Homework **4 - SVM & Kernels**

130 marks total

Question 1 (25 marks)

Let the mapping from the input space to a higher dimensional space be

$$x \in R \longrightarrow y \equiv \varphi(x) \in R^{2k+1}$$

where

$$\varphi(x) = \left[\frac{1}{\sqrt{2}}, \cos x, \cos 2x, \ldots\ldots, \cos kx, \sin x, \sin 2x, \ldots., \sin kx\right]^T$$

Then show that the corresponding inner product kernel is as following

$$k(x_i, x_j) = y_i^T y_j = \frac{\sin((k+0.5)(x_i - x_j))}{2\sin(\frac{x_i - x_j}{2})}$$
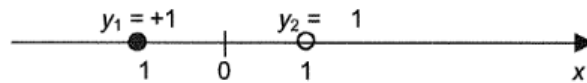
Question 2 (25 marks)

Two different one-dimensional classification tasks are given in the following tables. Draw the two-class data points in an (x, y) plane. (Draw two separate graphs.) Find analytically and sketch the optimal canonical hyperplanes belonging to these two classification tasks. Determine the equations for decision boundaries. (Hint: Identify the support vector first) (25 Mark)

a. x	y = d
2	1
−1	−1
−2	−1
1	1

b. x	y = d
3	1
1	−1
−1	−1

Question 3 (25 marks)

Two one-dimensional data shown in the following figure should be classified using SVM with the first-order polynomial kernel. Compute Kernel Matrix and Find $\alpha's$ and decision boundary using SVM Equations.

Question 4 (25 marks)

Using the KKT stationary conditions, find the minimum of the function f subject to the following constraints. Check your answer graphically

$$f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 2)^2$$

$$x_2 - x_1 = 1$$

$$x_2 + x_1 \leq 2$$

$$x_2 \geq 0 \quad x_1 \geq 0$$

Question 5 (30 marks) (Implementation)

Discuss the specification of the Sonar dataset from the following link:

http://archive.ics.uci.edu/dataset/151/connectionist+bench+sonar+mines+vs+rocks

(The dataset CSV file has been attached to the Homework)

Use SVM in linear form and also with polynomial and RBF kernel to classify this dataset with the lowest classification error for the test set. You can choose 80% of the dataset as the train set randomly and 20% of the remaining dataset as the test set. Report your results in a Table in the following form. You should also attach your code and screenshot of your results in the programming environment. You can check the correctness of your accuracy results with what is reported in the above link for SVM. What do you think about the reason of the difference between your result with what has been reported in that link?

SVM kernel	Kernel parameter Value	Classification Accuracy on Train Set	Classification Accuracy on Test Set
Linear	-		
polynomial			
RBF			

You can use the following links for using SVM in Scikit-learn

https://scikit-learn.org/stable/modules/svm.html

https://www.datacamp.com/tutorial/svm-classification-scikit-learn-python