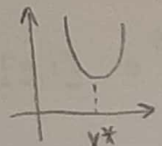
# Jednodimenzione numeričke metode

- Metode direktnog pretraživanja, gradijentne metode ; metode aproksimacije polinomom

## Metode direktnog pretraživanja

### Fibonačijev metod

- Nema izvoda, skraćuje se interval
- Mora biti unimodalna (rastuća, opadajuća)




- mana: mora se unapred znati br iteracija

$$F_n > \frac{|b_0 - a_0|}{\varepsilon} > F_{n-1}$$

$(a_0, b_0)$ - interval pretrage

$\varepsilon$ - tolerancija

① Naći min fje sa tačnošću $\varepsilon$ nad intervalom $[0,1]$

$f(x) = 2x^4 - 3x$

$\varepsilon = 10^{-5}$, $[\overset{a_0}{0}, \overset{b_0}{1}]$

$$F_n > \frac{1-0}{10^{-5}} > F_{n-1}$$

$$F_n > 100\,000 > F_{n-1}$$

broj iteracija

$F_{24} = 46\,368$

$F_{25} = 75025$

$F_{26} = 121\,393$

$\Big\}$ $75\,025 > 100\,000 > 121\,393 \to n = 26$

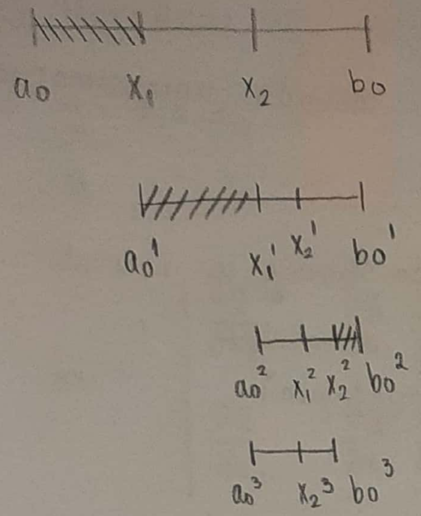
$$x_1 = a_0 + \frac{F_{n-2}}{F_n}(b_0 - a_0) = 0 + \frac{46\,368}{121\,393}(1-0) = 0.382$$

$$x_2 = a_0 + b_0 - x_1 = 0 + 1 - 0.382 = 0.618$$

$f(x_1) = -1.1033$

$f(x_2) = -1.5623$

$f(x_1) > f(x_2) \to$ skraćujemo interval




$f(x_1) > f(x_2)$

$a_0^1 = x_1 = 0.382$

$b_0^1 = b_0 = 1$

$x_1^1 = x_2 = 0.618$

$x_2^1 = a_0^1 + b_0^1 - x_1^1 = 0.764$

$f(x_1^1) = -1.5623$

$f(x_2^1) = -1.6406$

$f(x_1^1) > f(x_2^1)$

$a_0^2 = x_1^1 = 0.618$

$b_0^2 = b_0^1 = 1$

$x_1^2 = x_2^1 = 0.764$

$x_2^2 = a_0^2 + b_0^2 - x_1^2 = 0.854$

$f(x_1^2) = -1.6406$

$f(x_2^2) = -1.4981$

$f(x_1^2) < f(x_2^2)$

$a_0^3 = a_0^2 = 0.618$

$b_0^3 = x_2^2 = 0.854$

$x_2^3 = x_1^2 = 0.764$

$x_1^3 = a_0^3 + b_0^3 - x_2^3 = 0.708$

$\vdots$

$x^* = 0.708$

$y^* = -1.641$

Zlatni presek

$$\frac{F_n}{F_{n+2}} \approx c = \frac{3-\sqrt{5}}{2} \approx 0.38197$$

$|b_0 - a_0| < \varepsilon$

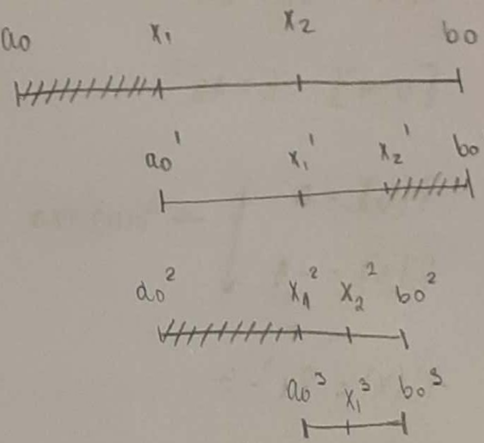
$x_1 = a_0 + c(b_0 - a_0)$

$x_2 = a_0 + b_0 - x_1$

(2) Naći minimum fje $f(x)$ nad intervalom $[0,1]$ u 4 iteracije

$f(x) = 8x^3 - 2x^2 - 7x + 3$

$\overset{a_0}{[0}, \overset{b_0}{1]}$ 4 iteracije




$x_1 = a_0 + c(b_0 - a_0) = 0 + \frac{3-\sqrt{5}}{2}(1-0) = 0.382$

$x_2 = a_0 + b_0 - x_1 = 0 + 1 - 0.382 = 0.618$

$f(x_1) = 0.4806$

$f(x_2) = -0.2016$

$\underline{f(x_1) > f(x_2)}$

$a_0^1 = x_1 = 0.382$

$b_0^1 = 1$

$x_1^1 = x_2 = 0.618$

$x_2^1 = a_0^1 + b_0^1 - x_1^1 = 0.763$

$f(x_1) = x_1^1 = -0.2016$

$f(x_2^1) = 0.0482$

$\underline{f(x_1^1) < f(x_2^1)}$

$a_0^2 = a_0^1 = 0.382$

$b_0^2 = x_2^1 = 0.762$

$x_2^2 = x_1^2 = 0.618$

$x_1^2 = a_0^2 + c(b_0^2 - a_0^2) = 0.527$

$f(x_1^2) = -0.0735$

$f(x_2^2) = -0.2016$

$\underline{f(x_1^2) > f(x_2^2)}$

$a_0^3 = x_1^2 = 0.527$

$b_0^3 = b_0^2 = 0.763$

$x_1^3 = x_2^3 = 0.618$

$x_2^3 = a_0^3 + b_0^3 - x_1^3 = 0.673$

$f(x_1^3) = -0.2016$

$f(x_2^3) = -0.1782$

$x^* = x_1^3 = 0.618$

$f^* = -0.2016$

Gradijentne metode - na osnovu izvoda traže ekstrem
exstrem

Njutn-Rapsonov metod

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$|x_{k+1} - x_k| < \varepsilon$$

(3) Naći min

$f(x) = 2x^4 - 3x$

$[0,1]$ $\varepsilon = 10^{-2}$

$\left.\begin{array}{l} f(0) = 0 \\ f(1) = -1 \end{array}\right\} \rightarrow$ tražimo min $\rightarrow x_0 = 1$

$f'(x) = 8x^3 - 3$

$f''(x) = 24x^2$

$1^\circ$ $x_1 = x_0 + \dfrac{f'(x_0)}{f''(x_0)} = 0.7917$

$|x_1 - x_0| = 0.2083 > \varepsilon$

$2^\circ$ $x_2 = x_1 + \dfrac{f'(x_1)}{f''(x_2)} = 0.7272$

$|x_2 - x_1| = 0.0645 > \varepsilon$

$3^\circ$ $x_3 = x_2 + \dfrac{f'(x_2)}{f''(x_2)} = 0.7212$

$|x_3 - x_2| = 0.006 < \varepsilon$ => KRAJ ALGORITMA

$x^* = 0.7212$

$f^* = -1.6225$

Metod sječice

$$f''(x_k) = \frac{f'(x_k) - f'(x_{k-1})}{x_k - x_{k-1}}$$

- Nema 2 izvoda nego se aproksimira pomoću 1 ali imamo 2 početne tačke

$$x_{k+1} = x_k - \frac{f'(x_k)}{f'(x_k) - f'(x_{k-1})}(x_k - x_{k-1})$$

(4) Naći min

$f(x) = 2x^4 - 3x$

$[0,1]$ $\varepsilon = 10^{-2}$

$f(0) = 0$, $f(1) = -1$ $\Big\}$ $x_0 = 0$, $x_1 = 1$

$f'(x) = 8x^3 - 3$

1° $x_2 = x_1 - \frac{f'(x_1)}{f'(x_1) - f'(x_0)} \cdot (x_1 - x_0) = 0.375$

$|x_2 - x_1| = 0.625 > \varepsilon$

2° $x_3 = x_2 - \frac{f'(x_2)}{f'(x_2) - f'(x_1)} (x_2 - x_1) = 0.5876$

$|x_3 - x_2| = 0.213 > \varepsilon$

3° $x_4 = 0.813$

$|x_4 - x_3| = 0.2437$

4° $x_5 = 0.7005$

$|x_5 - x_4| = 0.1318 > \varepsilon$

5° $x_6 = 0.7245$

$|x_6 - x_5| = 0.024 > \varepsilon$

6° $x_7 = 0.721$

$|x_7 - x_6| = 0.0035 < \varepsilon$ => KRAJ

$x^* = 0.721$

$f^* = -1.6225$

Metode aproksimacije ~~parabolom~~ polinomom

Metod parabole

(5) Naći min u 3 iteracije

$f(x) = 2x^4 - 3x$

$[0,2]$ , 3 iteracije

1° $x_1 = 0$

$x_3 = 2$ $\qquad x_1 < x_2 < x_3$

$x_2 = \frac{0+2}{2} = 1$

$f(x_1) = 0$

$f(x_2) = -1$ $\qquad f(x_1) > f(x_2) < f(x_3)$

$f(x_3) = 26$

$y_i = f(x_i) = a_i + b_i \cdot x_i + c x_i^2$

$$\left.\begin{array}{l} a + bx_1 + cx_1^2 = f(x_1) \\ a + bx_2 + cx_2^2 = f(x_2) \\ a + bx_3 + cx_3^2 = f(x_3) \end{array}\right\} \quad a = 0 \quad b = -15 \quad c = 14$$

$y(x) = -15x + 14x^2$

$y'(x) = -15 + 28x = 0 \rightarrow x^* = \frac{15}{28} = 0.5357$

$f^* = -1.4424$

$x_1 < x^* < x_2$ $\qquad f(x_1) < f(x^*) < f(x_2)$

2° $x_1 = 0$ $\qquad f(x_1) = 0$

$x_2 = 0.5357$ $\qquad f(x_2) = -1.4424$

$x_3 = 1$ $\qquad f(x_3) = -1$

$$\left.\begin{array}{l} a + bx_1 + cx_1^2 = f(x_1) \\ a + bx_2 + cx_2^2 = f(x_2) \\ a + bx_3 + cx_3^2 = f(x_3) \end{array}\right\} \quad a = 0,\ b = -4.6454 \quad c = 3.6454$$

$y(x) = -4.6454x + 3.6454x^2$

$y'(x) = 0 \quad \rightarrow \quad x^* = 0.06371$

$f(x^*) = -1.5818$

$x_2 < x^* < x_3 \qquad f(x_2) > f(x^*) < f(x_3)$

$3^o$ $x_1 = 0.5357 \qquad f(x_1) = -1.4424$

$x_2 = 0.6371 \qquad f(x_2) = -1.5818$

$x_3 = 1 \qquad f(x_3) = -1$

$$\left.\begin{array}{l} a + bx_1 + cx_1^2 = f(x_1) \\ a + bx_2 + cx_2^2 = f(x_2) \\ a + bx_3 + cx_3^2 = f(x_3) \end{array}\right\} \quad a = 1.4835,\ b = -8.8981,\ c = 6.4146$$

$y(x) = 1.4835 - 8.8981x + 6.4146x^2$

$y'(x) = 0 \qquad x^* = 0.6936$

$f^* = -1.6179$