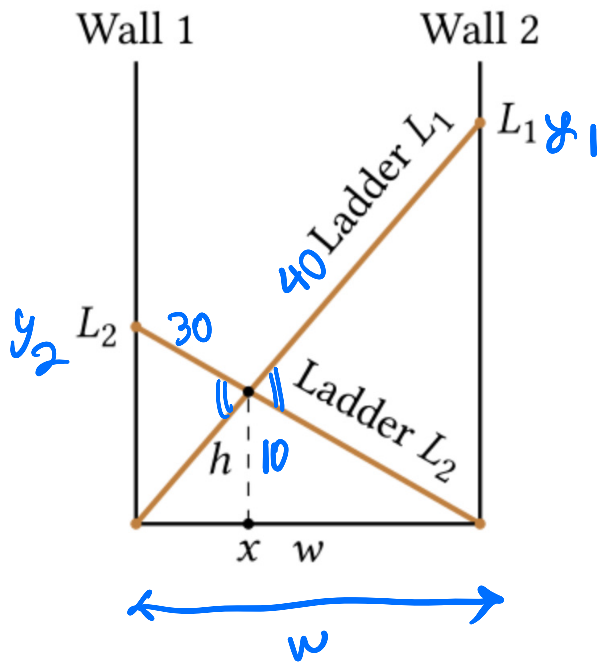
no numpy

no sqrt.



$$\begin{cases} y_1 = \sqrt{L_1^2 - w^2} \\ y_2 = \sqrt{L_2^2 - w^2} \end{cases}$$

$$h = \frac{y_1 y_2}{y_1 + y_2} \iff \frac{1}{\sqrt{L_1^2 - w^2}} + \frac{1}{\sqrt{L_2^2 - w^2}} = \frac{1}{h}$$

$x = w^2$

$$\Rightarrow \left(\frac{(L_1^2 - x)(L_2^2 - x)}{h^2} - (L_1^2 + L_2^2 - 2x) \right)^2 = 4(L_1^2 - x) \cdot (L_2^2 - x)$$

$$10^{-4}x^4 - \frac{23}{50}x^3 + 763x^2 - 537400x + 135850000 = 0$$

$$x = w^2 < \min(L_1^2, L_2^2) = 900$$

$$w^2 = 677.71071 \Rightarrow w = \underline{26.03\,ft}$$

Exercise 3.23. Solve the following equation in radicals, where a and b are positive parameters, for the unknown ξ:

$$\frac{1}{\sqrt{a-\xi}} + \frac{1}{\sqrt{b-\xi}} = 1$$

The solution of interest satisfies $\xi > 0$.

1. Prove that a solution exists iff $1/a + 1/b < 1$. Prove that the solution is unique.
2. Reduce the equation to a quartic equation

$$\xi^4 + A\xi^3 + B\xi^2 + C\xi + D = 0.$$

   You may consider using a CAS or Python package *Sympy*.
3. Using on-line resources, AI assistance or otherwise, express the solution in terms of radicals. The only radicals you need are $\sqrt{\cdot}$ and $\sqrt[3]{\cdot}$.
4. Implement the solution in software and compare run times to the bisection method and Regula Falsi, when solving the equation for 1000 randomly chosen parameters a and b.

□

Solution To Exercise 3.23:

(1) Use continuity and monotonicity (2) $A = -2\,b - 2\,a + 4$, $B = b^2 + 4\,a\,b - 6\,b + a^2 - 6\,a$, $C = -2\,a\,b^2 + 2\,b^2 - 2\,a^2\,b + 8\,a\,b + 2\,a^2$, $D = a^2\,b^2 - 2\,a\,b^2 + b^2 - 2\,a^2\,b - 2\,a\,b + a^2$. (3) Very complicated, have a CAS or AI write it out.