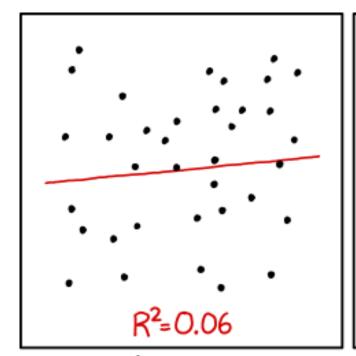
## MAT 395/495

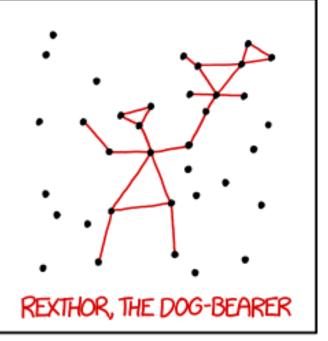
## Scientific Data Analysis and Computing Summer 2020

Topic 10 Least-Square Fitting

SIDDHA PIMPUTKAR







I DON'T TRUST LINEAR REGRESSIONS WHEN IT'S HARDER TO GUESS THE DIRECTION OF THE CORRELATION FROM THE SCATTER PLOT THAN TO FIND NEW CONSTELLATIONS ON IT.

## Least-Square Fitting

## Reading

Data Reduction and Error Analysis for the Physical Sciences by P.R. Bevington, D.K. Robinson, 3rd Ed.

Ch. 6 Least-Squares Fit to a Straight Line: pg. 98-114

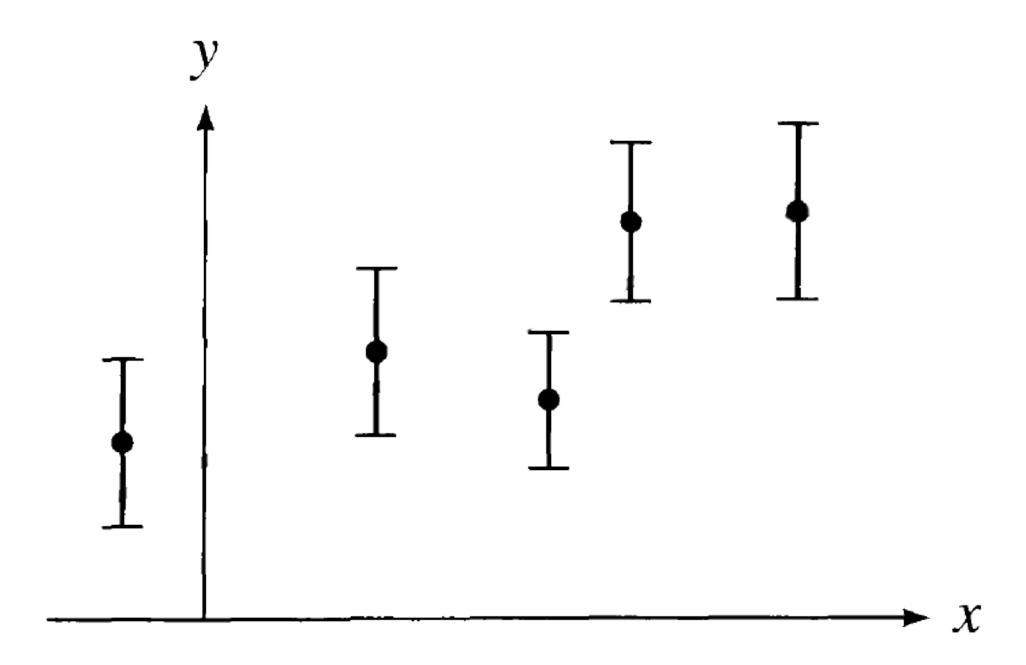
Ch. 7 Least-Squares Fit to a Polynomial: pg. 116-137

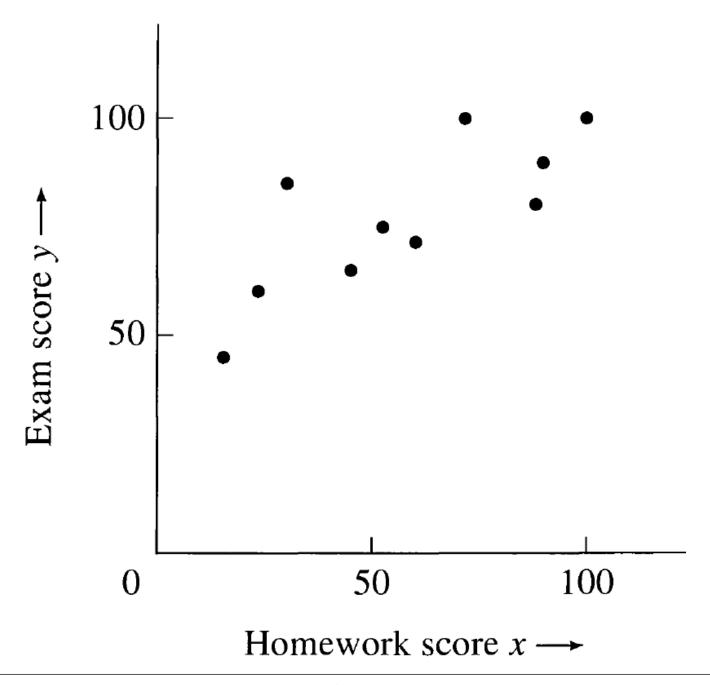
Ch. 11 Testing the Fit: pg. 194-201

An Introduction to Error Analysis by J.R. Taylor, 2nd Ed.

Ch. 8 Least-Square Fitting

Ch.9 Covariance and Correlation





**Table 9.4.** The probability  $Prob_N(|r| \ge r_0)$  that N measurements of two uncorrelated variables x and y would produce a correlation coefficient with  $|r| \ge r_0$ . Values given are percentage probabilities, and blanks indicate values less than 0.05%.

	$r_{ m o}$										
N	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
3	100	94	87	81	74	67	59	51	41	29	0
6	100	85	70	56	43	31	21	12	6	1	0
10	100	78	58	40	25	14	7	2	0.5		0
20	100	67	40	20	8	2	0.5	0.1			0
50	100	49	16	3	0.4						0

**Table C.** The percentage probability  $Prob_N(|r| \ge r_0)$  that N measurements of two uncorrelated variables give a correlation coefficient with  $|r| \ge r_0$ , as a function of N and  $r_0$ . (Blanks indicate probabilities less than 0.05%.)

N         0         0.1         0.2         0.3         0.4         0.5         0.6         0.7         0.8         0.9         1           3         100         94         87         81         74         67         59         51         41         29         0           4         100         90         80         70         60         50         40         30         20         10         0           5         100         87         75         62         50         39         28         19         10         3.7         0           6         100         85         70         56         43         31         21         12         5.6         1.4         0           7         100         83         67         51         37         25         15         8.0         3.1         0.6         0           8         100         81         63         47         33         21         12         5.3         1.7         0.2         0           9         100         80         61         43         29         17         8.8         3.6         1.0 <t< th=""><th></th><th></th><th></th><th></th><th></th><th></th><th><math>r_{\rm o}</math></th><th></th><th></th><th></th><th></th><th></th></t<>							$r_{\rm o}$					
4         100         90         80         70         60         50         40         30         20         10         0           5         100         87         75         62         50         39         28         19         10         3.7         0           6         100         85         70         56         43         31         21         12         5.6         1.4         0           7         100         83         67         51         37         25         15         8.0         3.1         0.6         0           8         100         81         63         47         33         21         12         5.3         1.7         0.2         0           9         100         80         61         43         29         17         8.8         3.6         1.0         0.1         0           10         100         78         58         40         25         14         6.7         2.4         0.5         0           11         100         75         51         32         18         8.2         3.9         1.1         0.2	N	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
5         100         87         75         62         50         39         28         19         10         3.7         0           6         100         85         70         56         43         31         21         12         5.6         1.4         0           7         100         83         67         51         37         25         15         8.0         3.1         0.6         0           8         100         81         63         47         33         21         12         5.3         1.7         0.2         0           9         100         80         61         43         29         17         8.8         3.6         1.0         0.1         0           10         100         78         58         40         25         14         6.7         2.4         0.5         0           11         100         75         56         37         22         12         5.1         1.6         0.3         0           12         100         76         53         34         20         9.8         3.9         1.1         0.2         0	3	100	94	87	81	74	67	59	51	41	29	0
6         100         85         70         56         43         31         21         12         5.6         1.4         0           7         100         83         67         51         37         25         15         8.0         3.1         0.6         0           8         100         81         63         47         33         21         12         5.3         1.7         0.2         0           9         100         80         61         43         29         17         8.8         3.6         1.0         0.1         0           10         100         78         58         40         25         14         6.7         2.4         0.5         0           11         100         75         56         37         22         12         5.1         1.6         0.3         0           12         100         76         53         34         20         9.8         3.9         1.1         0.2         0           13         100         75         51         32         18         8.2         3.0         0.8         0.1         0           14	4	100	90	80	70	60	50	40	30	20	10	0
7         100         83         67         51         37         25         15         8.0         3.1         0.6         0           8         100         81         63         47         33         21         12         5.3         1.7         0.2         0           9         100         80         61         43         29         17         8.8         3.6         1.0         0.1         0           10         100         78         58         40         25         14         6.7         2.4         0.5         0           11         100         76         53         34         20         9.8         3.9         1.1         0.2         0           13         100         75         51         32         18         8.2         3.0         0.8         0.1         0           14         100         73         49         30         16         6.9         2.3         0.5         0.1         0           15         100         71         46         26         12         4.9         1.4         0.3         0           17         100         70 <td>5</td> <td>100</td> <td>87</td> <td>75</td> <td>62</td> <td>50</td> <td>39</td> <td>28</td> <td>19</td> <td>10</td> <td>3.7</td> <td>0</td>	5	100	87	75	62	50	39	28	19	10	3.7	0
8       100       81       63       47       33       21       12       5.3       1.7       0.2       0         9       100       80       61       43       29       17       8.8       3.6       1.0       0.1       0         10       100       78       58       40       25       14       6.7       2.4       0.5       0         11       100       77       56       37       22       12       5.1       1.6       0.3       0         12       100       76       53       34       20       9.8       3.9       1.1       0.2       0         13       100       75       51       32       18       8.2       3.0       0.8       0.1       0         14       100       73       49       30       16       6.9       2.3       0.5       0.1       0         15       100       72       47       28       14       5.8       1.8       0.4       0         16       100       71       46       26       12       4.9       1.4       0.3       0         17       100		100										0
9 100 80 61 43 29 17 8.8 3.6 1.0 0.1 0 10 100 78 58 40 25 14 6.7 2.4 0.5 0  11 100 77 56 37 22 12 5.1 1.6 0.3 0 12 100 76 53 34 20 9.8 3.9 1.1 0.2 0 13 100 75 51 32 18 8.2 3.0 0.8 0.1 0 14 100 73 49 30 16 6.9 2.3 0.5 0.1 0 15 100 72 47 28 14 5.8 1.8 0.4 0 16 100 71 46 26 12 4.9 1.4 0.3 0 17 100 70 44 24 11 4.1 1.1 0.2 0 18 100 69 43 23 10 3.5 0.8 0.1 0 19 100 68 41 21 9.0 2.9 0.7 0.1 0 20 100 67 40 20 8.1 2.5 0.5 0.1 0 25 100 63 34 15 4.8 1.1 0.2 0 30 100 60 29 11 2.9 0.5 0.1 0 35 100 57 25 8.0 1.7 0.2 0 40 100 54 22 6.0 1.1 0.1 0.1 0 45 100 70 45 25 13 5.4 2.0 0.6 0 0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45  50 100 73 49 30 16 8.0 3.4 1.3 0.4 0.1  80 100 66 38 18 7.5 2.5 0.7 0.1 90 100 66 38 18 7.5 2.5 0.7 0.1 90 100 66 38 18 7.5 2.5 0.7 0.1 90 100 66 38 18 7.5 2.5 0.7 0.1 90 100 66 38 18 7.5 2.5 0.7 0.1	7	100	83	67	51	37	25	15	8.0	3.1	0.6	0
10         100         78         58         40         25         14         6.7         2.4         0.5         0           11         100         77         56         37         22         12         5.1         1.6         0.3         0           12         100         76         53         34         20         9.8         3.9         1.1         0.2         0           13         100         75         51         32         18         8.2         3.0         0.8         0.1         0           14         100         73         49         30         16         6.9         2.3         0.5         0.1         0           15         100         72         47         28         14         5.8         1.8         0.4         0           16         100         71         46         26         12         4.9         1.4         0.3         0           17         100         70         44         24         11         4.1         1.1         0.2         0           18         100         69         43         23         10         3.5         0.	8	100	81	63	47	33	21	12	5.3	1.7	0.2	0
11       100       77       56       37       22       12       5.1       1.6       0.3       0         12       100       76       53       34       20       9.8       3.9       1.1       0.2       0         13       100       75       51       32       18       8.2       3.0       0.8       0.1       0         14       100       73       49       30       16       6.9       2.3       0.5       0.1       0         15       100       72       47       28       14       5.8       1.8       0.4       0         16       100       71       46       26       12       4.9       1.4       0.3       0         17       100       70       44       24       11       4.1       1.1       0.2       0         18       100       69       43       23       10       3.5       0.8       0.1       0         19       100       68       41       21       9.0       2.9       0.7       0.1       0         25       100       63       34       15       4.8       1.1	9	100	80	61	43	29	17	8.8	3.6	1.0	0.1	0
12       100       76       53       34       20       9.8       3.9       1.1       0.2       0         13       100       75       51       32       18       8.2       3.0       0.8       0.1       0         14       100       73       49       30       16       6.9       2.3       0.5       0.1       0         15       100       72       47       28       14       5.8       1.8       0.4       0         16       100       71       46       26       12       4.9       1.4       0.3       0         17       100       70       44       24       11       4.1       1.1       0.2       0         18       100       69       43       23       10       3.5       0.8       0.1       0         19       100       68       41       21       9.0       2.9       0.7       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0         30       100       60       29       11       2.9       0.5       0       0	10	100	78	58	40	25	14	6.7	2.4	0.5		0
13       100       75       51       32       18       8.2       3.0       0.8       0.1       0         14       100       73       49       30       16       6.9       2.3       0.5       0.1       0         15       100       72       47       28       14       5.8       1.8       0.4       0         16       100       71       46       26       12       4.9       1.4       0.3       0         17       100       70       44       24       11       4.1       1.1       0.2       0         18       100       69       43       23       10       3.5       0.8       0.1       0         19       100       68       41       21       9.0       2.9       0.7       0.1       0         20       100       67       40       20       8.1       2.5       0.5       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0         30       100       60       29       11       2.9       0.5       0       0	11	100	77	56	37	22	12	5.1	1.6	0.3		0
14       100       73       49       30       16       6.9       2.3       0.5       0.1       0         15       100       72       47       28       14       5.8       1.8       0.4       0         16       100       71       46       26       12       4.9       1.4       0.3       0         17       100       70       44       24       11       4.1       1.1       0.2       0         18       100       69       43       23       10       3.5       0.8       0.1       0         19       100       68       41       21       9.0       2.9       0.7       0.1       0         20       100       67       40       20       8.1       2.5       0.5       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0       0         35       100       57       25       8.0       1.7       0.2       0       0         40       100       54       22       6.0       1.1       0.1       0       0         45	12	100	76	53	34	20	9.8	3.9	1.1	0.2		0
15       100       72       47       28       14       5.8       1.8       0.4       0         16       100       71       46       26       12       4.9       1.4       0.3       0         17       100       70       44       24       11       4.1       1.1       0.2       0         18       100       69       43       23       10       3.5       0.8       0.1       0         19       100       68       41       21       9.0       2.9       0.7       0.1       0         20       100       67       40       20       8.1       2.5       0.5       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0         30       100       60       29       11       2.9       0.5       0         35       100       57       25       8.0       1.7       0.2       0         40       100       54       22       6.0       1.1       0.1       0         45       100       51       19       4.5       0.6       0	13	100	75	51	32	18	8.2	3.0	0.8	0.1		0
16       100       71       46       26       12       4.9       1.4       0.3       0         17       100       70       44       24       11       4.1       1.1       0.2       0         18       100       69       43       23       10       3.5       0.8       0.1       0         19       100       68       41       21       9.0       2.9       0.7       0.1       0         20       100       67       40       20       8.1       2.5       0.5       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0         30       100       60       29       11       2.9       0.5       0         35       100       57       25       8.0       1.7       0.2       0         40       100       54       22       6.0       1.1       0.1       0         45       100       51       19       4.5       0.6       0.2       0.3       0.35       0.4       0.45         50       100       73       49       30       16 </td <td>14</td> <td>100</td> <td>73</td> <td>49</td> <td>30</td> <td>16</td> <td>6.9</td> <td>2.3</td> <td>0.5</td> <td>0.1</td> <td></td> <td>0</td>	14	100	73	49	30	16	6.9	2.3	0.5	0.1		0
17       100       70       44       24       11       4.1       1.1       0.2       0         18       100       69       43       23       10       3.5       0.8       0.1       0         19       100       68       41       21       9.0       2.9       0.7       0.1       0         20       100       67       40       20       8.1       2.5       0.5       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0         30       100       60       29       11       2.9       0.5       0         35       100       57       25       8.0       1.7       0.2       0         40       100       54       22       6.0       1.1       0.1       0         45       100       51       19       4.5       0.6       0       0         50       100       73       49       30       16       8.0       3.4       1.3       0.4       0.1         60       100       70       45       25       13       5.4       2.0	15	100	72	47	28	14	5.8	1.8	0.4			0
18       100       69       43       23       10       3.5       0.8       0.1       0         19       100       68       41       21       9.0       2.9       0.7       0.1       0         20       100       67       40       20       8.1       2.5       0.5       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0         30       100       60       29       11       2.9       0.5       0         35       100       57       25       8.0       1.7       0.2       0         40       100       54       22       6.0       1.1       0.1       0         45       100       51       19       4.5       0.6       0       0         50       100       73       49       30       16       8.0       3.4       1.3       0.4       0.1         60       100       70       45       25       13       5.4       2.0       0.6       0.2         70       100       68       41       22       9.7       3.7       1.2 <td>16</td> <td>100</td> <td>71</td> <td>46</td> <td>26</td> <td>12</td> <td>4.9</td> <td>1.4</td> <td>0.3</td> <td></td> <td></td> <td>0</td>	16	100	71	46	26	12	4.9	1.4	0.3			0
19       100       68       41       21       9.0       2.9       0.7       0.1       0         20       100       67       40       20       8.1       2.5       0.5       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0         30       100       60       29       11       2.9       0.5       0         35       100       57       25       8.0       1.7       0.2       0         40       100       54       22       6.0       1.1       0.1       0         45       100       51       19       4.5       0.6       0       0         50       100       73       49       30       16       8.0       3.4       1.3       0.4       0.1         60       100       70       45       25       13       5.4       2.0       0.6       0.2         70       100       68       41       22       9.7       3.7       1.2       0.3       0.1         80       100       66       38       18       7.5       2.5       0.7	17	100	70	44	24	11	4.1	1.1	0.2			0
20       100       67       40       20       8.1       2.5       0.5       0.1       0         25       100       63       34       15       4.8       1.1       0.2       0         30       100       60       29       11       2.9       0.5       0         35       100       57       25       8.0       1.7       0.2       0         40       100       54       22       6.0       1.1       0.1       0         45       100       51       19       4.5       0.6       0       0         50       100       73       49       30       16       8.0       3.4       1.3       0.4       0.1         60       100       70       45       25       13       5.4       2.0       0.6       0.2         70       100       68       41       22       9.7       3.7       1.2       0.3       0.1         80       100       66       38       18       7.5       2.5       0.7       0.1         90       100       64       35       16       5.9       1.7       0.4       0	18	100	69	43	23	10	3.5	0.8	0.1			0
25	19	100	68	41	21	9.0	2.9	0.7	0.1			0
30       100       60       29       11       2.9       0.5       0         35       100       57       25       8.0       1.7       0.2       0         40       100       54       22       6.0       1.1       0.1       0         45       100       51       19       4.5       0.6       0       0         50       100       73       49       30       16       8.0       3.4       1.3       0.4       0.1         60       100       70       45       25       13       5.4       2.0       0.6       0.2         70       100       68       41       22       9.7       3.7       1.2       0.3       0.1         80       100       66       38       18       7.5       2.5       0.7       0.1         90       100       64       35       16       5.9       1.7       0.4       0.1	20	100	67	40	20	8.1	2.5	0.5	0.1			0
35     100     57     25     8.0     1.7     0.2     0       40     100     54     22     6.0     1.1     0.1     0       45     100     51     19     4.5     0.6     0     0       50     100     73     49     30     16     8.0     3.4     1.3     0.4     0.1       60     100     70     45     25     13     5.4     2.0     0.6     0.2       70     100     68     41     22     9.7     3.7     1.2     0.3     0.1       80     100     66     38     18     7.5     2.5     0.7     0.1       90     100     64     35     16     5.9     1.7     0.4     0.1	25	100	63	34	15	4.8	1.1	0.2				0
40     100     54     22     6.0     1.1     0.1     0       45     100     51     19     4.5     0.6     0       0     0.05     0.1     0.15     0.2     0.25     0.3     0.35     0.4     0.45       50     100     73     49     30     16     8.0     3.4     1.3     0.4     0.1       60     100     70     45     25     13     5.4     2.0     0.6     0.2       70     100     68     41     22     9.7     3.7     1.2     0.3     0.1       80     100     66     38     18     7.5     2.5     0.7     0.1       90     100     64     35     16     5.9     1.7     0.4     0.1	30	100	60	29	11	2.9	0.5					0
45     100     51     19     4.5     0.6     0       0     0.05     0.1     0.15     0.2     0.25     0.3     0.35     0.4     0.45       50     100     73     49     30     16     8.0     3.4     1.3     0.4     0.1       60     100     70     45     25     13     5.4     2.0     0.6     0.2       70     100     68     41     22     9.7     3.7     1.2     0.3     0.1       80     100     66     38     18     7.5     2.5     0.7     0.1       90     100     64     35     16     5.9     1.7     0.4     0.1	35	100	57	25	8.0	1.7	0.2					0
0         0.05         0.1         0.15         0.2         0.25         0.3         0.35         0.4         0.45           50         100         73         49         30         16         8.0         3.4         1.3         0.4         0.1           60         100         70         45         25         13         5.4         2.0         0.6         0.2           70         100         68         41         22         9.7         3.7         1.2         0.3         0.1           80         100         66         38         18         7.5         2.5         0.7         0.1           90         100         64         35         16         5.9         1.7         0.4         0.1	40	100	54	22	6.0	1.1	0.1					0
50     100     73     49     30     16     8.0     3.4     1.3     0.4     0.1       60     100     70     45     25     13     5.4     2.0     0.6     0.2       70     100     68     41     22     9.7     3.7     1.2     0.3     0.1       80     100     66     38     18     7.5     2.5     0.7     0.1       90     100     64     35     16     5.9     1.7     0.4     0.1	45	100	51	19	4.5	0.6						0
60     100     70     45     25     13     5.4     2.0     0.6     0.2       70     100     68     41     22     9.7     3.7     1.2     0.3     0.1       80     100     66     38     18     7.5     2.5     0.7     0.1       90     100     64     35     16     5.9     1.7     0.4     0.1		0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	
60     100     70     45     25     13     5.4     2.0     0.6     0.2       70     100     68     41     22     9.7     3.7     1.2     0.3     0.1       80     100     66     38     18     7.5     2.5     0.7     0.1       90     100     64     35     16     5.9     1.7     0.4     0.1	50	100	73	49	30	16	8.0	3.4	1.3	0.4	0.1	
80 100 66 38 18 7.5 2.5 0.7 0.1 90 100 64 35 16 5.9 1.7 0.4 0.1	60	100	70	45	25	13	5.4	2.0	0.6	0.2		
90 100 64 35 16 5.9 1.7 0.4 0.1	70	100	68	41	22	9.7	3.7	1.2	0.3	0.1		
	80	100	66	38	18	7.5	2.5	0.7	0.1			
100 100 62 32 14 4.6 1.2 0.2	90	100	64	35	16	5.9	1.7	0.4	0.1			
	100	100	62	32	14	4.6	1.2	0.2				