Homework No. 4

Implementation of Water-Filling Method

In this homework, apply water-filling method to minimize aggregate transmit power and maximize aggregate throughput (data rate) in a simple wireless network with 1 user. In the problem of the aggregate transmit power minimization, user has a minimum target rate denoted by $R^{\min}$. Likewise, In the problem of the aggregate data rate maximization, user has a maximum transmit power budget denoted by $p^{\max}$. This homework has two sections namely applying water-filling method to minimize aggregate transmit power and maximize aggregate throughput (data rate) in a simple wireless network. The following parameters are fixed for both cases:

- Cell coverage area = $500 \text{ m} \times 500 \text{ m}$
- Background noise power σ^2= 10^{-10} W
- Path gain h^k = $x^k d^{-\alpha}$, where d is the distance from the user to the BS, x^k is a random parameter with Rayleigh distribution, and $\alpha = 3$ is the path loss exponent.

I. Applying Water-filling Method to Minimize Aggregate Transmit Power

1 Solve the following problem:

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{k=1}^{10} p^k \\ \text{subject to.} \quad & C1: \quad \sum_{k \in \mathcal{C}} \log_2\left(1 + \frac{a^k p^k h^k}{N^k}\right) \geq 1.5 \text{ bps}\backslash\text{Hz}, \\ & C2: \quad p^k \geq 0, \qquad k = 1, 2, \cdots, 10, \end{aligned} \tag{A}$$

where, $\boldsymbol{h} = \{h^k\}_{k=1}^{10} = [14, 10, 18, 8, 4, 20, 12, 3, 16, 6] * 10^{-10}$.

Now, solve the above problem in which maximum power (mask power) on each sub-channel is 10 mW.

Compare and discuss the achieved aggregate transmit power in two cases.

2 Simulate water-filling algorithm to solve problem P1.1 in course slides where the number of sub-channels and minimum target rate are set to 200 (i.e., $C = 200$) and $50 \text{ bps}\backslash\text{Hz}$ (i.e., $R^{\min} = 50 \text{ bps}\backslash\text{Hz}$), respectively.

3 Simulate water-filling algorithm to address problem P2.1 in course slides where the number of sub-channels is set to 200 (i.e., $C = 200$), minimum target rate is set to 50 bps\Hz (i.e., $R^{\min} = 50$ bps\Hz), and maximum power (mask power) on each sub-channel is set to 0.5 mW (i.e., $p^{\max,k} = 0.5$ mW).

In cases 2 and 3, please achieve your results by averaging from at least 100 independent snapshots with a different location for the user in each snapshot.

4 Compare and discuss the results in cases 2 and 3. Explain the impacts of considering the mask power at each sub-channel on obtained aggregate transmit power.

II. Applying Water-filling Method to Maximize Aggregate Data Rate

1 Solve the following problem:

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & \sum_{k=1}^{10} \log_2(1 + \frac{a^k p^k h^k}{N^k}) \\ \text{subject to.} \quad & C1: \quad \sum_{k=1}^{10} a^k p^k \leq 50 \text{ mW}, \\ & C2: \quad p^k \geq 0, \qquad k = 1, 2, \cdots, 10, \end{aligned} \tag{B}$$

where, $\boldsymbol{h} = h_{k=1}^{10} = [14, 10, 18, 8, 4, 20, 12, 3, 16, 6] * 10^{-10}$.

Now, solve the above problem in which mask power on each sub-channel is 7.5 mW.

Compare and discuss the achieved aggregate data rate in two cases.

2 Simulate water-filling algorithm to solve problem P2.1 in course slides where the number of sub-channels and maximum transmit power budget are set to 200 (i.e., $C = 200$) and 1 W (i.e., $p^{\max} = 1$ W), respectively.

3 Simulate water-filling algorithm to address problem P2.2 in course slides where the number of sub-channels is set to 200 (i.e., $C = 200$), maximum transmit power budget is set to 1 W (i.e., $p^{\max} = 1$ W), and mask power on each sub-channel is set to 1 mW (i.e., $p^{\max,k} = 1$ mW).

In cases 2 and 3, please achieve your results by averaging from at least 100 independent snapshots with a different location for the user in each snapshot.

4 Compare and discuss the results in cases 2 and 3. Explain the impacts of considering the mask power at each sub-channel on obtained aggregate data rate.

The aforementioned problems are given in Appendix.

III. Appendix

- Problem P1.1:

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{k\in\mathcal{C}} p^k \\ \text{subject to.} \quad & C1: \quad \sum_{k\in\mathcal{C}} \log_2(1+\frac{a^k p^k h^k}{N^k}) \geq R^{\min}, \\ & C2: \quad p^k \geq 0, \qquad \forall k \in \mathcal{C}. \end{aligned}$$

- Problem P1.2:

$$\begin{aligned} \min_{\boldsymbol{P}} \quad & \sum_{k\in\mathcal{C}} p^k \\ \text{subject to.} \quad & C1: \quad \sum_{k\in\mathcal{C}} \log_2(1+\frac{a^k p^k h^k}{\sigma^2}) \geq R^{min}, \\ & C2: \quad 0 \leq p^k \leq p^{\max,k}, \qquad \forall k \in \mathcal{C}. \end{aligned}$$

- problem P2.1;

$$\begin{aligned} \max_{\boldsymbol{P}} \quad & \sum_{k\in\mathcal{C}} \log_2(1+\frac{a^k p^k h^k}{N^k}) \\ \text{subject to.} \quad & C1: \quad \sum_{k\in\mathcal{C}} a^k p^k \leq p^{\max}, \\ & C2: \quad p^k \geq 0, \qquad \forall k \in \mathcal{C}. \end{aligned}$$

- Problem P2.2:

$$\begin{aligned} \max_{\boldsymbol{P}} \quad & \sum_{k\in\mathcal{C}} \log_2(1+\frac{a^k p^k h^k}{N^k}) \\ \text{subject to.} \quad & C1: \quad \sum_{k\in\mathcal{C}} a^k p^k \leq p^{\max}, \\ & C2: \quad 0 \leq p^k \leq p^{\max,k}, \qquad \forall k \in \mathcal{C}. \end{aligned}$$