

Prosumers' Cost Recovery in Peer-to-Peer Electricity Markets

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Abstract

Monetization of renewable energy sources (RESs) at the grid edge and boosting their proliferation depend on the economic properties of local electricity markets, in particular, peer-to-peer (P2P) electricity markets. In this regard, this paper proposes a mechanism design framework for investigating prosumers' incentive for participation in P2P electricity trading. To this end, we develop a game-theoretic model based on network games capable of capturing prosumers' interaction and preferences under the bilateral marginal pricing (BMP) mechanism. Then, we focus on individual rationality as a key desideratum for a market-clearing mechanism, which guarantees non-negative payoff for prosumers' participation in the market. By leveraging dual analysis, we prove that individual rationality in the BMP mechanism is contingent on prosumers' flexibility in the market. At the end, numerical case studies for reflecting prosumers' participation in the market and the role of flexibility in their payoffs are provided. Moreover, the results show that there are heterogeneous cut off points, based on the network charge, for prosumers' involvement in the market platform, and after that points, prosumers opt out of the market platform.

Keywords: Peer-to-peer energy trading, Local electricity markets, Individual rationality, Flexibility, Mechanism design

Nomenclature

Indices, Sets, and Vectors		V_i	Set of valuation functions for prosumer i
i, j, Ω	Indexes and set of prosumers		
Ω_i	Set of prosumer i 's neighboring peers	ξ_i	Prosumer i 's vector of bilateral trade coefficients with neighboring peers, denoting $(\xi_{ij})_{j \in \Omega_i}$
S_i	Domain of local decisions for prosumer i		
Θ_i	Set of private information or type of prosumer i , and $\theta_i \in \Theta_i$ is the announced type	λ_i	Prosumer i 's vector of P2P trading prices with neighboring peers, denoting $(\lambda_{ij})_{j \in \Omega_i}$

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\mathbf{t}_i Prosumer i 's vector of P2P trading power with neighboring peers, denoting $(t_{ij})_{j \in \Omega_i}$

Parameters

n Number of prosumers

$\underline{d}_i, \overline{d}_i$ Lower/ upper bound of prosumer i 's electricity demand

$\underline{g}_i, \overline{g}_i$ Lower/ upper bound of prosumer i 's generation capacity

Decision Variables

d_i Demand of prosumer i

g_i Generation of prosumer i

t_{ij} Trading power from prosumer i to prosumer j

s_i Decision variables of prosumer i , including d_i, g_i, \mathbf{t}_i

$\overline{\mu}_i^d, \underline{\mu}_i^d$ Dual variables corresponding to prosumer i 's demand upper/ lower bound

$\overline{\mu}_i^g, \underline{\mu}_i^g$ Dual variables corresponding to prosumer i 's generation upper/ lower bound

ρ_i Dual variable corresponding to prosumer i 's power balance constraint

λ_{ij} P2P trading price between prosumer i and j (dual variable)

Functions

w Social choice function

$v_i(s_i, \theta_i)$ Valuation function of prosumer i

$U_i(d_i)$ Prosumer i 's utility of using demand d_i

$C_i(g_i)$ Prosumer i 's cost of generating power g_i

$p_i(v_i, v_{-i})$ Payment function of prosumer i

$f_i(\cdot)$ Payoff function of prosumer i

$A_i(\cdot)$ Prosumer i 's aggregate of the trading partners' strategies

$\mathcal{L}_i(\cdot)$ Lagrangian function of prosumer i 's local optimization problem

1. Introduction

The recent developments of RESs technologies and public policies aiming to the transition toward clean energy have led to an unprecedented rise in RESs adoption by customers [1], [2]. This status quo has paved the way for designating an active role to customers and advent of local electricity markets [3]. In this context, previously passive consumers turn to prosumers acting rationally toward their interests by participating in local market platforms, in particular, P2P electricity markets [4], [5]. Indeed, P2P electricity trading is a promising

paradigm for unleashing the true potential of RESs at the grid edge. However, the success of P2P electricity markets is deeply rooted in the underlying economic mechanism.

A desired P2P market-clearing mechanism should be able to organically attract prosumers into the market and facilitate their transactions [6], [7]. In this regard, the main question motivating this paper is centered around the role of the P2P market-clearing mechanism on prosumers' incentive to participate in P2P electricity trading. Moreover, we try to demonstrate the role of the flexibility for voluntary participation of prosumers in the market platform. For addressing these questions, our paper provides a mechanism design framework and investigates the individual behavior of prosumers in P2P electricity trading.

2. Related Work

This paper is more leaned toward the research works on economics of P2P electricity trading. In this regard, we take a closer look of this research strand in the literature to highlight the research gap motivating this paper. A comparative analysis of auction mechanisms in P2P electricity markets, regarding the economic efficiency, is proposed in [8]. The analysis is particularly focused on discriminatory and uniform double auction mechanisms. In addition, the performance metrics of the analysis comprise of percentage of energy sold, percentage of energy bought, and percentage of cleared demand. In [9], a game-theoretic pricing approach is proposed for P2P electricity trading, and the existence of the Nash equilibrium is examined in a Stackelberg game. Moreover, the proposed model investigates the interactions between sellers and buyers, and also among sellers. Empirical studies to analyze the bidding behavior of consumers in P2P electricity trading and their engagement in the market are provided in [10–12]. A price discrimination technique based on a cake cutting game is provided in [13] for energy trading, and its main focus is to study fairness criteria. In [14], authors focus on the role of prosumers' battery flexibility in P2P electricity market design. They aim to address questions regarding the value of prosumers' battery in P2P electricity trading and market properties to unlock the true potential of the prosumers' batteries. In [15], a four-layer system architecture for P2P energy trading is proposed, and the bidding process among the peers is simulated by game theory. Reference [16] develops

a bilateral energy trading scheme and a decision strategy for choosing a bilateral price in a smart home community. The proposed method requires far fewer negotiation rounds for reaching a consensus in compare to traditional methods. The main issue addressed in [17] is about prosumers participation in P2P microgrids and ensuring microgrids sustainable development. Authors in [18] develop a P2P market structure based on multi-bilateral economic dispatch, which enables product differentiation for a central planner and also prosumers. Moreover, this paper proposes a consensus approach to solve the market-clearing problem in a fully decentralized manner. An auction-theoretic scheme for energy transactions between non-convex prosumers is provided in [19]. The paper concludes that the proposed scheme aligns prosumers' incentive toward social efficiency. In [20], a new scalable market design for P2P trading based on bilateral contract networks is proposed, including forward and real time markets. Moreover, the proposed model is capable of reflecting the preferences of different type of market participants. An energy sharing model for P2P microgrids, with price-based demand response, is developed in [21], and a dynamical internal pricing model is proposed for the operation of energy sharing zone. In [22], authors highlight the importance of the market model for boosting prosumers' contribution in electricity services. To this end, a framework is proposed for enabling prosumers' engagement in P2P electricity trading based on their preferences. Moreover, for ensuring the feasibility of the P2P trades, due to the network constraints, a flexibility market is provided to mitigate the network limits via prosumers' flexibility.

3. Summary of Contributions and Paper Organization

Despite the rich literature in P2P electricity trading, the market mechanism and its implications on prosumers' incentive for P2P electricity trading is still unexplored. Whereas, monetization of RESs at the grid edge and boosting their proliferation depend on the economic properties of the underlying market mechanisms [23], [24]. This paper aims to address this research gap by providing a mechanism design framework for prosumers interactions in P2P electricity trading. The main contributions of this paper can be summarized as the following:

- We analyze individual rationality of the BMP mechanism in a duality framework,
- We demonstrate the necessity of prosumers' flexibility for ensuring their voluntary participation in the market,
- We provide a unified model in a mechanism design framework for P2P electricity trading with the capability of price discrimination, modelling customers as prosumers, and capturing their preferences,
- We develop a game-theoretic model based on network games capable of capturing the prosumers' interaction and the equilibrium concept in their P2P electricity trading.

The rest of the paper is organized as follows. The standard format of an economic mechanism for P2P electricity markets is presented in section 4. Section 5 belongs to the game-theoretic analysis of prosumers interactions under the BMP mechanism. In section 6, we study individual rationality of the BMP mechanism. Then, in section 7, we provide numerical case studies for reflecting the theoretical properties of the mechanism in practice. Following that, the conclusion is presented in section 8.

4. The Model

We consider a P2P market-clearing problem setting with a finite set of prosumers $\Omega = \{1, \dots, n\}$. There is a set of potential social decisions $S = \prod_{i=1}^n S_i$, where $S_i \in \mathbb{R}^{|S_i|}$ is the set of prosumer i 's potential local decisions. In this setting, a mechanism socially chooses some alternative $s \in S$, as well as decides on prosumers' payments. Each prosumer is endowed with a private parameter, called type, that reflects its preferences over the set of potential decisions S . The type of prosumer $i \in \Omega$, including utility function, cost function, and bilateral trade preferences, is represented by $\theta_i \in \Theta_i$. Given a type, prosumer i 's preferences can be evaluated by a valuation function $v_i : S \times \theta_i \rightarrow \mathbb{R}$, where $v_i(s, \theta_i)$ denotes the value of alternative $s \in S$ for prosumer i with type θ_i . Moreover, since prosumer i 's valuation depends only on its local decisions, we can say $v_i(s, \theta_i) = v_i(s_i, \theta_i)$.

The valuation function $v_i(\cdot) \in V_i$ of prosumer i comprises of the utility of using demand d_i , cost of generating active power g_i , and its energy transactions t_{ij} with other prosumers. These trades can be as selling ($t_{ij} \geq 0$) or buying ($t_{ij} \leq 0$) transactions. Then, the set of decision variables in the strategy of each prosumer can be shown as $s_i = \{d_i, g_i, (t_{ij})_{j \in \Omega_i}\}$. In this regard, the value function $v_i(\cdot)$ of prosumer i can be denoted as

$$v_i(d_i, g_i, \mathbf{t}_i) = U_i(d_i) - C_i(g_i) - \boldsymbol{\xi}_i^T \mathbf{t}_i, \forall i \in \Omega. \quad (1)$$

The parameter $\boldsymbol{\xi}_i$ is a vector $(\xi_{ij})_{j \in \Omega_i}$ denoting the bilateral trade coefficients, which are used for the purpose of product differentiation. The interpretation of bilateral trading costs depends on its application. In this paper, these trading costs are imposed centrally by the platform operator for the cost allocation policy of the distribution utility.

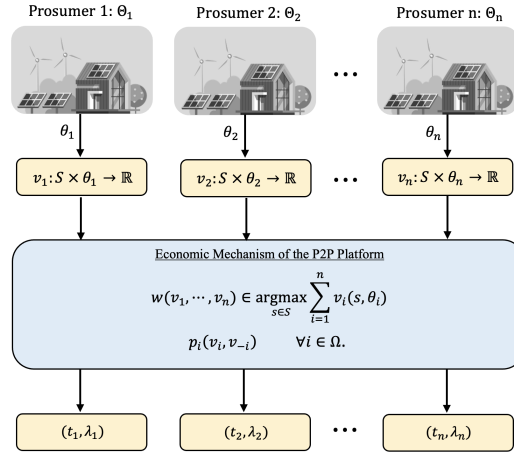


Figure 1: Abstract view of a P2P electricity trading platform.

In this context, a mechanism is a social choice function $w : V = \prod_{i=1}^n V_i \rightarrow S$ and a vector of payment functions $p = (p_1, \dots, p_n)$, where $p_i : V \rightarrow \mathbb{R}$ is the amount that player i pays [25]. Figure 1 provides an abstract view of a P2P electricity trading platform and the market-clearing mechanism as its core module. In the P2P electricity trading literature, the Bilateral Marginal Pricing (BMP) mechanism is usually used as the market-clearing mechanism. In addition, different decomposition techniques, such as Lagrangian relaxation, Alternative Method of Multipliers (ADMM), and consensus + innovation are used for its distributed

implementation in the negotiation procedure. However, the economic characteristics of this mechanism have not been fully scrutinized in the context of mechanism design theory. Thus, the focus of this paper is on the BMP mechanism for P2P electricity trading. In this regard, the BMP mechanism can be framed in the following standard form (w, p_1, \dots, p_n) :

$$w(v_1, \dots, v_n) \in \arg \max_{s \in S} \sum_{i=1}^n U_i(d_i) - C_i(g_i) - \boldsymbol{\xi}_i^T \mathbf{t}_i \quad (2a)$$

$$p_i(v_i, v_{-i}) = -\boldsymbol{\lambda}_i^T \mathbf{t}_i, \forall i \in \Omega, \quad (2b)$$

Where (2a) and (2b) represent the social choice function and payment of prosumers respectively. Moreover, $\boldsymbol{\lambda}_i$ is a vector denoting $(\lambda_{ij})_{j \in \Omega_i}$, and λ_{ij} is the price of P2P energy trading between prosumer i and j . These P2P prices $\{\boldsymbol{\lambda}_i\}_{i=1}^n$ are derived via the Lagrange multipliers associated with the reciprocity constraints in an optimal solution of the following social choice function $w(v_1, \dots, v_n)$, which encapsulates the P2P market-clearing problem:

$$\max_{(d_i, g_i, \mathbf{t}_i)_{i \in \Omega}} \sum_{i=1}^n U_i(d_i) - C_i(g_i) - \boldsymbol{\xi}_i^T \mathbf{t}_i \quad (3a)$$

$$\text{s.t. } \underline{d}_i \leq d_i \leq \bar{d}_i : \left(\bar{\mu}_i^d, \underline{\mu}_i^d \right), \quad \forall i \in \Omega \quad (3b)$$

$$\underline{g}_i \leq g_i \leq \bar{g}_i : \left(\bar{\mu}_i^g, \underline{\mu}_i^g \right), \quad \forall i \in \Omega \quad (3c)$$

$$g_i - d_i - \sum_{j \in \Omega_i} t_{ij} = 0 : (\rho_i), \quad \forall i \in \Omega \quad (3d)$$

$$t_{ij} + t_{ji} = 0 : (\lambda_{ij}), \quad \forall i \in \Omega, \forall j \in \Omega_i \quad (3e)$$

In the objective function (3a), the first term and the second term of the summation represent the individual utility and generation cost of prosumers respectively, and the last term belongs to the total bilateral trading costs of each prosumer. The constraint (3b) and (3c) reflect the active supply and demand limits of prosumers respectively. And the constraint (3d) relates to the power balance equation of each prosumer. Furthermore, the reciprocity constraints of P2P trades (3e) are included in the feasibility set of the market-clearing problem. Each constraint has a corresponding dual variable, which is denoted in

the parenthesis. To avoid complexity and easing the representation of the general ideas, the proposed model is based on a future market for a specific period that can be easily extended to multi-period model. Moreover, we assume that for each prosumer i , $U_i(d_i)$ and $C_i(g_i)$ are concave and convex functions respectively. Thus, all the optimization problems in this paper are considered to be convex. In the next section, we analyze the game and the equilibrium concept of prosumers' interaction under the BMP mechanism.

5. Game Theoretic Analysis

Due to the interaction of prosumers over a P2P network, the induced game can be defined in a network games setup. Same as previous section, we consider a finite set of prosumers $\Omega = \{1, \dots, n\}$ who are connected on a weighted network with adjacency matrix $G \in \mathbb{R}^{n \times n}$, represented by the market structure. The entries of G , representing the perceived price in bilateral trades, are rooted in the product differentiation functionality of P2P electricity markets. The element G_{ij} denotes the influence of prosumer j 's strategy on the payoff of prosumer i . We assume $G_{ii} = 0$ for all $i \in \Omega$ and say that prosumer j is a neighbor of prosumer i if $G_{ij} \neq 0$. We denote by Ω_i the set of neighbors of prosumer i that it can trade with. Figure 2 illustrates an arbitrary P2P trading platform including n prosumers across an energy community. For visual simplicity, just the network edges related to prosumer 1 are depicted.

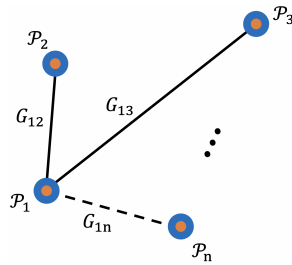


Figure 2: An underlying network of an arbitrary P2P platform with n prosumers $\mathcal{P}_i, i \in \Omega$.

Given the payment function of the BMP mechanism, each prosumer $i \in \Omega$ aims at selecting a vector strategy $s_i \in S_i$ to maximize its own payoff $f_i(s_i, A_i(s))$, which depends on its own strategy s_i and on the aggregate of the trading partners' strategies A_i , obtained

as the sum of bilateral transactions as follows:

$$A_i(s) = \sum_{j=1}^n G_{ij} t_{ij}, \quad (4)$$

Where $s = (s_1, \dots, s_n)$ is a vector that its i^{th} block component is the strategy of prosumer i . Furthermore, t_{ij} , which is part of the prosumer i 's strategy, denotes its P2P trading with neighbouring prosumers $j \in \Omega_i$.

5.1. Best Response Formulation of Prosumers

Each prosumer i tries to maximize its own payoff comprised of its valuation function $v_i(d_i, g_i, \mathbf{t}_i)$ and monetary transfers resulted by energy transactions t_{ij} with prosumer j , $j \in \Omega_i$. In this regard, the optimization problem of prosumer i for choosing the best response to the trading partners aggregate strategy, $A_i(s)$, is as the following:

$$\max_{s_i} f_i(s_i, A_i(s)) = U_i(d_i) - C_i(g_i) - \boldsymbol{\xi}_i^T \mathbf{t}_i + \boldsymbol{\lambda}_i^T \mathbf{t}_i \quad (5a)$$

$$\text{s.t. } \underline{d}_i \leq d_i \leq \overline{d}_i : \left(\overline{\mu}_i^d, \underline{\mu}_i^d \right) \quad (5b)$$

$$\underline{g}_i \leq g_i \leq \overline{g}_i : \left(\overline{\mu}_i^g, \underline{\mu}_i^g \right) \quad (5c)$$

$$g_i - d_i - \sum_{j \in \Omega_i} t_{ij} = 0 : (\rho_i) \quad (5d)$$

As we mentioned, the first term of the objective function (5a) refers to the prosumer i 's utility of using demand d_i , the second term refers to the prosumer i 's incurred cost of generating power g_i , the third term reflects the cost of bilateral trades, and the last term monetizes all the energy trades t_{ij} with other prosumers. Note that $A_i(s)$ is deployed implicitly through these terms: $-\boldsymbol{\xi}_i^T \mathbf{t}_i + \boldsymbol{\lambda}_i^T \mathbf{t}_i$. Furthermore, $\boldsymbol{\lambda}_i$ is a given parameter in the individual optimization problem of prosumer i . Indeed, prosumers cannot anticipate their impact in the P2P price $\boldsymbol{\lambda}_i$ formation and are assumed to be price-taker. The constraints (5b) to (5d) are the individual version of the same constraint in (3b) to (3d). Then, the same descriptions can be applied here.

5.2. Pricing Scheme and Market-Clearing

The pricing scheme should ensure the equilibrium of the market and the reciprocity constraints of P2P trades, which are the supply-demand balance constraints in each trade. Indeed, prosumers should have consensus on the quantity of the trade ($|t_{ij}| = |t_{ji}|$) and its price ($\lambda_{ij} = \lambda_{ji}$). The reciprocity constraints can be denoted as the following:

$$t_{ij} + t_{ji} = 0, \forall i \in \Omega, \forall j \in \Omega_i, j \neq i. \quad (6)$$

As we can see, prosumers have no unilateral control on the reciprocity constraints. Indeed, these constraints couple the individual optimization problems of prosumers. In the following, we present an optimization problem, which is equivalent with reciprocity constraints (6):

$$\max_{\lambda_{ij}, i \in \Omega, j \in \Omega_j} \sum_{i \in \Omega, j \in \Omega_j, i \neq j} -\lambda_{ij}(t_{ij} + t_{ji}). \quad (7)$$

By deriving the KKT conditions of this unconstrained optimization problem, we reach to the same reciprocity constraints (6). This transformation helps us to define a virtual agent as a price setter to set P2P prices and enforce the reciprocity constraints by maximizing the objective function (7). Figure 3 depicts the linkage of prosumers' optimization problems caused by the pricing scheme.

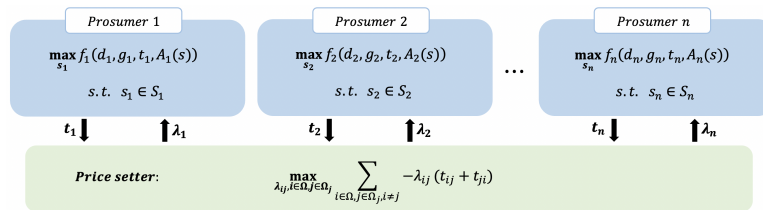


Figure 3: Linkage of prosumers' optimization problems under the BMP mechanism.

As illustrated in Figure 3, the optimization problems of the prosumers and price setter, as a virtual agent, are linked and cannot be solved individually. Because, each P2P market-clearing price is a variable for the price setter, but a parameter for other prosumers. Then, this setup of optimization problems leads to an equilibrium problem. And, if a solution exists

for this equilibrium problem, it will be a Nash equilibrium. Intuitively, Nash equilibrium implies that there is a strategy profile $(s_1^*, s_2^*, \dots, s_n^*)$ such that no prosumer i has incentive to unilaterally deviate from s_i^* . Indeed, Nash equilibrium guarantee the authenticity of market-clearing outcomes. Formally, a strategy profile $S = (s_1^*, s_2^*, \dots, s_n^*)$ is a Nash equilibrium if for all prosumer i and for all strategies $s'_i \neq s_i^*$:

$$f_i(s_i^*, A_i(s_i^*, s_{-i}^*)) \geq f_i(s'_i, A_i(s'_i, s_{-i}^*)). \quad (8)$$

6. Individual Rationality

One of the most desired properties of market-clearing mechanisms is individual rationality [25], [26]. This property implies voluntary participation of prosumers in the market and their cost recovery. Indeed, market designers try to design a mechanism that individuals organically prefers participation in the market to not participating. In the following, we evaluate the BMP mechanism based on this property. In this regard, we first provide the formal definition of individual rationality.

Definition 1(Individual Rationality). A mechanism is (ex-post) individually rational if players always get a non-negative payoff. Formally if for every v_1, \dots, v_n we have that $v_i(w(v_1, \dots, v_n)) - p_i(v_1, \dots, v_n) \geq 0$ [25].

Based on this definition, the BMP mechanism is individually rational if, at the optimal solution, all the prosumers have non-negative profit. Mathematically, it means:

$$f_i = U_i(d_i^*) - C_i(g_i^*) - \boldsymbol{\xi}_i^T \mathbf{t}_i^* + \boldsymbol{\lambda}_i^T \mathbf{t}_i^* \geq 0, \forall i \in \Omega, \quad (9)$$

Where $s_i^* = \{d_i^*, g_i^*, (t_{ij}^*)_{j \in \Omega_i}\}$ is the set of optimal decision variables and f_i denotes the payoff of prosumer i . To prove these conditions, we derive the dual problem corresponding to the optimization problem of each prosumer. Then, we leverage the strong duality condition for reaching an equivalent expression of f_i . In this regard, we should first derive Lagrangian

of the optimization problem (5):

$$\begin{aligned} \mathcal{L}_i(d_i, g_i, \mathbf{t}_i, \bar{\mu}_i^d, \underline{\mu}_i^d, \bar{\mu}_i^g, \underline{\mu}_i^g, \rho_i) = & U_i(d_i) - C_i(g_i) - \boldsymbol{\xi}_i^T \mathbf{t}_i + \boldsymbol{\lambda}_i^T \mathbf{t}_i - \\ & \bar{\mu}_i^d(d_i - \bar{d}_i) - \underline{\mu}_i^d(\underline{d}_i - d_i) - \bar{\mu}_i^g(g_i - \bar{g}_i) - \underline{\mu}_i^g(\underline{g}_i - g_i) \\ & - \rho_i(g_i - d_i - \sum_{j \in \Omega_i} t_{ij}). \end{aligned} \quad (10)$$

Then, we get the Lagrange dual function by determining the supremum of the Lagrangian in (10) due to the primal variables as the following:

$$G(\bar{\mu}_i^d, \underline{\mu}_i^d, \bar{\mu}_i^g, \underline{\mu}_i^g, \rho_i) = \sup_{d_i, g_i, \mathbf{t}_i} \mathcal{L}_i(d_i, p_i, \mathbf{t}_i, \bar{\mu}_i^d, \underline{\mu}_i^d, \bar{\mu}_i^g, \underline{\mu}_i^g, \rho_i). \quad (11)$$

For solving (11), we take the derivatives of (10) with respect to the primal variables and put them equal to zero:

$$\nabla_{d_i} \mathcal{L}_i = \bar{\mu}_i^d - \underline{\mu}_i^d - \rho_i - \nabla_{d_i} U_i(d_i) = 0 \quad (12a)$$

$$\nabla_{g_i} \mathcal{L}_i = \bar{\mu}_i^g - \underline{\mu}_i^g + \rho_i + \nabla_{g_i} C_i(g_i) = 0 \quad (12b)$$

$$\nabla_{\mathbf{t}_i} \mathcal{L}_i = \boldsymbol{\xi}_i^T - \boldsymbol{\lambda}_i^T - \rho_i \mathbf{1}^T = 0. \quad (12c)$$

Thus, Lagrange dual function is equal to:

$$\begin{aligned} G(\bar{\mu}_i^d, \underline{\mu}_i^d, \bar{\mu}_i^g, \underline{\mu}_i^g, \rho_i) = & \bar{\mu}_i^d \bar{d}_i - \underline{\mu}_i^d \underline{d}_i + \bar{\mu}_i^g \bar{g}_i - \underline{\mu}_i^g \underline{g}_i \\ & + U_i(\hat{d}_i) - \hat{d}_i \left(\nabla_{d_i} U_i(\hat{d}_i) \right) - C_i(\hat{g}_i) + \hat{g}_i \left(\nabla_{g_i} C_i(\hat{g}_i) \right), \end{aligned} \quad (13)$$

Where $\hat{s}_i = \left\{ \hat{d}_i, \hat{g}_i, (\hat{t}_{ij})_{j \in \Omega_i} \right\}$ is the solution of the equation system in (12). Thus, the dual problem corresponding to the primal optimization problem of each prosumer in (5) is given

by:

$$\min_{\rho_i, \bar{\mu}_i^d, \underline{\mu}_i^d, \bar{\mu}_i^g, \underline{\mu}_i^g \geq 0} \quad \bar{\mu}_i^d \bar{d}_i - \underline{\mu}_i^d \underline{d}_i + \bar{\mu}_i^g \bar{g}_i - \underline{\mu}_i^g \underline{g}_i + U_i(\hat{d}_i) - \hat{d}_i \left(\nabla_{d_i} U_i(\hat{d}_i) \right) - C_i(\hat{g}_i) + \hat{g}_i \left(\nabla_{g_i} U_i(\hat{g}_i) \right). \quad (14)$$

The required conditions of the strong duality hold in our model, and we can leverage it to investigate individual rationality of the mechanism. So, we get the following:

$$f_i^* = \underbrace{\bar{\mu}_i^{d^*} \bar{d}_i - \underline{\mu}_i^{d^*} \underline{d}_i + \bar{\mu}_i^{g^*} \bar{g}_i - \underline{\mu}_i^{g^*} \underline{g}_i}_{(A)} + \underbrace{U_i(\hat{d}_i) - \hat{d}_i \left(\nabla_{d_i} U_i(\hat{d}_i) \right)}_{(B)} - \underbrace{C_i(\hat{g}_i) + \hat{g}_i \left(\nabla_{g_i} C_i(\hat{g}_i) \right)}_{(C)}. \quad (15)$$

Now, we should take a closer look at the right-hand side of the (15) and check the sign of its partitioned terms. First, we check the sign of (C). As we mentioned, $C_i(\cdot)$ is a convex function. Thus, due to the first-order condition of convexity, the following inequality should hold for all $x, y \in \text{domain}(C_i)$:

$$C_i(y) \geq C_i(x) + \nabla C_i(x)^T (y - x). \quad (16)$$

If we put $y = 0$ and $x = \hat{g}_i$, then we get:

$$-C_i(\hat{g}_i) + \hat{g}_i \left(\nabla_{g_i} C_i(\hat{g}_i) \right) \geq 0, \quad (17)$$

Which guarantees that term (C) is always non-negative. A similar approach can be leveraged for term (B). Note that $U_i(\cdot)$ is a concave function, and $-U_i(\cdot)$ is applied in the first-order condition of convexity. Thus, we get:

$$U_i(\hat{d}_i) - \hat{d}_i \left(\nabla_{d_i} U_i(\hat{d}_i) \right) \geq 0, \quad (18)$$

Which in the same manner guarantees that term (B) is always non-negative.

Examining term (A) is more straightforward because variables $\bar{\mu}_i^{d*}$, $\underline{\mu}_i^{d*}$, $\bar{\mu}_i^{g*}$, $\underline{\mu}_i^{g*}$, and parameters \bar{d}_i , \underline{d}_i , \bar{g}_i , \underline{g}_i are all non-negative by definition. If we assume that the parameters \underline{d}_i and \underline{g}_i are equal to zero, we can guarantee that term (A) is always non-negative. Thus, the mechanism is individually rational, and prosumers do not walk away from the mechanism. Nonetheless, putting \underline{d}_i and \underline{g}_i equal to zero seems like a strong assumption, and the situation for term (A) is still vague in real world applications. Hence, we should put the subtractions by $\underline{\mu}_i^{d*} \underline{d}_i$ and $\underline{\mu}_i^{g*} \underline{g}_i$ on the spotlight, and check the critical conditions may occur for individual rationality of the mechanism. In this regard, we leverage the complementary slackness conditions corresponding to prosumer i 's minimum demand and generation capacity boundaries, which are derived from its KKT conditions:

$$0 \leq (d_i^* - \underline{d}_i) \perp \underline{\mu}_i^{d*} \geq 0 \quad (19a)$$

$$0 \leq (\bar{d}_i - d_i^*) \perp \bar{\mu}_i^{d*} \geq 0 \quad (19b)$$

$$0 \leq (g_i^* - \underline{g}_i) \perp \underline{\mu}_i^{g*} \geq 0 \quad (19c)$$

$$0 \leq (\bar{g}_i - g_i^*) \perp \bar{\mu}_i^{g*} \geq 0. \quad (19d)$$

Due to above conditions, we can see that the individual rationality constraint in (15) is more likely to be negative when $d_i^* = \underline{d}_i$ and $g_i^* = \underline{g}_i$, because the corresponding dual variables $\underline{\mu}_i^{d*}$ and $\underline{\mu}_i^{g*}$ get positive values, which enforces $\bar{\mu}_i^{d*}$ and $\bar{\mu}_i^{g*}$ to be zero. Due to Figure 4, if the optimal decisions of prosumer i lies in the green area, prosumer i 's payoff is non-negative. Meanwhile, if the optimal decisions of prosumer i lies in the critical boundaries, highlighted with black lines, the prosumer may incur a negative payoff. This situation reflects the necessity of prosumers' flexibility for guaranteeing individual rationality. Indeed, the more parameters \underline{d}_i and \underline{g}_i decrease for prosumers, the more chance prosumers have for ensuring their cost recovery and reaching individual rationality.

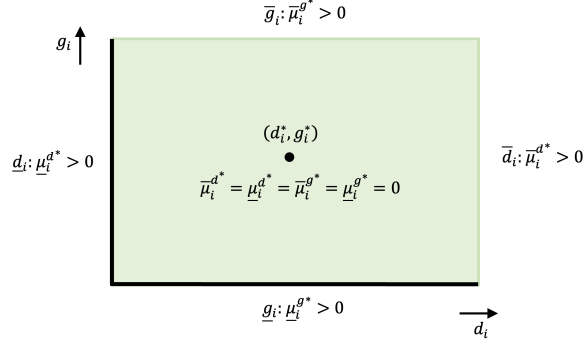


Figure 4: Critical boundaries for individual rationality of prosumers.

7. Numerical Results

7.1. Simulation Setup

This section presents numerical case studies to reflect the theoretical properties of the prosumers' behavior in practice. Toward that, a testbed comprised of six prosumers interacting across a P2P electricity market platform is provided. In the following case studies, the product differentiation of P2P trades stems from the cost allocation policy of the distribution utility. The leveraged policy is based on the electrical distance between the prosumers. Based on this policy, the network charge imposed on each trade is proportional to the corresponding electrical distance, and both sides of a trade are considered to have an equal contribution. Thus, the bilateral trade coefficient ξ_{ij} between prosumer i and j is based on the corresponding electrical distance and can be computed as the following:

$$\xi_{ij} = \frac{u^{dist} d_{ij}}{2}, \quad (20)$$

Where u^{dist} is the network charge expressed in (\$/kWh.km) and d_{ij} is the electrical distance between prosumer i and j in (km). The electrical distance between the prosumers in our setup are reflected in Figure 5.

The prosumers' cost functions and utility functions are in the quadratic format $C_i(\cdot) = a_i^g g_i^2 + b_i^g g_i$ and $U_i(\cdot) = a_i^u d_i^2 + b_i^u d_i$ respectively, where $a_i^g, b_i^g, a_i^u, b_i^u, \forall i \in \{1, 2, 3\}$ are given in Table 1, and the physical characteristics of prosumers are denoted in Table 2. The data

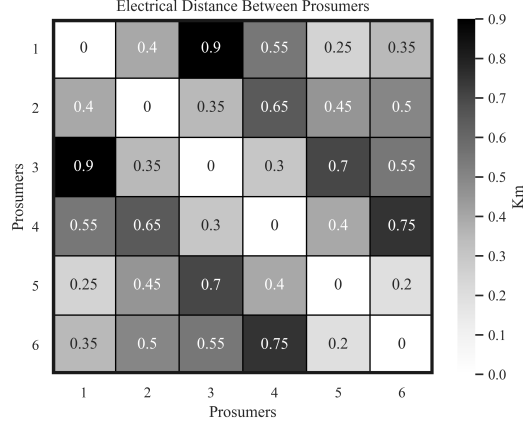


Figure 5: Matrix of electrical distance between prosumers.

used in our numerical experiments are from a case study in paper [27]. In the following sections, we first provide a high-level view of the P2P transactions across the platform. Then, we focus on individual rationality in the BMP mechanism.

Table 1: Cost and utility coefficients of prosumers

Prosumers	a_i^g (\$/kWh ²)	b_i^g (\$/kWh)	a_i^u (\$/kWh ²)	b_i^u (\$/kWh)
1	0.015	0.038	-0.008	0.8
2	0.008	0.047	-0.014	0.5
3	0.011	0.056	-0.009	0.4
4	0.008	0.065	-0.007	0.8
5	0.006	0.057	-0.008	0.1
6	0.01	0.04	-0.004	0.2

7.1.1. P2P Transactions Pattern

This section tries to give a high-level view of P2P electricity trading across the described platform. Figure 6 depicts the heat map of P2P electricity trading in three scenarios based on the network charge: 0, 0.05, 0.1 (\$/kWh.km). The corresponding cell to each non-zero transaction contains the amount of electricity trading (positive for selling, negative for buying). Note that the diagonal cells do not represent any P2P trading and are always equal to zero. By increasing the network charge, the cost of bilateral trades increases, which leads

Table 2: Physical limits of prosumers

Prosumers	\underline{g}_i (kW)	\overline{g}_i (kW)	\underline{d}_i (kW)	\overline{d}_i (kW)
1	0	20	5	15
2	0	25	7	18
3	0	30	10	25
4	0	10	0	7
5	0	15	0	5
6	0	20	0	13

to a decline in prosumers' willingness to participate in the market. Hence, the quantity of P2P electricity trading decreases for each prosumer. We can see that the P2P electricity trading between peers has a negative correlation with their corresponding electrical distance in Figure 5.

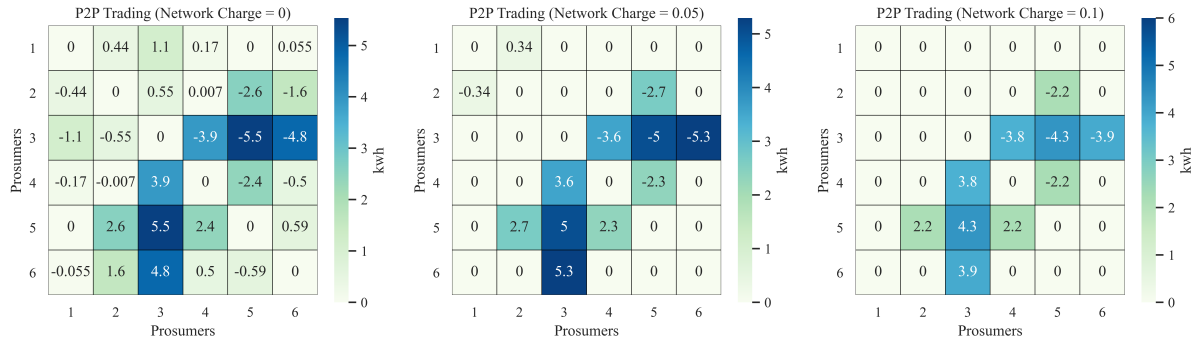


Figure 6: P2P electricity trading pattern of prosumers.

Figure 7 depicts the corresponding heat map of P2P prices for P2P electricity tradings in Figure 6. In the first scenario, when the platform operator does not impose any network charge, there is no price discrimination. Then, the P2P electricity market turns to a pool market with a uniform price. When the platform operator sets a non-zero network charge for market participation, e.g., 0.05 \$/kWh.km, the P2P prices are not uniform anymore, and the P2P electricity tradings are differentiated based on the electrical distance between prosumers. As we can see, increasing the network charge leads to the higher P2P prices. Because, due to the increased cost of bilateral trades, prosumers need higher prices to cover their costs of participation in the market.

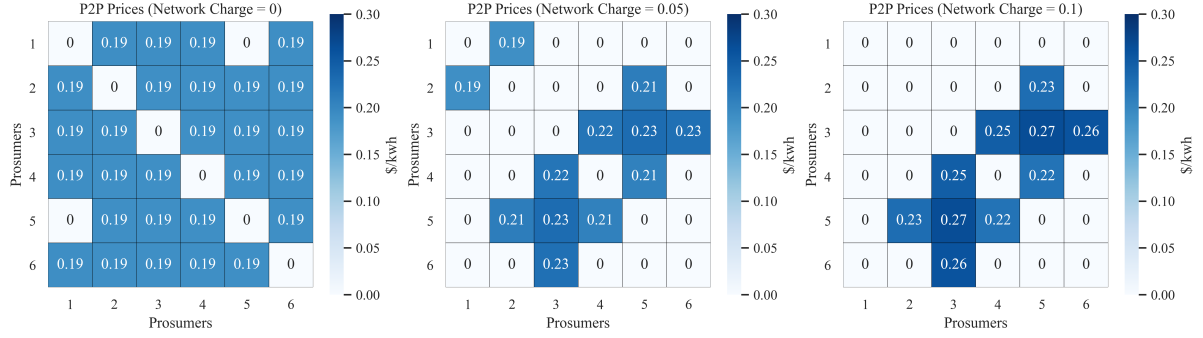


Figure 7: Corresponding pattern of P2P prices in prosumers' P2P electricity trading.

Table 3 summarizes the market-clearing results for prosumers, including prosumer i 's electricity demand d_i , generation g_i , payment p_i , and payoff f_i . We can see that the social welfare of prosumers decreases as the network charge imposed by the market operator increases. In addition, by increasing the network charge, prosumers' electricity generation converges to their electricity demand, which reflects their departure from the market platform. Hence, lowering the electrical distance between prosumers by applying low cost grid expansion approaches, e.g., non-wire alternatives, increases the prosumers' participation in P2P electricity trading and sharing of RESs at the grid edge.

Table 3: Summary of market-clearing results for prosumers

No.	Network charge = 0				Network charge = 0.05				Network charge = 0.1			
	d_i	g_i	p_i	f_i	d_i	g_i	p_i	f_i	d_i	g_i	p_i	f_i
1	6.75	8.54	-0.34	1.67	7.52	7.86	-0.06	1.6	7.7	7.7	0	1.59
2	12.97	8.96	0.76	3.16	12.52	9.53	0.62	3.13	12.16	9.98	0.5	3.11
3	22.98	7.01	3.04	6.32	21.89	8	3.13	5.7	20.91	8.89	3.1	5.25
4	7	7.82	-0.15	5.43	7	8.25	-0.3	5.5	7	8.54	-0.43	5.56
5	0	11.1	-2.11	2	0	10	-2.19	2.02	0	8.72	-2.14	1.94
6	1.24	7.5	-1.19	1.17	1.93	7.22	-1.19	1.26	2.91	6.83	-1.01	1.26
social welfare = 14.9 \$					social welfare = 14.36 \$				social welfare = 13.91 \$			

7.2. Analyzing Individual Rationality

We proved that individual rationality in the BMP mechanism is conditional, and only in extreme scenarios, where prosumers' flexibility to adjust their contribution in the market is limited, they may incur negative payoff. Thus, in most of the cases, typical prosumers get

non-negative payoff regardless of their matching pairs, other P2P trades, and the network charge. To visualize this property, Figure 8 depicts the payoff of each prosumer versus the network charge. We can see that the payoffs are all non-negative, and the prosumers have incentive to participate in the market.

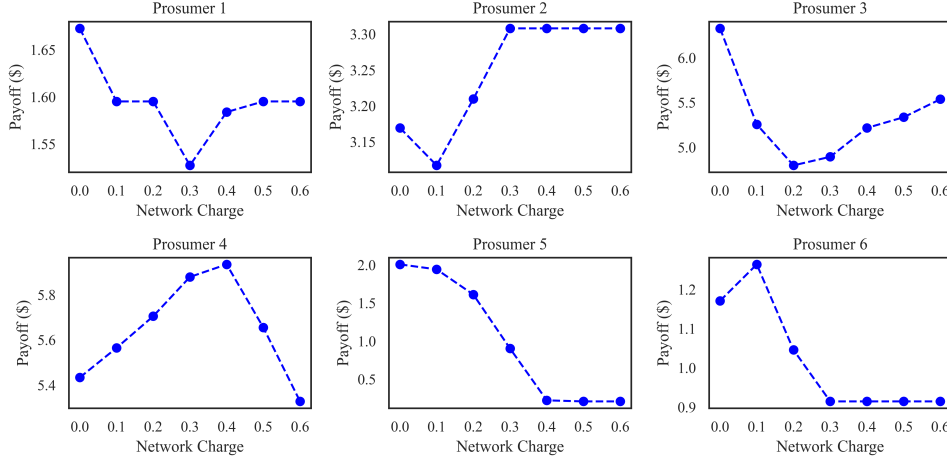


Figure 8: Prosumers' payoff versus the network charge.

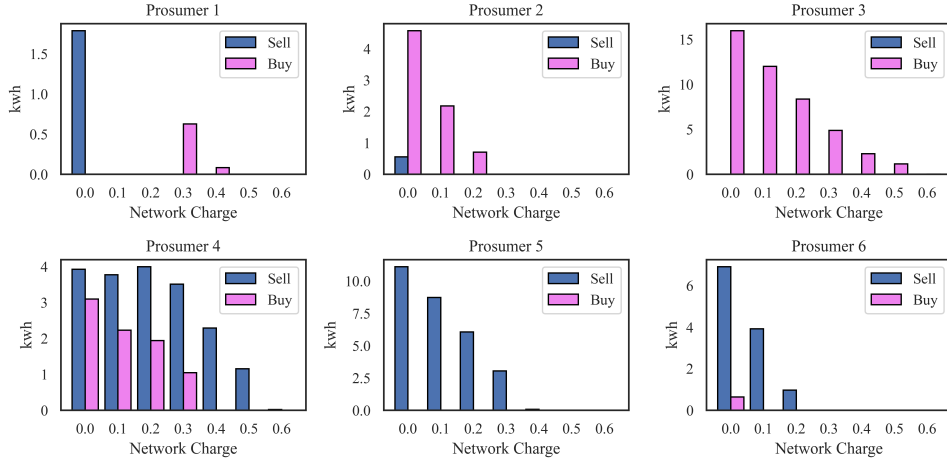


Figure 9: Aggregated P2P electricity trading of prosumers.

The aggregated P2P electricity trading versus the network charge is shown in Figure 9 for each prosumer. Due to Figure 9, the prosumers' P2P electricity trading, either buying or selling in the platform, reduces by increasing the network charge. In other words, increasing the network charge steers prosumers toward self sufficiency. Indeed, there is a specific cut

off point for each prosumer based on the network charge, and, when the network charge reaches to that point, the prosumer opts out of the platform. For instance, the cut off point for prosumer 2 and 5 are around 0.3 \$/kwh.km and 0.4 \$/kwh.km, respectively.

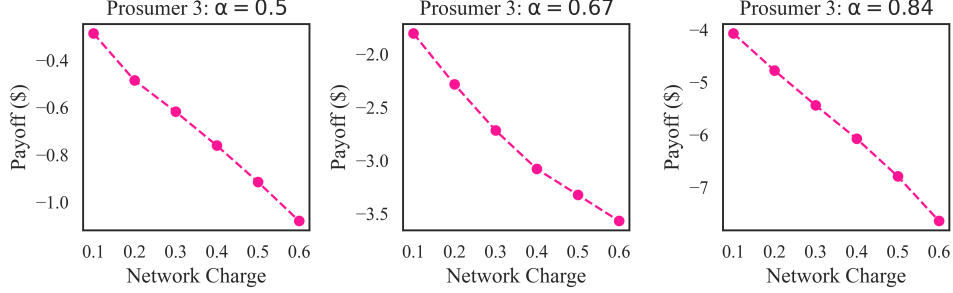


Figure 10: Payoffs of Prosumer 3 under different flexibility levels.

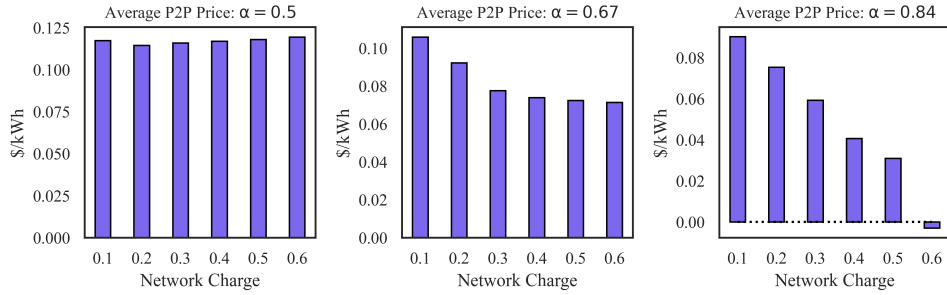


Figure 11: Average P2P prices for Prosumer 3 under different flexibility levels.

In the following, we provide a scenario in which a prosumer experiences negative payoff in the BMP mechanism. This would happen when a prosumer's flexibility to decrease the amount of electricity it consumes or generates is limited. That is, the lower bounds of the production capacity \underline{g}_i and consumption level \underline{d}_i of prosumer i is considerably high. In our scenario, we focus on prosumer 3's production flexibility and set its consumption level equal to zero, $\underline{d}_3 = \overline{d}_3 = 0$. For reflecting the production flexibility of prosumer 3, we consider $\underline{g}_3 = \alpha \overline{g}_3$, where $0 \leq \alpha \leq 1$, and higher α implies lower flexibility. Figure 10 depicts the payoffs of prosumer 3 for different flexibility levels versus the network charge. We can see in Figure 10 that the payoffs of prosumer 3 get worse as α increases. Because, due to Figure 11, the average P2P prices decrease for prosumer 3, and it cannot recover the

cost of P2P electricity trading with others. Besides, in some scenarios (e.g., $\alpha = 0.84$ and network charge = 0.6) P2P prices for prosumer 3 become negative, and it pays money to other prosumers for selling electricity. Therefore, the lack of flexibility forces prosumers to participate in P2P electricity tradings that may incur them negative payoffs.

8. Conclusion

This paper proposed a mechanism design framework for investigating prosumers' incentive for P2P electricity trading. In this regard, we focused on individual rationality as a built-in incentive in the P2P market-clearing mechanism that guarantees non-negative payoffs for prosumers. We proved that individual rationality in the BMP mechanism depends on prosumers' flexibility for market participation. That is, a prosumer may experience a negative payoff if its flexibility to decrease the amount of electricity it consumes or generates is limited. As we mentioned, individual rationality is essential for incentivizing prosumers, but there are also other properties that have major role in steering prosumers to participate in P2P electricity trading. For instance, incentive compatibility is a property that guarantees truthful behavior is the dominant strategy for prosumers in P2P electricity trading, regardless of other prosumers' behavior. In the future work, we will develop a P2P market-clearing mechanism that is incentive compatible by design.

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