

# Modeling\_King\_County\_House\_Sales

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## Final Project Submission

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## 1 Introduction (DONE)

In this note, we are trying to help homeowners to buy or sell homes by predicting the price of the property in **King County, WA**. We will give them some suggestions how they could increase the value of the property and what are the main features they should consider. We use some available data from the housing prices in this county to present a model to predict the price of a house. By using the

In order to do so, we use regression methods to find an appropriate model to fit housing price data so that we can predict the price of different houses with different features.

This notebook is organized as follows:

**2. Importing data.** In this part we import the data and we will introduce which columns it contains.

**3. Functions.** This section contains the functions we defined to perform special computations for us. These functions are:

3.1 `corr`

3.2 `summary_of_results`

3.3 `concatenate`

**4. Some Insight Into Data.** In this section we are trying to identify the categorical and numerical features. By plotting some graphs, we will find the outliers and how to clean the data. This section has the following subsections:

4.1 `Scatter Plots for Categorical Features`

4.2 `Scatter Plots for Numerical data`

4.3 `Cleaning data`

**5. Categorical.** In this section, we are converting the categorical data into numerical values to be able to use them in the model. This section contains the following subsections:

5.1 `Dealing with Null Values`

5.2 `Converting multi categorical columns to numerical values`

**6. Preprocessing.** In this section, we are going to see the effects of containing different categorical and numerical variables on R2 score to see which features we need to keep. This section contains:

- 6.1 First Model: Putting `grade`, `condition` and `zipcode` into the model.
- 6.2 Second Model: Putting only `condition` into the model.
- 6.3 Third Model: Putting `grade` and `condition` into the model.
- 6.5 Forth Model-Part 1: Considering only `grade` into the modeling.
- 6.5 Forth Model-Part 2: Considering only `grade` into the modeling.

**7. Features Selection.** In this section, based on the dataframe that we found in the previous section, we will try to find the features that we have more information but low collinearity and high R2 score. We use different approaches to decide which features we need to keep. These approaches are used in different subsections which are:

- 7.1 First Approach By using p-values, R2 scores and Condition number.

**8. Final Model.** In this section, we find the baseline model to compare the model we found in the previous section with. This section contains the following subsections:

- 8.1 Baseline Model
- 8.2 Final Model
- 8.3 Interpretation of Coefficients

**9. Prediction.** We will use the model introduced in the section 9 to predict some data.

**10. Assumption Checking.** In this section, we are going to check the regression model's assumptions to see if they are satisfied or not. This section contains the following subsections:

- 11.1 Normality of Residuals
- 11.2 Investigating Multicollinearity (Independence Assumption)
- 11.3 Investigating Homoscedasticity
- 11.4 Investigating Linearity

**11. Summary and Suggestions.** In this section we discuss the model and we will we will give some suggestions as an answer to our business question.

## 2 Importing Data (DONE)

First we are going to import the data and save them into a dataframe called `data_initial`. After that we select some columns as the features of our model. The `dataframe` that we are using in the rest of the work contains the following type of variables:

### 1. Numerical Columns

- `price`
- `bedrooms`
- `bathrooms`
- `sqft_living`

- sqft\_lot
- floors
- yr\_built
- lat
- long

## 2. Categorical Columns

- waterfront
- condition
- grade
- zipcode

However, we may not use all of these columns in our model and we need to choose among them as features of our final model.

```
[1]: import os
import numpy as np
from glob import glob
import pandas as pd
import seaborn as sns
from matplotlib import pyplot as plt
import seaborn as sns

%matplotlib inline

data_initial = pd.read_csv("./data/kc_house_data.csv")

to_drop_initial = ["date", "view", "sqft_above", "sqft_basement",
↪ "yr_renovated",
                    "sqft_living15", "sqft_lot15", "id"]

df = data_initial.drop(columns = to_drop_initial, axis = 1)
```

## 3 Functions we use (DONE)

In this section, the functions we used to get some information about data (`corr` function) or create a final dataframe to use (`concatenate` function) or will return the R2score and condition number (`summary_of_results` function). These functions are:

1. `corr`
2. `summary_of_results`
3. `concatenate`

Each of these functions is introduced below.

### 3.1 `corr`

In order to get the correlation coefficients, we define a function that takes two inputs which are data and then the minimum value of the correlation. This minimum value will be used to find

features with correlation more than or equal to this minimum value.

```
[2]: def corr(data, value):
    corr_table = data.corr().abs().stack().reset_index().sort_values(0,
                                                                    ascending=False)

    corr_table["pairs"]=list(zip(corr_table["level_1"],corr_table["level_0"]))

    corr_table.drop(columns = ["level_1", "level_0"], inplace = True)
    corr_table.reset_index(inplace = True, drop = True)

    corr_1 = corr_table.iloc[0:len(data.columns)].index
    corr_table.drop(index = corr_1, inplace = True)
    corr_table.reset_index(inplace = True, drop = True)
    table1 = corr_table.iloc[range(0, len(corr_table), 2)]
    table = table1.loc[table1[0]>value]
    table.reset_index(inplace = True, drop = True)

    return table
```

### 3.2 summary\_of\_results

In order to update the text automatically and get R2 score and collinearity, we are going to convert results\_summary obtained from statsmodels library into Pandas DataFrame to find R2-scores, coefficients and P-values for different models.

```
[3]: def summary_of_results(data, to_drop, pval):
    import statsmodels.api as sm

    features = df_final.drop(columns = to_drop, axis = 1)
    X = sm.add_constant(features)
    model = sm.OLS(df_final["price"], X)
    results = model.fit()
    results_summary = results.summary()

    ### Converting results_summary to pandas dataframe
    results_R2 = results_summary.tables[0].as_html()
    R2_df = pd.read_html(results_R2, header=0, index_col=0)[0]
    R2_df.reset_index(inplace = True)
    R2_df = R2_df.columns.to_frame().T.append(R2_df, ignore_index=True)
    R2_df.columns = range(len(R2_df.columns))

    results_coeff = results_summary.tables[1].as_html()
    coeff_df = pd.read_html(results_coeff, header=0, index_col=0)[0]
```

```

coeff_df.reset_index(inplace = True)
coeff_df = coeff_df.columns.to_frame().T.append(coeff_df,
                                                ignore_index=True)
coeff_df.columns = range(len(coeff_df.columns))

results_collin = results_summary.tables[2].as_html()
collin_df = pd.read_html(results_collin, header=0, index_col=0)[0]
collin_df.reset_index(inplace = True)
collin_df = collin_df.columns.to_frame().T.append(collin_df,
                                                ignore_index=True)
collin_df.columns = range(len(collin_df.columns))

R2 = R2_df.iloc[0, 3]

collinearity_num = collin_df.iloc[3, 3]

coeff = coeff_df.iloc[1:,[0,4, 1]]

coeff.columns = ["feature", "P-value", "coefficient"]

coeff["coefficient_absolute_value"] = np.abs(coeff["coefficient"])
coeff.sort_values(by = "coefficient_absolute_value",
                  ascending = True, inplace = True)

critical_pval = coeff.loc[coeff["P-value"]>= pval]

return [R2, collinearity_num, critical_pval, coeff]

```

### 3.3 concatenate

This functions will return the final dataframe that we use for our analysis.

```

[4]: def concatenate(
        include_grade = False,
        include_zipcode = False,
        include_condition = False
    ):

    data      = df_no_outliers
    sub_df1   = grade_num_df
    sub_df2   = zipcode_num_df
    sub_df3   = condition_num_df

```

```

include_sub_df1 = include_grade
include_sub_df2 = include_zipcode
include_sub_df3 = include_condition

if (include_sub_df1 == True
    and include_sub_df2 == False
    and include_sub_df3 == False):

    df_final = pd.concat([data, sub_df1], axis = 1)

elif (include_sub_df1 == False
    and include_sub_df2 == True
    and include_sub_df3 == False):

    df_final = pd.concat([data, sub_df2], axis = 1)

elif (include_sub_df1 == False
    and include_sub_df2 == False
    and include_sub_df3 == True):

    df_final = pd.concat([data, sub_df3], axis = 1)

elif include_sub_df1 == True and include_sub_df2 == True:

    df_final = pd.concat([data, sub_df1, sub_df2], axis = 1)

elif include_sub_df1 == True and include_sub_df3 == True:

    df_final = pd.concat([data, sub_df1, sub_df3], axis = 1)

elif include_sub_df2 == True and include_sub_df3 == True:

    df_final = pd.concat([data, sub_df2, sub_df3], axis = 1)

else:

    df_final = pd.concat([data, sub_df1, sub_df2, sub_df3], axis = 1)

return df_final

```

## 4 Some Insight Into Data (DONE)

In order to find the outliers and how the data looks like, we will check the scatter plots of both categorical and numerical data as well as histogram for numerical data. In order to do so, first we need to find categorical and numerical data. To do this we use `select_dtypes` method as shown below:

```
[5]: y = df["price"]
numerical = df.drop(columns = ["price", "zipcode"], axis = 1).select_dtypes(
    include=["float64", "int64"])
l = list(numerical.columns)
l.append("price")
categorical = df.drop(columns = l, axis = 1)
```

## 4.1 Scatter Plots for Categorical Features

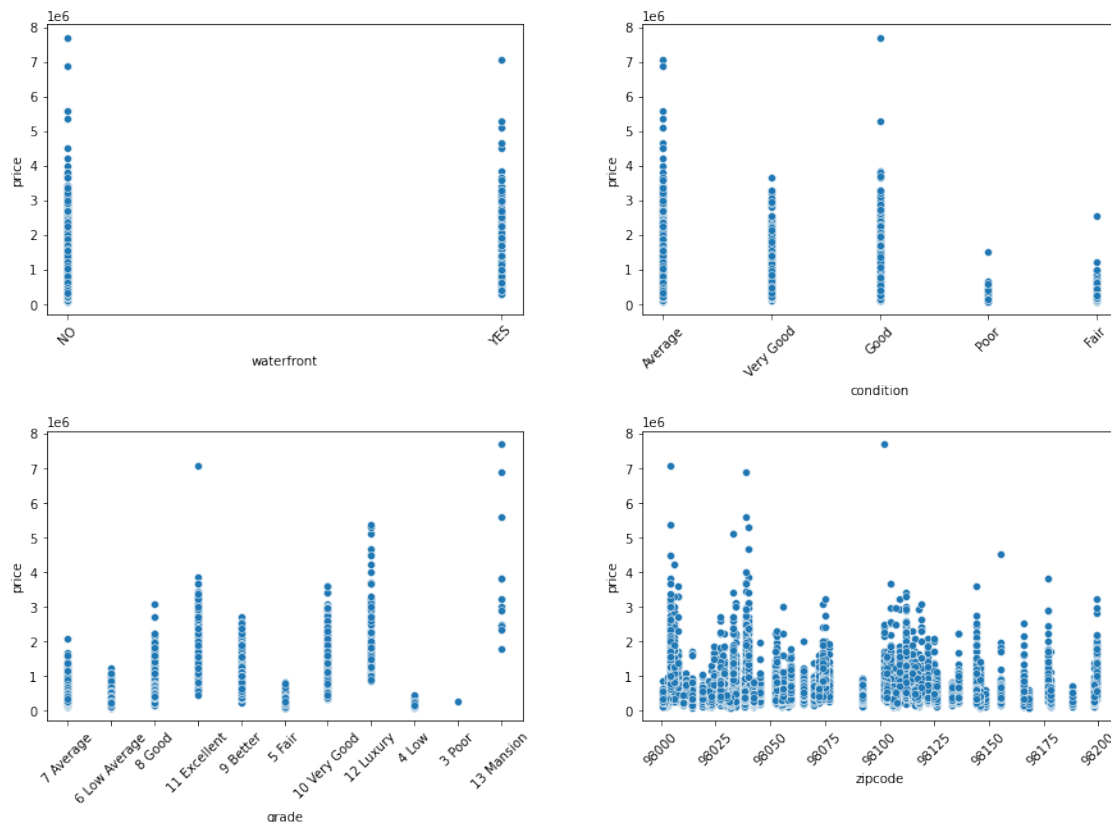
By studying the scatter plots of different categorical variable vs. price, we may find some points that can be considered not that much useful. For example, we see that we only have one data point for 3 Poor in grade column, so we may drop this because we cannot use it.

```
[6]: fig, axes = plt.subplots(nrows = 2, ncols = 2, figsize = (15,10))

fig.subplots_adjust(hspace=0.4, wspace=0.25)

to_pick = list(categorical.columns)

for i,col in enumerate(to_pick):
    ax = axes[i//2][i%2]
    sns.scatterplot(x = df[col], y = df["price"], ax = ax)#, label = col)
    ax.tick_params(axis='x', labelrotation = 45)
```



```
[7]: len(df.loc[df["grade"] == "3 Poor"])
```

```
[7]: 1
```

## 4.2 Scatter Plots for Numerical data

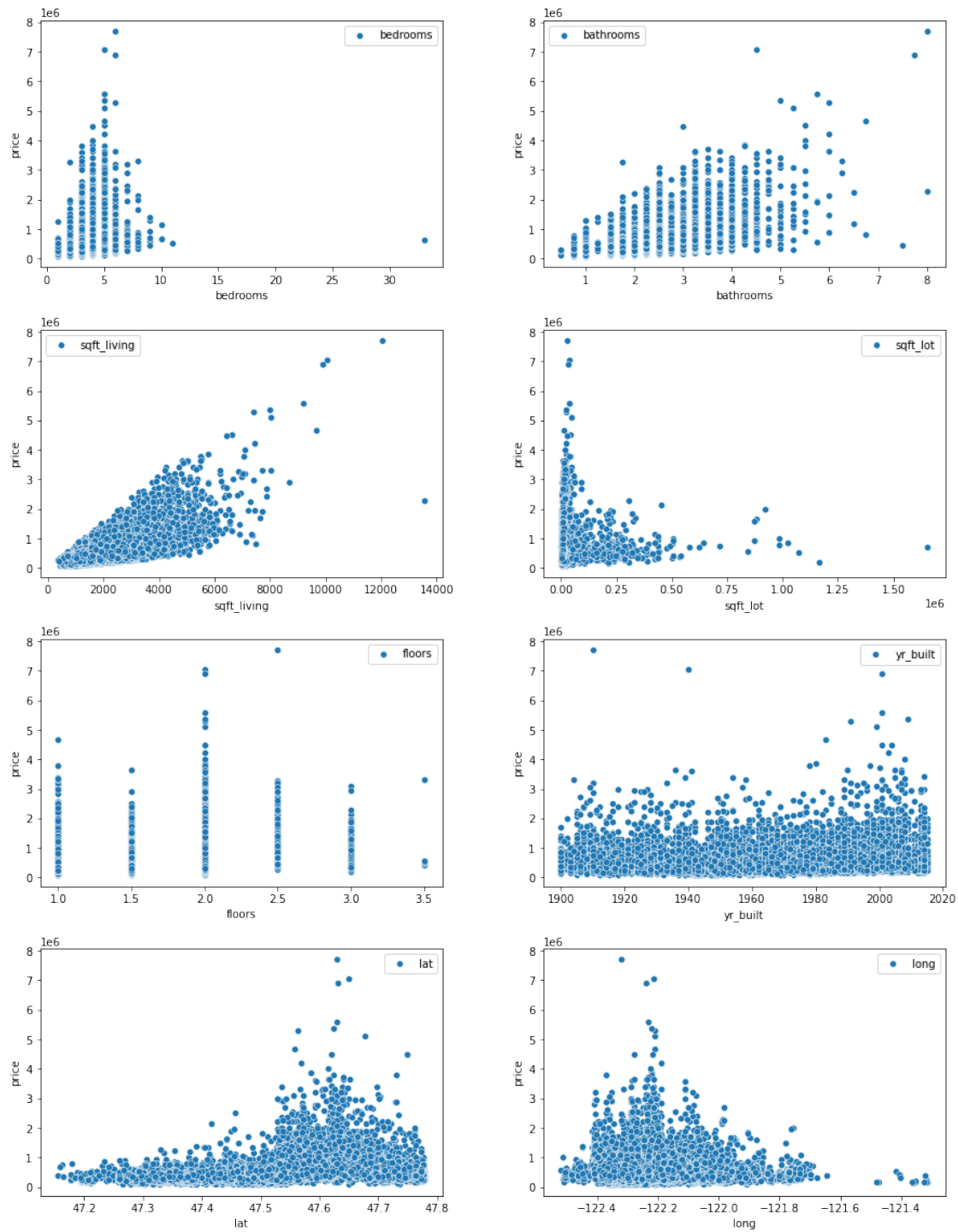
In this section, by checking the scatter plot of numerical data vs. price, we may be able to find some outliers. For example we see that there is a point with more than 30 rooms. Also, we see that there are houses with less than a bathroom. So, checking these scatter plots will help us with finding some outliers and not that useful data points.

```
[8]: fig, axes = plt.subplots(nrows = 4, ncols = 2, figsize = (15,20))
fig.subplots_adjust(hspace=0.25, wspace=0.25)

to_pick = list(numerical.columns)

for i,col in enumerate(to_pick):
    ax = axes[i//2][i%2]
    sns.scatterplot(x = df[col], y = df["price"], ax = ax, label = col)
```





```
[9]: len(df.loc[df["floors"] == 3.5])
```

```
[9]: 7
```

### 4.3 Cleaning data

First we are going to drop the outliers for highest and lowest prices. In order to do so, we are going to drop the row in `price` column that are more/less than 3 standard deviations from average price. In order to do that, we are going to use the function `stats.zscore` from `scipy.stats` library.

```
[10]: import scipy.stats as stats

import warnings
warnings.filterwarnings("ignore")

z = stats.zscore(df["price"], ddof=0)
df["z_score"] = stats.zscore(df["price"], ddof=0)
df_no_outliers = df.loc[(df["z_score"] < 3) & (df["z_score"] > -3)]
df_no_outliers.drop(columns = "z_score", axis = 1, inplace = True)
```

As we can see there are some outliers in the `bedrooms`, `sqft_lot` and `sqft_living`. So, we are going to drop houses with more than 8 bedrooms, 7 bathrooms, and 3.5 floors as well as houses with less than 1 bathrooms. Moreover, we are going to drop the houses with the highest values of `sqft_lot` and `sqft_living` and the only data whose grade is 3 Poor.

```
[12]: max_b = 8
max_bath = 7
min_bath = 1
max_floors = 3.5
max_lot = max(df_no_outliers["sqft_lot"])
max_liv = max(df_no_outliers["sqft_living"])

condition = ((df_no_outliers["bedrooms"] == max_b)
             | (df_no_outliers["bathrooms"] == max_bath)
             | (df_no_outliers["bathrooms"] < min_bath)
             | (df_no_outliers["sqft_lot"] == max_lot)
             | (df_no_outliers["sqft_living"] == max_liv)
             | (df_no_outliers["grade"] == "3 Poor")
             | (df_no_outliers["floors"] == max_floors))

ind = df_no_outliers.loc[condition].index
df_no_outliers.drop(index = ind, inplace = True)
```

```
[13]: y = df_no_outliers["price"]
numerical = df_no_outliers.drop(columns = ["price", "zipcode"],
                                , axis = 1).select_dtypes(include=["float64", "int64"])
l = list(numerical.columns)
l.append("price")
categorical = df_no_outliers.drop(columns = l, axis = 1)
```

Now we are going to convert (scaling and normalizing) the data in the columns `price`, `lat`, `long`, `yr_built`, `sqft_living` and `sqft_lot` to make the data more normal. However, if we check we realize that the values of the column `long` are all negative and we need to first multiply them with

a minus sign to be able to convert them by a logarithmic function.

```
[14]: sum(df_no_outliers["long"] > 0)
df_no_outliers["long"] = -1 * df_no_outliers["long"]
to_convert = ["price", "long", "lat", 'sqft_living', 'sqft_lot', "yr_built" ]
for item in to_convert:
    if item == "price":
        df_no_outliers[item] = np.log(df_no_outliers[item])
    else:
        df_no_outliers[item] = np.log(df_no_outliers[item])
        numerical[item] = np.log(numerical[item])
```

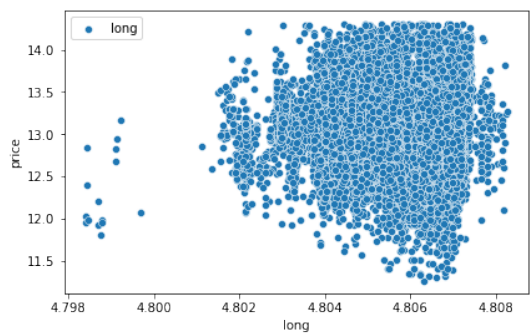
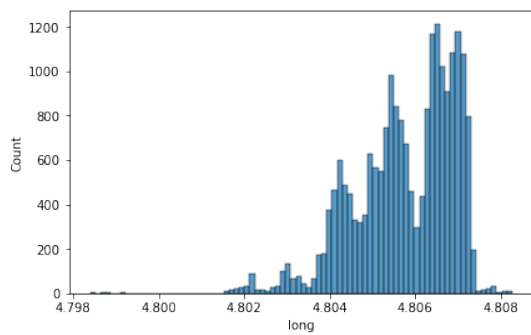
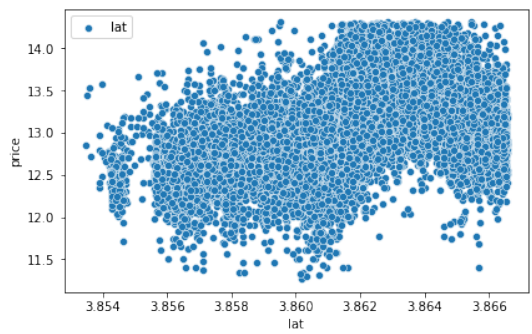
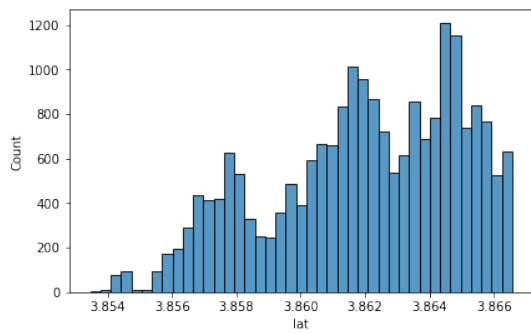
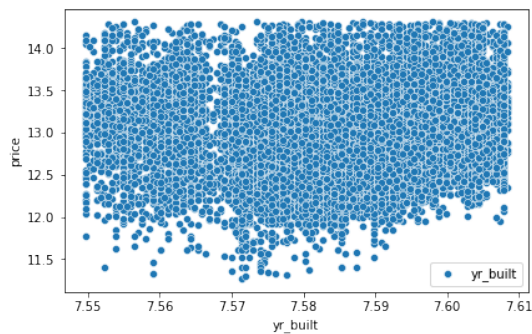
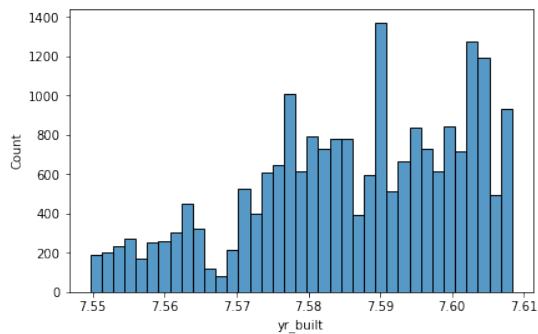
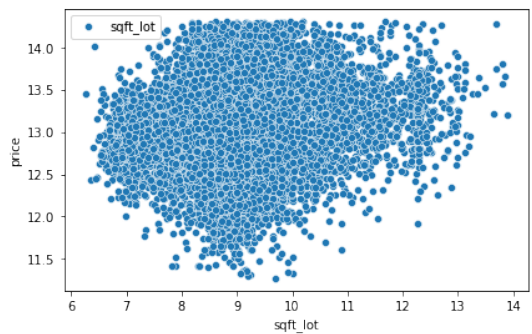
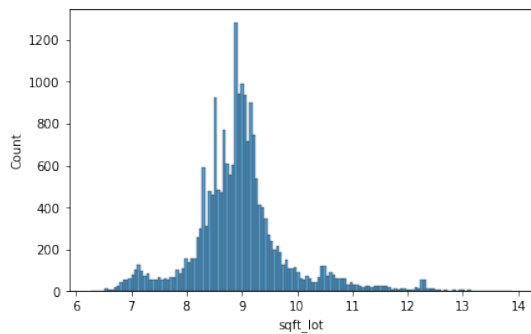
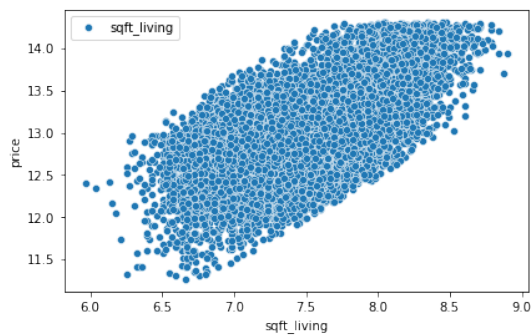
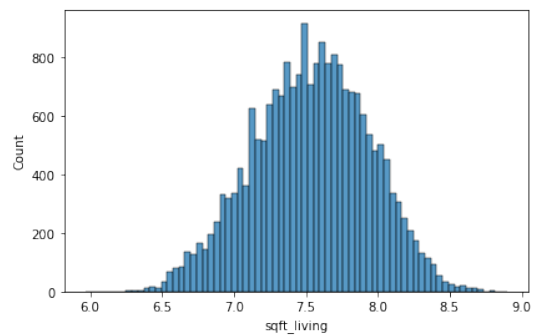
After cleaning, we are going to check the histogram and scatter plots of the data that are recently converted.

```
[15]: to_pick = list(numerical.columns)

to_pick.remove("bedrooms")
to_pick.remove("bathrooms")
to_pick.remove("floors")

r = len(to_pick)
c = 2
fig, axes = plt.subplots(nrows = r, ncols = c, figsize = (15,25))
fig.subplots_adjust(hspace=0.25, wspace=0.25)

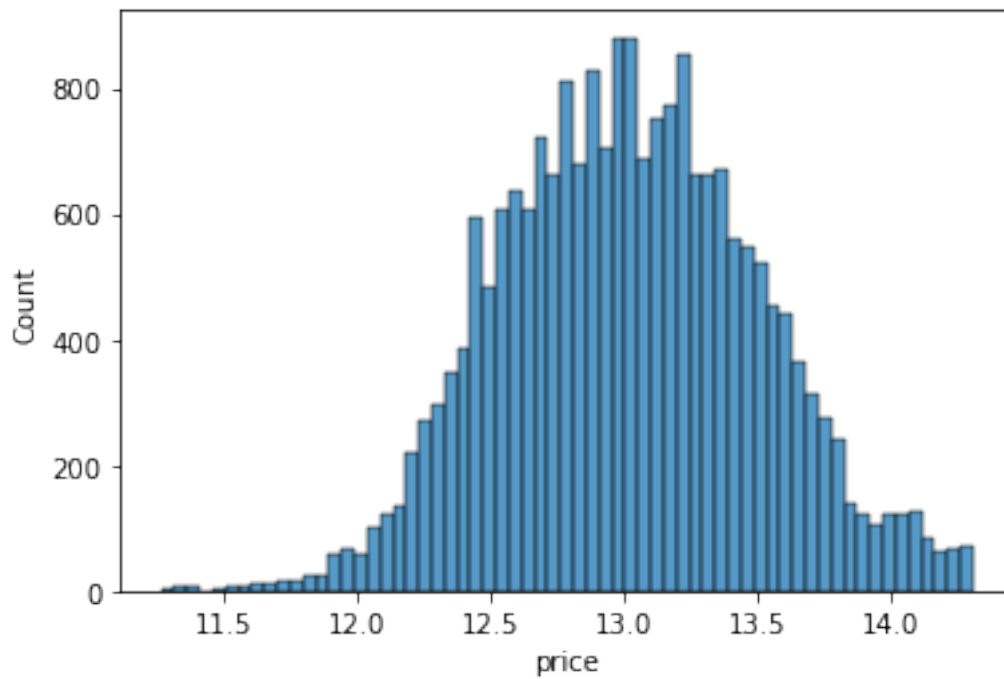
for i,col in enumerate(to_pick):
    axh = axes[i%r][0]
    axs = axes[i%r][1]
    sns.histplot(x = df_no_outliers[col],ax = axh, label = col)
    sns.scatterplot(x = df_no_outliers[col],y = df_no_outliers["price"]
                    , ax = axs, label = col)
```

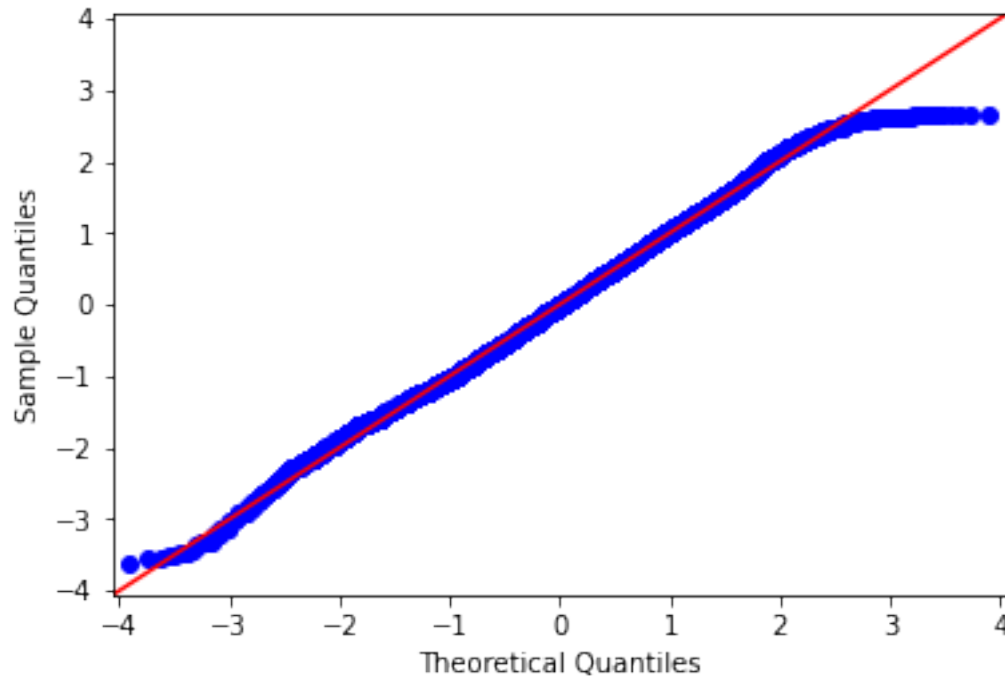


Let's also check the histogram and Q-Q plot of `price` to see if `price` is normal or not.

```
[16]: import scipy.stats as stats
import statsmodels.api as sm

sns.histplot(x = df_no_outliers["price"], label = "price");
sm.graphics.qqplot(df_no_outliers["price"],
                  dist=stats.norm, line='45', fit=True);
```





## 5 Categorical (DONE)

We need to convert the categorical data to numerical values so that we can use them in our model. First we are going to find the columns that have missing data.

```
[17]: a = categorical.isna().sum().to_frame().reset_index()
```

We can see that only `waterfront` has missing values and the the total number of values that are missed is 2325.

### 5.1 Dealing with Null Values

First we are going to create a new column for missing values. We know that the column `waterfront` has 2325 missing values. Therefore, we are going to create a column called `waterfront_null` to indicate where data is missing. In order to do so, we take the following steps.

First we use `MissingIndicator` from `sklearn.impute` to create a column in `df` called `waterfront_null` as shown below:

```
[18]: ### Missing Indicator for waterfront
import warnings
from sklearn.impute import MissingIndicator
warnings.filterwarnings("ignore")
missing_indicator = MissingIndicator()
```

```

null_val = df_no_outliers[["waterfront"]]
missing_indicator.fit(null_val)

df_no_outliers["waterfront_null"] = missing_indicator.transform(null_val)

```

After creating a new column for the missing values, we are going to impute the missing values in the column `waterfront` by using `SimpleImputer` from `sklearn.impute` as done below

```

[19]: ### Imputing Missing Values for waterfront

import warnings
warnings.filterwarnings("ignore")

from sklearn.impute import SimpleImputer

imputer = SimpleImputer(strategy="most_frequent")
imputer.fit(df_no_outliers[["waterfront"]])
df_no_outliers["waterfront_impute"] = imputer.transform(
    df_no_outliers[["waterfront"]])

```

At the end, we will use `OrdinalEncoder` from `sklearn.preprocessing` to convert the binary values into numerical values. To do so, first we need to convert the Column `waterfront_impute` to numerical value as:

```

[20]: ### Converting the Column waterfront_impute to numerical value

import warnings
warnings.filterwarnings("ignore")

from sklearn.preprocessing import OrdinalEncoder

encoder_waterfront = OrdinalEncoder()
encoder_waterfront.fit(df_no_outliers[["waterfront_impute"]])

encoder_waterfront_transform = encoder_waterfront.transform(
    df_no_outliers[["waterfront_impute"]]).flatten()

df_no_outliers["waterfront_impute"] = encoder_waterfront_transform

df_no_outliers.drop(columns = ['waterfront'], inplace = True, axis = 1)

```

Now we are going to convert the column `waterfront_null` to Numerical value.

```

[21]: ### Converting the Column waterfront_null to Numerical value

import warnings

```

```
warnings.filterwarnings("ignore")

from sklearn.preprocessing import OrdinalEncoder

encoder_waterfront_null = OrdinalEncoder()
encoder_waterfront_null.fit(df_no_outliers[["waterfront_null"]])

encoder_waterfront_null_transform = encoder_waterfront_null.transform(
    df_no_outliers[["waterfront_null"]]).flatten()

df_no_outliers["waterfront_null"] = encoder_waterfront_null_transform
```

## 5.2 Converting multi categorical columns to numerical values

The categorical multiple values are stored in columns `condition`, `grade` and `zipcode`. In order to convert them to numerical values we should use `OneHotEncoder` from `sklearn.preprocessing`.

First we are going to convert the categorical variable `condition` to numerical values in the following cell.

```
[22]: import warnings
warnings.filterwarnings("ignore")

from sklearn.preprocessing import OneHotEncoder

condition_cat = df_no_outliers[["condition"]]

ohe = OneHotEncoder(categories='auto', sparse=False, handle_unknown='ignore')
ohe.fit(condition_cat)

condition_num = ohe.transform(condition_cat)
condition_num_df = pd.DataFrame(condition_num,
                                columns = ohe.categories_[0],
                                index = df_no_outliers.index)
df_no_outliers.drop(columns = ["condition"], inplace = True, axis = 1)
```

In the following cell, we are going to do the same thing to convert `grade` to numerical values

```
[23]: import warnings
warnings.filterwarnings("ignore")

from sklearn.preprocessing import OneHotEncoder

grade_cat = df_no_outliers[["grade"]]

ohe = OneHotEncoder(categories='auto', sparse=False, handle_unknown='ignore')
```



```

ohe.fit(grade_cat)

grade_num = ohe.transform(grade_cat)
grade_num_df = pd.DataFrame(grade_num,
                             columns = ohe.categories_[0],
                             index = df_no_outliers.index)
df_no_outliers.drop(columns = ["grade"], inplace = True, axis = 1)

```

And finally, we are going to converting zipcode to numerical values in the following cell

```

[24]: import warnings
warnings.filterwarnings("ignore")

from sklearn.preprocessing import OneHotEncoder

zipcode_cat = df_no_outliers[["zipcode"]]

ohe = OneHotEncoder(categories='auto', sparse=False, drop= "first")
ohe.fit(zipcode_cat)
zipcode_num = ohe.transform(zipcode_cat)

zipcode_num_df = pd.DataFrame(zipcode_num,
                               columns = ohe.categories_[0][1:],
                               index = df_no_outliers.index)
df_no_outliers.drop(columns = ["zipcode"], inplace = True, axis = 1)

```

## 6 Preprocessing (DONE)

In order to create a model, first we need to decide what features we want to consider in our model. Initially, we will try to calculate R2 score and Cond. No. for different combinations of categorical features to decide which combination we want to choose.

### 6.1 First Model: Putting grade, condition and zipcode into the model.

First we want to see how different features affect the data. If we include grade\_num\_df, zipcode\_num\_df and condition\_num\_df, we get R2-score and collinearity as:

```

[25]: df_final = concatenate(include_grade = True, include_zipcode = True,
                             include_condition = True)

to_drop = ["price"]
l = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2      : ", l[0])
print("Cond. No. : ", l[1])

```

```

R2      : 0.854
Cond. No. : 2380000000000000.0

```

the R2 score which is 0.854 is really good but Cond. No. which is 23800000000000000.0. The value of R2 score is really good, so we may consider it as a strong candidate for our model. In the next part we compare it with the baseline model.

### 6.1.1 Comparison with the Baseline Model

We will create a baseline model with which we can compare the results of the first model. We consider the baseline model as the linear regression model between `price` and the variable that has the highest correlation with `price`. The following code, gives us the name of the variable that has the highest correlation with `price`

```
[26]: numerical_II = df_no_outliers.drop(columns = "price", axis = 1).select_dtypes(
      include=["float64", "int64"])
corr_column = list(numerical_II.columns)
corr_dict = {}
for item in corr_column:
    corr_dict[df_no_outliers["price"].corr(df_no_outliers[item])] = item
high_corr = corr_dict[max(corr_dict)]
high_corr
```

```
[26]: 'sqft_living'
```

Now, we are going to find the R2 of the base line model for different test train sets

```
[27]: from sklearn.model_selection import train_test_split
      from sklearn.linear_model import LinearRegression
      from sklearn.model_selection import cross_validate, ShuffleSplit

      df_final
      X = df_final[[high_corr]]
      y = df_final["price"]
      X_train_b, X_test_b, y_train_b, y_test_b = train_test_split(X, y,
      ↪random_state=42)

      baseline_model = LinearRegression()
      splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)

      baseline_scores = cross_validate(
          estimator=baseline_model,
          X=X_train_b[[high_corr]],
          y=y_train_b,
          return_train_score=True,
          cv=splitter
      )
```

Now we will make our model and will compare the result of the model with the baseline model

```
[29]: from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_validate, ShuffleSplit

X = df_final.drop(columns = to_drop, axis = 1)
y = df_final["price"]
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42)

second_model = LinearRegression()
splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)

secondmodel_scores = cross_validate(
    estimator=second_model,
    X=X_train,
    y=y_train,
    return_train_score=True,
    cv=splitter
)

print("Train score(mean):      ", secondmodel_scores["train_score"].mean())
print("Validation score(mean): ", secondmodel_scores["test_score"].mean())

print()

print("Train score:      ", baseline_scores["train_score"].mean())
print("Validation score:", baseline_scores["test_score"].mean())
```

```
Train score(mean):      0.8537218955040682
Validation score(mean): 0.8526895649895243
```

```
Train score:      0.409409242793934
Validation score: 0.41724912602012776
```

The R2 for the train set and cross validation set is really good so now we need to check the collinearity condition as done in the next part.

### 6.1.2 Final Decision

In order for us to consider the model, we need to check the collinearity condition. If we calculate the variance-inflation factors we will notice that including zipcode\_num\_df into the final dataframe will cause a lot of collinearities as shown below

```
[30]: from statsmodels.stats.outliers_influence import variance_inflation_factor
import statsmodels.api as sm

df_final = concatenate(include_grade = True, include_zipcode = True,
                        include_condition = True)
```

```

to_drop = ["price"]

XX = df_final.drop(columns = to_drop, axis = 1)

XX_constant_added = sm.add_constant(XX)

vif = [
    variance_inflation_factor(XX_constant_added.values, i)
    for i in
        range(XX_constant_added.shape[1])
]
variance_inf_fact = pd.Series(vif, index=XX_constant_added.columns,
                             name="Variance Inflation Factor")

```

We can see that the number of coefficients with Variance Inflation Factor more than 5 is around

```
[31]: print(sum(variance_inf_fact>5))
```

49

Therefore, we just exclude `zipcode` from rest of the work and we can use `lat` and `long` as a method to check the location of different houses.

## 6.2 Second Model: Putting only condition into the model.

If we only consider `condition` in the model, we find R2 and Cond. No. as

```

[32]: df_final = concatenate(include_grade = False, include_zipcode = False,
                             include_condition = True)

to_drop = ["price"]
c = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2      : ", c[0])
print("Cond. No. : ", c[1])

```

```

R2      : 0.659
Cond. No. : 1.17e+16

```

Therefore, considering only `condition` in the model will result in R2 score equals to 0.659 and Cond. No. equals 1.17e+16 which are, respectively, lower and higher than previous cases. Therefore, we should ignore this model. However, in the next part, we compare it with the baseline model and we will check the collinearity by using Variance Inflation Factor as:

### 6.2.1 Creating Baseline Model and Checking this Model

We will create a baseline model with which we can compare the results of the first model. We consider the baseline model as the linear regression model between `price` and the variable that has the highest correlation with `price`. The following code, gives us the name of the variable that has the highest correlation with `price`

```
[33]: numerical_II = df_no_outliers.drop(columns = "price", axis = 1).select_dtypes(
        include=["float64", "int64"])
corr_column = list(numerical_II.columns)
corr_dict = {}
for item in corr_column:
    corr_dict[df_no_outliers["price"].corr(df_no_outliers[item])] = item
high_corr = corr_dict[max(corr_dict)]
high_corr
```

```
[33]: 'sqft_living'
```

Now, we are going to find the R2 of the base line model for different test train sets

```
[117]: from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_validate, ShuffleSplit

df_final
X = df_final[[high_corr]]
y = df_final["price"]
X_train_b, X_test_b, y_train_b, y_test_b = train_test_split(X, y,
                                                            random_state=42)

baseline_model = LinearRegression()
splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)

baseline_scores = cross_validate(
    estimator=baseline_model,
    X=X_train_b[[high_corr]],
    y=y_train_b,
    return_train_score=True,
    cv=splitter
)
```

Now we will make our model and will compare the result of the model with the baseline model

```
[34]: from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_validate, ShuffleSplit

X = df_final.drop(columns = to_drop, axis = 1)
y = df_final["price"]
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42)
```

```

second_model = LinearRegression()

splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)

secondmodel_scores = cross_validate(
    estimator=second_model,
    X=X_train,
    y=y_train,
    return_train_score=True,
    cv=splitter
)

print("Train score(mean):      ", secondmodel_scores["train_score"].mean())
print("Validation score(mean): ", secondmodel_scores["test_score"].mean())

print()

print("Train score:      ", baseline_scores["train_score"].mean())
print("Validation score:", baseline_scores["test_score"].mean())

```

```

Train score(mean):      0.6565464191775292
Validation score(mean): 0.659017570712206

```

```

Train score:      0.409409242793934
Validation score: 0.41724912602012776

```

### 6.2.2 Final Decision

Even though, the R2 score is very low, we will check the collinearity condition as shown below:

```

[35]: from statsmodels.stats.outliers_influence import variance_inflation_factor
import statsmodels.api as sm

df_final = concatenate(include_grade = False, include_zipcode = False,
                        include_condition = True)
to_drop = ["price"]

XX = df_final.drop(columns = to_drop, axis = 1)

XX_constant_added = sm.add_constant(XX)

vif = [
    variance_inflation_factor(XX_constant_added.values, i)
    for i in
        range(XX_constant_added.shape[1])
]

```

```
variance_inf_fact = pd.Series(vif, index=XX_constant_added.columns,
                             name="Variance Inflation Factor")
```

```
[36]: variance_inf_fact
```

```
[36]: const          0.000000
      bedrooms      1.668676
      bathrooms     3.036552
      sqft_living    3.258498
      sqft_lot       1.642850
      floors        1.757526
      yr_built       2.065105
      lat           1.092481
      long          1.476223
      waterfront_null 1.000600
      waterfront_impute 1.016641
      Average        inf
      Fair           inf
      Good           inf
      Poor           inf
      Very Good      inf
      Name: Variance Inflation Factor, dtype: float64
```

From this we realize that there are some collinearities in the model, also R2 score is not good so we will ignore this model.

### 6.3 Third Model: Putting grade and condition into the model.

Now let's check how `grade_num_df` and `condition_num_df` affect the modeling. Therefore, we only consider these two features and we will get R2-score and collinearity as:

```
[39]: df_final = concatenate(include_grade = True, include_zipcode = False,
                             include_condition = True)
      to_drop = ["price"]
      gc = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
      print("R2      : ", gc[0])
      print("Cond. No. : ", gc[1])
```

```
R2      : 0.729
Cond. No. : 7.86e+16
```

Again, we see that the the R2 score which is 0.729 is good but Cond. No. which is 7.86e+16 is really high. However, both condition and grade might be responsible for the quality of the house and we can just exclude one of them and use the other one. Before deciding which one to drop, let's check the coefficients with p-values greater than 0.05.

```
[40]: gc[2]
```

```
[40]:
```

|    | feature         | P-value | coefficient | coefficient_absolute_value |
|----|-----------------|---------|-------------|----------------------------|
| 10 | waterfront_null | 0.443   | -0.0042     | 0.0042                     |

We can see that `waterfront_null` has p-values more than the critical value of 0.05. Therefore, we are going to drop this columns. On the other hand, if a house has a view to a water fall, they would not miss the data and they will state that the property has that view. So, `waterfall_null` might be just those houses that they do not have the view which they are already in the model. Therefore, we are going to include these categorical variables in the model and we will try to find the best features which will reduce the collinearity of the model's features.

```
[41]: df_final.drop(columns = ["waterfront_null", "long"], axis = 1, inplace = True)
```

First we are going to drop `waterfront_null` and `long` from the dataframe and will check the R2 and Cond. No.. At the same time we are going to calculate the correlation table to find the highly correlated features as

```
[42]: df_final = concatenate(include_grade = True, include_zipcode = False,
                             include_condition = True)
to_drop = ["price", "waterfront_null", "long"]
gc = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2          : ", gc[0])
print("Cond. No.  : ", gc[1])
```

```
R2          : 0.729
Cond. No.   : 9.34e+16
```

Also we may check the correlation coefficients as:

```
[43]: print(corr(df_final, value = 0.8))
```

```
0          pairs
0  0.814806  (Good, Average)
```

As we can see, `Average` and `Good` are highly correlated so we are going to drop `Average` and will keep only `Good`.

```
[44]: df_final.drop(columns = ["Average"], axis = 1, inplace = True)
```

By checking the coefficients, we notice that there are several groups of features that almost have similar coefficients which can we group them to reduce the number of features. In order to do so, we define new columns that contains linear combination of these features. These columns are:

1. `df_final["MLE"] = df_final["13 Mansion"] + df_final["12 Luxury"] + df_final["11 Excellent"]`
2. `df_final["BV"] = df_final["10 Very Good"] + df_final["9 Better"]`
3. `df_final["LF"] = df_final["4 Low"] + df_final["5 Fair"]`
4. `df_final["LA"] = df_final["7 Average"] + df_final["6 Low Average"]`



Moreover, we notice that by ignoring the following features, we can significantly reduce the collinearity from  $9.34e+16$  to 24200.0

3 Poor, 8 Good, floors, bedrooms, Good, Very Good, floors, bedrooms, bathrooms.

```
[46]: df_final = concatenate(include_grade = True, include_zipcode = False,
                           include_condition = True)
to_drop = ["price", "waterfront_null", "long#", "13 Mansion"
           , "12 Luxury"
           , "11 Excellent", "10 Very Good", "9 Better"
           , "4 Low", "5 Fair", "7 Average", "6 Low Average"
           , "8 Good", "Average", "Good", "Very Good"
           , "floors", "bedrooms", "bathrooms"]

df_final["MLE"] = (df_final["11 Excellent"] + df_final["12 Luxury"])
#                  + df_final["13 Mansion"])

df_final["BV"] = df_final["10 Very Good"] + df_final["9 Better"]
df_final["LF"] = df_final["4 Low"] + df_final["5 Fair"]
df_final["LA"] = df_final["7 Average"] + df_final["6 Low Average"]

gc_new = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2          : ", gc_new[0])
print("Cond. No. : ", gc_new[1])
print()

# gc_new[3]
```

```
R2          : 0.707
Cond. No.   : 24200.0
```

However, we could not reduce the collinearity more than what we obtained. The reason might be because of including both categorical variables `condition` and `grade`. Therefore, we are just going to pick one of them and we will drop the other one.

```
[47]: df_final = concatenate(include_grade = True, include_zipcode = False,
                           include_condition = True)
to_drop = ["price", "waterfront_null", "long"
           , "12 Luxury", "11 Excellent"
           , "10 Very Good", "9 Better"
           , "4 Low", "5 Fair"
           , "7 Average", "6 Low Average"
           , "8 Good"
           , "Average", "Good", "Very Good"
           , "floors", "bedrooms", "bathrooms"]
```

```

df_final["EL"] = df_final["11 Excellent"] + df_final["12 Luxury"]
df_final["BV"] = df_final["9 Better"] + df_final["10 Very Good"]
df_final["LF"] = df_final["4 Low"] + df_final["5 Fair"]
df_final["LA"] = df_final["6 Low Average"] + df_final["7 Average"]

gc = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2 : ", gc[0])
print("Cond. No. : ", gc[1])
# gc[3]

```

```

R2 : 0.707
Cond. No. : 24200.0

```

### 6.3.1 Creating Baseline Model and Checking this Model

We will create a baseline model with which we can compare the results of the first model. We consider the baseline model as the linear regression model between price and the variable that has the highest correlation with price. The following code, gives us the name of the variable that has the highest correlation with price

```

[48]: numerical_II = df_no_outliers.drop(columns = "price", axis = 1).select_dtypes(
        include=["float64", "int64"])
corr_column = list(numerical_II.columns)
corr_dict = {}
for item in corr_column:
    corr_dict[df_no_outliers["price"].corr(df_no_outliers[item])] = item
high_corr = corr_dict[max(corr_dict)]
high_corr

```

```
[48]: 'sqft_living'
```

Now, we are going to find the R2 of the base line model for different test train sets

```

[49]: from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_validate, ShuffleSplit

df_final
X = df_final[[high_corr]]
y = df_final["price"]
X_train_b, X_test_b, y_train_b, y_test_b = train_test_split(X, y,
    random_state=42)

```

```

baseline_model = LinearRegression()
splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)
baseline_scores = cross_validate(
    estimator=baseline_model,
    X=X_train_b[[high_corr]],
    y=y_train_b,
    return_train_score=True,
    cv=splitter
)

```

Now we will make our model and will compare the result of the model with the baseline model

```

[51]: from sklearn.model_selection import train_test_split
      from sklearn.linear_model import LinearRegression
      from sklearn.model_selection import cross_validate, ShuffleSplit

X = df_final.drop(columns = to_drop, axis = 1)
y = df_final["price"]
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42)

second_model = LinearRegression()
splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)

secondmodel_scores = cross_validate(
    estimator=second_model,
    X=X_train,
    y=y_train,
    return_train_score=True,
    cv=splitter
)

print("Train score(mean):      ", secondmodel_scores["train_score"].mean())
print("Validation score(mean):", secondmodel_scores["test_score"].mean())

print()

print("Train score:      ", baseline_scores["train_score"].mean())
print("Validation score:", baseline_scores["test_score"].mean())

```

```

Train score(mean):      0.7073804014324245
Validation score(mean): 0.710140564682148

```

```

Train score:      0.409409242793934
Validation score: 0.41724912602012776

```

### 6.3.2 Final Decision

Now to check the collinearity we will use variance inflation factor to see which coefficient has a value more than 5.

```
[52]: from statsmodels.stats.outliers_influence import variance_inflation_factor
import statsmodels.api as sm

to_drop = to_drop

XX = df_final.drop(columns = to_drop, axis = 1)

XX_constant_added = sm.add_constant(XX)

vif = [
    variance_inflation_factor(XX_constant_added.values, i)
    for i in
        range(XX_constant_added.shape[1])
]
variance_inf_fact = pd.Series(vif, index=XX_constant_added.columns,
                             name="Variance Inflation Factor")
```

```
[53]: variance_inf_fact
```

```
[53]: const                2.659392e+06
sqft_living            2.053082e+00
sqft_lot               1.213843e+00
yr_built               1.361471e+00
lat                   1.104297e+00
waterfront_impute     1.006973e+00
Fair                  1.013881e+00
Poor                  1.013996e+00
EL                    1.123583e+00
BV                    1.518346e+00
LF                    1.167318e+00
LA                    1.815532e+00
Name: Variance Inflation Factor, dtype: float64
```

We can see that all of the coefficients are under 5 which is a good sign because it shows that the collinearity is minimized in this model. However, the issue is that the coefficients of the features EL, BV, LF, and LA are not clear and it is hard to interpret them. Therefore, we may ignore this model and move forward to find another model.

### 6.4 Forth Model-Part 1: Putting only grade into the model.

Now, if we only consider **grade** as the only categorical variable in the model, we find

```
[54]: df_final = concatenate(include_grade = True, include_zipcode = False,
                             include_condition = False)
to_drop = ["price"]
g = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2          : ", g[0])
print("Cond. No. : ", g[1])
```

```
R2          : 0.724
Cond. No.   : 5.25e+16
```

Relative to the case where we only considered `condition`, we see that R2 score has increased from 0.659 to 0.724 while Cond. No. has changed from 1.17e+16 to 5.25e+16. So, we may consider it as the proposed model. We will check it with the baseline model and we will check the collinearity of features in the next part.

#### 6.4.1 Creating Baseline Model and Checking this Model

We will create a baseline model with which we can compare the results of the first model. We consider the baseline model as the linear regression model between `price` and the variable that has the highest correlation with `price`. The following code, gives us the name of the variable that has the highest correlation with `price`

```
[55]: numerical_II = df_no_outliers.drop(columns = "price", axis = 1).select_dtypes(
      include=["float64", "int64"])
corr_column = list(numerical_II.columns)
corr_dict = {}
for item in corr_column:
    corr_dict[df_no_outliers["price"].corr(df_no_outliers[item])] = item
high_corr = corr_dict[max(corr_dict)]
high_corr
```

```
[55]: 'sqft_living'
```

Now, we are going to find the R2 of the base line model for different test train sets

```
[56]: from sklearn.model_selection import train_test_split
      from sklearn.linear_model import LinearRegression
      from sklearn.model_selection import cross_validate, ShuffleSplit

df_final
X = df_final[[high_corr]]
y = df_final["price"]
X_train_b, X_test_b, y_train_b, y_test_b = train_test_split(X, y,
                                                             random_state=42)

baseline_model = LinearRegression()
splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)
```

```

baseline_scores = cross_validate(
    estimator=baseline_model,
    X=X_train_b[[high_corr]],
    y=y_train_b,
    return_train_score=True,
    cv=splitter
)

```

Now we will make our model and will compare the result of the model with the baseline model

```

[57]: from sklearn.model_selection import train_test_split
      from sklearn.linear_model import LinearRegression
      from sklearn.model_selection import cross_validate, ShuffleSplit

X = df_final.drop(columns = to_drop, axis = 1)
y = df_final["price"]
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42)

second_model = LinearRegression()

splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)

secondmodel_scores = cross_validate(
    estimator=second_model,
    X=X_train,
    y=y_train,
    return_train_score=True,
    cv=splitter
)

print("Train score(mean):      ", secondmodel_scores["train_score"].mean())
print("Validation score(mean):", secondmodel_scores["test_score"].mean())

print()

print("Train score:      ", baseline_scores["train_score"].mean())
print("Validation score:", baseline_scores["test_score"].mean())

```

```

Train score(mean):      0.7221015846013794
Validation score(mean): 0.7254736526441403

```

```

Train score:      0.409409242793934
Validation score: 0.41724912602012776

```

### 6.4.2 Final Decision

We need to check the collinearity condition by calculating the variance-inflation factors as shown below:

```
[58]: from statsmodels.stats.outliers_influence import variance_inflation_factor
import statsmodels.api as sm

df_final = concatenate(include_grade = True, include_zipcode = False,
                        include_condition = False)

XX = df_final.drop(columns = to_drop, axis = 1)

XX_constant_added = sm.add_constant(XX)

vif = [
    variance_inflation_factor(XX_constant_added.values, i)
    for i in
        range(XX_constant_added.shape[1])
]
variance_inf_fact = pd.Series(vif, index=XX_constant_added.columns,
                             name="Variance Inflation Factor")

[59]: print(sum(variance_inf_fact>5))
```

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In this case we have a lot of collinearity but we have a higher R2 score. So, we may be able to make some changes in this combination and get a better results. So, this combination is one of the candidate to be used for making a model.

### 6.5 Forth Model-Part 2: Putting only grade into the model.

As we saw, including both conditions and grades will result in a high collinearity. Therefore, we are going to only include grade in our data sets. By doing so, we get R2 equals to 0.724 and Cond. No. equals to 5.25e+16 as shown below

```
[60]: df_final = concatenate(include_grade = True, include_zipcode = False,
                            include_condition = False)
to_drop = ["price"]
gg = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2          : ", gg[0])
print("Cond. No.  : ", gg[1])
# gg[2]
```

```
R2          : 0.724
Cond. No.   : 5.25e+16
```

However, we notice that `waterfront_null` has P-Values more than 0.05. Therefore, we are going to drop this feature, as a result we find R2 and Cond. No. to be equal to 0.724 and 5.13e+16, respectively, as shown below:

```
[62]: df_final = concatenate(include_grade = True, include_zipcode = False,
                             include_condition = False)
to_drop = ["price"
           , "waterfront_null", "long"]
ggg = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2          : ", ggg[0])
print("Cond. No.  : ", ggg[1])
```

```
R2          : 0.724
Cond. No.   : 5.13e+16
```

Now, we notice that there are some features whose coefficients are close to each other. Therefore, we can group them and create new columns (as we did in the previous section) to reduce these features. Moreover, we see that if we drop some features we can reduce the collinearity significantly as shown below

```
[63]: df_final = concatenate(include_grade = True, include_zipcode = False,
                             include_condition = False)
to_drop = ["price"
           , "waterfront_null", "long"
           , "5 Fair", "4 Low"
           , "9 Better"
           , "12 Luxury", "11 Excellent"
           , "6 Low Average", "7 Average", "8 Good"
           , "sqft_lot", "bedrooms", "floors", "bathrooms"]

df_final["LF"] = df_final["4 Low"] + df_final["5 Fair"]

df_final["MLE"] = df_final["11 Excellent"] + df_final["12 Luxury"]

df_final["LAG"] = (df_final["6 Low Average"] + df_final["7 Average"]
                  + df_final["8 Good"])

g4 = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2          : ", g4[0])
print("Cond. No.  : ", g4[1])
# g4[3]
```

```
R2          : 0.677
Cond. No.   : 18200.0
```

The problem is that we almost lost all the numerical features and we have a much lower R2. Therefore, we need to drop some categorical features and not combine them so that we can keep some of the numerical features. However, before that, let's check the `variance_inflation_factor`



```
[65]: from statsmodels.stats.outliers_influence import variance_inflation_factor
import statsmodels.api as sm

to_drop = to_drop

XX = df_final.drop(columns = to_drop, axis = 1)

XX_constant_added = sm.add_constant(XX)

vif = [
    variance_inflation_factor(XX_constant_added.values, i)
    for i in
        range(XX_constant_added.shape[1])
]
variance_inf_fact = pd.Series(vif, index=XX_constant_added.columns,
                             name="Variance Inflation Factor")
variance_inf_fact
```

```
[65]: const                2.419926e+06
sqft_living             1.602726e+00
yr_built                1.229283e+00
lat                    1.047787e+00
waterfront_impute      1.004437e+00
10 Very Good           1.341163e+00
LF                     1.233100e+00
MLE                    1.128681e+00
LAG                    1.884318e+00
Name: Variance Inflation Factor, dtype: float64
```

These numbers are below 5 and we can conclude that collinearity between features is low. However, because we combined several variables, it is hard to interpret the coefficients.

## 7 Features Selection (DONE)

In this section, we are going to find the features we want in the model, by checking R2 score, Cond. No. and variance inflation factor of each model that we find by trial and error. At the end, we will present the final model.

Now we will try to drop categorical features to improve R2 and Cond. No.. This process is shown below:

1. First we drop `waterfront_null`, `long` because the `p-values` for them are more than 0.05. After dropping these columns, we find R2 and Cond. No. as 0.752 and 2.58e+16, respectively. Now we go to the next step.
2. We see that the coefficients of the features 12 `Luxury`, 11 `Excellent` and 10 `Very Good` are close to each other, so we may be able to drop two of them and keep one of them. We see if

we drop one of them and keep the others, we find that R2 score and Cond. No. will become 0.752 and 26200. This is an improvement in the collinearity. Also, we tried and notice that if we drop two of them and keep one of them, these numbers do not change. So, we are going to keep 10 Very Good and drop the others. Now we go to the next step.

3. Now we are checking the p-value and we see that 3 Poor has a high p value. So, we are going to drop this feature. This will not change the desired scores. Now we go to the next step.
4. We can see that sqft\_lot has the lowest coefficient compares to the rest of the features. Therefore, we are going to drop this feature. By doing so, we get R2 equals to 0.751 and Cond. No. equals to 20900.0. However, we notice that after dropping this feature, the other coefficients changed significantly and they got closer to each other.

```
[64]: df_final = concatenate(include_grade = True, include_zipcode = False,
                             include_condition = False)
to_drop = ["price", "waterfront_null", "long"
           , "12 Luxury" , "11 Excellent", "sqft_lot"]

l = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
print("R2          : ", l[0])
print("Cond. No. : ", l[1])
```

```
R2          : 0.724
Cond. No.   : 20900.0
```

Now, to make a better judgment, we are going to calculate the variance\_inflation\_factor of each feature and we will drop the features with variance\_inflation\_factor more than 5

```
[65]: from statsmodels.stats.outliers_influence import variance_inflation_factor
import statsmodels.api as sm

df_final = concatenate(include_grade = True, include_zipcode = False,
                       include_condition = False)
to_drop = to_drop

XX = df_final.drop(columns = to_drop, axis = 1)

XX_constant_added = sm.add_constant(XX)

vif = [
    variance_inflation_factor(XX_constant_added.values, i)
    for i in
        range(XX_constant_added.shape[1])
]
variance_inf_fact = pd.Series(vif, index=XX_constant_added.columns,
                             name="Variance Inflation Factor")
```

```
variance_inf_fact
```

```
[65]: const          2.817772e+06
      bedrooms       1.726101e+00
      bathrooms      3.010277e+00
      sqft_living     3.804743e+00
      floors          1.579067e+00
      yr_built        1.756940e+00
      lat             1.089289e+00
      waterfront_impute 1.008662e+00
      10 Very Good    4.027724e+00
      4 Low           1.076944e+00
      5 Fair          2.102718e+00
      6 Low Average    9.255176e+00
      7 Average        2.088862e+01
      8 Good           1.600198e+01
      9 Better         8.361958e+00
      Name: Variance Inflation Factor, dtype: float64
```

From this calculation, we are going to drop either 9 Better or 6 Low Average or both and we see how R2 changes. After that we again will check the `variance_inflation_factors` of coefficients to make sure that all the values are below 5.

We notice that if we drop both 9 Better and 6 Low Average, we will get R2 as 0.670 and if we only drop 9 Better while keeping 6 Low Average in the model we find R2 to be 0.716 and if we drop 9 Better and keep 6 Low Average, we find R2 as 0.689. Therefore, we will drop 9 Better and keep 6 Low Average in the model. Moreover, we noticed that if we keep 12 Luxury and 11 Excellent in the model, the R2 will change to 0.720 which is an improvement.

```
[66]: df_final = concatenate(include_grade = True, include_zipcode = False,
                             include_condition = False)
      to_drop = ["price", "waterfront_null", "long"
                  , "6 Low Average", "sqft_lot", "bathrooms"]

      final = summary_of_results(data = df_final, to_drop = to_drop, pval = 0.05)
      print("R2          : ", final[0])
      print("Cond. No. : ", final[1])
```

```
R2          : 0.720
Cond. No.   : 20300.0
```

We see that R2 decreased significantly and Cond. No. also has increased. So, we are going to check the variance inflation factor method of each feature as:

```
[67]: from statsmodels.stats.outliers_influence import variance_inflation_factor
      import statsmodels.api as sm
```

```

df_final = concatenate(include_grade = True, include_zipcode = False,
                        include_condition = False)
to_drop = to_drop

XX = df_final.drop(columns = to_drop, axis = 1)

XX_constant_added = sm.add_constant(XX)

vif = [
    variance_inflation_factor(XX_constant_added.values, i)
    for i in
        range(XX_constant_added.shape[1])
]
variance_inf_fact = pd.Series(vif, index=XX_constant_added.columns,
                             name="Variance Inflation Factor")
variance_inf_fact

```

```

[67]: const                2.749039e+06
      bedrooms            1.685363e+00
      sqft_living         3.093639e+00
      floors              1.492391e+00
      yr_built            1.609789e+00
      lat                 1.088182e+00
      waterfront_impute   1.008531e+00
      10 Very Good        2.378075e+00
      11 Excellent        1.495305e+00
      12 Luxury           1.064299e+00
      4 Low                1.008811e+00
      5 Fair               1.107675e+00
      7 Average           3.729062e+00
      8 Good               4.462711e+00
      9 Better            3.604938e+00
      Name: Variance Inflation Factor, dtype: float64

```

We see that the values are below 5 so we should accept these values. So, we will define a function to give us the final dataframe as defined below:

```

[68]: def final_dataframe():
      df_final = concatenate(include_grade = True, include_zipcode = False,
                             include_condition = False)
      to_drop = ["waterfront_null", "long"
                  , "6 Low Average", "sqft_lot", "bathrooms"]
      df_final.drop(columns = to_drop, axis = 1, inplace = True)
      return df_final

```

## 8 Final Model (DONE)

In this section we are going to introduce baseline model and the final model and we will compare the results of the final model with the base line model. We will use split our data to train and test sets one for modeling and the other for checking the results.

### 8.1 Baseline Model

We will introduce a baseline model to compare the results of the model we propose with the result of the baseline model. In order to find the baseline model, we will find a variable that has the highest correlation with the price.

```
[69]: numerical_II = df_no_outliers.drop(columns = "price", axis = 1).select_dtypes(
        include=["float64", "int64"])
corr_column = list(numerical_II.columns)
corr_dict = {}
for item in corr_column:
    corr_dict[df_no_outliers["price"].corr(df_no_outliers[item])] = item
high_corr = corr_dict[max(corr_dict)]
high_corr
```

```
[69]: 'sqft_living'
```

Now, we are going to find the R2 of the base line model for different test train sets

```
[70]: from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_validate, ShuffleSplit

df_final = final_dataframe()
X = df_final[[high_corr]]
y = df_final["price"]
X_train_b, X_test_b, y_train_b, y_test_b = train_test_split(X, y,
    random_state=42)

baseline_model = LinearRegression()
splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)

baseline_scores = cross_validate(
    estimator=baseline_model,
    X=X_train_b[[high_corr]],
    y=y_train_b,
    return_train_score=True,
    cv=splitter
)

print("Train score:      ", baseline_scores["train_score"].mean())
```

```
print("Validation score:", baseline_scores["test_score"].mean())
```

Train score: 0.409409242793934  
Validation score: 0.41724912602012776

## 8.2 Final Model

Now we are going to make the final model and will split the data into train and test sets so that we can compare the R2 score of the final model with that of the baseline model.

```
[83]: from sklearn.model_selection import train_test_split
      from sklearn.linear_model import LinearRegression
      from sklearn.model_selection import cross_validate, ShuffleSplit

      df_final = final_dataframe()
      X = df_final.drop(columns = "price", axis = 1)
      y = df_final["price"]
      X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42)

      second_model = LinearRegression()
      splitter = ShuffleSplit(n_splits=5, test_size=0.25, random_state=0)

      secondmodel_scores = cross_validate(
          estimator=second_model,
          X=X_train,
          y=y_train,
          return_train_score=True,
          cv=splitter
      )

      print("Train score(mean): ", secondmodel_scores["train_score"].mean())
      print("Validation score(mean): ", secondmodel_scores["test_score"].mean())

      print()

      print("Train score: ", baseline_scores["train_score"].mean())
      print("Validation score:", baseline_scores["test_score"].mean())
```

Train score(mean): 0.7187064792939413  
Validation score(mean): 0.7215312885070686

Train score: 0.409409242793934  
Validation score: 0.41724912602012776

As we can see, the final model has higher R2 score compared to the baseline model. The features we will consider in our model and their coefficients are:

```
[72]: coeffs=final[3][["feature", "P-value", "coefficient"]].reset_index(drop = True)
      coeffs
```

```
[72]:
```

|    | feature           | P-value | coefficient |
|----|-------------------|---------|-------------|
| 0  | bedrooms          | 0       | -0.0208     |
| 1  | floors            | 0       | 0.054       |
| 2  | 5 Fair            | 0       | -0.118      |
| 3  | 7 Average         | 0       | 0.1789      |
| 4  | 4 Low             | 0.002   | -0.2152     |
| 5  | 8 Good            | 0       | 0.3805      |
| 6  | sqft_living       | 0       | 0.4858      |
| 7  | waterfront_impute | 0       | 0.5539      |
| 8  | 9 Better          | 0       | 0.5932      |
| 9  | 10 Very Good      | 0       | 0.7328      |
| 10 | 11 Excellent      | 0       | 0.8616      |
| 11 | 12 Luxury         | 0       | 1.0057      |
| 12 | yr_built          | 0       | -7.6531     |
| 13 | lat               | 0       | 62.5216     |
| 14 | const             | 0       | -174.355    |

### 8.3 Interpretation of Coefficients

In order to interpret the model, we know that there are two types of features in the model. One of these features is numerical features and the others are categorical variables. It is easy to interpret the numerical features. For example, consider `sqft_living` which has the coefficient

feature

P-value

coefficient

6

`sqft_living`

0

0.4858

this coefficient is positive meaning that by increasing `sqft_living` the price of the property goes up. On the other hand, we calculate the logarithm of both price and this variable. Therefore, if we ignore all other variables and we just consider `price` and `sqft_living` we have

$$\ln(p) = c_l \ln(l) + f \quad (1)$$

in which,  $p$  and  $l$  denote `price` and `sqft_living`, respectively,  $c_l$  is the coefficient of `sqft_living` in our model which is equal to 0.4858.  $f$  is rest of the models which we assume is fixed here for rest of the analysis.

Now if `sqft_living` is increased from  $l_0$  to  $l_1 = 2.72 \times l_0$ , the price will change from  $p_0$  to  $p_1$  as:

$$\begin{aligned}
\ln(p_1) &= c_l \ln(2.72 \times l_0) + f \\
&= c_l + c_l \ln(l_0) + f \\
&= c_l + \ln(p_0)
\end{aligned}$$

as a result

$$\ln\left(\frac{p_1}{p_0}\right) = c_l \implies p_1 = p_0 e^{c_l} \quad (2)$$

in which  $e = 2.718281828459045$  is called Neper number. Therefore, if we increase `sqft_living` from one value  $l_0$  to  $l_1 = 2.72 \times l_0$ , the `price` will change from  $p_0$  to  $1.6254748687259315 \times p_0$ .

Interpreting the coefficient of categorical variables is more complicated than the numerical variables. In order to interpret these coefficients, first we need to choose a base coefficient to compare all other categorical coefficients with. In order to do so, we are going to first create another dataframe in which we only have categorical features and their coefficients.

```
[73]: to_drop_coeff = ["bedrooms", "floors", "sqft_living",
                      "yr_built", "lat", "const"]
to_drop_index = []
for item in to_drop_coeff:
    ind = list(coeffs.loc[coeffs["feature"] == item].index)[0]
    to_drop_index.append(ind)

cat_coeff = coeffs.drop(index = to_drop_index, axis = 0)

cat_coeff
```

```
[73]:
```

|    | feature           | P-value | coefficient |
|----|-------------------|---------|-------------|
| 2  | 5 Fair            | 0       | -0.118      |
| 3  | 7 Average         | 0       | 0.1789      |
| 4  | 4 Low             | 0.002   | -0.2152     |
| 5  | 8 Good            | 0       | 0.3805      |
| 7  | waterfront_impute | 0       | 0.5539      |
| 8  | 9 Better          | 0       | 0.5932      |
| 9  | 10 Very Good      | 0       | 0.7328      |
| 10 | 11 Excellent      | 0       | 0.8616      |
| 11 | 12 Luxury         | 0       | 1.0057      |

Let's pick 7 **Average** as the base coefficient and then we divide all other coefficients with the coefficient of 7 **Average**

```
[74]: base_variable = "7 Average"
base_coeff = list(cat_coeff.loc[cat_coeff["feature"] == base_variable,
                              "coefficient"])[0]
cat_coeff["divided by "+base_variable] = cat_coeff["coefficient"]/base_coeff
cat_coeff
```



```
[74]:
```

|    | feature           | P-value | coefficient | divided by 7 | Average   |
|----|-------------------|---------|-------------|--------------|-----------|
| 2  | 5 Fair            | 0       | -0.118      |              | -0.659586 |
| 3  | 7 Average         | 0       | 0.1789      |              | 1         |
| 4  | 4 Low             | 0.002   | -0.2152     |              | -1.20291  |
| 5  | 8 Good            | 0       | 0.3805      |              | 2.12689   |
| 7  | waterfront_impute | 0       | 0.5539      |              | 3.09614   |
| 8  | 9 Better          | 0       | 0.5932      |              | 3.31582   |
| 9  | 10 Very Good      | 0       | 0.7328      |              | 4.09614   |
| 10 | 11 Excellent      | 0       | 0.8616      |              | 4.8161    |
| 11 | 12 Luxury         | 0       | 1.0057      |              | 5.62158   |

From this dataframe we can realize that 5 Fair and 4 Low as negative effect on the price of the property since they have negative coefficients. In addition to that, we realize that if we improve/change the grade from 7 Average to 12 Luxury we should expect high change in the logarithm of the price since the coefficient of 12 Luxury is 5.62157629960872 times of that of 7 Average.

## 9 Prediction (DONE)

We want to use the model we presented in the previous sections to predict the price of properties with the data saved in `X_test` and then we want to compare the real values with the predicted ones. In order to do so, we do the following:

```
[76]: from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error

## Test-Train Split
df_final = final_dataframe()
X = df_final.drop(columns = "price", axis = 1)
y = df_final["price"]
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42)

## Fitting to the Model
final_model = LinearRegression()
final_model.fit(X_train, y_train)
final_model.score(X_test, y_test)
prediction = final_model.predict(X_test)

## Calculating Mean Squared Error
MSE = mean_squared_error(np.exp(y_test),
                          np.exp(final_model.predict(X_test)),
                          squared = False)

MSE
```

[76]: 143365.36020504768

We can see that the mean squared error is very high. This means that for a property, this model will be off by about \$143365.36. Therefore, we would definitely want to have a person to check the features we chose for the model and compare it with the situation of the house and adjust these prices rather than just allowing the algorithm to set them.

## 10 Assumption Checking (DONE)

In order to be able to use regression model, we need to check if the following assumptions are satisfied.

1. Normality of Residuals
2. Multicollinearity (Independence Assumption)
3. Homoscedasticity
4. Linearity

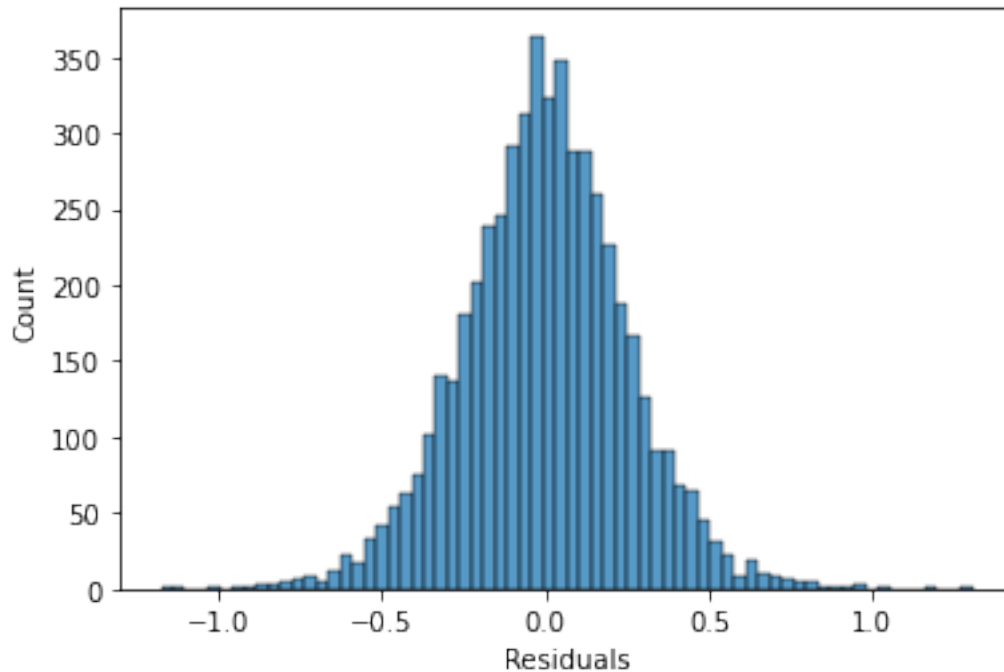
In the following subsections, we will check each of these assumptions.

### 10.1 Normality of Residuals

We will check if the residuals are normal by visually checking histogram and QQ-Plot of the residuals.

```
[159]: import scipy.stats as stats

residuals = (y_test - prediction)
sns.histplot(residuals)
plt.xlabel('Residuals');
# sm.graphics.qqplot(residuals, dist=stats.norm, line='45', fit=True);
```



It seems that the histogram of residuals follows the normal distribution so, we might accept that the residuals are normally distributed.

## 10.2 Investigating Multicollinearity (Independence Assumption)

In order to check if the features are independent or not, we will check the `variance_inflation_factor` and see if the values of each coefficient is below 5 or not. If the values are below 5, we may accept that the features are independent.

```
[160]: from statsmodels.stats.outliers_influence import variance_inflation_factor
X_train_constant_added = sm.add_constant(X_train)
X_test_constant_added = sm.add_constant(X_test)

vif = [
    variance_inflation_factor(X_train_constant_added.values, i)
    for i in
        range(X_train_constant_added.shape[1])
]
variance_inf_fact = pd.Series(vif, index=X_train_constant_added.columns,
                             name="Variance Inflation Factor")
variance_inf_fact
```

```
[160]: const          2.759625e+06
bedrooms      1.680581e+00
sqft_living   3.069384e+00
```

```

floors          1.495836e+00
yr_built        1.619488e+00
lat             1.090620e+00
waterfront_impute 1.010457e+00
10 Very Good    2.374400e+00
11 Excellent    1.520302e+00
12 Luxury       1.058862e+00
4 Low           1.010572e+00
5 Fair          1.103537e+00
7 Average       3.757455e+00
8 Good          4.474132e+00
9 Better        3.622918e+00
Name: Variance Inflation Factor, dtype: float64

```

We can see that all the values are below 5 so we may conclude that the features are independent from one another and we do not have collinearity in our model.

### 10.3 Investigating Homoscedasticity

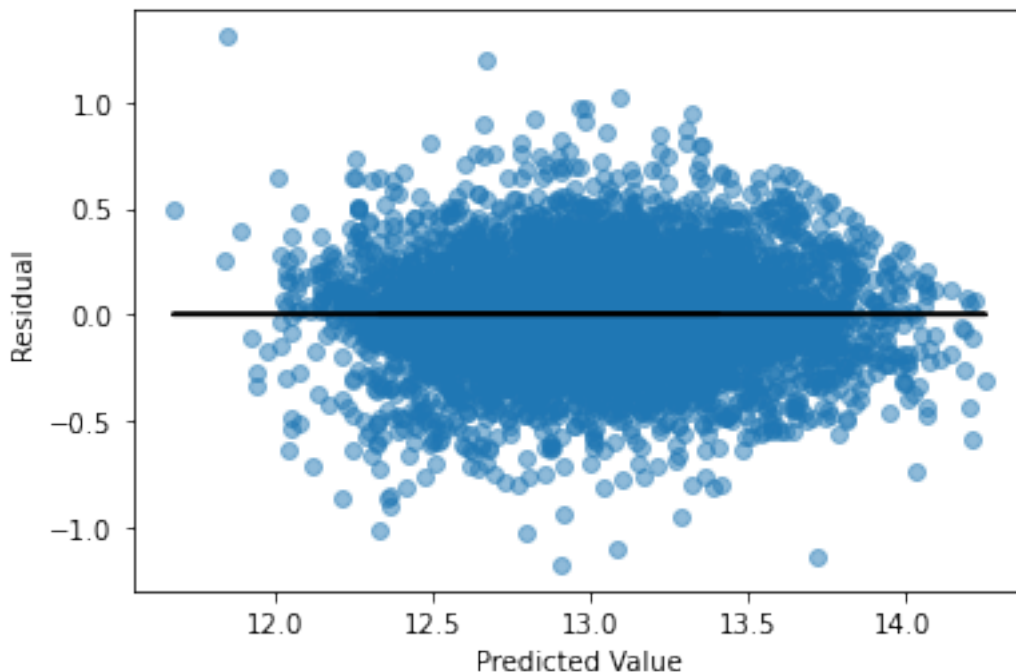
We need to check if the residuals satisfy the “Homoscedasticity” assumptions. In order to check the if this is the case or not, we check the scatter plot of predicted values and the residuals.

```

[161]: fig, ax = plt.subplots()

ax.scatter(prediction, residuals, alpha=0.5)
ax.plot(prediction, [0 for i in range(len(X_test))], color = "black")
ax.set_xlabel("Predicted Value")
ax.set_ylabel("Residual");

```



It can be seen that the scatter plot of predicted values and the residuals is almost satisfying the homoscedasticity assumption except some individual points.

## 10.4 Investigating Linearity

At the end, we should make sure that the predicted values and the actual values are linear. In order to do so, we draw a line and we check the scatter plot of actual values vs. predicted value as:

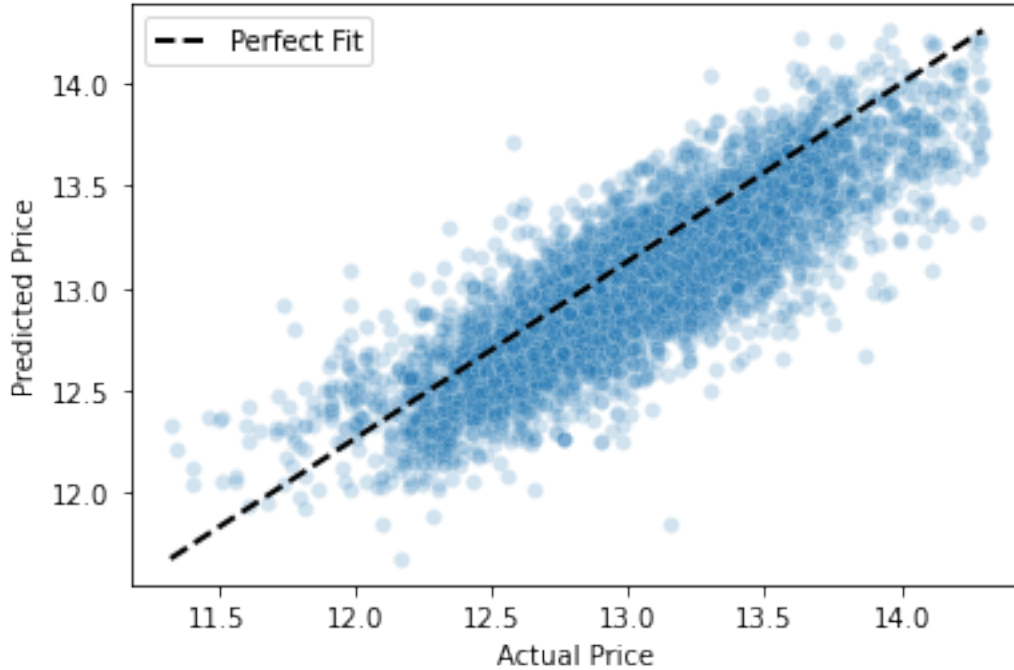
```
[162]: fig, ax = plt.subplots()

prediction = final_model.predict(X_test)

test_line = np.linspace(y_test.min(), y_test.max(), 200)
prediction_line = np.linspace(prediction.min(), prediction.max(), 200)
sns.lineplot(x = test_line, y = prediction_line
              , linestyle="--",
              color="black", label="Perfect Fit",
              lw = 2);
sns.scatterplot(x = y_test, y = prediction, alpha=0.2);
ax.set_xlabel("Actual Price")
ax.set_ylabel("Predicted Price")
ax.legend();
print(y_test.min(), y_test.max())
print(prediction.min(), prediction.max())
```

```
11.326595886778735 14.297936707208567
```

```
11.67919343736753 14.259572840391826
```



It can be seen that the diagram is pretty linear and we can conclude that the linearity assumption is satisfied.

## 11 Summary and Suggestions

Given that we cannot find a perfect model, each model has its own pros and cons. The model we proposed try to predict the price of a property in King County, WA by using `bedrooms`, `sqft_living`, `floors`, `yr_built`, `lat` as numerical features and `grade` of a house and its view toward a waterfall as the categorical value. This model has a mean of the cross validation score of 0.722.

We realized that `lat` has the highest coefficient with respect to other numerical features which means that this feature might have the highest impact on the price of a property. Since the latitude and longitude of a property represent the coordinate of the property on the earth, these columns contain the information about the location and zip code of the property. Therefore, it makes sense that `lat` should have a highest coefficient among others since it represents to location of a property. After `lat`, `sqft_living` has the second highest impact on the price of a property.

Among the categorical variables, we realize that improving the grade of a property to *Luxury* will increase the price of the property since this feature has the highest coefficient among other categorical variable. Therefore, we strongly suggest to improve the grade of a property because in turn the price of the house will increase greatly.

Since one can not change the location or square footage of the living room by that much, it makes sense to increase the grade of the property. By doing so, the owner may be able to sell the property with a higher price.

It is important to mention that we would get different results by considering different features in the

model. For example, we could add additional features such as `'sqft_basement` and `yr_renovated` to find a different model. Choosing different features will give different models and in turn different predictions.