Deep Reinforcement Learning

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Most slides are based on Bhiksha Raj, 11-785, CMU 2019,
some slides from Fei Fei Li and colleagues lectures, cs231n, Stanford 2018,
and some from Surguy Levin lectures, cs294-112, Berkeley 2016.

Q-Learning

- Currently most-popular RL algorithm
- Topics not covered:
  - Value function approximation
  - Continuous state spaces
  - Deep-Q learning

Scaling up the problem..

- We've assumed a discrete set of states
- And a discrete set of actions

- Value functions can be stored as a table
  - One entry per state
- Action value functions can be stored as a table
  - One entry per state-action combination
- Policy can be stored as a table
  - One probability entry per state-action combination

- None of this is feasible if
  - The state space grows too large (e.g. chess)
  - Or the states are continuous valued

Problem

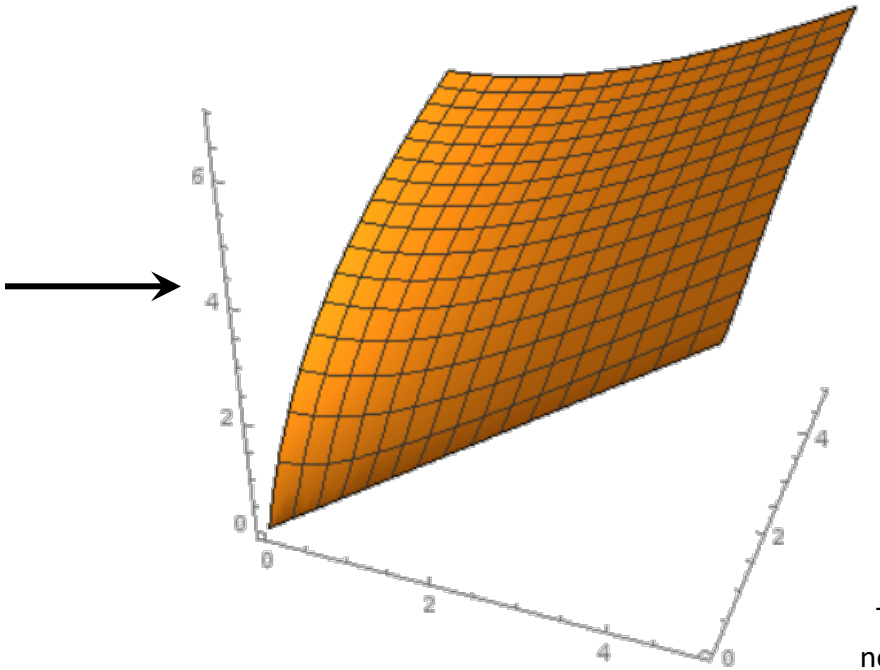
- Not scalable.
  - Must compute Q(s,a) for every state-action pair.
    - it computationally infeasible to compute for entire state space!

- Solution: use a function approximator to estimate Q(s,a).
  - E.g. a neural network!

Continuous State Space

- Tabular methods won't work if our state space is infinite or huge
- E.g. position on a [0, 5] x [0, 5] square, instead of a 5x5 grid.

4.4	4.5	4.8	5.3	5.9
3.9	4.0	4.4	4.9	5.6
3.2	3.4	3.8	4.0	5.1
2.2	2.4	3.0	3.7	4.6
0	1.0	2.0	3.0	4.0



The graphs show the negative value function

Parameterized Functions

- Instead of using a table of Q-values, we use a parametrized function:

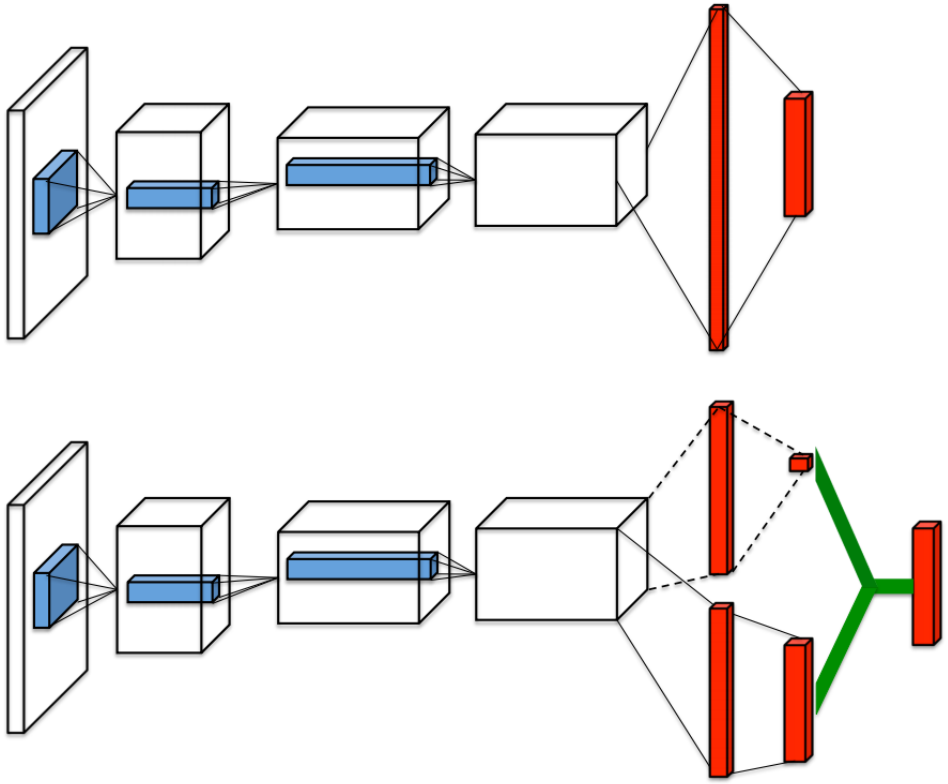
$$Q(s, a|\theta)$$

- If the function approximator is a deep network => Deep RL

- Instead of writing values to the table, we fit the parameters to minimize the prediction error of the “Q function”

$$\theta_{k+1} \leftarrow \theta_k - \eta \nabla_\theta \left(Loss(Q(s, a|\theta_k), Q_{\mathrm{s,a}}^{\mathrm{target}}) \right)$$

Parameterized Functions



Case Study: Playing Atari Games (seen before)

Objective: Complete the game with the highest score

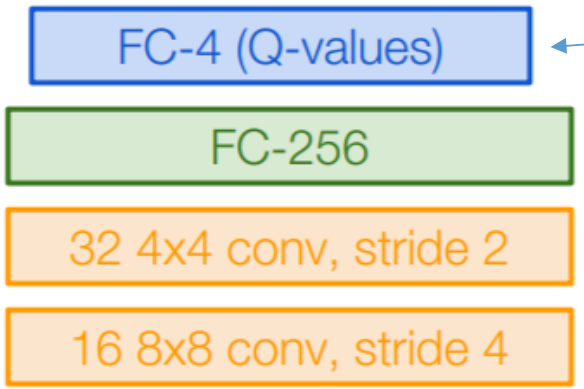
State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ

A single feedforward pass to compute Q-values for all actions from the current state => efficient!



Last FC layer has 4-d output (if 4 actions)
Q(st ,a1), Q(st,a2), Q(st,a3), Q(st,a4)

Number of actions between 4-18 depending on Atari game

Current state s_t: 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

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Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)} \left[(y_i - Q(s,a;\theta_i))^2 \right]$

Iteratively try to make the Q-value close to the target value (yi) it should have (according to Bellman Equations).

$$y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) | s, a \right]$$

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Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[(r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) - Q(s,a;\theta_i)) \nabla_{\theta_i} Q(s,a;\theta_i) \right]$$

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Target Q

$$\theta_{i+1} \leftarrow \theta_i - \eta \nabla_\theta \left(Loss(Q(s, a|\theta_i), Q_{\text{s,a}}^{\text{target}}) \right)$$

→ What is $Q_{\text{s,a}}^{\text{target}}$?

As in TD, use bootstrapping for the target :

$$Q_{\text{s,a}}^{\text{target}} = r + \gamma \underset{a' \in \mathcal{A}}{\text{argmax}}\, Q(s', a'|\theta_i)$$

And $Loss$ can be L2 distance

DQN (v0)

- Initialize θ_1
- For each episode e
  - Initialize s_1
  - For $t = 1 \ldots Termination$
    - Choose action a_t using ε –greedy policy obtained from θ_t
    - Observe r_t, s_{t+1}
    - $Q_{target} = r_t + \gamma max_a Q(s_{t+1}, a|\theta_t)$
    - $\theta_{t+1} = \theta_t - \eta \nabla_\theta \left\| Q_{target} - Q(s_t, a_t|\theta_t) \right\|_2^2$

Deep Q Network

- Note : $\nabla_\theta \left\| Q_{target} - Q(s_t, a_t|\theta_t) \right\|_2^2$ does **not** consider Q_{target} as depending of θ_t (although it does). Therefore this is **semi-gradient descent**.

Training the Q-network: Experience Replay

- Learning from batches of **consecutive samples** is problematic:
  - Samples are correlated => inefficient learning
  - Current Q-network parameters determines next training samples
    - can lead to bad feedback loops
    - e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand side => can lead to bad feedback loops

- Address these problems using **experience replay**
  - Continually update a replay memory table of transitions (s_t, a_t, r_t, s_{t+1})
  - Train Q-network on random minibatches of transitions from the replay memory
    - ✓ Each transition can also contribute to multiple weight updates => greater data efficiency
    - ✓ Smoothing out learning and avoiding oscillations or divergence in the parameters

Parameterized Functions

- Fundamental issue: limited capacity
  - A table of Q values will never forget any values that you write into it
  - But, modifying the parameters of a Q-function will affect its *overall* behavior
    - Fitting the parameters to match one (s, a) pair can change the function's output at (s', a').
    - If we don't visit (s', a') for a long time, the function's output can diverge considerably from the values previously stored there.

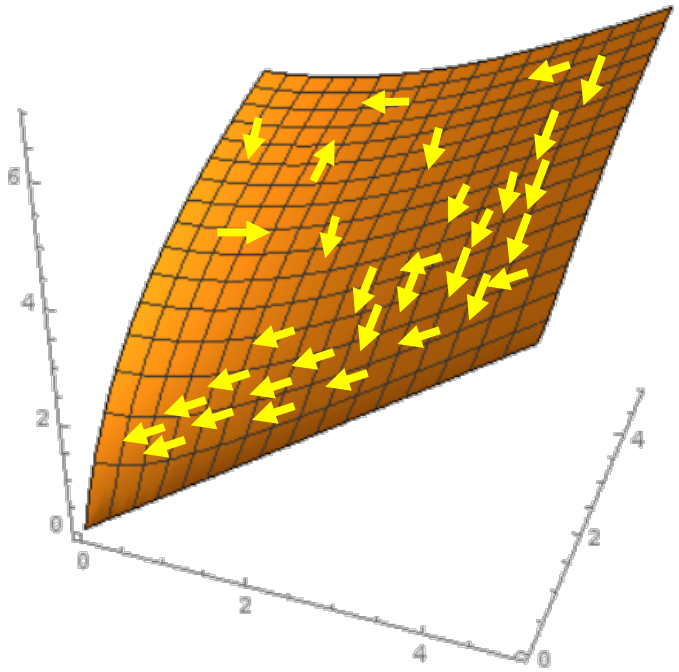
Tables have full capacity

- Q-learning works well with Q-tables
  - The sample data is going to be heavily biased toward optimal actions $\big(s, \pi^*(s)\big)$, or close approximations thereof.
  - But still, ϵ-greedy policy will ensure that we will visit all state-action pairs arbitrarily many times if we explore long enough.
  - The action-value for uncommon inputs will still converge, just more slowly.

Limited Capacity of $Q(s, a|\theta)$

- The Q-function will fit more closely to more common inputs, even at the expense of lower accuracy for less common inputs.

- Just exploring the whole state-action space isn't enough. We also need to visit those states often enough so the function computes accurate Q-values before they are "forgotten".



Experience Replay

- The raw data obtained from Q-learning is:
  - Highly correlated: current data can look very different from data from several episodes ago if the policy changed significantly.
  - Very unevenly distributed: only ϵ probability of choosing suboptimal actions.

- Instead, create a *replay buffer* holding past experiences, so we can train the Q-function using this data.

Experience Replay

- We have control over how the experiences are added, sampled and deleted.
  - Can make the samples look independent
  - Can emphasize old experiences more
  - Can change frequency depending on accuracy

- What is the best way to sample? (A trade off!)
  - On the one hand, our function has limited capacity, so we should let it optimize more strongly for the common case
  - On the other hand, our function needs explore uncommon examples just enough to compute accurate action-values, so it can avoid missing out on better policies

DQN (with Experience Replay)

- Initialize θ_0
- Initialize buffer with some random episodes
- For each episode e
  - Initialize s_1, a_1
  - For $t = 1 \ldots Termination$
    - Choose action a_t using ε –greedy policy obtained from θ_t
    - Observe r_t, s_{t+1}
    - Add s_t, a_t, r_t, s_{t+1} to the buffer
    - Sample from the buffer a batch of tuples s, a, r, s_{new}
    - $Q_{target} = r + \gamma max_a Q(s_{\text{new}}, a|\theta_t)$
    - $\theta_{t+1} = \theta_t - \eta \nabla_\theta \left\| Q_{target} - Q(s, a|\theta_t) \right\|_2^2$

Moving target

- We already have moving targets in Q-learning itself

- The problem is much worse with Q-functions though. Optimizing the function at one state-action pair affects *all other state-action pairs*.
  - The target value is fluctuating at all inputs in the function's domain, and all updates will shift the target value across the entire domain.

Frozen target function

- Solution : Create two copies of the Q-function.
  - The "target copy" is frozen and used to compute the target Q-values.
  - The "learner copy" will be trained on the targets.

$$Q_{\text{learner}}(s_t, a_t) \leftarrow_{\text{fit}} r_t + \gamma \max_{\text{a}} \left(Q_{\text{target}}(s_{t+1}, a) \right)$$

- Just need to periodically update the target copy to match the learner copy.

Fixed target DQN

- Initialize $\theta_0, \theta^* = \theta_0$
- Initialize buffer with some random episodes
- For each episode e
  - Initialize S_1, A_1
    - For $t = 1 \ldots Termination$
    - If $t\%k = 0$ then update $\theta^* = \theta_t$
    - Choose action a_t using ε –greedy policy obtained from θ_t
    - Observe r_t, s_{t+1}
    - Add s_t, a_t, r_t, s_{t+1} to the buffer
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Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
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    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

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Initialize replay memory, Q-network

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Play M episodes (full games)

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

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Initialize state (starting game screen pixels) at the beginning of each episode

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

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For each time-step of game

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

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With small probability, select a random action (explore), otherwise select greedy action from current policy

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

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Take the selected action observe the reward and next state

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

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Store transition in replay memory

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

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Sample a random minibatch of transitions and perform a gradient descent step

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Performance

	Breakout	R. Raid	Enduro	Sequest	S. Invaders
DQN	**316.8**	**7446.6**	**1006.3**	**2894.4**	**1088.9**
Naive DQN	3.2	1453.0	29.1	275.8	302.0
Linear	3.0	2346.9	62.0	656.9	301.3

Replay	○	○	×	×
Target	○	×	○	×
Breakout	**316.8**	240.7	10.2	3.2
River Raid	**7446.6**	4102.8	2867.7	1453.0
Seaquest	**2894.4**	822.6	1003.0	275.8
Space Invaders	**1088.9**	826.3	373.2	302.0

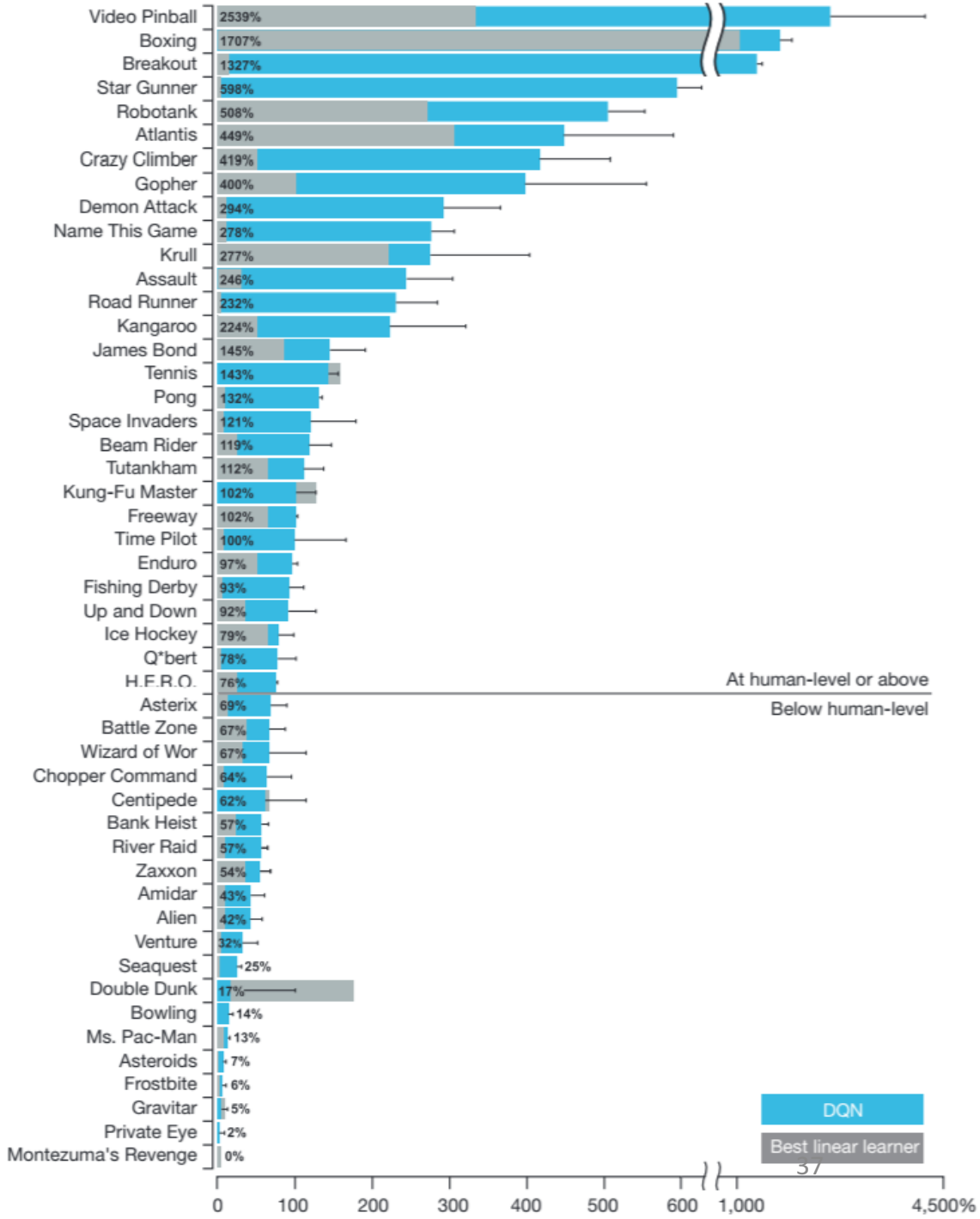
https://www.youtube.com/watch?v=V1eYniJ0Rnk

https://www.youtube.com/watch?v=V1eYniJ0Rnk

Video by Károly Zsolnai-Fehér. Reproduced with permission.

Results on 49 Games

- The architecture and hyperparameter values were the same for all 49 games.
- DQN achieved performance comparable to or better than an experienced human on 29 out of 49 games.



[V. Mnih et al., Human-level control through deep reinforcement learning, Nature 2015]

Policy Gradients

- What is a problem with Q-learning?
  - The Q-function can be very complicated!
- Hard to learn exact value of every (state, action) pair
- But the policy can be much simple
- Can we learn a policy directly, e.g. finding the best policy from a collection of policies?

Direct Policy Estimation

- It's also possible to make a deep neural network that directly produces a distribution over actions given a state
  - Also known as a policy network, or the policy gradient method
  - Useful when **the action space is also large or continuous**

Policy Network

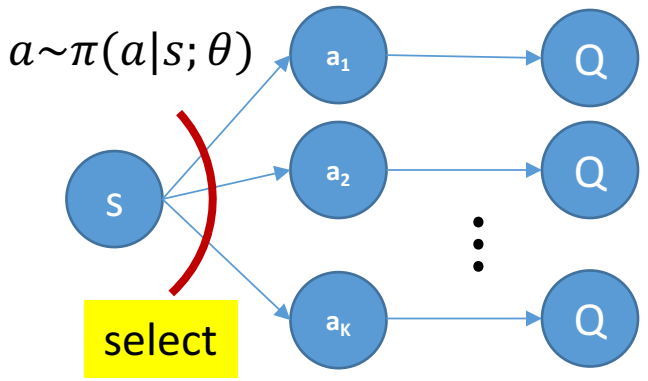
- Train a neural network to prescribe actions at each state:

$$\pi(a|s;\theta)$$

  - Input is s, output is probability distribution over a
  - Could be deterministic
- **Problem** : how to train such a network?
- No golden truth
  - Unlike *value* functions, where there is a *target* value for the value at each state
    - Against which we can compute a loss

Maximizing return

- ***Learn policy to maximize expected return!***



- **Problem:** For discrete action space, the return is not differentiable with respect to policy function parameters
  - Selection is not a differentiable operation

How to choose policy

- In any run starting at a state S we get
  - $(s_1 = s,)\ a_1\ r_1\ s_2\ a_2\ r_2\ s_3\ a_3\ r_3\ \ldots$
- The trajectory T associated with the run is
  - $\tau = s\ a_1\ r_1\ s_2\ a_2\ r_2\ s_3\ a_3\ r_3\ \ldots$
- The total return over the run (at t=1) is
  - $G = r_1 + \gamma r_2 + \gamma^2 r_3 \ldots.$
- The choice of θ in $\pi(a|s;\theta)$ will modify the trajectory and thereby the return

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

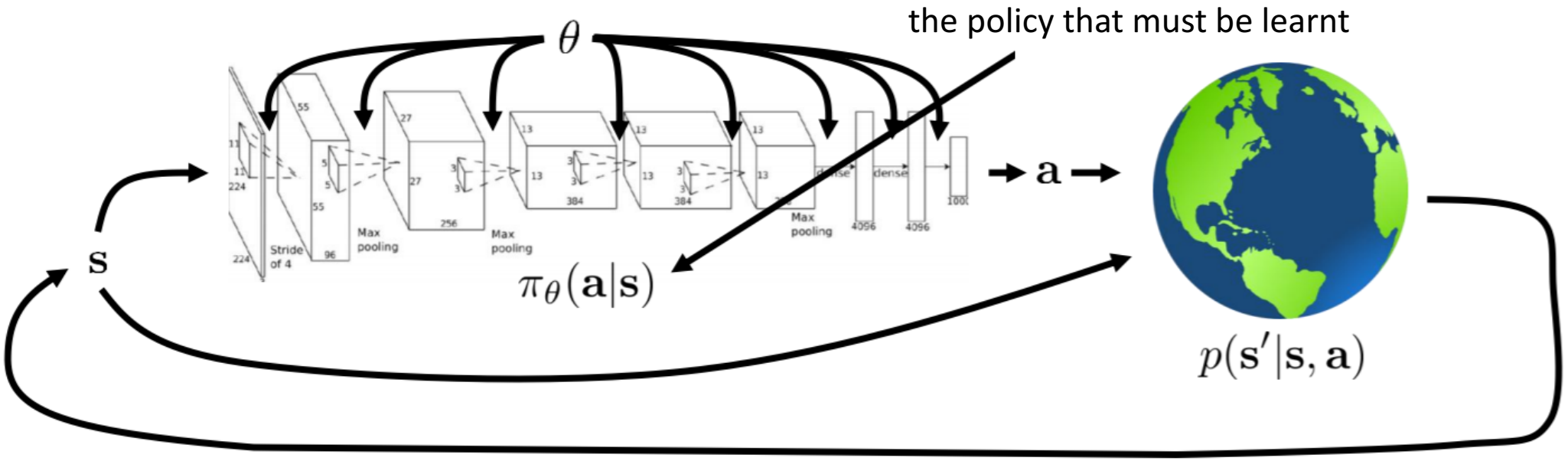
$$J(\theta) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi_\theta\right]$$

We want to find the optimal policy $\theta^* = \arg\max_\theta J(\theta)$

How can we do this?

Gradient ascent on policy parameters!

The goal of RL



$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)}_{p_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\theta^\star = \arg\max_\theta E_{\tau \sim p_\theta(\tau)} \left[\sum_t \gamma^t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

The objective

- The probability of a trajectory τ is a function of $\pi(a|s;\theta)$ and hence of θ
  - $\tau \sim P(\tau;\theta)$
- The probability of a return G is a function of the trajectory τ
  - $G(\tau)$
- Objective: to maximize expected return

$$\underset{\theta}{\operatorname{argmax}} J(\theta) = \underset{\theta}{\operatorname{argmax}} \sum_{\tau} P(\tau;\theta)\, G(\tau)$$

REINFORCE algorithm

Expected reward: $$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[G(\tau)\right]$$
$$= \int_{\tau} G(\tau) p(\tau;\theta) \mathrm{d}\tau$$

Now let's differentiate this: $$\nabla_\theta J(\theta) = \int_{\tau} G(\tau) \nabla_\theta p(\tau;\theta) \mathrm{d}\tau$$

Intractable! Gradient of an expectation is problematic when p depends on θ

Solution

- Recast differentiation as an *expectation* operation
  - Can now be approximated by sampling
  - Policy gradient method

- Compute expected returns using an action-value function approximator
  - Actor-critic methods

Gradient of the objective

$$J(\theta) = E_{\tau \sim P(\tau;\theta)}[G(\tau)] = \sum_{\tau} P(\tau;\theta)\, G(\tau)$$

$$\nabla_\theta J(\theta) = \sum_{\tau} \nabla_\theta P(\tau;\theta)\, G(\tau)$$

- A simple trick:

$$\nabla_\theta P(\tau;\theta) = P(\tau;\theta)\frac{\nabla_\theta P(\tau;\theta)}{P(\tau;\theta)} = P(\tau;\theta)\nabla_\theta \log P(\tau;\theta)$$

$$\nabla_\theta J(\theta) = \sum_{\tau} P(\tau;\theta)\nabla_\theta \log P(\tau;\theta)\, G(\tau)$$

$$\nabla_\theta J(\theta) = E_{\tau \sim P(\tau;\theta)} \nabla_\theta \log P(\tau;\theta)\, G(\tau)$$

We can estimate it with Monte Carlo sampling

The trajectory

- The trajectory: $\tau = s\ a_1\ r_1\ s_2\ a_2\ r_2\ s_3\ a_3\ r_3\ \ldots$
- The probability of τ, under the policy function $\pi(a|s;\theta)$ is

$$P(\tau;\theta) = \prod_{t\geq 1} P(s_t|s_{t-1})\pi(a_t|s_t;\theta)$$
$$= P(s_1)\pi(a_1|s_1;\theta)P(s_2|s_1,a_1)\pi(a_2|s_2;\theta)\ldots$$

- Taking logs

$$\log P(\tau;\theta) = \sum_{t\geq 1} \log P(s_{t+1}|s_t,a_t) + \sum_{t\geq 1} \log \pi(a_t|s_t;\theta)$$

- Giving us the deriviative

$$\nabla_\theta \log P(\tau;\theta) = \sum_t \nabla_\theta \log \pi(a_t|s_t;\theta)$$

Does not depend on transition probabilities

Gradient of the objective

$$\nabla_\theta J(\theta) = E_{\tau \sim P(\tau;\theta)} \left[\left(\sum_t \nabla_\theta \log \pi(a_t | s_t; \theta) \right) G(\tau) \right]$$

- This is a simple expectation that can be approximated by sampling!

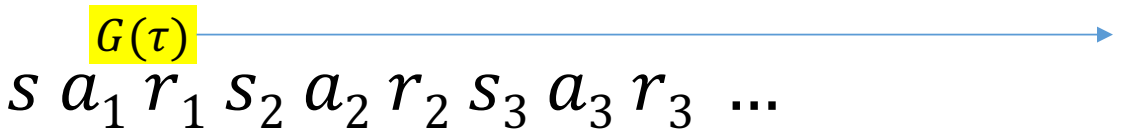
Policy Gradients

- Record an episode (or episodes)

$$s\ a_1\ r_1\ s_2\ a_2\ r_2\ s_3\ a_3\ r_3\ \ldots$$

Policy Gradients

- Episode



- Compute returns at each time

Policy Gradients

- Episode

$G(\tau)$ →

$$s\ a_1\ r_1\ s_2\ a_2\ r_2\ s_3\ a_3\ r_3\ \ldots$$

$\log \pi(a_1|s_1;\theta)$ $\log \pi(a_2|s_2;\theta)$ $\log \pi(a_3|s_3;\theta)$

- Compute returns at each time
- Compute log policy at each time

Policy Gradients

- Episode

$G(\tau)$ →

$$s\ a_1\ r_1\ s_2\ a_2\ r_2\ s_3\ a_3\ r_3\ \ldots$$

$\log \pi(a_1|s_1;\theta)$ $\log \pi(a_2|s_2;\theta)$ $\log \pi(a_3|s_3;\theta)$

$$\nabla_\theta J(\theta) \approx \sum_t \nabla_\theta \log \pi(a_t|s_t;\theta) G(\tau)$$

- Compute returns at each time
- Compute log policy at each time
- Compute gradient

Policy Gradients

- Episode

$G(\tau)$

$$s\ a_1\ r_1\ s_2\ a_2\ r_2\ s_3\ a_3\ r_3\ \ldots$$

$\log \pi(a_1|s_1;\theta)$ $\log \pi(a_2|s_2;\theta)$ $\log \pi(a_3|s_3;\theta)$

$$\nabla_\theta J(\theta) \approx \sum_t \nabla_\theta \log \pi(a_t|s_t;\theta) G(\tau)$$

$$\theta = \theta + \eta \nabla_\theta J(\theta)$$

- Compute returns at each time
- Compute log policy at each time
- Compute gradient
- Update network parameters
  - Ideally $\nabla_\theta J(\theta)$ is averaged over many episodes

Its like Maximum Likelihood

- The gradient actually looks like the derivative of a log likelihood function

$$\nabla_\theta J(\theta) = E_{\tau \sim P(\tau;\theta)}[\nabla_\theta \log P(\tau;\theta)\, G(\tau)]$$

- Can be written as

$$\nabla_\theta J(\theta) = E_{\tau \sim P(\tau;\theta)}\left[\nabla_\theta \log P(\tau;\theta)^{G(\tau)}\right]$$

- Maximization increases the probability of trajectories with greater return
  - If you see a trajectory you increase its probability

Its like Maximum Likelihood

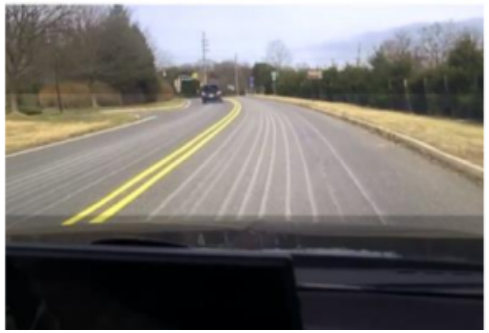
- The gradient actually looks like the derivative of a log likelihood function

$$\nabla_\theta J(\theta) \approx \sum_t \nabla_\theta \log \pi(a_t|s_t;\theta)^{G(\tau)}$$

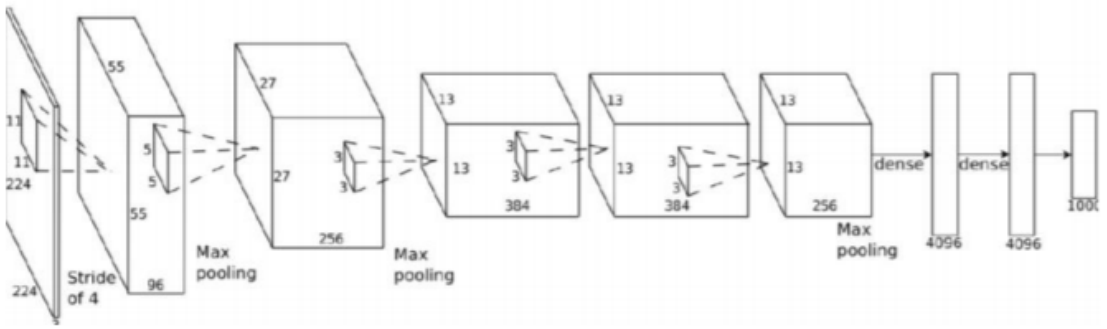
- Maximization increases the probability of all seen actions
  - At the cost of the probability of unseen actions
  - Usual ML estimator

Evaluating the policy gradient

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\underbrace{\sum_{t\geq 0}\gamma^t r\left(s_t^{(n)}, a_t^{(n)}\right)}_{G\left(\tau^{(n)}\right)}\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$



s_t



$\pi_\theta(a_t|s_t)$

a_t

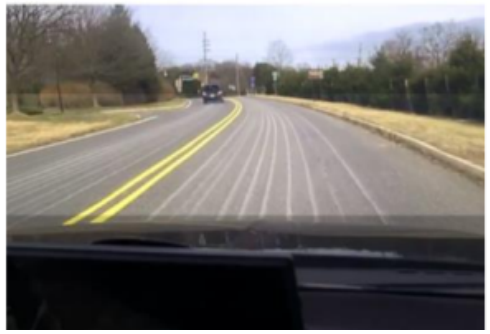
s_t

- Policy gradient:

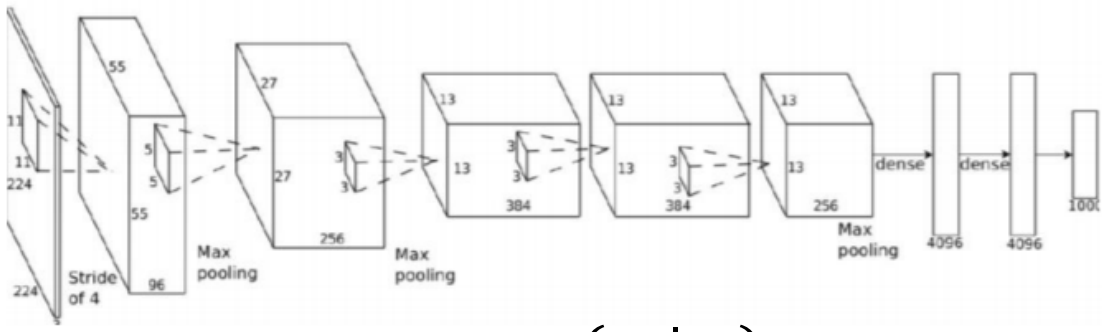
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} G\left(\tau^{(n)}\right) \sum_{t\geq 0} \nabla_\theta \log \pi_\theta \left(a_t^{(n)} | s_t^{(n)}\right)$$

- Maximum Likelihood:

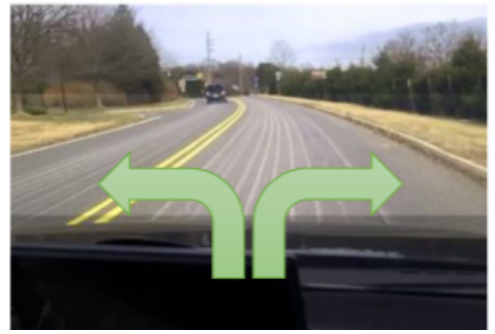
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} \sum_{t\geq 0} \nabla_\theta \log \pi_\theta \left(a_t^{(n)} | s_t^{(n)}\right)$$



s_t



$\pi_\theta(a_t|s_t)$



a_t

$\mathbf{s}_t$

$\mathbf{a}_t$

$\pi_\theta(a_t|s_t)$

What did we just do?

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} G\left(\tau^{(n)}\right)\nabla_\theta \log p_\theta\left(\tau^{(n)}\right)$$

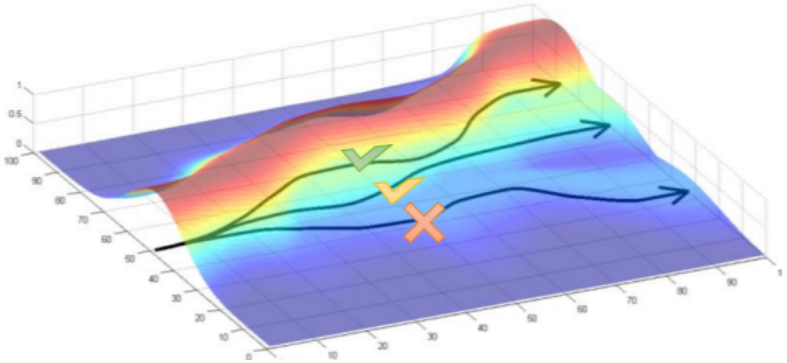
$$\sum_{t\geq 0} \nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$$

$$\nabla_\theta J_{ML}(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} \nabla_\theta \log p_\theta\left(\tau^{(n)}\right)$$

good stuff is made more likely

bad stuff is made less likely

simply formalizes the notion of “trial and error”!



Intuition

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} G\left(\tau^{(n)}\right) \sum_{t\geq 0} \nabla_\theta \log \pi_\theta \left(a_t^{(n)} | s_t^{(n)}\right)$$

Interpretation:

- If r(τ) is high, push up the probabilities of the actions seen
- If r(τ) is low, push down the probabilities of the actions seen

Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

- However, this also suffers from high variance
  - because credit assignment is really hard.

A simple extension

$$\nabla_\theta J(\theta) = E_{\tau \sim P(\tau;\theta)} \left[\left(\sum_t \nabla_\theta \log \pi(a_t | s_t; \theta) \right) G(\tau) \right]$$

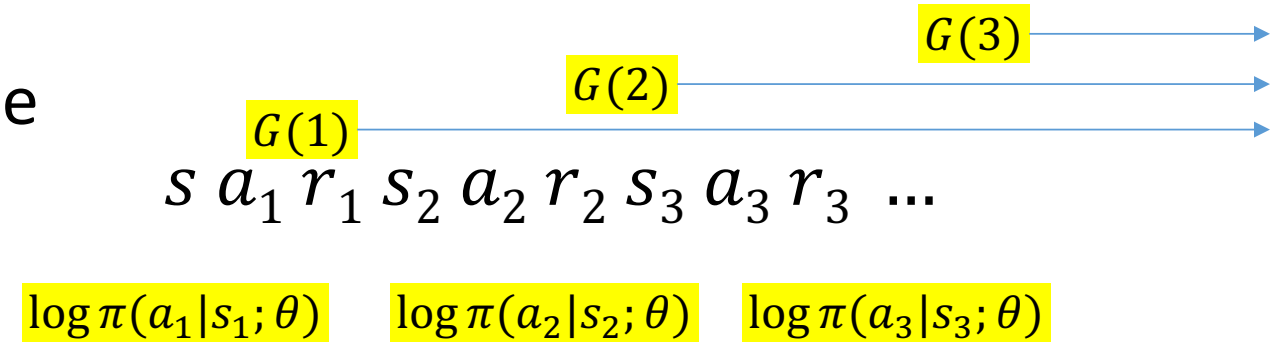
- Better to compute the above instead as follows

$$\nabla_\theta J(\theta) = E_{\tau \sim P(\tau;\theta)} \left[\sum_t \nabla_\theta \log \pi(a_t | s_t; \theta)\, G(t) \right]$$

- This too can be estimated by sampling

Policy Gradients: A simple extension

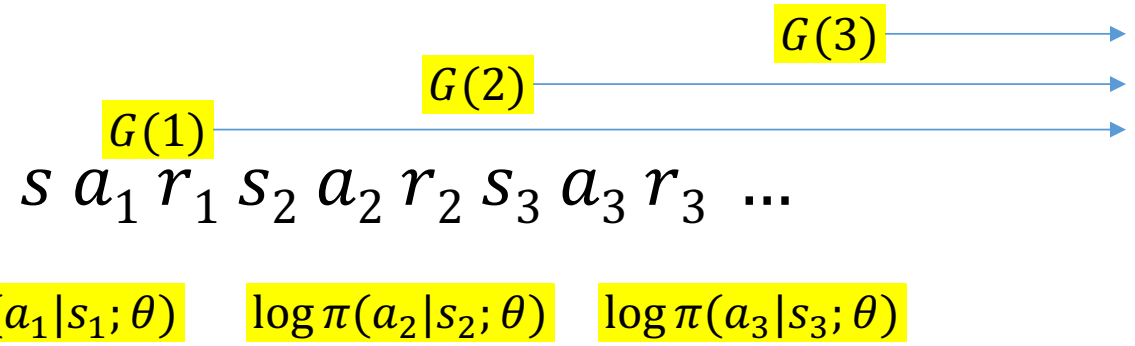
- Episode



- Compute returns at each time
- Compute log policy at each time

Policy Gradients: A simple extension

- Episode

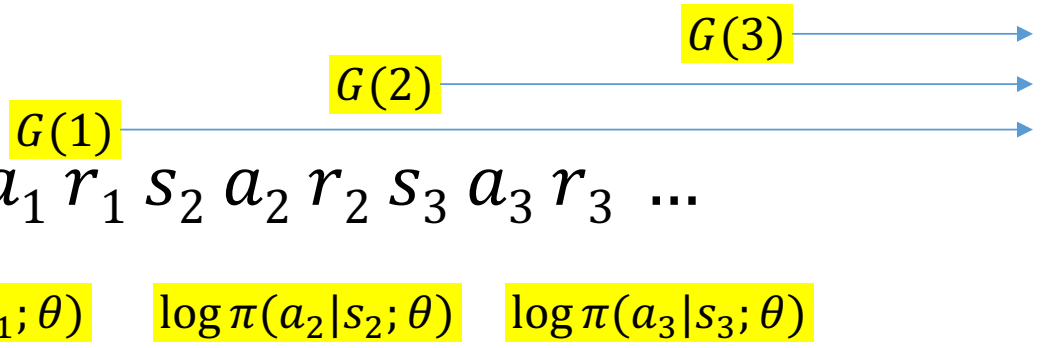


$$\nabla_\theta J(\theta) \approx \frac{1}{\tau}\sum_t \nabla_\theta \log \pi(a_t|s_t;\theta) G(t)$$

- Compute returns at each time
- Compute log policy at each time
- Compute gradient

Policy Gradients: A simple extension

- Episode



$$\nabla_\theta J(\theta) \approx \frac{1}{\tau}\sum_t \nabla_\theta \log \pi(a_t|s_t;\theta) G(t)$$

$$\theta = \theta + \eta \nabla_\theta J(\theta)$$

- Compute returns at each time
- Compute log policy at each time
- Compute gradient
- Update network parameters
  - Ideally $\nabla_\theta J(\theta)$ is averaged over many episodes

Reducing variance

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\gamma^t r\left(s_t^{(n)}, a_t^{(n)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

$G\left(\tau^{(n)}\right)$

- Causality:

policy at time t' cannot affect reward at time t when $t < t'$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\left(\sum_{t'\geq t}\gamma^t r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right)\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Variance reduction

Gradient estimator: $$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0} r\left(s_t^{(n)}, a_t^{(n)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\sum_{t'\geq t} r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Second idea: Use discount factor γ to ignore delayed effects

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\sum_{t'\geq t} \gamma^{t'-t} r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Merely seeing a trajectory isn't good

- We want to emphasize trajectories with high return and *reduce* the probability of low-return trajectories

- If an action results in more returns than the current average return for the state, we must improve its probability
  - If it results in less, we must decrease it

Variance reduction: Baseline

- **Problem:** The raw value of a trajectory isn't necessarily meaningful.
  - For example, if rewards are all positive, you keep pushing up probabilities of actions.
- **What is important then**?
  - Whether a reward is better or worse than what you expect to get

- **Idea**: Introduce a baseline function dependent on the state.

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\left(\overbrace{\sum_{k=0}^{T-t}\gamma^k r\left(s_{t+k}^{(n)}, a_{t+k}^{(n)}\right)}^{G\left(s_t^{(n)}\right)} - V\left(s_t^{(n)}\right)\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Simple baseline: $V\left(s_t^{(n)}\right) = \bar{G}(s_t)$

A simple baseline: constant moving average of rewards experienced so far from all trajectories

Its like Maximum Likelihood

- Subtract the *expected* return for the current state

$$\nabla_\theta J(\theta) \approx \sum_t \nabla_\theta \log \pi(a_t|s_t;\theta)^{G(t)-V(S_t)}$$

- $A(t) = G(t) - V(S_t)$ is the *advantage function*
  - How much advantage the current action has over the average
- Train $\pi(a_t|s_t;\theta)$ to maximize advantage

Reinforce

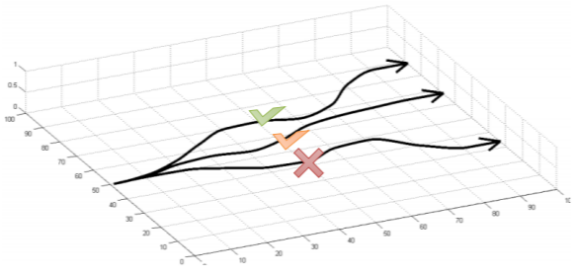
- Initialize θ
- For each episode e
  - Initialize s_1
  - For $t = 1 \ldots Termination$
    - Choose action a_t using policy obtained from θ
    - Observe r_t, s_{t+1}
    - Compute the returns $G(S_t)$, then the advantages A_t
    - Compute $J(\theta) = \frac{1}{T}\sum_t \log\big(\pi_\theta(a_t|s_t)\big) A_t$
    - $\theta \leftarrow \theta + \eta \nabla_\theta J(\theta)$

REINFORCE algorithm: Summary

$$\nabla_\theta J(\theta) = \int_\tau \left(G(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\mathrm{d}\tau$$
$$= \mathbb{E}_{\tau\sim p(\tau;\theta)}\left[G(\tau)\nabla_\theta \log p(\tau;\theta)\right]$$

$$\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} G(\tau^{(n)})\, \nabla_\theta \log p\left(\tau^{(n)};\theta\right)$$



$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\left(G(s_t^{(n)}) - V(s_t^{(n)})\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Instability

- In Reinforce, the estimator for the expected return has high variance: rewards on one episode act as estimates for state action value functions.

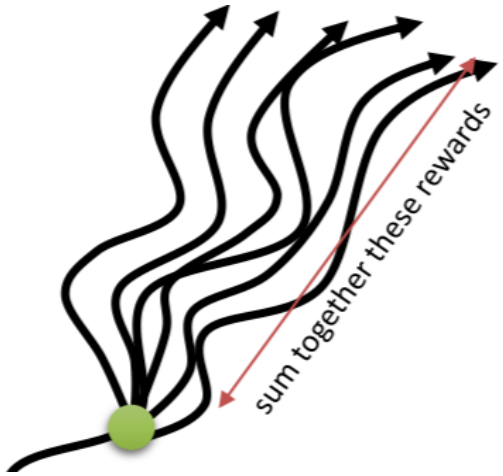
$$G(S_t) = \sum_{t' \geq t} \gamma^{t'-t} r_{t'}$$

- It also requires entire runs of episodes
  - Not online
- It can be made more stable through function approximation of the value function

Actor-Critic

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\left(\sum_{k=0}^{T-t}\gamma^k r\left(s_{t+k}^{(n)}, a_{t+k}^{(n)}\right)\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Instead of this we can use the temporal difference to estimate this average and also the baseline



Actor-Critic

- In actor-critic methods, two networks are used :
- The **actor** is the policy network : $\pi(s, a|\theta) = \pi_\theta(a|s)$ and is used to predict the next action
- The **critic** is a state value network : $g(s|\phi) = V_\phi(s)$ and is used to guide the optimization direction of the actor

- To estimate the expected return based on an episode, we use a one-step lookahead:

$$G(S_t) = r_t + \gamma V_\phi(s_{t+1})$$

- Or by M-step lookahead :

$$G(S_t) = \sum_{0 \le k \le M-1} \gamma^k r_{t+k} + \gamma^M V_\phi(s_{t+N})$$

Advantage Actor Critic (A2C)

Rethink the advantages

The critic can also be used as the "baseline" when computing the advantages:

$$A_t = G(S_t) - V_\phi(S_t)$$

The trajectory's probability is improved if it is better than the trajectories previously followed.

The critic is trained on how well it predicted the return.

Another view

- $E[G_t] = Q(s_t, a_t)$
- To push up the probability of an action from a state:
  - if this action was better than the expected value of what we should get from that state.

$$Q(s_t, a_t) - V(s_t)$$

- We are happy with an action a_t in a state s_t if it is large
- we are unhappy with an action if it's small

Another view

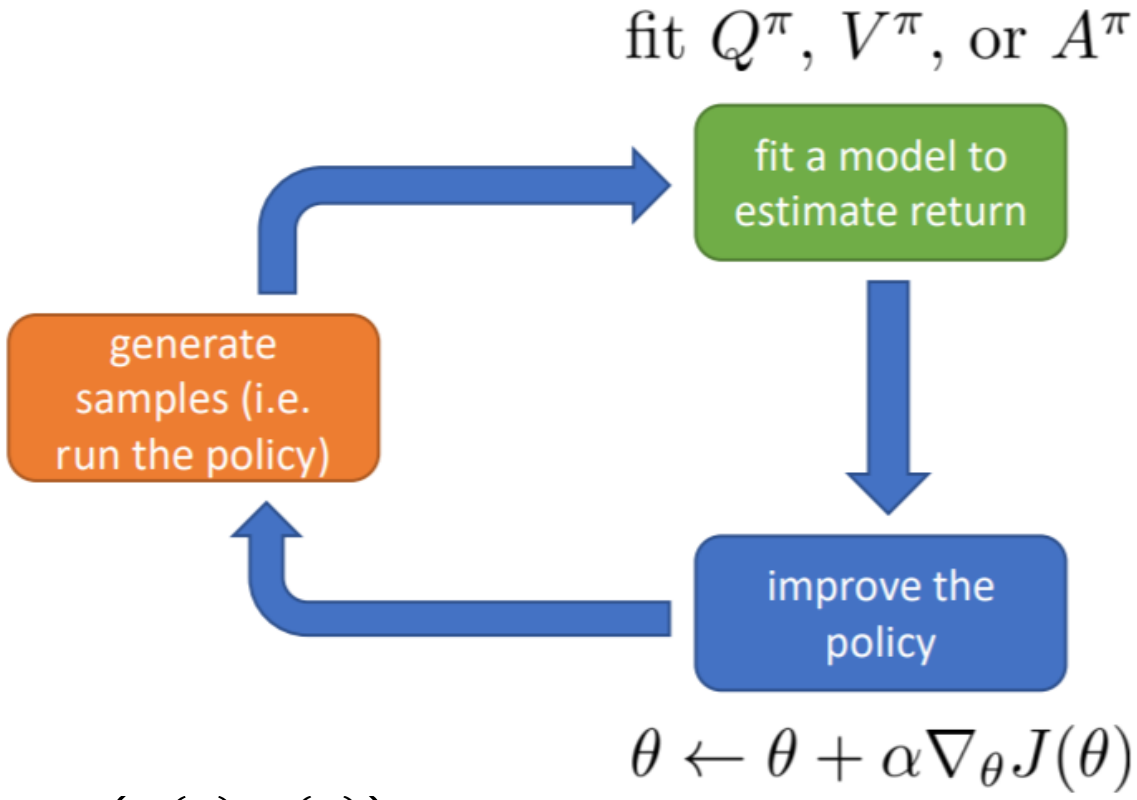
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\left(\underbrace{\sum_{k=0}\gamma^k r\left(s_{t+k}^{(n)}, a_{t+k}^{(n)}\right)}_{\text{Reward to go } \hat{Q}_t^{(n)}}\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

- $\hat{Q}_t^{(n)}$: estimate of expected reward if we take action $a_t^{(n)}$ in state $s_t^{(n)}$
- $Q(s_t, a_t) = \sum_{k=0}^{T-t} E_{p_\theta}[\gamma^k r(s_{t+k}, a_{t+k})|s_t, a_t]$
  - True expected reward to go
- $V(s_t) = E_{a_t \sim \pi_\theta(a_t|s_t)} Q(s_t, a_t)$
- $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\left(Q\left(s_t^{(n)}, a_t^{(n)}\right) - V\left(s_t^{(n)}\right)\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$

Another view

- $Q^{\pi}(s_t, a_t) = \sum_{k=0}^{T-t} E_{p_\theta}[\gamma^k r(s_{t+k}, a_{t+k}) | s_t, a_t]$
  - True expected reward to go
- $V^{\pi}(s_t) = E_{a_t \sim \pi_\theta(a_t|s_t)} Q(s_t, a_t)$
  - Total reward from s_t
- $A^{\pi}(s_t, a_t) = Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$
  - How much better a_t is

Remark: we can define by the advantage function how much an action was better than expected



- $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t \geq 0} A^{\pi}\left(s_t^{(n)}, a_t^{(n)}\right) \nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$

Value function fitting

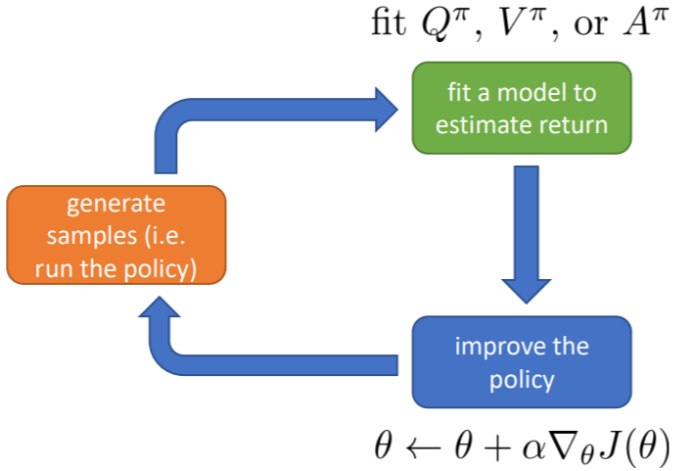
fit *what* to *what*?

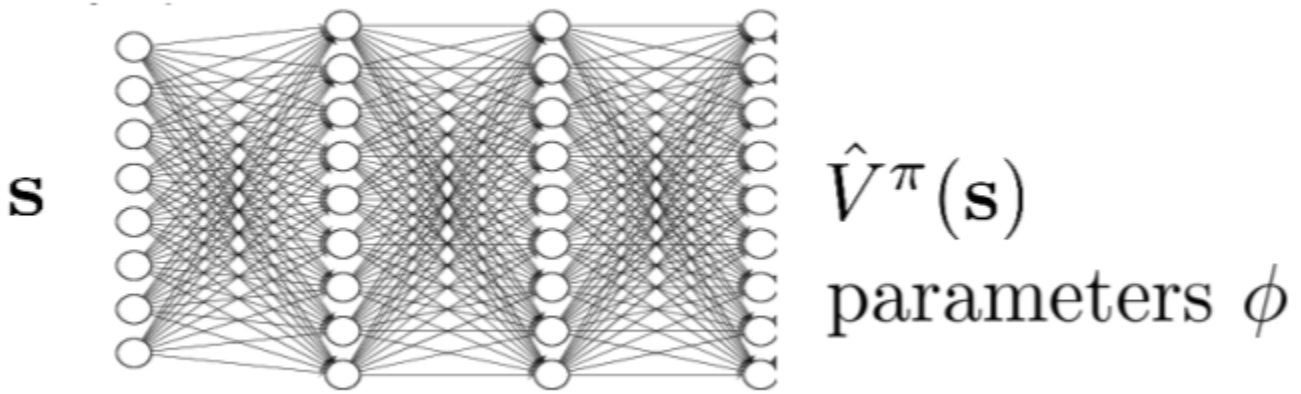
Q^π, V^π, A^π?

$$Q^\pi(s_t, a_t) = E[r_{t+1} + \gamma V^\pi(s_{t+1})]$$
$$\approx r(s_t, a_t) + \gamma V^\pi(s_{t+1})$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + \gamma V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

let's just fit $V^\pi(\mathbf{s})$!



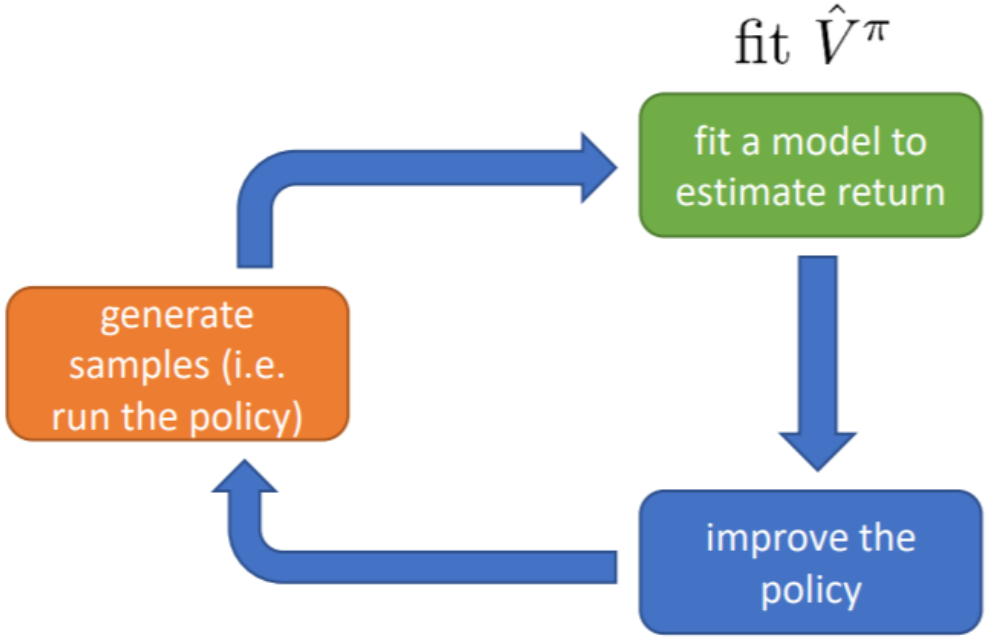


An actor-critic algorithm

batch actor-critic algorithm:

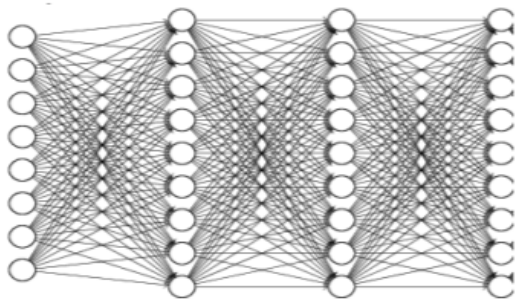
repeat

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



supervised regression $L_c(\phi) = \sum_t \left(\hat{V}^\pi_\phi(s_t) - y_t\right)^2$

$y_t \approx r(s_t, a_t) + \gamma \hat{V}^\pi_\phi(s_{t+1})$



$\hat{V}^\pi(\mathbf{s})$
parameters ϕ

$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]\gamma^{t'-t}$

A2C

- Initialize θ_a, θ_c
- For each episode e
  - Initialize s_1
  - For $t = 1 \ldots Termination$
    - Choose action a_t using policy obtained from θ
    - Observe r_t, s_{t+1}
  - Compute the returns $G(s_t) = \sum_{0 \le k \le M-1} \gamma^k r_{t+k} + \gamma^M V_\theta(s_{t+M})$ if t + $M < T$, else $\sum_{0 \le k \le T-t-1} \gamma^k r_{t+k}$
  - Compute the advantages $A_t = G(S_t) - V_\theta(S_t)$
  - Compute $L_a(\theta) = \frac{1}{T}\sum_t \log\left(\pi_\theta(a_t|s_t)\right) A_t$, $L_c(\phi) = \frac{1}{T}\sum_t \left(G(s_t) - V_\phi(s_t)\right)^2$
  - $\theta \leftarrow \theta + \eta_a \nabla_\theta L_a(\theta)$, $\phi \leftarrow \phi - \eta_c \nabla_\phi L_c(\phi)$,

M-step look ahead

Extensions

- A2C can be applied in a multi-thread environment on several episodes simultaneously, with a final mini-batch update
- **Asynchronous Advantage Actor-Critic** (A3C) (Deepmind, 2016): Each thread performs its updates without waiting for the others to end → **each thread keeps its own version of the parameters**. They upload their gradients asynchronously to a master server that performs batch updates
- Experience Replay can be adapted to A2C → ACER algorithm (Deepmind 2017)

Policy gradient in practice

- Remember that the gradient has high variance
  - This isn't the same as supervised learning!
  - Gradients will be really noisy!
- Consider using much larger batches
- Tweaking learning rates is very hard
  - Adaptive step size rules like ADAM can be OK-ish
  - policy gradient-specific learning rate adjustment methods

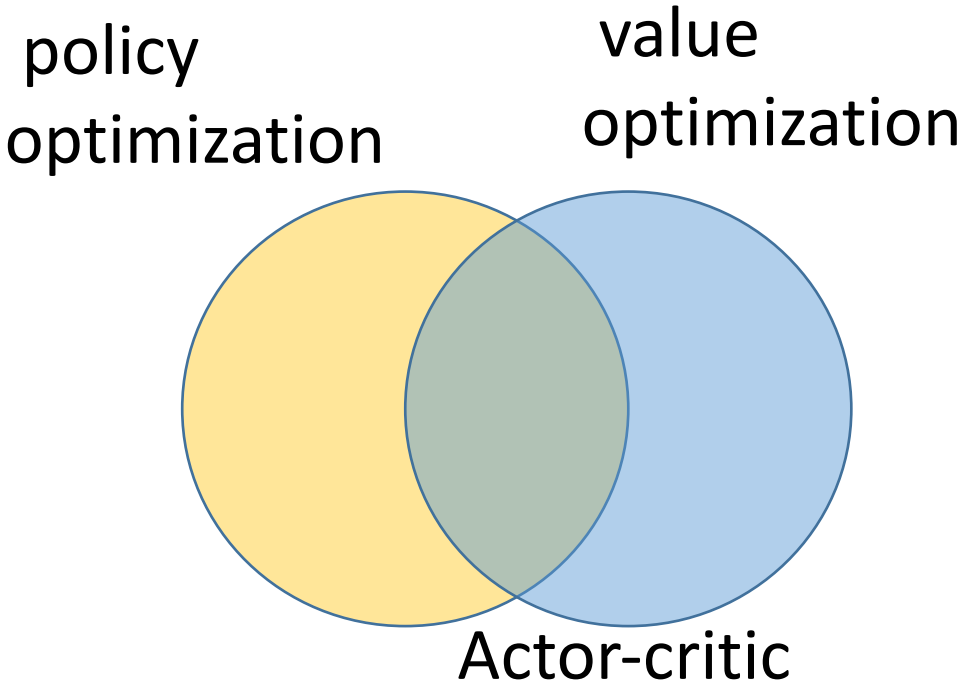
Continuous action space

- Action probabilities $\pi_\theta(a_t|s_t)$: We have seen the discrete action space case (n labels + softmax) → Very large or **continuous space** ?

- You can use a network that **predict the parameters of a distribution** and sample an action from it. Ex : $a_t \sim \mathrm{N}(\mu, \sigma)$ with $\mu, \sigma = \pi(s_t|\theta)$ (similar to the encoder of a VAE) → Reinforce/A2C can be used (with the reparametrization trick).

- Most general case : $f(s_t|\theta) = a_t$. What algorithm can I use ?

Actor-critic methods



Advantages of Policy-based RL

- Advantages
  - Better convergence properties
  - Effective in high dimensional or continuous action spaces
  - Can learn stochastic policies

- Disadvantages
  - Typically converges to a local rather than global optimum
  - Evaluating a policy is typically inefficient and high variance

Example: RL in Other ML Problems

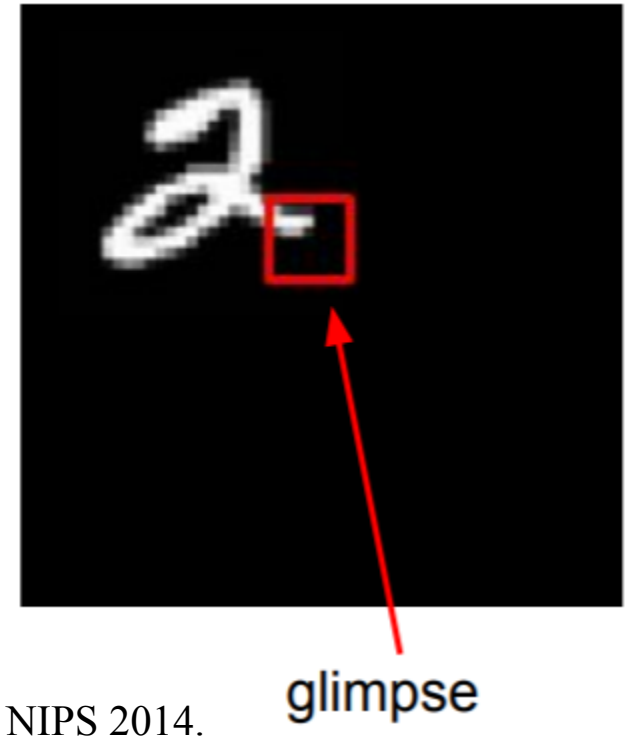
- Hard Attention
  - Observation: current image window
  - Action: where to look
  - Reward: classification

V. Mnih et al., “Recurrent models of visual attention”, NIPS 2014.

REINFORCE in action: Recurrent Attention Model (RAM)

- **Objective**: Image Classification

- Take a sequence of "glimpses" selectively focusing on regions of the image, to predict class
  - Inspiration from human perception and eye movements
  - Saves computational resources => scalability
  - Able to ignore clutter / irrelevant parts of image



V. Mnih et al., "Recurrent models of visual attention", NIPS 2014.

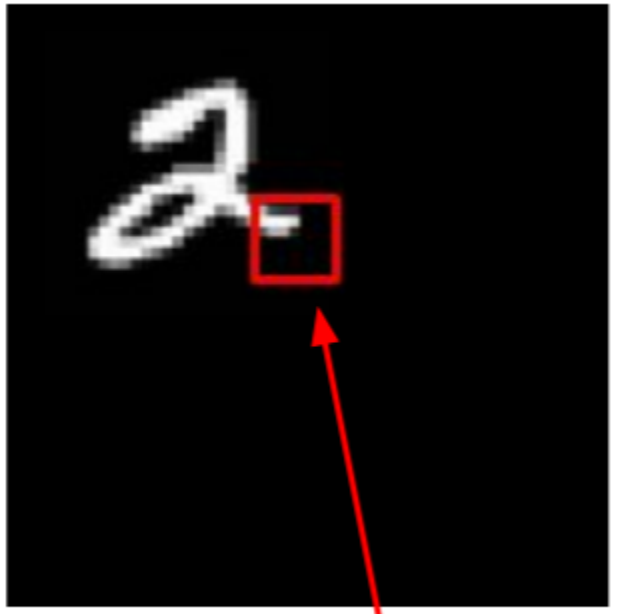
REINFORCE in action: Recurrent Attention Model (RAM)

- **Objective**: Image Classification

- **State**: Glimpses seen so far
- **Action**: (x,y) coordinates (center of glimpse) of where to look next in image
- **Reward**: 1 at the final timestep if image correctly classified, 0 otherwise

- Glimpsing is a non-differentiable operation

  => learn policy for how to take glimpse actions using REINFORCE



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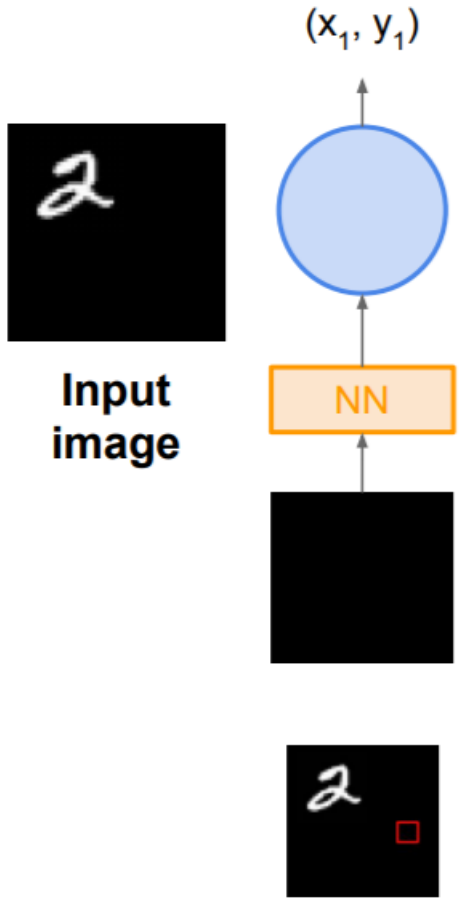
REINFORCE in action: Recurrent Attention Model (RAM)

- Given state of glimpses seen so far, use RNN to model the state and output next action



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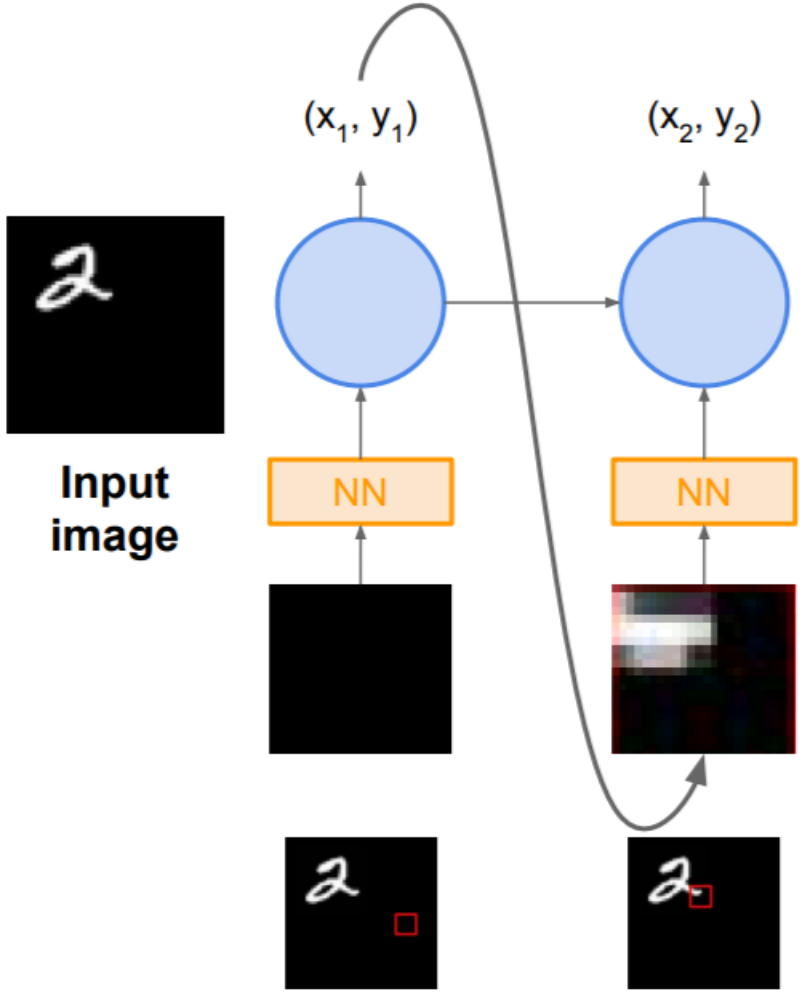

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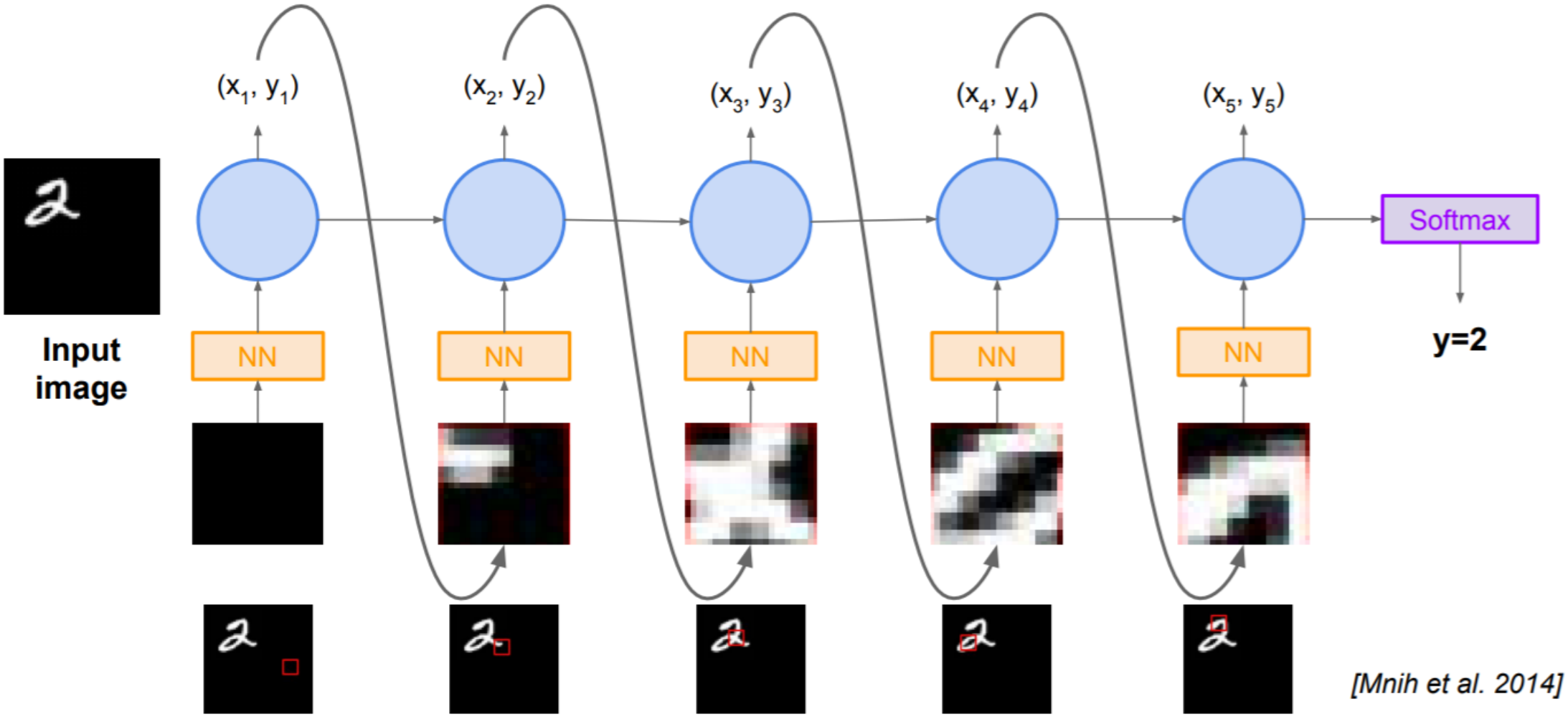
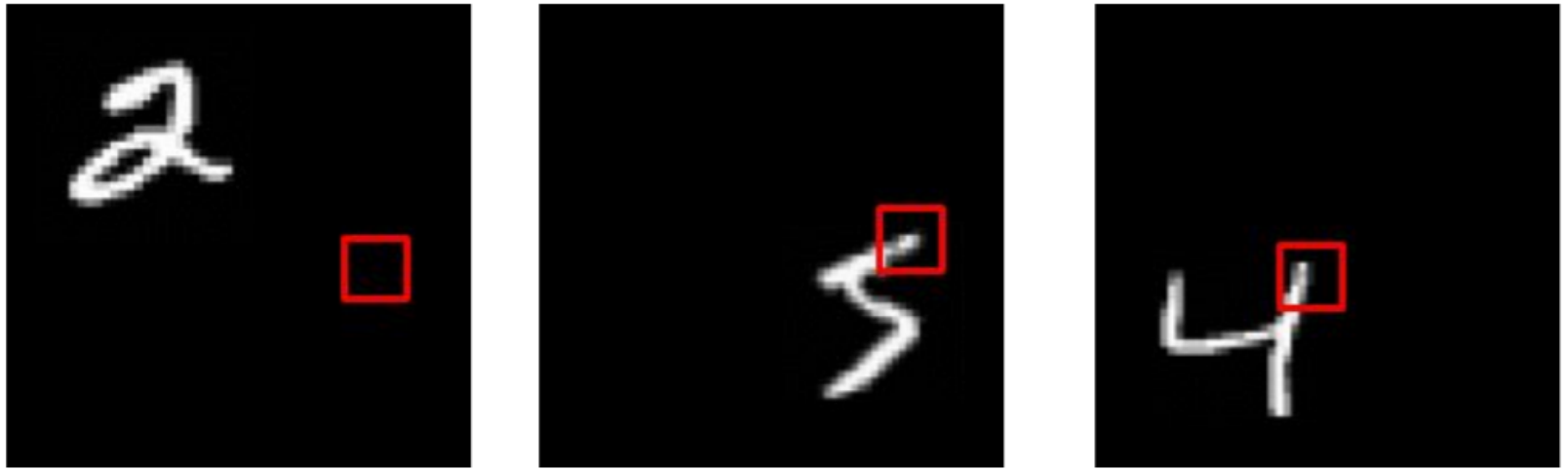


[Mnih et al. 2014]

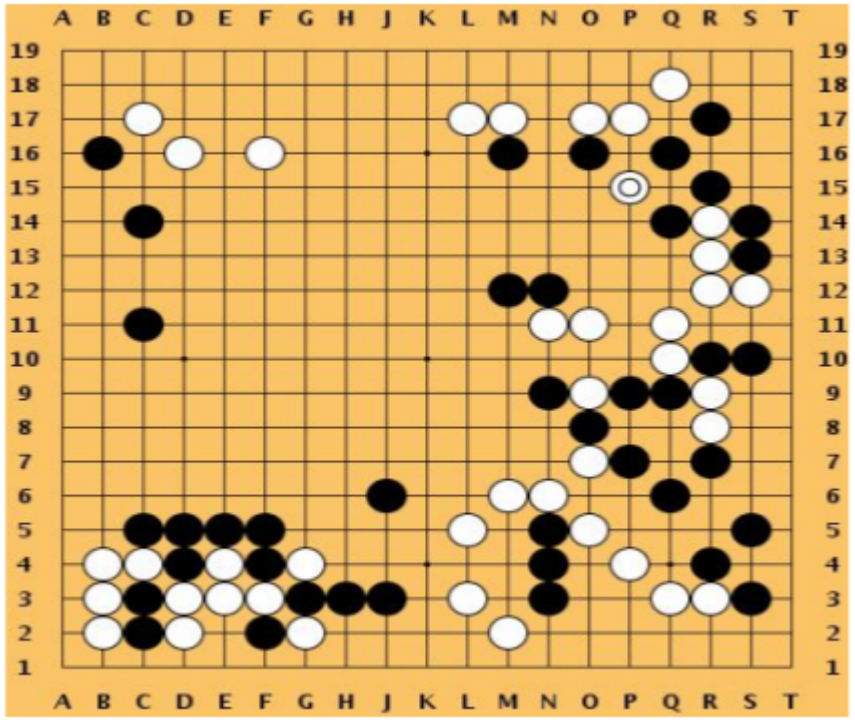
REINFORCE in action: Recurrent Attention Model (RAM)

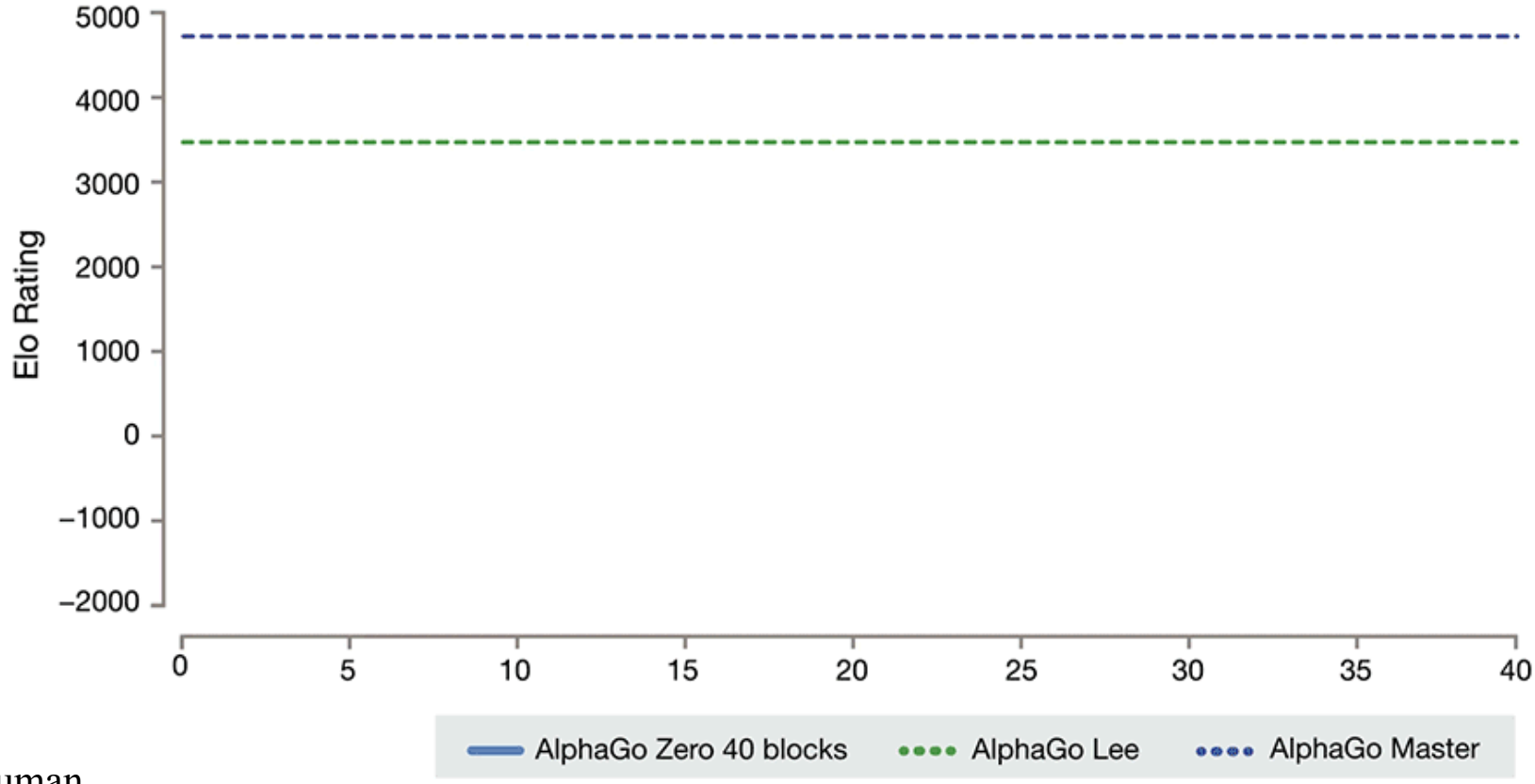


Has also been used in many other tasks including fine-grained image recognition, image captioning, and visual question-answering!

[Mnih et al. 2014]

More policy gradients: AlphaGo Zero





Silver et al., Mastering the game of Go without human knowledge, Nature, 2017.

https://deepmind.com/blog/article/alphago-zero-starting-scratch

Summary

- Policy gradients: very general but suffer from high variance so requires a lot of samples.
  - Challenge: sample-efficiency
- Q-learning: does not always work but when it works, usually more sample-efficient.
  - Challenge: exploration
- Guarantees:
  - Policy Gradients: Converges to a local minima of $J(\theta)$, often good enough!
  - Q-learning: Zero guarantees since you are approximating Bellman equation with a complicated function approximator

Summary

- Parameterized Functions
- Deep Q Networks (DQNs)
  - Experience-replay
  - Target functions
- Policy gradients
  - Reinforce
  - Actor-Critic