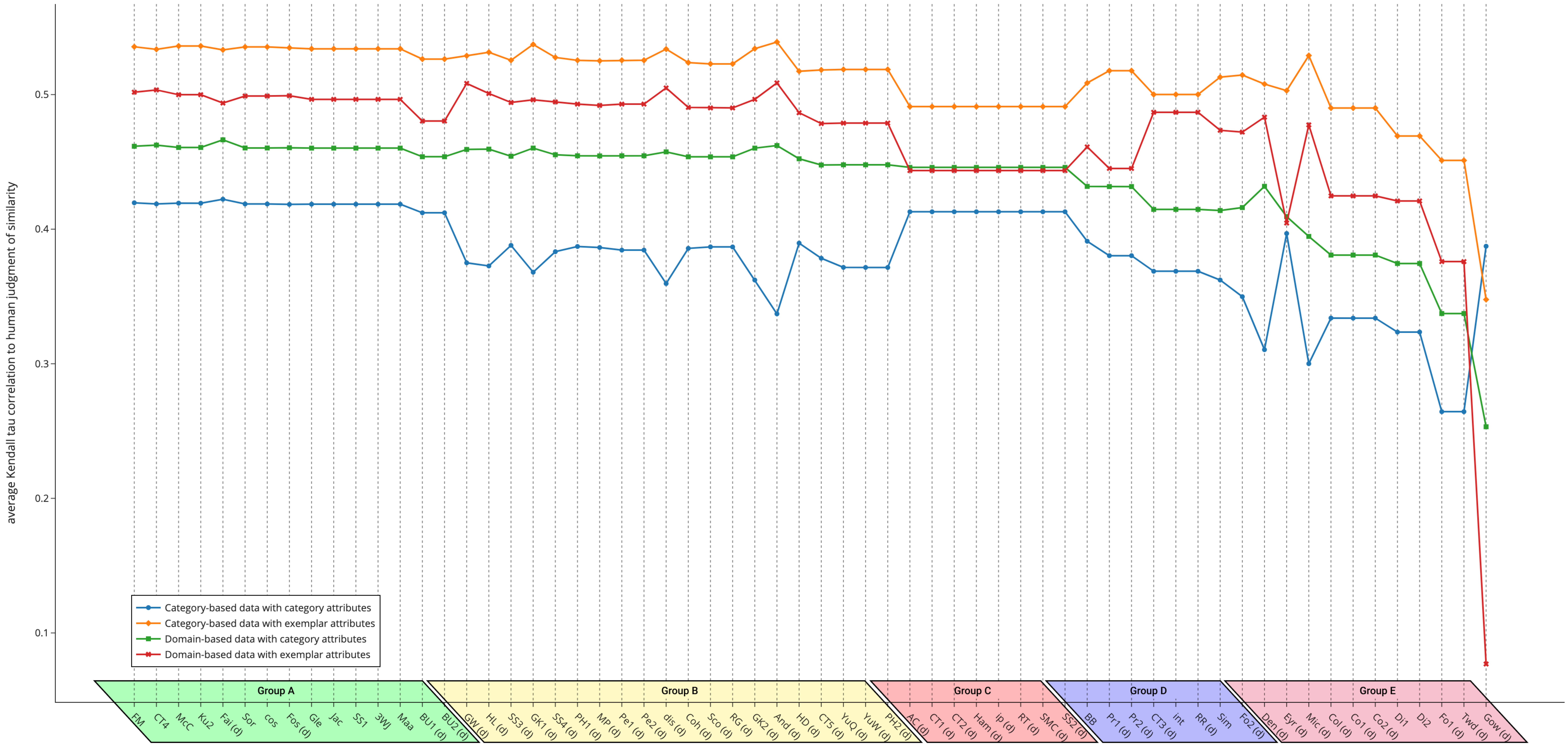

has feathers
sparrow
...
penguin
Bird
>= 2 threshold
69 similarity matrices
Jac
...
HJ
(sparrow, sparrow)
1.0
(sparrow, penguin)
...
(penguin, penguin)
Kendall tau-b correlation to HJ
Jac
...
Vehicle
Tool
...
Bird
Fish



average Kendall tau correlation to human judgment of similarity
0.1
0.2
0.3
0.4
0.5
Category-based data with category attributes
Category-based data with exemplar attributes
Domain-based data with category attributes
Domain-based data with exemplar attributes
Group A
Group B
Group C
Group D
Group E
FM
CT4
McC
Ku2
Fai (d)
Sor
cos
Fos (d)
Gle
Jac
SS1
3WJ
Maa
BU1 (d)
BU2 (d)
GW (d)
HL (d)
SS3 (d)
GK1 (d)
SS4 (d)
PH1 (d)
MP (d)
Pe1 (d)
Pe2 (d)
dis (d)
Coh (d)
Sco (d)
RG (d)
GK2 (d)
And (d)
HD (d)
CT5 (d)
YuQ (d)
YuW (d)
PH2 (d)
AC (d)
CT1 (d)
CT2 (d)
Ham (d)
ip (d)
RT (d)
SMC (d)
SS2 (d)
BB
Pr1 (d)
Pr2 (d)
CT3 (d)
int
RR (d)
Sim
Fo2 (d)
Den (d)
Eyr (d)
Mic (d)
Col (d)
Co1 (d)
Co2 (d)
Di1
Di2
Fo1 (d)
Twd (d)
Gow (d)

Do Dutch categories form formal concepts?

- Experimental evaluation of categories defined in Dutch data form formal concepts w.r.t. formal concept analysis.

- Surprisingly, mostly yes.

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Attribute type	Domain	Category	≥ 1	≥ 2	≥ 3	= 4
Category	Animal	Bird	yes	yes	yes	yes
		Fish	yes	yes	yes	yes
		Insect	yes	yes	yes	yes
		Mammal	yes	yes	yes	no
		Reptile	yes	yes	yes	no
	Artifact	Clothing	yes	yes	yes	no
		Kitchen utensil	yes	yes	yes	no
		Musical instrument	yes	yes	yes	yes
		Tool	yes	yes	no	no
		Vehicle	no	no	no	no
		Weapon	no	no	no	no
Exemplar	Animal	Bird	yes	yes	yes	yes
		Fish	yes	yes	yes	yes
		Insect	yes	yes	yes	yes
		Mammal	yes	yes	yes	yes
		Reptile	yes	yes	yes	no
	Artifact	Clothing	yes	yes	yes	yes
		Kitchen utensil	yes	yes	yes	no
		Musical instrument	yes	yes	yes	yes
		Tool	yes	yes	no	no
		Vehicle	yes	no	no	no
		Weapon	no	no	no	no

Results summary

1. Formalized of Rosch and Mervis’ scheme.

2. Established a relationship between similarity and R&M’s scheme.

3. A new attribute weight was proposed which improves the typicality prediction.

4. Overview of similarity measures and their performance w.r.t. human respondents.

5. Interesting results relating the formal concepts and real-life conceptual data.

Rosch ~~staff~~ mervis
Basic levels norms

FURNITURE

Woven attributes

1

attribute	frequency	chair	sofa	table	dresser	desk	bed	bookcase	footstool	lamp	piano	cushion	mirror	rug	radio	stove	clock	picture	closet	vase	telephone
base, stand	12	x	x	x	x	x	x	x	x	x					x	x	x				
frame	11	x	x	x	x	x	x	x	x	x	x								x		
legs	9	x	x	x	x	x	x	x	x		x										
backing	8	x	x		x	x		x					x	x				x			
coasters	7	x	x	x	x	x	x				x										
top to put things on	5			x	x	x		x			x										
electronic parts	5									x					x	x	x				x
stuffing, cushions	4	x	x						x			x									
springs	3	x	x				x														
control knobs	3														x	x	x				
back	2	x	x																		
seat	2	x	x																		
arms	2	x	x																		
drawers	2				x	x															
switch	2									x					x						
receiver	2														x						x
feet	1																				x
leaves	1			x																	
mattress	1						x														
headboard	1						x														

prof. Dr. Gert Storms (1/3)

- **The smallworldofwords.org project created by Dr. Simon De Deyne and Prof. Gert Storms.**
  - I was doing preliminary experiments with similarities derived from association, and I agree it is definitely an interesting research direction.
- **Cross-validation of typicality via other similarity-based cognitive tasks.**
  - I agree that cross-validation is the next natural step to make our results even more relevant for the field of psychology. Especially in the case of newly proposed attribute weights used for typicality calculation.

prof. Dr. Gert Storms (2/3)

- **Do you think the findings generalize to categories defined at the superordinate and subordinate levels?**
  - This was addressed in the original 1975 paper from Rosch and Mervis, in which the typicality influence was verified across multiple levels.
  - I can imagine that different sets of similarity measures would work best on different levels of categories.
  - Connection to basic level research done by Belohlavek and Trnecka
  - In general, to verify these, more comprehensive datasets are needed; large LLM like ChatGPT may suit this well.

prof. Dr. Gert Storms (3/3)

- **In the psychological literature, many authors made a distinction between “defining” and “characteristic” features. Does that distinction show up in your formal analysis of the exemplar by feature matrices?**
  - From our perspective, we always thought of intent as the defining attributes and opened the research question of what the characteristic attributes could be.
  - In fact, we proposed the characteristicness weights for attributes; future research to address their role in FCA is needed. Note, that these weights can be applied on attributes outside of an intent.

prof. Dr. Richard Emilion (1/4)

- **A question which arises is whether all the very interesting notions presented in Chapter 1, such as long-range memory, empirical concept, stability and so on, are taken in account in the concept formalism presented in Chapter 3.**

  - For the majority of the time, we were focused on modeling typicality in its bare minimum form. As mentioned, we started moving forward to more general ideas with the concept of characteristic attributes.

  - In general, the main motivation behind Chapter 1 was to show that modeling the human conceptual system is a tremendous task.

prof. Dr. Richard Emilion (2/4)

- **Note that the robust Kendall rank correlation $\widetilde{\gamma}$ depends on a T-norm that is not mentioned in the paper.**
  - Thank you for pointing it out; the Gödel T-norm was used.
  - We used fuzzy correlation experimentally once; future research on these methods is needed (out of our scope). We preferred the classic Kendall tau-b correlation in all other experiments.
  - In general, how to compare the predicted typicality ratings with the ground truth is one of the fundamental problems for our research.

prof. Dr. Richard Emilion (3/4)

- **A question which arises is whether typicality only depends on the marginal distributions or rather depends on the joint distribution of a set of attributes.**

  - Psychologists were examining the interconnected attributes and their influence on category. We have not done any experiments related to this question yet.

prof. Dr. Richard Emilion (4/4)

- A second question is whether there is any performance difference between the two schemes.

$$typ_{sim}(x, A) = \frac{\sum_{x_1 \in A} sim(x, x_1)}{|A|}.$$

$$typ^{a^+, a^-}(x, A) = a^+. \sum_{y \in Y : y \in \{x\}^{\uparrow}} w(y, A) + a^-. \sum_{y \in Y : y \notin \{x\}^{\uparrow}} w(y, A).$$

prof. Dr. Richard Emilion (4/4)

Several examples of attribute weights are given:

- $a^+ = 1, a^- = 0, \quad w(y, A) = |A \cap \{y\}^\downarrow|$
- $a^+ = 1, a^- = 0, \quad w(y, A) = \frac{|A \cap \{y\}^\downarrow|}{|A|} \frac{|A \cap \{y\}^\downarrow|}{|\{y\}^\downarrow|}$
- $a^+ = 1, a^- = 0, w(y, A) = \frac{|A \cap \{y\}^\downarrow|}{|A|} \frac{|(X \setminus A) \setminus \{y\}^\downarrow|}{|X \setminus A|}$
- $w(y, A) = |A \cap \{y\}^\downarrow|$ if $y \in \{x\}^\uparrow$ and $w(y, A) = |A \setminus \{y\}^\downarrow|$ if $y \notin \{x\}^\uparrow$.

Mutual relationship of the schemes

- Rosch and Mervis' scheme is equal to scaled similarity scheme:

$$typ_{\mathrm{RM}^{\pm}}^{a^+,a^-}(x,A) = |A| \cdot |Y| \cdot typ_{\mathrm{SMC}^{a^+,a^-}}(x,A)$$

where:

$$\mathrm{SMC}^{a^+,a^-}(x_1,x_2) = \frac{a^+ \cdot |\{x_1\}^{\uparrow} \cap \{x_2\}^{\uparrow}| + a^- \cdot |Y - (\{x_1\}^{\uparrow} \cup \{x_2\}^{\uparrow})|}{|Y|}$$

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Mutual relationship of the schemes

- In fact the $\mathrm{SMC}^{a^+,a^-}(x_1, x_2)$ with adequately selected parameters is equal to two well-known similarity coefficients:

$$\mathrm{RR}(x_1, x_2) = \mathrm{SMC}^{1,0}(x_1, x_2) = \frac{|\{x_1\}^{\uparrow} \cap \{x_2\}^{\uparrow}|}{|Y|}$$

$$\mathrm{SMC}(x_1, x_2) = \mathrm{SMC}^{1,1}(x_1, x_2) = \frac{|\{x_1\}^{\uparrow} \cap \{x_2\}^{\uparrow}| + |Y - (\{x_1\}^{\uparrow} \cup \{x_2\}^{\uparrow})|}{|Y|}$$

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Theorem 1. *For the function* $sim_{\mathrm{rm}}(x_1, x_2) = \frac{|\{x_1\}^{\uparrow} \cap \{x_2\}^{\uparrow}|}{|Y|}$ *we have*

$$typ_{\mathrm{RM}}(x, \langle A, B\rangle) = |A| \cdot |Y| \cdot typ_{\mathrm{rm}}(x, \langle A, B\rangle)$$

where $typ_{\mathrm{rm}}(x, \langle A, B\rangle)$ *is determined by* sim_{rm} *according to (2).*

Proof. Since

$$|A| \cdot |Y| \cdot typ_{\mathrm{rm}}(x, \langle A, B\rangle) = |A| \cdot |Y| \cdot \frac{\sum_{x_1 \in A} sim_{\mathrm{rm}}(x, x_1)}{|A|}$$

$$= |A| \cdot |Y| \cdot \frac{\sum_{x_1 \in A} \frac{|\{x\}^{\uparrow} \cap \{x_1\}^{\uparrow}|}{|Y|}}{|A|} = \sum_{x_1 \in A} |\{x\}^{\uparrow} \cap \{x_1\}^{\uparrow}|,$$

we clearly need to verify

$$typ_{\mathrm{RM}}(x, \langle A, B\rangle) = \sum_{x_1 \in A} |\{x\}^{\uparrow} \cap \{x_1\}^{\uparrow}|.$$

Denoting $||\varphi||$ the truth value of φ (e.g. $||y \in \{x_1\}^{\uparrow}|| = 1$ iff $y \in \{x_1\}^{\uparrow}$), we obtain

$$\sum_{x_1 \in A} |\{x\}^{\uparrow} \cap \{x_1\}^{\uparrow}| = \sum_{x_1 \in A} \sum_{y \in \{x\}^{\uparrow}} ||y \in \{x_1\}^{\uparrow}||$$

$$= \sum_{x_1 \in A} \sum_{y \in \{x\}^{\uparrow}} ||x_1 \in \{y\}^{\downarrow}|| = \sum_{y \in \{x\}^{\uparrow}} \sum_{x_1 \in A} ||x_1 \in \{y\}^{\downarrow}||$$

$$= \sum_{y \in \{x\}^{\uparrow}} |\{x_1 \in A \mid x_1 \in \{y\}^{\downarrow}\}| = \sum_{y \in \{x\}^{\uparrow}} w(y, \langle A, B\rangle) = typ_{\mathrm{RM}}(x, \langle A, B\rangle),$$

completing the proof. $\square$