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Modeling and analysis of the multiperiod effects of social relationship on supply chain networks

Jose M. Cruz [a,*], Zugang Liu [b]

[a] Department of Operations and Information Management, School of Business, University of Connecticut, Storrs, CT 06269-2041, USA
[b] Department of Business and Economics, Pennsylvania State University, Hazleton, PA 18202, USA

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ABSTRACT

In this paper, we analyze the effects of levels of social relationship on a multiperiod supply chain network with multiple decision-makers (suppliers, manufacturers, and retailers) associated at different tiers. The model incorporates the individual attitudes towards disruption and opportunism risks and allows us to investigate the interplay of the heterogeneous decision-makers and to compute the resultant network equilibrium pattern of production, transactions, prices, and levels of social relationship over the multiperiod planning horizon. In our analysis, we focus on the following questions: (1) how do the evolving relationships affect the profitability and risks of supply chain firms as well as the prices and demands of the product in the market? (2) how do the relationships with the upstream supply chain firms affect the relationships with the downstream firms, and how these relationships influence the profitability and risks of the supply chain firms? (3) how do the supply disruption risks interact with the opportunism risks through supply chain relationships, and how these risks influence the profitability of the firms? The results show that high levels of relationship can lead to lower supply chain overall cost, lower risk, lower prices, higher product transaction and therefore higher profit.

1. Introduction

In recent years supply chain partnerships and alliance relationships have become increasingly important, as companies need to minimize their costs and maximize their opportunities on the market. Supply chains are embedded in a complex network of relationships with suppliers, customers as well as a number of other stakeholders. According to Croom et al. (2000), without a foundation of effective supply chain organizational relationships, any effort to manage the flow of information or materials across the supply chain is likely to be unsuccessful. To help the understanding of the issue of relationship in supply chain, in this paper, we analyze the effects of levels of social relationship on a multiperiod supply chain network in the presence of disruption risk and opportunism risk. In our analysis, we focus on the following questions: (1) how do the evolving relationships affect the profitability and risks of supply chain firms as well as the prices and demands of the product in the market? (2) how do the relationships with the upstream supply chain firms affect the relationships with the downstream firms, and how these relationships influence the profitability and risks of the supply chain firms? (3) how do the supply disruption risks interact with the opportunism risks through supply chain relationships, and how these risks influence the profitability of the firms?

Relationship issues surrounding supply chains have been a topic of high interest in the disciplines of sociology, marketing; specifically, relationship marketing, and economics. For example, embeddedness theory (cf. Granovetter, 1985; Uzzi, 1996 among others) attempts to explain the effects that relationships play in different economic actions, including financial transactions (see, e.g., Uzzi, 1998). Uzzi (1997) suggest that the most important features of companies' embedded ties are trust, information exchange and joint problem-solving arrangements. Jones et al. (1997), in turn, stressed that it is necessary to further concretize the results of the embeddedness theory. They described the conditions under which interfirm coordination can emerge by integrating transaction cost economics and social network theory. In the context of relationship marketing (cf. Ganesan, 1994; Bagozzi, 1995), on the other hand, researchers have tried to illuminate the motivation of sellers and buyers who actively seek relationships in the context of B2B (see, e.g., Wilson, 1995) or B2C commerce (see, e.g., Sheth and Parvatiyar, 1995). Different attempts to classify relationship structures have been made (see, e.g., Donaldson and Toole, 2000). Economists are especially concerned about determining the importance of the economic characteristics that characterize specific buyer and seller relationships and the role of transaction costs in

* Corresponding author. Tel.: +1 860 486 2317; fax: +1 860 486 4839.
E-mail addresses: jcruz@business.uconn.edu, jose.cruz@business.uconn.edu (J.M. Cruz), zugangliu@psu.edu (Z. Liu).

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determining the cost-minimizing governance structure for exchange (Joskow, 1988).

The value of relationship is not only economical but also technical and social (Gadde and Snehota, 2000). Strong supply chain relationships enable firms to react to changes in the market, create customer value and loyalty, which lead to improve profit margins (Flint, 2004). The benefits are reduction of production, transportation and administrative costs. On the technical development the greatest benefit is the possibility of sharing the resources of suppliers and shortening the lead-times. Spekman and Davis, 2004 found that supply chain networks that exhibit collaborative behaviors tend to be more responsive and that supply chain-wide costs are, hence, reduced. These results are also supported by Dyer, 2000 who demonstrated empirically that a higher level of trust (relationship) lowers transaction costs (costs associated with negotiating, monitoring, and enforcing contracts). Baker and Faulkner, 2004 present an overview of papers by economic sociologists that show the important role of relationships due to their potential to reduce risk and uncertainty. Uzzi, 1997 and Gadde and Snehota, 2000 suggest that multiple relationships can help companies deal with the negative consequences related to dependence on supply chain partners. Krause et al., 2007 found that buyer commitment and social capital accumulation with key suppliers can improve buying company performance. However, Christopher and Jüttner, 2000 indicate that the value of the relationship depends on the substitutability of the buyers or sellers, the indispensability of goods, savings resulting from partner's practices and the degree of common interest. In this paper we analyze the effects of relationships on a multitiered, multiperiod supply chain network in the presence of disruption risk and opportunism risk.

Supply chain disruptions and the associated risk are major topics in theoretical and applied research, as well as in practice, since this risk can affect the entire supply chain network. Craighead et al., 2007 have argued that supply chain disruptions and the associated operational and financial risks are the most pressing issue faced by firms in today's competitive global environment. The results of Hendricks and Singhal, 2005 analysis of 800 instances of supply chain disruptions illustrated the impact of supply chain disruptions. They found that the companies that suffered supply chain disruptions experienced share price returns 33 percent to 40 percent lower than the industry and the general market benchmarks. Furthermore, share price volatility was 13.5 percent higher in these companies in the year following a disruption than in the prior year. Indeed, supply chain disruptions may have impacts that propagate not only locally but globally and, hence, a holistic, system-wide approach to supply chain network modeling and analysis is essential in order to be able to capture the complex interactions among decision-makers. To-date, however, most supply disruption studies have focused on a local point of view, in the form of a single-supplier problem (see, e.g., Gupta, 1996; Parlar, 1997) or a two-supplier problem (see, e.g., Parlar and Perry, 1996). Very few papers have examined supply chain disruption risk management in an environment with multiple decision-makers (cf. Tomlin, 2006) while taking in consideration relationship issues. For a comprehensive review of supply chain risk management models to that date, please refer to Tang, 2006. We believe that it is imperative to study how the supply disruption risks interact with the opportunism risks through supply chain relationships, and how these risks influence the profitability of the firms. Towards that end, in this paper, we take an entirely different perspective, and we consider, for the first time, supply chain disruptions risk and opportunism risk in the context of multiple period, multi-tiered supply chain networks with multiple decision-makers under equilibrium conditions.

In terms of opportunism risk, according to Vandenbosch and Sapp, 2010 today's complex supply chains are vulnerable to opportunistic behavior. They point out that supply chain decision makers far from the end consumers tend to optimize their local goals rather than the entire supply chain. They conclude that that the longer the supply chain, the higher is the risk of opportunism. Wathne and Heide, 2000 suggested that opportunism behavior include falsification of expense reports (Phillips 1982), breach of distribution contracts (Dutta et al., 1994), bait-and-switch tactics (Wilkie et al., 1998), quality shirking (Hadfield, 1990), and violation of promotion agreements (Murry and Heide, 1998). For example, opportunism in the form of quality shirking means that a party is withholding efforts, or passively failing to honor an agreement (Wathne and Heide, 2000). According to Williamson, 1985,Williamson, 1996, if the risk of opportunism in a particular relationship is sufficiently high, considerable resources must be spent on control and monitoring, resources that could have been deployed more productively for other purposes. In addition, the risk of opportunism may produce substantial opportunity costs in the form of "valuable deals that won't be done" (p.164 Calfee and Rubin, 1993). Moreover, Wathne and Heide, 2000 suggested that risk of opportunism between exchange partners creates trading difficulties. In this paper we focus on opportunism risks through supply chain relationships and how these risks influence the profitability of the firms.

This paper models the multicriteria decision-making behavior of the various decision-makers in a multiperiod supply chain network, which includes the maximization of profit and the minimization of risk through the inclusion of the social relationship, in the presence of both business-to-business (B2B) and business-to-consumer (B2C) transactions. Wakolbinger and Nagurney, 2004 and Cruz et al., 2006 developed a framework for the modeling and analysis of supply chains networks that included the role that relationships play. Their contribution was apparently the first to introduce relationship levels in terms of flows on networks, along with logistical flows in terms of product transactions, combined with pricing. However, their models were a single period models and hence did not considerer multiple period effects of relationship levels on supply chains network and their disruption and opportunism risks. In terms of multiple period supply chain network models and single period supply chain dynamics Cruz and Wakolbinger, 2008 and Cruz, 2008,Cruz, 2009, respectively, studied the effects of corporate social responsibility on risk. Nagurney et al., 2005 develop a single period supply chain networks model with supply side and demand side risk. However, these preview published research did not consider multiple period effects of relationship on supply chain network opportunism and disruption risks.

We describes the role of relationships in supply chain networks over time. We assume that the firms in the same tier (e.g. all manufacturers in the second tier) compete in a non-cooperative manner. The firms in different tiers (e.g. suppliers in the first tier and manufacturers in the second tier) need to cooperate in order to complete transactions and establish relationship. Decision-makers in a given tier of the network can decide on the relationship levels that they want to achieve with decision-makers associated with the other tiers of the network in other to maximize profit and minimize risk. Establishing/maintaining a certain relationship level induces some costs, but may also lower the risk. We explicitly describe the role of relationships in influencing risk. Both the risk functions and the relationship cost functions are allowed to depend on the relationship levels. Hence, we truly capture the timing and location of investments in social relationships, and their impact on product flows, price, risk and profit.

This paper is organized as follows. In Section 2, we develop the multitiered, multiperiod supply chain network model. We describe decision-makers' optimizing behavior and establish the governing equilibrium conditions along with the corresponding variational inequality formulation. We conclude the paper with Section 4 in which we summarize our results and suggest directions for future research.

2. The multiperiod supply chain network model

In this section, we develop the multiperiod supply chain network model with risk management. We assume that all decision-makers consider a fixed planning horizon which is discretized into periods: $1,\ldots,t,\ldots,T$. The model consists of I suppliers, J manufacturers, and K retailers at demand markets as depicted in Fig. 1. We denote a typical supplier by i, a typical manufacturer by j, and a typical retailer by k. The links between the tiers represent economic transaction links. The variables for this model are given in Table 1. The equilibrium solution is denoted by *. All vectors are assumed to be column vectors, except where noted.

The top-tiered nodes in Fig. 1 represent the I suppliers in the T time periods with node (i,t) denoting supplier i in time period t. The suppliers are the decision-makers who produce parts/materials and sell them to the manufacturers in the second tier of nodes in the supply chain network in Fig. 1. Note that our model can be easily expanded to consider suppliers of different parts and materials. However, in order to keep the model focused and relatively simple in this paper we assume that the suppliers produce a substitutable part. A node (j, t) corresponds to manufacturer j in time

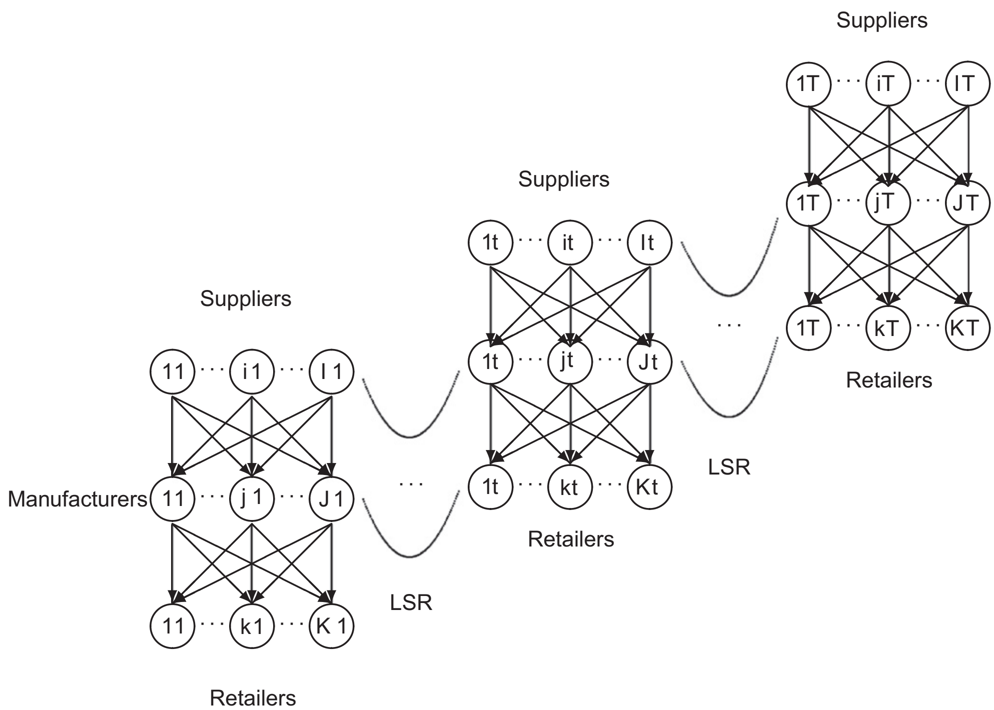

Fig. 1. Time evolution of the supply chain network with levels of social relationship (LSR).

Table 1
Variables in the supply chain network.

Notation	Definition
Q^1	IJT-dimensional vector of part/material flows transacted between each supplier and each manufacturer at each time period with component ijt denoted by q^i_{jt}
Q^2	JKT-dimensional vector of product flows transacted between each manufacturer and each retailer at each time period with component jkt denoted by q^j_{kt}
Q^3	KT-dimensional vector of product flows sold at retailers at each time period with component kt denoted by q_{kt}
Y^1	IT-dimensional vector of part/material production quantities of suppliers at each time period with component it denoted by y_{it}
Y^2	JT-dimensional vector of product production quantities of manufacturers each time period with component jt denoted by y_{jt}
V^1	IT-dimensional vector of suppliers' inventory at each time period with component it denoted by v_{it}
V^2	JT-dimensional vector of manufacturers' inventory at each time period with component jt denoted by v_{jt}
V^3	KT-dimensional vector of retailers' inventory at each time period with component kt denoted by v_{kt}
η^1	IJT-dimensional vector of levels of social relationship between manufacturers and suppliers at each time period with component ijt denoted by η^i_{jt}
η^{1+}	IJT-dimensional vector of the increases of social relationship levels between manufacturers and suppliers at each time period with component ijt denoted by η^{i+}_{jt}
η^{1-}	IJT-dimensional vector of the reductions of social relationship levels between manufacturers and suppliers at each time period with component ijt denoted by η^{i-}_{jt}
η^2	JKT-dimensional vector of levels of social relationship between manufacturers and retailers at each time period with component jkt denoted by η^j_{kt}
η^{2+}	JKT-dimensional vector of the increases of social relationship levels between manufacturers and retailers at each time period with component jkt denoted by η^{j+}_{kt}
η^{2-}	JKT-dimensional vector of the reductions of social relationship levels between manufacturers and retailers at each time period with component jkt denoted by η^{j-}_{kt}

period t, where $j = 1,\ldots,J$ and $t = 1,\ldots,T$. The retailers are represented by the nodes in the bottom tier of the supply chain network. They acquire the product from the manufacturers and sell it to consumers. Retailer k at time period t is denoted by node (k,t) with $k = 1,\ldots,K$ and $t = 1,\ldots,T$.

We now turn to the description of the functions. We first discuss the production cost, transaction cost, handling, and unit transaction cost functions given in Table 2. At each time period t, each supplier or manufacturer is faced with a certain production cost function that depend on the production output. Furthermore, each supplier and each manufacturer are faced with transaction and transportation costs which are determined based on the amount of the product transacted. In addition, the suppliers and manufacturers incur inventory holding costs which depend on the quantities of the goods held between the periods.

We assume that the production cost, the transaction cost, and the handling cost functions are continuously differentiable and that the unit cost functions are continuous.

We now turn to the description of the relationship production cost, the risk functions and the demand functions. We assume that the relationship production cost functions as well as the risk functions are continuously differentiable.The demand functions are assumed to be continuous.

We start by describing the relationship production cost that are given in Table 3. We assume that each supplier may actively try to achieve a certain relationship level with a manufacturer. Furthermore, each manufacturer may actively try to achieve a certain relationship level with a supplier and/or a retailer. The relationship cost functions reflect how much money, for example, in the form additional personnel, assets, time or service a supplier, manufacturer or retailer has to spend in order to maintain and improve a particular relationship level with a supply chain partner. These relationship production cost functions may be distinct for each such combination. For example, maintaining and improving the relationship with a domestic supply chain firm may be less costly than with a foreign firm in another country. Crosby and Stephens, 1987 indicate that the relationship strength changes with the amount of buyer–seller interaction and communication. We assume that these levels of social relationship (cf. Table 1) take on a value that lies in the range [0, 1]. No social relationship is indicated by a level of zero and the strongest possible level of social relationship is indicated by a level of one. The levels of social relationship, along with the product flows, are endogenously determined in the model.

The concept of relationship levels was inspired by a paper by Golicic et al., 2003 who introduced the concept of relationship magnitude. That research strongly suggested that different relationship magnitudes lead to different benefits and that different levels of relationship magnitudes can be achieved by putting more or less time and effort into the relationship. The idea of a continuum of relationship strength is also supported by several theories of relationship marketing that suggest that business relationships vary on a continuum from transactional to highly relational (cf. Gabarino and Johnson, 1999). The model by Wakolbinger and Nagurney, 2004 operationalized the frequently mentioned need to create a portfolio of relationships (cf. Cannon and Perreault, 1999; Golicic et al., 2003). The optimal portfolio balanced out the various costs and the risk, against the profit and the relationship value and included the individual decision-makers preferences and risk aversions.

We now describe the supply chain firms' risk functions as presented in Table 4. We note that the risk functions in our model are functions of both the product transactions and the relationship levels. Jüttner et al., 2003 suggest that supply chain-relevant risk sources falls into three categories: environmental risk sources (e.g., fire, social-political actions, or acts of God), organizational risk sources (e.g., production uncertainties), and network-related risk sources. Johnson, 2001 and Norrman and Jansson, 2004 argue that network-related risk arises from the interaction between organizations within the supply chain, e.g., due to insufficient interaction and cooperation. In this paper, we assume a manufacturer is faced with supply disruption risk caused by environmental and/or organizational risk sources, and with opportunism risk which is due to insufficient cooperation and commitment between the partners in the supply chain network. Note that a better supply chain relationship can improve the trust and commitment between supply chain firms, and, hence, reduce the opportunism risk. However, the supply disruption risk may not be significantly influenced by the relationship since the disruption risk is mainly caused by suppliers' technological characteristics and/or "acts of God". Of course, in

Table 2
Production and transaction cost functions.

$f_{it}(y_{it})$	production cost of supplier i at time period t
$f_{jt}(y_{jt})$	production cost of manufacturer j at time period t
$c^i_{jt}(q^i_{jt})$	transaction cost of supplier i with manufacturer j at time t
$c^j_{it}(q^i_{jt})$	transaction cost of manufacturer j with supplier i at time t
$c^j_{kt}(q^j_{kt})$	transaction cost of manufacturer j with retailer k at time t
$c^k_{jt}(q^j_{kt})$	transaction cost of retailer k with manufacturer j at time t
$h_{it}(v_{it})$	inventory cost of supplier i at time t
$h_{jt}(v_{jt})$	inventory cost of manufacturer j at time period t
$h_{kt}(v_{kt})$	inventory cost of retailer k at time period t

Table 3
Cost functions for social relationship.

Notation	Definition
$b^i_{jt}(\eta^i_{jt})$	cost functions of supplier i for maintaining social relationship with manufacturer j in time period t
$b^{i+}_{jt}(\eta^{i+}_{jt})$	cost functions of supplier i for improving social relationship with manufacturer j in time period t
$b^j_{it}(\eta^i_{jt})$	cost functions of manufacturer j for maintaining social relationship with supplier i in time period t
$b^{j+}_{it}(\eta^{i+}_{jt})$	cost functions of manufacturer j for improving social relationship with supplier i in time period t
$b^j_{kt}(\eta^j_{kt})$	cost functions of manufacturer j for maintaining social relationship with retailer k in time period t
$b^{j+}_{kt}(\eta^{j+}_{kt})$	cost functions of manufacturer j for improving social relationship with retailer k in time period t
$b^k_{jt}(\eta^j_{kt})$	cost functions of retailer k for maintaining social relationship with manufacturer j in time period t
$b^{j+}_{kt}(\eta^{j+}_{kt})$	cost functions of retailer k for improving social relationship with manufacturer j in time period t

Table 4
Risk functions.

Notation	Definition
$r^{i1}_{jt}\left(q^i_{jt}\right)$	supply disruption risk incurred at supplier i associated with the transaction with manufacturer j at period t
$r^{i2}_{jt}\left(q^i_{jt}, \eta^i_{jt}\right)$	opportunism risk incurred at supplier i associated with the transaction with manufacturer j at period t
$r^{j1}_{it}\left(q^i_{jt}\right)$	supply disruption risk incurred at manufacturer j associated with the transaction with supplier i at period t
$r^{j2}_{it}\left(q^i_{jt}, \eta^i_{jt}\right)$	opportunism risk incurred at manufacturer j associated with the transaction with supplier i at period t
$r^{j1}_{kt}\left(q^j_{kt}\right)$	supply disruption risk incurred at manufacturer j associated with the transaction with retailer k at period t
$r^{j2}_{kt}\left(q^j_{kt}, \eta^j_{kt}\right)$	opportunism risk incurred at manufacturer j associated with the transaction with retailer k at period t
$r^{k1}_{jt}\left(q^j_{kt}\right)$	supply disruption risk incurred at retailer k associated with the transaction with manufacturer j at period t
$r^{k2}_{jt}\left(q^j_{kt}, \eta^j_{kt}\right)$	opportunism risk incurred at retailer k associated with the transaction with manufacturer j at period t

certain situations, the opportunism risk may be adversely affected by higher levels of relationships (Granovetter, 1985). Nevertheless, the functions in Table 4 explicitly include relationship levels and product transactions as inputs into the risk functions and reflect this dependence.

The inverse demand functions as given in Table 5 are associated with the retailers of the supply chain network.

We now turn to describing the behavior of the various economic decision-makers. The model is presented, for ease of exposition, for the case of a single substitutable product. It can also handle multiple products/parts through a replication of the nodes, links and added notation. We first focus on the suppliers. We will then turn to the manufacturers and the retailers.

2.1. Decision-making behavior of the suppliers and their optimality conditions

Let ρ^{i*}_{1jt} denote the price charged for the part/material by supplier i in transacting with manufacturer j in period t. The price ρ^{i*}_{1jt} is an endogenous variable and will be determined once the entire multiperiod supply chain network equilibrium model is solved. Supplier i's production, inventory, and transactions in time period t must satisfy the following conservation of flow equations:

$$y_{it} \leqslant CAP_{it}; \tag{1}$$

$$y_{i1} \geqslant \sum_{j=1}^{J} q^i_{j1} + v_{i1}; \tag{2}$$

$$y_{it} + v_{it-1} \geqslant \sum_{j=1}^{J} q^i_{jt} + v_{it}, \quad t = 2, \ldots, T; \tag{3}$$

where constraint (1) states that the quantity of the part produced by supplier i in time period t must be less than or equal to the production capacity; and constraints (2) and (3) model the balance between the inventory, production and sales in each period.

We also use the following constraints to track the relationship levels between the manufacturers and retailers.

$$\eta^i_{j1} = \dot{\eta}^i_j; \tag{4}$$

$$\eta^i_{jt} + \eta^{i+}_{jt} - \eta^{i-}_{jt} = \eta^i_{jt+1}, \quad t = 2, \ldots, T; \tag{5}$$

where $\dot{\eta}^i_j$ in (4) denotes the initial relationship level; and constraint (5) states that the relationship level in the next period is equal to the relationship in the current period plus the relationship improvement.

The first objective of the suppliers is to maximize the total profit over the planning horizon T. The decision variables for supplier i are the production quantity in each period, y_{it}, the distribution quantities in each period, q^i_{jt}, the inventory quantity in each period, v^i_t, the levels of social relationship in each period, η^i_{jt}, and the improvements of social relationship in each period, η^{i+}_{jt}. Thus, supplier i maximizes its profit over the T time periods.

$$\text{Maximize} \sum_{t=1}^{T}\sum_{j=1}^{J}\left[\rho^{i*}_{1jt}q^i_{jt} - c^i_{jt}\left(q^i_{jt}\right) - b^i_{jt}\left(\eta^i_{jt}\right) - b^{i+}_{jt}\left(\eta^{i+}_{jt}\right)\right] - \sum_{t=1}^{T} f_{it}(y_{it}) - \sum_{t=1}^{T} h_{it}(v_{it}). \tag{6}$$

Table 5
Inverse Demand function.

Notation	Definition
$\rho_{3t}(d_t)$	inverse demand function in the retail markets

The first term in (6) represents the revenue and the subsequent four terms represent the transaction costs, the costs for social relationship for supplier i, the production costs, and the inventory costs. Note that we allow the specifications of all the cost functions to be time-dependent.

In addition to the criterion of profit maximization, we also assume that each supplier is concerned with risk minimization associated with dealing with the manufacturers. Here, for the sake of generality, we assume, as given, a supply disruption risk function r^{1i}_{jt} and an opportunism risk function r^{2i}_{jt}, for supplier i in dealing with manufacturer j in time period t. The second criterion of each supplier can be expressed mathematically as:

$$\text{Minimize} \sum_{t=1}^{T}\sum_{j=1}^{J}\left[\omega^{1i}_j r^{1i}_{jt}\left(q^i_{jt}\right) + \omega^{2i}_j r^{2i}_{jt}\left(q^i_{jt}, \eta^i_{jt}\right)\right]. \tag{7}$$

In particular, $r^{1i}_{jt}\left(q^i_{jt}\right)$ and $r^{2i}_{jt}\left(q^i_{jt}, \eta^i_{jt}\right)$ are defined as the *expected material quantities*subject to the supply disruption risk and the opportunism risk as follows:

$$r^{1i}_{jt}\left(q^i_{jt}\right) \equiv \int_{s=0}^{q^i_{jt}} \left(q^i_{jt} - s\right) PDF^i_j(s)ds, \tag{8}$$

$$r^{2i}_{jt}\left(q^i_{jt}, \eta^i_{jt}\right) \equiv \phi^i_{jt}\left(1 - \eta^i_{jt}\right)q^i_{jt}, \tag{9}$$

where $PDF^i_j(s)$ is the probability density function that supplier i is able to deliver s units of the materials/parts to manufacturer j without any disruptions and delays. ϕ^i_{jt} can be interpreted as the probability that manufacturer j conducts opportunism behaviors, such as, cancellation of orders, failure or delay of payments, etc., in the transactions with supplier i when the relationship level between the two parties is 0. Note that the opportunism risk can be lowered by an increase of the relationship level, and that a full relationship $\left(\eta^i_{jt} = 1\right)$ means that the opportunism risk can be completely eliminated. In addition, supplier i assigns the nonnegative weights ω^{1i}_j and ω^{2i}_j to the risks where the weights can be interpreted as the unit penalty costs when the associated risks occur. Hence, $\omega^1_i r^{1i}_{jt}\left(q^i_{jt}\right)$ and $\omega^2_i r^{2i}_{jt}\left(q^i_{jt}, \eta^i_{jt}\right)$ represent the *expected costs* associated with the two types of risks, respectively. Note that if certain type of risk has no impact on supplier i then the associated unit penalty cost ω is zero.

We are now ready to construct the multicriteria decision-making problem faced by a supplier, which combines with the criteria of profit maximization given by (6) and risk minimization given by (7). Let supplier i's multicriteria decision-making problem be expressed as:

$$P_i = \sum_{t=1}^{T}\sum_{j=1}^{J}\left[\rho^{i*}_{1jt}q^i_{jt} - c^i_{jt}\left(q^i_{jt}\right) - b^i_{jt}\left(\eta^i_{jt}\right) - b^{i+}_{jt}\left(\eta^{i+}_{jt}\right)\right] - \sum_{t=1}^{T} f_{it}(y_{it}) - \sum_{t=1}^{T} h_{it}(v_{it}) - \sum_{t=1}^{T}\sum_{j=1}^{J}\left[\omega^{1i}_j r^{1i}_{jt}\left(q^i_{jt}\right) + \omega^{2i}_j r^{2i}_{jt}\left(q^i_{jt}, \eta^i_{jt}\right)\right] \tag{10}$$

subject to (1)–(5) and the non-negativity constraints:$y_{it} \geqslant 0, q^i_{jt} \geqslant 0, v_{it} \geqslant 0, 0 \leqslant \eta^i_{jt} \leqslant 1,$ and $0 \leqslant \eta^{i+}_{jt} \leqslant 1, \quad \forall j, t.$

We let θ^{1*} denote the vector of the Lagrangian multipliers of constraint (1), μ^{1*} denote the vector of the Lagrangian multipliers of constraints (2) and (3), and $\hat{\lambda}^{1*}$ denote the vector of the Lagrangian multipliers of constraints (4) and (5). We assume that suppliers compete in a noncooperative manner in the sense of Nash, 1950,Nash, 1951, and the cost and risk functions are linear or convex. The optimality conditions for all suppliers i; $i = 1,\ldots,I$, simultaneously, can then be expressed as the following variational inequality (cf. Bazaraa and Shetty, 1979; Cruz et al., 2006): determine

$$(Y^{1*}, Q^{1*}, V^{1*}, \eta^{1*}, \eta^{1+*}, \mu^{1*}, \theta^{1*}, \hat{\lambda}^{1*}) \in \mathcal{K}^1 \text{ satisfying:}$$

$$
\begin{aligned}
&\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\partial c_{jt}^{i*}}{\partial q_{jt}^{i}}+\omega_j^{1i}\frac{\partial r_{jt}^{1i*}}{\partial q_{jt}^{i}}+\omega_j^{2i}\frac{\partial r_{jt}^{2i*}}{\partial q_{jt}^{i}}+\mu_{it}^{*}-\rho_{1jt}^{i*}\right]\times\left[q_{jt}^{i}-q_{jt}^{i*}\right]\\
&+\sum_{t=1}^{T}\sum_{i=1}^{I}\left[\frac{\partial f_{it}^{*}}{\partial y_{it}}+\theta_{it}^{*}-\mu_{it}^{*}\right]\times\left[y_{it}-y_{it}^{*}\right]\\
&+\sum_{t=1}^{T-1}\sum_{j=1}^{I}\left[\frac{\partial h_{it}^{*}}{\partial v_{it}}+\mu_{it}^{*}-\mu_{it+1}^{*}\right]\times\left[v_{it}-v_{it}^{*}\right]\\
&+\sum_{j=1}^{I}\left[\frac{\partial h_{iT}^{*}}{\partial v_{iT}}+\mu_{iT}^{*}\right]\times\left[v_{iT}-v_{iT}^{*}\right]\\
&+\sum_{t=1}^{T}\sum_{i=1}^{I}\left[\frac{\partial b_{jt}^{i+*}}{\partial \eta_{jt}^{i+}}-\hat{\lambda}_{jt}^{i*}\right]\times\left[\eta_{jt}^{i+}-\eta_{jt}^{i+*}\right]\\
&+\sum_{t=2}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\partial b_{jt}^{i*}}{\partial \eta_{jt}^{i}}+\omega_j^{1i}\frac{\partial r_{it}^{2j*}}{\partial \eta_{jt}^{i}}+\hat{\lambda}_{jt-1}^{i*}-\hat{\lambda}_{jt}^{i*}\right]\times\left[\eta_{jt}^{i}-\eta_{jt}^{i*}\right]\\
&+\sum_{i=1}^{I}\sum_{t=1}^{T}\left[CAP_{it}-y_{it}^{*}\right]\times\left[\theta_{it}-\theta_{it}^{*}\right]\\
&+\sum_{i=1}^{I}\sum_{t=2}^{T}\left[y_{it}^{*}+v_{it-1}^{*}-\sum_{j=1}^{J}q_{jt}^{i*}-v_{it}^{*}\right]\times\left[\mu_{it}-\mu_{it}^{*}\right]\\
&+\sum_{i=1}^{I}\left[y_{i1}^{*}-\sum_{j=1}^{J}q_{j1}^{i*}-v_{i1}^{*}\right]\times\left[\mu_{i1}-\mu_{i1}^{*}\right]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\dot{\eta}_{j}^{i}+\eta_{j1}^{i+*}-\eta_{j2}^{i*}\right]\times\left[\hat{\lambda}_{j1}^{i}-\hat{\lambda}_{j1}^{i*}\right]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T-1}\left[\eta_{jt}^{i*}+\eta_{jt}^{i+*}-\eta_{jt+1}^{i*}\right]\times\left[\hat{\lambda}_{jt}^{i}-\hat{\lambda}_{jt}^{i*}\right]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\eta_{jT}^{i*}+\eta_{jT}^{i+}\right]\times\left[\hat{\lambda}_{jT}^{i}-\hat{\lambda}_{jT}^{i*}\right]\geqslant 0,\\
&\forall(Y^1,Q^1,V^1,\eta^1,\eta^{1+},\mu^1,\theta^1,\hat{\lambda}^1)\in\mathcal{K}^1, \qquad (11)
\end{aligned}
$$

where $\mathcal{K}^1 \equiv [(Y^1,Q^1,V^1,\eta^1,\eta^{1+},\mu^1,\theta^1,\hat{\lambda}^1)|y_{it} \geqslant 0, q_{jt}^i \geqslant 0, v_{it} \geqslant 0, 0 \leqslant \eta_{jt}^i \leqslant 1, 0 \leqslant \eta_{jt}^{i+} \leqslant 1, \mu_{it} \geqslant 0,\ \theta_{it} \geqslant 0,\ \hat{\lambda}_{jt}^i \geqslant 0, \forall i,j,t]$ and * represents the evaluation of the functions at the equilibrium.

The variational inequality (11) has a meaningful economic interpretation. Note that the dual price associated with the production and inventory constraints (2) and (3), μ_{it}^* can be interpreted as the implicit marginal supply cost of the product. From the first term in (11) we can see that, if there is a positive amount of the material transacted from a supplier to a manufacturer at time period t, then the sum of the marginal transaction cost, the weighted marginal risks, and the marginal supply cost must be equal to the price that the manufacturer is willing to pay for the product. If the first sum, in turn, exceeds the second one then there will be no product transacted. The second term in (11), in turn, show that if the production quantity of a supplier in a period is positive the marginal production cost plus the shadow price of the production capacity is equal to the marginal supply cost. The third term in (11) means that if there is a positive inventory level between two consecutive periods then the marginal supply cost of the previous period plus the marginal inventory cost must be equal to the marginal supply cost of the next period. The fourth term is simply a special case of the third term for period 1.

Moreover, note that the dual price associated with the relationship constraints (4) and (5),$\hat{\lambda}_{jt}^{i*}$, can be interpreted as the marginal value of the relationship between supplier i and manufacturer j at the end of period t. Hence, the fifth term in (11) indicates that if there is an increase in the relationship level then the marginal cost of the relationship improvement is equal to $\hat{\lambda}_{jt}^{i*}$.

The sixth term in (11), in turn, means that if the relationship level between the supplier and the manufacturer is positive then $\hat{\lambda}_{jt-1}^{i*}$ plus the marginal cost of maintaining the relationship and the marginal reduction of the risk is equal to the $\hat{\lambda}_{jt}^{i*}$. Equivalently, the sixth term means that the marginal value of the relationship evaluated at the end of period t is equal to the marginal benefit of the relationship in period $t+1$ plus the marginal value of the relationship at the end of period $t+1$. The remaining terms in (11) associate constraints with the corresponding Lagrangian multipliers.

2.2. Multicriteria decision-making behavior of the manufacturers and their optimality conditions

The manufacturers, in turn, are involved in transactions both with the suppliers since they wish to obtain the parts/materials for their production, as well as with the retailers. The manufacturers are assumed to be multicriteria decision-makers who seek to maximize profits, and to minimize their risk associated with their transactions with suppliers.

We assume that the objective of a manufacturer is to maximize his total profit over the planning horizon T. The decision variables of manufacturer j include the production quantity in each period, y_{jt}, the transactions with suppliers at each period, $q_{jt}^i; i=1,\ldots,I; t=1,\ldots,T$, the transactions with retailers at each period, $q_{kt}^j; k=1,\ldots,K; t=1,\ldots,T$, the inventory level in each period, v_{jt}, $t=1,\ldots,T$, the levels of social relationship with supplier i at each period, $\eta_{jt}^i; t=1,\ldots,T$, the improvement of social relationship level with supplier i at each period, $\eta_{jt}^{i+}; t=1,\ldots,T$, the levels of social relationship with retailer k at each period, η_{kt}^j, and the improvement of social relationship level with retailer k at each period, $\eta_{kt}^{j+}; t=1,\ldots,T$. Hence, the profit maximization problem faced by manufacturer j is given by:

$$
\begin{aligned}
\text{Maximize}\quad &\sum_{t=1}^{T}\left[\sum_{k=1}^{K}\rho_{2kt}^{j*}q_{kt}^{j}-f_{jt}(y_{jt})-h_{jt}(v_{jt})-\right.\\
&\sum\nolimits_{i=1}^{I}\left(c_{it}^{j}\left(q_{jt}^{i}\right)+b_{it}^{j}\left(\eta_{jt}^{i}\right)+b_{it}^{j+}\left(\eta_{j}^{i+}\right)\right)\\
&\left.-\sum_{k=1}^{K}\left(c_{kt}^{j}\left(q_{kt}^{j}\right)+b_{kt}^{j}\left(\eta_{kt}^{j}\right)+b_{kt}^{j+}\left(\eta_{k}^{j+}\right)\right)-\sum_{i=1}^{I}\rho_{1jt}^{i*}q_{jt}^{i}\right]
\end{aligned}
\tag{12}
$$

$$\text{subject to}: \; y_{jt} \leqslant CAP_{jt} \quad \forall t, \tag{13}$$

$$\sum_{i=1}^{I} q_{jt}^{i} \geqslant y_{jt} \quad \forall t, \tag{14}$$

$$y_{jt}+v_{jt-1} \geqslant \sum_{k=1}^{K} q_{kt}^{j}+v_{jt} \quad \forall t, \tag{15}$$

$$y_{j1} \geqslant \sum_{k=1}^{K} q_{k1}^{j}+v_{j1}; \tag{16}$$

$$\eta_{j1}^{i}=\dot{\eta}_{j}^{i}; \tag{17}$$

$$\eta_{jt}^{i}+\eta_{jt}^{i+}-\eta_{jt}^{i-}=\eta_{jt+1}^{i}, \quad t=2,\ldots,T; \tag{18}$$

$$\eta_{k1}^{j}=\dot{\eta}_{k}^{j}; \tag{19}$$

$$\eta_{kt}^{j}+\eta_{kt}^{j+}-\eta_{kt}^{i-}=\eta_{kt+1}^{j}, \quad t=2,\ldots,T; \tag{20}$$

and the non-negativity constraints $y_j^i \geqslant 0, q_{jt}^i \geqslant 0, q_{kt}^j \geqslant 0, v_{jt} \geqslant 0, 0 \leqslant \eta_{jt}^i \leqslant 1, 0 \leqslant \eta_{jt}^{i+} \leqslant 1, 0 \leqslant \eta_{jt}^{i-} \leqslant 1, 0 \leqslant \eta_{kt}^j \leqslant 1, 0 \leqslant \eta_{kt}^{j+} \leqslant 1, 0 \leqslant \eta_{kt}^{j-} \leqslant 1, \forall i,k,t$.

The first term in the objective function (12) represents the revenue of manufacturer j, whereas the next six terms represent various costs (see Table 2), and the last term represents the payout to the suppliers. Constraint (14) models the material requirements of production. Note that for the purpose of easy exposition, we assume one unit of the product requires one unit of the part. Constraints (15) and (16) models the balance of production, inventory, and sales in each time period t. Constraints (17)–(20) track the relationship levels with suppliers and with the retailers where $\dot{\eta}_j^i$ and $\dot{\eta}_k^j$ represent the initial relationship levels.

In addition to the criterion of profit maximization, we also assume that each manufacturer is concerned with risk minimization associated with dealing with the suppliers and the retailers. Here, for the sake of generality, we assume, as given, a supply disruption risk function r_{it}^{1j} and an opportunism risk function r_{it}^{2j}, for manufacturer j in dealing with supplier i in time period t. In addition, we assume a supply disruption risk function r_{kt}^{1j} and a opportunism risk function r_{kt}^{2j}, for manufacturer j in dealing with retailer k in period t. The second criterion of each manufacturer can be expressed mathematically as:

$$\text{Minimize} \sum_{t=1}^{T}\sum_{i=1}^{I}\left[\omega_i^{1j} r_{it}^{1j}\left(q_{jt}^i\right)+\omega_i^{2j} r_{it}^{2j}\left(q_{jt}^i,\eta_{jt}^i\right)\right] + \sum_{t=1}^{T}\sum_{k=1}^{K}\left[\omega_k^{1j} r_{kt}^{1j}\left(q_{kt}^j\right)+\omega_k^{2j} r_{kt}^{2j}\left(q_{kt}^j,\eta_{kt}^j\right)\right]. \tag{21}$$

In the first term of the risk minimization problem (21), $r_{it}^{1j}\left(q_{jt}^i\right)$ and $r_{it}^{2j}\left(q_{jt}^i,\eta_{jt}^i\right)$ are defined as the *expected material quantities*subject to the supply disruption risk and the opportunism risk as follows:

$$r_{jt}^{1i}\left(q_{jt}^i\right) \equiv \int_{s=0}^{q_{jt}^i}\left(q_{jt}^i - s\right) PDF_j^i(s)ds, \tag{22}$$

$$r_{jt}^{2i}\left(q_{jt}^i,\eta_{jt}^i\right) \equiv \phi_{jt}^i\left(1-\eta_{jt}^i\right)q_{jt}^i, \tag{23}$$

where $PDF_j^i(s)$ is the probability density function that supplier i is able to deliver s units of the materials/parts to manufacturer j without any disruptions and delays; and ϕ_{it}^j is the probability that supplier i conducts opportunism behaviors, such as, the failure to conform to the quality or safety standards, in the transactions with manufacturer j when the relationship level between the two parties is 0. Note that the opportunism risk can be lowered by an improvement of the relationship level, and that a full relationship $\left(\eta_{jt}^i = 1\right)$ implies that the opportunism risk can be completely eliminated. In addition, manufacturer j assigns the nonnegative weights ω_i^{1j} and ω_i^{2j} to the risks which can be interpreted as the unit penalty costs when the associated risks occur. Hence, $\omega_i^{1j} r_{it}^{1j}\left(q_{jt}^i\right)$ and $\omega_i^{2j} r_{it}^{2j}\left(q_{jt}^i,\eta_{jt}^i\right)$ represent the *expected costs* associated with the two types of risks, respectively.

In the second term of the risk minimization problem (21), $r_{kt}^{1j}\left(q_{kt}^j\right)$ and $r_{kt}^{2j}\left(q_{kt}^j,\eta_{kt}^j\right)$ are defined as the *expected product quantities*subject to the supply disruption risk and the opportunism risk as follows:

$$r_{kt}^{1j}\left(q_{kt}^j\right) \equiv \int_{s=0}^{q_{kt}^j}\left(q_{kt}^j - s\right) PDF_k^j(s)ds, \tag{24}$$

$$r_{kt}^{2j}\left(q_{kt}^j,\eta_{kt}^j\right) \equiv \phi_{kt}^j\left(1-\eta_{kt}^j\right)q_{kt}^j, \tag{25}$$

where $PDF_k^j(s)$ is the probability density function that manufacturer j is able to deliver s units of the product to retailer k without any disruptions and delays; and ϕ_{kt}^j is the probability that retailer i conducts opportunism behaviors in the transactions with manufacturer j when the relationship level between the two parties is 0. Note that the opportunism risk can be reduced by an increase of the relationship level, and that a full relationship $\left(\eta_{kt}^j = 1\right)$ implies that the opportunism risk can be completely eliminated. In addition, manufacturer j assigns the nonnegative weights ω_k^{1j} and ω_k^{2j} to the risks which represent the unit penalty costs when the associated risks occur. Hence, $\omega_k^{1j} r_{kt}^{1j}\left(q_{kt}^j\right)$ and $\omega_k^{2j} r_{kt}^{2j}\left(q_{kt}^j,\eta_{kt}^j\right)$ represent the *expected costs* associated with the two types of risks, respectively.

Note that if certain type of risk has no impact on manufacturer j the associated unit penalty cost, ω, will be equal to zero.

We are now ready to construct the multicriteria decision-making problem faced by a manufacturer, which combines the criteria of profit maximization given by (12) and risk minimization given by (21). Let manufacturer j's multicriteria decision-making problem be expressed as:

$$\begin{aligned} P_j:\ \text{Maximize} \quad & \sum_{t=1}^{T}\left[\sum_{k=1}^{K}\rho_{2kt}^{j*}q_{kt}^j - f_{jt}(y_{jt}) - h_{jt}(v_{jt})\right. \\ & - \sum_{i=1}^{I}\left(c_{it}^j(q_{jt}^i) + b_{it}^j(\eta_{jt}^i) + b_{it}^{j+}(\eta_j^{i+})\right) \\ & \left. - \sum_{k=1}^{K}\left(c_{kt}^j\left(q_{kt}^j\right) + b_{kt}^j\left(\eta_{kt}^j\right) + b_{kt}^{j+}\left(\eta_k^{j+}\right)\right) - \sum_{i=1}^{I}\rho_{1jt}^{i*}q_{jt}^i\right] \\ & - \sum_{t=1}^{T}\sum_{i=1}^{I}\left[\omega_i^{1j} r_{it}^{1j}\left(q_{jt}^i\right)+\omega_i^{2j} r_{it}^{2j}\left(q_{jt}^i,\eta_{jt}^i\right)\right] \\ & - \sum_{t=1}^{T}\sum_{k=1}^{K}\left[\omega_k^{1j} r_{kt}^{1j}\left(q_{kt}^j\right)+\omega_k^{2j} r_{kt}^{2j}\left(q_{kt}^j,\eta_{kt}^j\right)\right] \end{aligned} \tag{26}$$

subject to: (13)–(20) and the non-negativity constraints.

We let θ^{2*} denote the vector of the Lagrangian multipliers of constraint (13), v^{2*} denote the vector of Lagrangian multipliers of constraints (14), μ^{2*} denote the vector of the Lagrangian multipliers of constraints (15) and (16), $\check{\lambda}^{1*}$ denote the vector of the Lagrangian multipliers of constraints (17) and (18), and $\hat{\lambda}^{2*}$ denote the vector of the Lagrangian multipliers of constraints (19) and (20). We assume that the manufacturers also compete in a noncooperative manner, and the cost and risk functions are linear or convex. The optimality conditions for all manufacturers simultaneously can be expressed as the variational inequality: determine $(Q^{1*}, Q^{2*}, Y^{2*}, V^{2*}, \eta^{1*}, \eta^{2*}, \eta^{1+*}, \eta^{2+*}, \mu^{2*}, v^{2*}, \theta^{2*}, \check{\lambda}^{1*}, \hat{\lambda}^{2*}) \in \mathcal{K}^2$ satisfying:

$$\begin{aligned} & \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\partial c_{it}^{j*}}{\partial q_{jt}^i} + \omega_i^{1j}\frac{\partial r_{it}^{1j*}}{\partial q_{jt}^i} + \omega_i^{2j}\frac{\partial r_{it}^{2j*}}{\partial q_{jt}^i} + \rho_{1jt}^{i*} - v_{jt}^*\right] \times \left[q_{jt}^i - q_{jt}^{i*}\right] \\ & + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K}\left[\frac{\partial c_{kt}^{j*}}{\partial q_{kt}^j} + \omega_k^{1j}\frac{\partial r_{kt}^{1j*}}{\partial q_{kt}^j} + \omega_k^{2j}\frac{\partial r_{kt}^{2j*}}{\partial q_{kt}^j} + \mu_{jt}^* - \rho_{2kt}^{j*}\right] \times \left[q_{kt}^j - q_{kt}^{j*}\right] \\ & + \sum_{t=1}^{T}\sum_{j=1}^{J}\left[\frac{\partial f_{jt}^*}{\partial y_{jt}} + \theta_{jt}^* + v_{jt}^* - \mu_{jt}^*\right] \times \left[y_{jt} - y_{jt}^*\right] \\ & + \sum_{j=1}^{J}\sum_{t=1}^{T}\left[CAP_{jt} - y_{jt}^*\right] \times \left[\theta_{jt} - \theta_{jt}^*\right] \\ & + \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\partial b_{it}^{j+*}}{\partial \eta_{jt}^{i+}} - \check{\lambda}_{jt}^{i*}\right] \times \left[\eta_{jt}^{i+} - \eta_{jt}^{i+*}\right] \\ & + \sum_{t=2}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\partial b_{it}^{j*}}{\partial \eta_{jt}^i} + \omega_i^{1j}\frac{\partial r_{jt}^{2i*}}{\partial \eta_{jt}^i} + \check{\lambda}_{jt-1}^{i*} - \check{\lambda}_{jt}^{i*}\right] \times \left[\eta_{jt}^i - \eta_{jt}^{i*}\right] \\ & + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K}\left[\frac{\partial b_{kt}^{j+*}}{\partial \eta_{kt}^{j+}} - \hat{\lambda}_{kt}^{j*}\right] \times \left[\eta_{kt}^{j+} - \eta_{kt}^{j+*}\right] \\ & + \sum_{t=2}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K}\left[\frac{\partial b_{kt}^{j*}}{\partial \eta_{kt}^j} + \omega_k^{1j}\frac{\partial r_{kt}^{2j*}}{\partial \eta_{kt}^j} + \hat{\lambda}_{kt-1}^{j*} - \hat{\lambda}_{kt}^{j*}\right] \times \left[\eta_{kt}^j - \eta_{kt}^{j*}\right] \\ & + \sum_{t=1}^{T-1}\sum_{j=1}^{J}\left[\frac{\partial h_{jt}^*}{\partial v_{jt}} + \mu_{jt}^* - \mu_{jt+1}^*\right] \times \left[v_{jt} - v_{jt}^*\right] \\ & + \sum_{j=1}^{J}\left[\frac{\partial h_{jT}^*}{\partial v_{jT}} + \mu_{jT}^*\right] \times \left[v_{jT} - v_{jT}^*\right] + \sum_{j=1}^{J}\sum_{t=1}^{T}\left[\sum_{i=1}^{I}q_{jt}^{i*} - y_{jt}^*\right] \times \left[v_{jt} - v_{jt}^*\right] \end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{J}\sum_{t=2}^{T}\left[y_{jt}^{*}+v_{jt-1}^{*}-\sum_{k=1}^{K}q_{kt}^{j*}-v_{jt}^{*}\right]\times\left[\mu_{jt}-\mu_{jt}^{*}\right]\\
&+\sum_{j=1}^{J}\left[y_{j1}^{*}-\sum_{k=1}^{K}q_{k1}^{j*}-v_{j1}^{*}\right]\times\left[\mu_{j1}-\mu_{j1}^{*}\right]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\dot{\eta}_{j}^{i}+\eta_{j1}^{i+*}-\eta_{j2}^{i*}\right]\times\left[\check{\lambda}_{j1}^{i}-\check{\lambda}_{j1}^{i*}\right]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=2}^{T-1}\left[\eta_{jt}^{i*}+\eta_{jt}^{i+*}-\eta_{jt+1}^{i*}\right]\times\left[\check{\lambda}_{jt}^{i}-\check{\lambda}_{jt}^{i*}\right]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\eta_{jT}^{i*}+\eta_{jT}^{i+*}\right]\times\left[\check{\lambda}_{jT}^{i}-\check{\lambda}_{jT}^{i*}\right]\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\left[\dot{\eta}_{k}^{j}+\eta_{k1}^{j+*}-\eta_{k2}^{j*}\right]\times\left[\hat{\lambda}_{k1}^{j}-\hat{\lambda}_{k1}^{j*}\right]\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=2}^{T-1}\left[\eta_{kt}^{j*}+\eta_{kt}^{j+*}-\eta_{kt+1}^{j*}\right]\times\left[\hat{\lambda}_{kt}^{j}-\hat{\lambda}_{kt}^{j*}\right]\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\left[\eta_{kT}^{j*}+\eta_{kT}^{j+*}\right]\times\left[\hat{\lambda}_{kT}^{j}-\hat{\lambda}_{kT}^{j*}\right]\geqslant 0,\\
&\forall(Q^1,Q^2,Y^2,V^2,\eta^1,\eta^2,\eta^{1+},\eta^{2+},\mu^2,v^2,\theta^2,\check{\lambda}^1,\hat{\lambda}^2)\in\mathcal{K}^1, \qquad (27)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{K}^2\equiv\Big[&(Q^1,Q^2,Y^2,V^2,\eta^1,\eta^2,\eta^{1+},\eta^{2+},\mu^2,v^2,\theta^2,\check{\lambda}^1,\hat{\lambda}^2)|y_{jt}\\
&\geqslant 0,q_{jt}^{i}\geqslant 0,q_{kt}^{j}\geqslant 0,v_{jt}\geqslant 0,0\leqslant\eta_{jt}^{i}\leqslant 1,0\leqslant\eta_{kt}^{j}\leqslant 1,0\\
&\leqslant\eta_{jt}^{i+}\leqslant 1,0\leqslant\eta_{kt}^{j+}\leqslant 1,\theta_{jt}\geqslant 0,\ \mu_{jt}\geqslant 0,v_{jt}\geqslant 0,\check{\lambda}_{jt}^{i}\\
&\geqslant 0,\hat{\lambda}_{kt}^{j}\geqslant 0,\forall i,j,k,t\Big].
\end{aligned}
$$

Note that we can interpret the manufacturers' variational inequality (27) in the same manner as we did for the suppliers' variational inequality (11).

2.3. Multicriteria decision-making behavior of the retailers and their optimality conditions

The retailers, in turn, are involved in transactions both with the manufacturers, since they wish to obtain the products for their production, as well as with the customers in demand markets. The retailers are assumed to be multicriteria decision-makers who seek to maximize profits, and to minimize their risk associated with their transactions with manufacturers.

We assume that the objective of a retailer is to maximize his total profit over the planning horizon T. The decision variables of retailer k include the sales in each period, q_{kt}; $t=1,\ldots,T$, the transaction amounts with manufacturers in each period, $q_{kt}^{j}; j=1,\ldots,J; t=1,\ldots,T$, the inventory level in each period, v_{kt}, $t=1,\ldots,T$, the levels of social relationship with manufacturer j at each period, $\eta_{kt}^{j}; t=1,\ldots,T$, and the improvement of social relationship level with manufacturer j at each period, $\eta_{kt}^{j+}; t=1,\ldots,T$. Hence, the profit maximization problem faced by retailer k is given by:

$$
\text{Maximize}\quad\sum_{t=1}^{T}\left[\rho_{3kt}^{*}q_{kt}-f_{kt}(q_{kt})-h_{kt}(v_{kt})-\sum_{j=1}^{J}\left(c_{jt}^{k}\left(q_{kt}^{j}\right)+b_{jt}^{k}\left(\eta_{jt}^{k}\right)+b_{jt}^{k+}\left(\eta_{k}^{j+}\right)\right)-\sum_{i=1}^{I}\rho_{2kt}^{j*}q_{kt}^{j}\right] \qquad (28)
$$

$$
\text{subject to}:\ \sum_{j=1}^{J}q_{kt}^{j}+v_{k(t-1)}\geqslant q_{kt}+v_{kt}\quad t=2,\ldots,T;, \qquad (29)
$$

$$
\sum_{j=1}^{J}q_{kt}^{j}\geqslant q_{k1}+v_{k1}; \qquad (30)
$$

$$
\eta_{k1}^{j}=\dot{\eta}_{k}^{j}; \qquad (31)
$$

$$
\eta_{kt}^{j}+\eta_{kt}^{j+}-\eta_{kt}^{j-}=\eta_{kt+1}^{j},\quad t=2,\ldots,T \qquad (32)
$$

and the non-negativity constraints $q_{kt}^{j}\geqslant 0, q_{kt}\geqslant 0, v_{kt}\geqslant 0, 0\leqslant\eta_{kt}^{j}\leqslant 1, 0\leqslant\eta_{kt}^{j+}\leqslant 1, 0\leqslant\eta_{kt}^{j-}\leqslant 1, \forall j,t$. The first term in the objective function (28) represents the revenue of retailer k, whereas the next five terms represent various costs (see Table 2), and the last term represents the payout to the manufacturers.

Constraints (29) and (30) models the balance of inventory, and sales in each time period. Constraints (31) and (32), in turn, track the relationship levels with manufacturers where $\dot{\eta}_{k}^{j}$ s represent the initial relationship levels.

In addition to the criterion of profit maximization, we also assume that each retailer is concerned with risk minimization associated with dealing with the manufacturers. Here, for the sake of generality, we assume, as given, a supply disruption risk function r_{jt}^{1k} and an opportunism risk function r_{kt}^{2j}, for retailer k in dealing with manufacturer j in period t. The second criterion of each retailer can be expressed mathematically as:

$$
\text{Minimize}\ \sum_{t=1}^{T}\sum_{j=1}^{J}\left[\omega_{j}^{1k}r_{jt}^{1k}\left(q_{kt}^{j}\right)+\omega_{j}^{2k}r_{jt}^{2k}\left(q_{kt}^{j},\eta_{kt}^{j}\right)\right]. \qquad (33)
$$

In the risk minimization problem (33), $r_{jt}^{1k}\left(q_{kt}^{j}\right)$ and $r_{jt}^{2k}\left(q_{kt}^{j},\eta_{kt}^{j}\right)$ are defined as the *expected product quantities*subject to the supply disruption risk and the opportunism risk as follows:

$$
r_{jt}^{1k}\left(q_{kt}^{j}\right)\equiv\int_{s=0}^{q_{kt}^{j}}\left(q_{kt}^{j}-s\right)PDF_{k}^{j}(s)ds, \qquad (34)
$$

$$
r_{jt}^{2k}\left(q_{kt}^{j},\eta_{kt}^{j}\right)\equiv\phi_{kt}^{j}\left(1-\eta_{kt}^{j}\right)q_{kt}^{j}, \qquad (35)
$$

where $PDF_{k}^{j}(s)$ is the probability density function that manufacturer j is able to deliver s units of the product to retailer k without any disruptions and delays; and ϕ_{jt}^{k} is the probability that manufacturer j conducts opportunism behaviors in the transactions with retailer k when the relationship level between the two parties is 0. Note that the opportunism risk can be lowered by an increase of the relationship level, and that a full relationship $\left(\eta_{jt}^{k}=1\right)$ means that the opportunism risk can be completely eliminated. In addition, retailer k assigns the nonnegative weights ω_{j}^{1k} and ω_{j}^{2k} to the risks which represent the unit penalty costs when the associated risks occur. Hence,$\omega_{j}^{1k}r_{jt}^{1k}\left(q_{kt}^{j}\right)$ and $\omega_{j}^{2k}r_{jt}^{2k}\left(q_{kt}^{j},\eta_{kt}^{j}\right)$ represent the *expected costs* associated with the two types of risks, respectively. Note that if certain risk has no impact on retailer k the associated ω is equal to zero.

We are now ready to construct the multicriteria decision-making problem faced by a retailer, which combines the criteria of profit maximization given by (28) and risk minimization given by (33). Let retailer k's multicriteria decision-making problem be expressed as:

$$
\begin{aligned}
\text{Maximize}\ P_k=&\sum_{t=1}^{T}\left[\rho_{3kt}^{*}q_{kt}-f_{kt}(q_{kt})-h_{kt}(v_{kt})-\sum_{j=1}^{J}\left(c_{jt}^{k}\left(q_{kt}^{j}\right)+b_{jt}^{k}\left(\eta_{jt}^{k}\right)+b_{jt}^{k+}\left(\eta_{k}^{j+}\right)\right)-\sum_{i=1}^{I}\rho_{2kt}^{j*}q_{kt}^{j}\right]\\
&-\left(\sum_{t=1}^{T}\sum_{j=1}^{J}\omega_{j}^{1k}r_{jt}^{1k}\left(q_{kt}^{j}\right)+\omega_{j}^{2k}r_{jt}^{2k}\left(q_{kt}^{j},\eta_{kt}^{j}\right)\right) \qquad (36)
\end{aligned}
$$

subject to: (29)–(32) and the non-negativity constraints.

We let μ^{3*} denote the vector of the Lagrangian multipliers of constraints (29) and (30), and $\check{\lambda}^{2*}$ denote the vector of the Lagrangian multipliers of constraints (31) and (32). We assume that the retailers also compete in a noncooperative manner, and the cost and risk functions are linear or convex. The optimality conditions for all retailers simultaneously can be expressed as the variational inequality: determine $(Q^{2*}, Q^{3*}, V^{3*}, \eta^{2*}, \eta^{2+*}, \mu^{3*}, \breve{\lambda}^{2*}) \in \mathcal{K}^3$ satisfying:

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K}\left[\frac{\partial c_{jt}^{k*}}{\partial q_{kt}^{j}} + \omega_j^{1k}\frac{\partial r_{jt}^{1k*}}{\partial q_{kt}^{j}} + \omega_j^{2k}\frac{\partial r_{jt}^{2k*}}{\partial q_{kt}^{j}} + \rho_{2kt}^{j*} - \mu_{kt}^{*}\right] \times \left[q_{kt}^{j} - q_{kt}^{j*}\right]$$
$$+ \sum_{t=1}^{T}\sum_{k=1}^{K}\left[\frac{\partial f_{kt}^{*}}{\partial q_{kt}} + \mu_{kt}^{*} - \rho_{3kt}^{*}\right] \times \left[q_{kt} - q_{kt}^{*}\right]$$
$$+ \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K}\left[\frac{\partial b_{jt}^{k*}}{\partial \eta_{kt}^{j}} + \omega_j^{2k}\frac{\partial r_{jt}^{2k*}}{\partial \eta_{kt}^{j}} + \breve{\lambda}_{jt-1}^{i*} + \breve{\lambda}_{jt}^{i*}\right] \times \left[\eta_{kt}^{j} - \eta_{kt}^{j*}\right]$$
$$+ \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K}\left[\frac{\partial b_{jt}^{k+*}}{\partial \eta_{kt}^{j+}} - \breve{\lambda}_{jt}^{i*}\right]\Bigg] \times \left[\eta_{kt}^{j+} - \eta_{kt}^{j+*}\right]$$
$$+ \sum_{t=1}^{T-1}\sum_{j=1}^{J}\left[\frac{\partial h_{jt}^{*}}{\partial v_{jt}} + \mu_{jt}^{*} - \mu_{jt+1}^{*}\right] \times \left[v_{jt} - v_{jt}^{*}\right]$$
$$+ \sum_{j=1}^{J}\left[\frac{\partial h_{jT}^{*}}{\partial v_{jT}} + \mu_{jT}^{*}\right] \times \left[v_{jT} - v_{jT}^{*}\right]$$
$$+ \sum_{k=1}^{K}\sum_{t=2}^{T}\left[\sum_{j=1}^{J} q_{kt}^{j*} + v_{kt-1}^{*} - q_{kt}^{*} - v_{qt}^{*}\right] \times \left[\mu_{kt} - \mu_{kt}^{*}\right]$$
$$+ \sum_{k=1}^{K}\left[\sum_{j=1}^{J} q_{k1}^{j*} - q_{k1}^{*} - v_{k1}^{*}\right] \times \left[\mu_{k1} - \mu_{k1}^{*}\right]$$
$$+ \sum_{j=1}^{J}\sum_{k=1}^{K}\left[\dot{\eta}_{k}^{j} + \eta_{k1}^{j+*} - \eta_{k2}^{j*}\right] \times \left[\breve{\lambda}_{k1}^{j} - \breve{\lambda}_{k1}^{j*}\right]$$
$$+ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=2}^{T-1}\left[\eta_{kt}^{j*} + \eta_{kt}^{j+*} - \eta_{kt+1}^{j*}\right] \times \left[\breve{\lambda}_{kt}^{j} - \breve{\lambda}_{kt}^{j*}\right]$$
$$+ \sum_{j=1}^{J}\sum_{k=1}^{K}\left[\eta_{kT}^{j*} + \eta_{kT}^{j+*}\right] \times \left[\breve{\lambda}_{kT}^{j} - \breve{\lambda}_{kT}^{j*}\right] \geqslant 0,$$
$$\forall (Q^2, Q^3, V^3, \eta^2, \eta^{2+}, \mu^3, \breve{\lambda}^2) \in \mathcal{K}^3, \tag{37}$$

where

$$\mathcal{K}^3 \equiv [(Q^2, Q^3, V^3, \eta^2, \eta^{2+}, \mu^3, \breve{\lambda}^2) | q_{kt}^{j} \geqslant 0, q_{kt} \geqslant 0, v_{kt} \geqslant 0, q_{kt}^{j} \geqslant 0, 0 \leqslant \eta_{kt}^{j} \leqslant 1, 0 \leqslant \eta_{kt}^{j+} \leqslant 1, \mu_{kt} \geqslant 0, \breve{\lambda}_{kt}^{j} \geqslant 0, \ \forall j,k,t].$$

Note that we can interpret the retailers' variational inequality (37) in the same manner as we did for the suppliers' variational inequality (11).

2.4. Equilibrium conditions for the demand markets

We now describe the behavior of the consumers located at the demand markets. The consumers take into account in making their consumption decisions not only the prices charged for the product by the retailers, $\rho_{3kt}^{*}; k = 1,\ldots,K; t = 1,\ldots,T$, but also the unit transaction costs to obtain the product. The equilibrium conditions for consumers at demand market k, (cf. Nagurney, 1999) take the form: for all retailers k; $k = 1,\ldots,K$ and time periods t; $t = 1,\ldots,T$:

$$\rho_{3kt}^{*} + c_{kt}\begin{cases} = \rho_{3t}(d_t^{*}), & \text{if } q_{kt}^{*} > 0, \\ \geqslant \rho_{3t}(d_t^{*}), & \text{if } q_{kt}^{*} = 0 \end{cases} \tag{38}$$

and

$$d_t = \sum_{k=1}^{K} q_{kt}. \tag{39}$$

Conditions (38) state that, in equilibrium, at each time period, if the consumers at demand market purchase the product from retailer k, then the price charged by the retailer for the product at that time period plus the unit transaction cost is equal to the price that the consumers are willing to pay for the product at that time period. If the price plus the unit transaction cost is higher than the price the consumers are willing to pay at the demand market then there will be no transaction between the retailer and the demand market at that time period.

In equilibrium, conditions (38) can be expressed as an inequality analogous to those in (11), (27) and (37) and given by: determine $(Q^{2*}) \in \mathcal{K}^4$, such that

$$\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\rho_{3kt}^{*} + c_{kt} - \rho_{3t}\left(\sum_{k=1}^{K} q_{kt}^{*}\right)\right] \times \left[q_{kt} - q_{kt}^{*}\right] \geqslant 0,$$
$$\forall (Q^2) \in \mathcal{K}^4, \ \textit{where } \mathcal{K}^4 \equiv [(Q^3) | q_{kt}^{j} \geqslant 0, \quad \forall k,t]. \tag{40}$$

2.5. The equilibrium conditions of the multiperiod supply chain network

In equilibrium, the optimality conditions for all suppliers, the optimality conditions for all manufacturers, the optimality conditions for all retailers, and the equilibrium conditions for all demand markets must hold simultaneously so that no decision-maker can be better off by altering his decisions. Also, the shipments that the suppliers ship to the manufacturers must be equal to the shipments that the manufacturers accept from the suppliers; and the shipments that the manufacturers ship to the retailers must be equal to the shipments that the retailers accept from the manufacturers. Similarly, the quantities of the product obtained by the consumers at the demand markets must coincide with the amounts sold by the retailers.

Definition 1. Multiperiod supply chain network equilibriumThe equilibrium state of the multiperiod supply chain network is one where the sum of (11), (27), (37) and (40) is satisfied, so that no decision-maker has any incentive to alter his decisions.

Theorem 1 (Variational inequality formulation). *The equilibrium conditions governing the multiperiod supply chain network model are equivalent to the solution of the variational inequality problem given by: determine*

$$(Q^{1*}, Q^{2*}, Q^{3*}, Y^{1*}, Y^{2*}, V^{1*}, V^{2*}, V^{3*}, \eta^{1*}, \eta^{2*}, \eta^{1+*}, \eta^{2+*}, \mu^{1*}, \mu^{2*}, \mu^{3*}, v^{*}, \theta^{1*}, \theta^{2*}, \lambda^{1*}, \lambda^{2*}) \in \mathcal{K}^5$$

satisfying:

$$\langle F(X^{*}), X - X^{*}\rangle \geqslant 0, \quad \forall X \in \mathcal{K}^5, \tag{41}$$

where $X \equiv (Q^1, Q^2, Q^3, Y^1, Y^2, V^1, V^2, V^3, \eta^1, \eta^2, \eta^{1+}, \eta^{2+}, \mu^1, \mu^2, \mu^3, v, \theta^1, \theta^2, \lambda^1, \lambda^2)$ *and* $F(X) \equiv \big(F_{Q^1}, F_{Q^2}, F_{Q^3}, F_{Y^1}, F_{Y^2}, F_{V^1}, F_{V^2}, F_{V^3}, F_{\eta^1}, F_{\eta^2}, F_{\eta^{1+}}, F_{\eta^{2+}}, F_{\mu^1}, F_{\mu^2}, F_{\mu^3}, F_{v}, F_{\theta^1}, F_{\theta^2}, F_{\lambda^1}, F_{\lambda^2}\big)$, *with the specific components of Fgiven by the functional terms preceding the multiplication signs in (41), respectively* (see Appendix *for complete formulation). The term* $\langle \cdot, \cdot \rangle$ *denotes the inner product in N-dimensional Euclidean space.*

The complete formulation of the variational inequality (41) and the proof of existence of its solution, the algorithm used, and the closed form solutions of the supply chain prices are provided in Appendix.

3. Computational studies

In this computational study, we analyze the effects of relationship parameters in the multiperiod supply chain network model on equilibrium product flows, prices, relationship levels, supply chain profitability, and risk. The supply chain model consists of 2 suppliers ($I = 2$); 2 manufacturers ($J = 2$); 2 retailers ($K = 2$); and 5 time periods ($T = 5$). The algorithm that is proposed is the modified projection method (see Appendix).The functions and parameters are specified in Table 6. The parameters not specified in Table 6 are assumed to be zero.

Suppliers mainly incur production costs and inventory costs. Manufacturers incur production costs, inventory costs, transaction/transportation costs, costs related to relationship maintenance and improvement, expected costs for supply disruptions, and expected costs for opportunism risk. Retailers incur handling cost, inventory costs, transaction/transportation costs, costs related to relationship maintenance and improvement, expected costs for supply disruptions, and expected costs for opportunism risk.

Fixed production and handling costs are assumed sunk costs and are not considered. In the production and handling cost functions, the linear terms reflect the unit costs and the minor quadratic terms reflect the fact that the marginal production increases as the production quantity approaches the maximum capacity.

The inverse demand (price) function of the consumers is linear decreasing with the total supply in the market. The modified projection method (see Appendix) was implemented in Matlab to solve these numerical examples and a_τ in the Euler method is set to 0.02. The parameter for convergence is set to 0.00001.

In our analysis we focus on the following questions: First, how do the evolving relationships affect the profitability and risks of supply chain firms as well as the prices and demands of the product in the market? Second, how do the relationships with the upstream supply chain firms affect the relationships with the downstream firms, and how these relationships influence the profitability and risks of the supply chain firms? Third, how do the supply disruption risks interact with the opportunism risks through supply chain relationships, and how these risks influence the profitability of the firms? Questions 1 to 3 represent our Cases 1 to 3.

3.1. Discussion of results

Functions and parameters used in Case 1 are specified in Table 6. The results of Case 1 are shown in Table 7.

We can see that the relationships are being established over time where the levels of relationships between the supply chain firms increase from 0 in period 1 to 0.275 in period 5. The direct impact of stronger relationships is that the average expected costs associated with opportunism risks, $\frac{r_{it}^{2j*}}{q_{jt}^{i*}}$ s and $\frac{r_{jt}^{2k*}}{q_{kt}^{j*}}$ s, are reduced from 2 to 1.451. The decline of the opportunism risks also leads to the reduction of the total risks. For example, the average total risks, $\frac{r_{it}^{1j*}+r_{it}^{2j*}}{q_{jt}^{i*}}$ s and $\frac{r_{jt}^{1k*}+r_{jt}^{2k*}}{q_{kt}^{j*}}$ s, decrease from 2.250 to 1.722. Due to the reduction of the risks, we observe an increase of product flows in the supply chain over time. For example, the transaction quantities, q_{jt}^{i*} s and q_{kt}^{j*} s, increase from 2.613 in period 1 to 2.835 in period 5. Consequently, the total supply to the consumers increases by 8.5% from 10.452 to 11.340, and the product price for the consumers decreased from 19.548 to 18.660.

The results of Case 1 we also observe that the profits of the suppliers, the manufacturers, and the retailers, P_i^*, P_j^*, and P_k^*, all increase as the relationships improve over time. In particular, P_i^*, P_j^*, and P_k^*, increase from 1.366, 2.618 and 2.618 in period 1 to 1.609, 3.066 and 3.066 in period 5, respectively.

It also worth noting that all the inter-period inventory levels are equal to zero. The reason is that since in general the cost of the product becomes lower over time due to the improved relationship and reduced risks, the products carried in the inventory from previous periods are almost always costlier than the products made in the current periods. In addition, since this paper focuses on how the relationships influence network related risks we assume deterministic demand, and do not consider safety inventory.

In summary, the analysis of Case 1 shows that the relationships between the firms will improve over time, which will reduce the opportunism risks and the total risks in the supply chain. The

Table 6
Functions for computational study.

Notation	Definition
$f_{it}(y_{it}) = 5y_{it} + 0.05y_{it}^2$	Production costs faced by supplier i at period t
$f_{jt}(y_{jt}) = 5y_{jt} + 0.05y_{jt}^2$	Production costs faced by manufacturer j at period t
$f_{kt}(q_{kt}) = 5q_{kt} + 0.05q_{kt}^2$	Handling costs faced by retailer k at period t
$h_{it}(v_{it}) = 1v_{it}$	Inventory costs faced by supplier i at period t
$h_{jt}(v_{jt}) = 1v_{jt}$	Inventory costs faced by manufacturer j at period t
$h_{kt}(v_{kt}) = 1v_{kt}$	Inventory costs faced by retailer k at period t
$c_{it}^j\left(q_{jt}^i\right) = 1q_{jt}^i$	Transportation/Transaction costs faced by manufacturer j transacting with supplier i at period t
$c_{jt}^k\left(q_{kt}^j\right) = 1q_{kt}^j$	Transportation/Transaction costs faced by retailer k transacting with manufacturer j at period t
$b_{it}^j\left(\eta_{jt}^i\right) = 10\left(\eta_{jt}^i\right)^2$	Cost for maintaining social relationship associated with manufacturer j transacting with supplier i in time period t
$b_{jt}^k\left(\eta_{kt}^j\right) = 10\left(\eta_{kt}^j\right)^2$	Cost for maintaining social relationship associated with retailer k transacting with manufacturer j in time period t
$b_{it}^{j+}\left(\eta_{jt}^{i+}\right) = 10\left(\eta_{jt}^{i+}\right)^2$	Cost for improving social relationship associated with manufacturer j transacting with supplier i in time period t
$b_{jt}^{k+}\left(\eta_{kt}^{j+}\right) = 10\left(\eta_{kt}^{j+}\right)^2$	Cost for improving social relationship associated with retailer k transacting with manufacturer j in time period t
$\rho_{kt}(d_t) = 30 - d_t$	Inverse demand functions for consumers at period t
$\omega_{it}^{1j} = 4$	Unit cost of manufacturer j if supply disruption risk with supplier i occurs at period t
$\omega_{jt}^{1k} = 4$	Unit cost of retailer k if supply disruption risk with manufacturer j occurs at period t
$\omega_{it}^{2j} = 10$	Unit cost of manufacturer j if opportunism risk with supplier i occurs at period t
$\omega_{jt}^{2k} = 10$	Unit cost of retailer k if opportunism risk with manufacturer j occurs at period t
$\phi_{jt}^i = 0.2$	Probability of opportunism risk faced by manufacturer j transacting with supplier i when relationship level is zero
$\phi_{kt}^j = 0.2$	Probability of opportunism risk faced by retailer k transacting with manufacturer j when relationship level is zero
$PDF_j^i = \frac{1}{20}e^{\frac{-1}{20}q_{ij}}$	Probability density function of supply disruption between supplier i and manufacturer j
$PDF_k^j = \frac{1}{20}e^{\frac{-1}{20}q_{jk}}$	Probability density function of supply disruption between manufacturer j and retailer k

Table 7
Computational study results (Case 1).

		t = 1		t = 2		t = 3		t = 4		t = 5	
q_{jt}^{i*}	$i = 1; j = 1, 2$	2.613	2.613	2.752	2.752	2.806	2.806	2.828	2.828	2.835	2.835
	$i = 2; j = 1, 2$	2.613	2.613	2.752	2.752	2.806	2.806	2.828	2.828	2.835	2.835
q_{kt}^{j*}	$j = 1; k = 1, 2$	2.613	2.613	2.752	2.752	2.806	2.806	2.828	2.828	2.835	2.835
	$j = 2; k = 1, 2$	2.613	2.613	2.752	2.752	2.806	2.806	2.828	2.828	2.835	2.835
η_{jt}^{i*}	$i = 1; j = 1, 2$	0.000	0.000	0.171	0.171	0.239	0.239	0.266	0.266	0.275	0.275
	$i = 2; j = 1, 2$	0.000	0.000	0.171	0.171	0.239	0.239	0.266	0.266	0.275	0.275
η_{kt}^{j*}	$j = 1; k = 1, 2$	0.000	0.000	0.171	0.171	0.239	0.239	0.266	0.266	0.275	0.275
	$j = 2; k = 1, 2$	0.000	0.000	0.171	0.171	0.239	0.239	0.266	0.266	0.275	0.275
$\frac{r_{it}^{1j*}}{q_{jt}^{i*}}$	$i = 1; j = 1, 2$	0.250	0.250	0.263	0.263	0.268	0.268	0.270	0.270	0.271	0.271
	$i = 2; j = 1, 2$	0.250	0.250	0.263	0.263	0.268	0.268	0.270	0.270	0.271	0.271
$\frac{r_{it}^{2j*}}{q_{jt}^{i*}}$	$i = 1; j = 1, 2$	2.000	2.000	1.657	1.657	1.522	1.522	1.469	1.469	1.451	1.451
	$i = 2; j = 1, 2$	2.000	2.000	1.657	1.657	1.522	1.522	1.469	1.469	1.451	1.451
$\frac{r_{it}^{2j*}+r_{it}^{1j*}}{q_{jt}^{i*}}$	$i = 1; j = 1, 2$	2.250	2.250	1.920	1.920	1.790	1.790	1.739	1.739	1.722	1.722
	$i = 2; j = 1, 2$	2.250	2.250	1.920	1.920	1.790	1.790	1.739	1.739	1.722	1.722
$\frac{r_{jt}^{1k*}}{q_{kt}^{j*}}$	$j = 1; k = 1, 2$	0.250	0.250	0.263	0.263	0.268	0.268	0.270	0.270	0.271	0.271
	$j = 2; k = 1, 2$	0.250	0.250	0.263	0.263	0.268	0.268	0.270	0.270	0.271	0.271
$\frac{r_{jt}^{2k*}}{q_{kt}^{j*}}$	$j = 1; k = 1, 2$	2.000	2.000	1.657	1.657	1.522	1.522	1.469	1.469	1.451	1.451
	$j = 2; k = 1, 2$	2.000	2.000	1.657	1.657	1.522	1.522	1.469	1.469	1.451	1.451
$\frac{r_{jt}^{2k*}+r_{jt}^{1k*}}{q_{kt}^{j*}}$	$j = 1; k = 1, 2$	2.250	2.250	1.920	1.920	1.790	1.790	1.739	1.739	1.722	1.722
	$j = 2; k = 1, 2$	2.250	2.250	1.920	1.920	1.790	1.790	1.739	1.739	1.722	1.722
P_i^*	$i = 1, 2$	1.366	1.366	1.515	1.515	1.576	1.576	1.600	1.600	1.609	1.609
P_j^*	$j = 1, 2$	2.618	2.618	2.892	2.892	3.006	3.006	3.051	3.051	3.066	3.066
P_k^*	$k = 1, 2$	2.618	2.618	2.902	2.902	3.015	3.015	3.059	3.059	3.075	3.075
ρ_{3t}^*		19.548		18.993		18.774		18.689		18.660	
d_t^*		10.452		11.007		11.226		11.311		11.340	

evolving relationship will increase the product flows, and the total supply to consumers, and will reduce the price of the product for consumers. The improving relationships will also increase the profits of the suppliers, the manufacturers, and the retailers.

In Case 2, we focus on the second question, that is, how the relationships with upstream supply chain firms affect the relationships with downstream supply chain firms? We use the same function parameters as those in Case 1 except that we now assume that manufacturer 1 has lower cost parameter associated with maintaining the relationships with suppliers than manufacturer 2. In particular, we let $b_{it}^1 = 9(\eta_{it}^1)^2$ and $b_{it}^2 = 11(\eta_{it}^2)^2$. The results of Case 2 are shown in Table 8.

First, we can observe that the relationship levels between manufacturer 2 and the suppliers, η_{1t}^{i*} s, grow faster than the relationship levels between manufacturer and the suppliers,η_{2t}^{i*}. For example,η_{1t}^{i*} s increase from 0 in period 1 to 0.332 in period 5 while η_{2t}^{i*} s increase from 0 in period 1 to 0.227 in period 2. In addition, the transaction quantities between the suppliers and manufacturer 1 increase from 2.613 to 3.122 while the transaction quantities between the suppliers and manufacturer 2 decrease from 2.613 to 2.552. Such results are not surprising since manufacturer 1 has lower cost parameter in the relationship cost functions than manufacturer 2 does. Moreover, although all the cost functions associated with the relationships between the manufacturers and the

Table 8
Computational study results (Case 2).

		t = 1		t = 2		t = 3		t = 4		t = 5	
q_{jt}^{i*}	$i = 1; j = 1, 2$	2.613	2.613	2.871	2.634	3.014	2.602	3.090	2.570	3.122	2.552
	$i = 2; j = 1, 2$	2.613	2.613	2.871	2.634	3.014	2.602	3.090	2.570	3.122	2.552
q_{kt}^{j*}	$j = 1; k = 1, 2$	2.613	2.613	2.871	2.871	3.014	3.014	3.090	3.090	3.123	3.123
	$j = 2; k = 1, 2$	2.613	2.613	2.634	2.634	2.602	2.602	2.570	2.570	2.552	2.552
η_{jt}^{i*}	$i = 1; j = 1, 2$	0.000	0.000	0.196	0.151	0.281	0.204	0.318	0.222	0.332	0.227
	$i = 2; j = 1, 2$	0.000	0.000	0.196	0.151	0.281	0.204	0.318	0.222	0.332	0.227
η_{kt}^{j*}	$j = 1; k = 1, 2$	0.000	0.000	0.181	0.181	0.257	0.257	0.289	0.289	0.301	0.301
	$j = 2; k = 1, 2$	0.000	0.000	0.162	0.162	0.221	0.221	0.242	0.242	0.249	0.249
$\frac{r_{it}^{1j*}}{q_{jt}^{i*}}$	$i = 1; j = 1, 2$	0.250	0.250	0.274	0.252	0.287	0.249	0.294	0.246	0.297	0.245
	$i = 2; j = 1, 2$	0.250	0.250	0.274	0.252	0.287	0.249	0.294	0.246	0.297	0.245
$\frac{r_{it}^{2j*}}{q_{jt}^{i*}}$	$i = 1; j = 1, 2$	2.000	2.000	1.608	1.698	1.438	1.592	1.364	1.556	1.337	1.545
	$i = 2; j = 1, 2$	2.000	2.000	1.608	1.698	1.438	1.592	1.364	1.556	1.337	1.545
$\frac{r_{it}^{2j*}+r_{it}^{1j*}}{q_{jt}^{i*}}$	$i = 1; j = 1, 2$	2.250	2.250	1.882	1.951	1.725	1.841	1.658	1.802	1.633	1.790
	$i = 2; j = 1, 2$	2.250	2.250	1.882	1.951	1.725	1.841	1.658	1.802	1.633	1.790
$\frac{r_{jt}^{1k*}}{q_{kt}^{j*}}$	$j = 1; k = 1, 2$	0.250	0.250	0.274	0.274	0.287	0.287	0.294	0.294	0.297	0.297
	$j = 2; k = 1, 2$	0.250	0.250	0.252	0.252	0.249	0.249	0.246	0.246	0.245	0.245
$\frac{r_{jt}^{2k*}}{q_{kt}^{j*}}$	$j = 1; k = 1, 2$	2.000	2.000	1.637	1.637	1.486	1.486	1.422	1.422	1.399	1.399
	$j = 2; k = 1, 2$	2.000	2.000	1.677	1.677	1.557	1.557	1.515	1.515	1.503	1.503
$\frac{r_{jt}^{2k*}+r_{jt}^{1k*}}{q_{kt}^{j*}}$	$j = 1; k = 1, 2$	2.250	2.250	1.911	1.911	1.772	1.772	1.716	1.716	1.695	1.695
	$j = 2; k = 1, 2$	2.250	2.250	1.929	1.929	1.807	1.807	1.762	1.762	1.747	1.747
P_i^*	$i = 1, 2$	1.366	1.366	1.515	1.515	1.577	1.577	1.601	1.601	1.610	1.610
P_j^*	$j = 1, 2$	2.618	2.618	3.147	2.659	3.461	2.596	3.638	2.529	3.722	2.486
P_k^*	$k = 1, 2$	2.618	2.618	2.900	2.900	3.020	3.020	3.069	3.069	3.087	3.087
ρ_{3t}^*		19.548		18.990		18.768		18.681		18.651	
d_t^*		10.452		11.010		11.232		11.319		11.349	

retailers are identical, the relationships of the retailers with manufacturer 1 still improves faster than those with manufacturer 2. For example,η_{kt}^{1*} s increase from 0 in period 1 to 0.301 in period 5 while η_{kt}^{2*}s increase from 0 in period 1 to 0.249 in period 2. Such results indicate that if a supply chain company has better relationships with upstream firms then the downstream firms in the supply chain will be more willing to improve relationships with the company. The reason is that the company with better relationships with upstream supply chain firms will have lower average cost associated with total risks. For example, for the transactions between the suppliers and manufacturer 1, the average cost related to the total risk is 1.882 in period 2 and 1.663 in period 5 while for the transactions between the suppliers and manufacturer 2, the average cost related to the total risk is 1.951 in period 2 and 1.790 in period 5. Such cost advantages make manufacturer 1 more competitive than manufacturer 2 so that the retailers are willing to purchase more products from manufacturer 1, and develop better relationships with it. The better relationship between retailers and manufacturer 1 also lowered the average cost related to the total risk in their transactions. For example, for the transactions between the retailers and manufacturer 1, the average cost related to the total risk is 1.911 in period 2 and 1.695 in period 5 while for the transactions between the retailers and manufacturer 2, the average cost related to the total risk is 1.929 in period 2 and 1.747 in period 5.

Consequently, the profit of manufacturer 1, $\left(P_j^*,\ j=1\right)$, increases from 2.618 to 3.722 while the profit of manufacturer 2, $\left(P_j^*,\ j=2\right)$, decreases from 2.618 to 2.486.

The analysis of Case 2 shows that if a manufacturer is more cost effective in maintaining better relationships with its upstream suppliers the firm will have lower costs related to supply risks, and become more competitive than other manufacturers. Such competitive advantage will also make its downstream supply chain partners more willing to establish stronger relationships with the manufacturer. In summary, better relationships with upstream supply chain firms will induce better relationships with downstream supply chain partners. Moreover, the manufacturer with better relationships will also have higher profits than its competitors.

Case 3 focuses on the third question, that is, how the supply disruption risks interact with the opportunism risks through supply chain relationships, and how these risks influence the profitability of the supply chain firms. We use the same function parameters as those in Case 1 except that we now assume that manufacturer 2 is less reliable and has higher supply disruption rate. In particular, we let $PDF_k^j = \frac{1}{15}e^{\frac{-1}{15}q_{jk}}\ j=1,\ k=1,2$. Note that the probability density function implies that the average supply quantity without any disruption is 15. The results of Case 3 are shown in Table 9.

First, we can see that since manufacturer 1 is more reliable the product flows from manufacturer 1 to the retailers are higher than those from manufacturer 2. For example, q_{kt}^{1*} s increases from 2.728 in period 1 and to 3.225 in period 5, and q_{kt}^{2*} s decrease from 2.469 in period 1 and to 2.415 in period 5. Because of higher transaction quantities with manufacturer 1, the retailers are willing to develop stronger relationship with manufacturer 1. For instance, the relationships between the retailers and manufacturer 1,η_{kt}^{1*} s are equal to 0.189 in period 1 and equal to 0.311 in period 5 while the retailer's relationships with manufacturer 2,η_{kt}^{2*} s are equal to 0.152 in period 1 and equal to 0.235 in period 5. Due to the higher sales to the retailers, manufacturer 1 also order more materials from the suppliers and develop stronger relationships with the suppliers. From Table 9, we can observe that $q_{1t}^{i*} \geqslant q_{2t}^{i*}$, and $\eta_{1t}^{i*} \geqslant \eta_{2t}^{i*}$ over all periods.

Since manufacturer 1 has strong relationships with suppliers and retailers the opportunism risks related to manufacturer 1 are lower than those related to manufacturer 2. For example, $fracr_{it}^{2j*}q_{jt}^{i*}s(j=1)$ are equal to 1.622 in period 2 and equal to 1.378 in period 5 while $\frac{r_{jt}^{2j*}}{q_{jt}^{i*}s}(j=2)$ are equal to 1.696 in period 2 and equal to 1.530 in period 5. For another instance, $\frac{r_{jt}^{2k*}}{q_{kt}^{j*}s}(j=1)$ are equal to 1.622 in period 2 and equal to 1.378 in period 5 while $\frac{r_{jt}^{2k*}}{q_{kt}^{j*}s}(j=2)$ are equal to 1.696 in period 2 and equal to 1.530 in period 5. On the other hand, since manufacturer 1 has higher transaction quantities with suppliers and retailers the disruption risks related to manufacturer 1 are also higher than those related to

Table 9
Computational study results (Case 3).

		t = 1		t = 2		t = 3		t = 4		t = 5	
q_{jt}^{i*}	i = 1; j = 1, 2	2.728	2.469	3.003	2.470	3.136	2.447	3.199	2.426	3.225	2.415
	i = 2; j = 1, 2	2.728	2.469	3.003	2.470	3.136	2.447	3.199	2.426	3.225	2.415
q_{kt}^{j*}	j = 1; k = 1, 2	2.728	2.728	3.003	3.003	3.136	3.136	3.199	3.199	3.225	3.225
	j = 2; k = 1, 2	2.469	2.469	2.470	2.470	2.447	2.447	2.426	2.426	2.415	2.415
η_{jt}^{i*}	i = 1; j = 1, 2	0.000	0.000	0.189	0.152	0.267	0.208	0.299	0.229	0.311	0.235
	i = 2; j = 1, 2	0.000	0.000	0.189	0.152	0.267	0.208	0.299	0.229	0.311	0.235
η_{kt}^{j*}	j = 1; k = 1, 2	0.000	0.000	0.189	0.189	0.267	0.267	0.299	0.299	0.311	0.311
	j = 2; k = 1, 2	0.000	0.000	0.152	0.152	0.208	0.208	0.229	0.229	0.235	0.235
$\frac{r_{jt}^{1j*}}{q_{jt}^{i*}}$	i = 1; j = 1, 2	0.261	0.237	0.286	0.237	0.298	0.235	0.303	0.233	0.306	0.232
	i = 2; j = 1, 2	0.261	0.237	0.286	0.237	0.298	0.235	0.303	0.233	0.306	0.232
$\frac{r_{jt}^{2j*}}{q_{jt}^{i*}}$	i = 1; j = 1, 2	2.000	2.000	1.622	1.696	1.465	1.583	1.401	1.543	1.378	1.530
	i = 2; j = 1, 2	2.000	2.000	1.622	1.696	1.465	1.583	1.401	1.543	1.378	1.530
$\frac{r_{jt}^{2j*}+r_{jt}^{1j*}}{q_{jt}^{i*}}$	i = 1; j = 1, 2	2.261	2.237	1.907	1.934	1.763	1.818	1.705	1.776	1.684	1.762
	i = 2; j = 1, 2	2.261	2.237	1.907	1.934	1.763	1.818	1.705	1.776	1.684	1.762
$\frac{r_{jt}^{1k*}}{q_{kt}^{j*}}$	j = 1; k = 1, 2	0.261	0.261	0.286	0.286	0.298	0.298	0.304	0.304	0.306	0.306
	j = 2; k = 1, 2	0.312	0.312	0.312	0.312	0.309	0.309	0.307	0.307	0.305	0.305
$\frac{r_{jt}^{2k*}}{q_{kt}^{j*}}$	j = 1; k = 1, 2	2.000	2.000	1.622	1.622	1.465	1.465	1.401	1.401	1.378	1.378
	j = 2; k = 1, 2	2.000	2.000	1.696	1.696	1.583	1.583	1.543	1.543	1.530	1.530
$\frac{r_{jt}^{2k*}+r_{jt}^{1k*}}{q_{kt}^{j*}}$	j = 1; k = 1, 2	2.261	2.261	1.907	1.907	1.763	1.763	1.705	1.705	1.684	1.684
	j = 2; k = 1, 2	2.312	2.312	2.008	2.008	1.892	1.892	1.849	1.849	1.835	1.835
P_i^*	i = 1, 2	1.351	1.351	1.498	1.498	1.558	1.558	1.582	1.582	1.590	1.590
P_j^*	j = 1, 2	2.848	2.343	3.437	2.345	3.745	2.296	3.901	2.251	3.962	2.231
P_k^*	k = 1, 2	2.760	2.760	3.044	3.044	3.161	3.161	3.207	3.207	3.224	3.224
ρ_{3t}^*		19.605		19.054		18.835		18.750		18.720	
d_t^*		10.395		10.946		11.165		11.250		11.280	

manufacturer 2. For example, $\frac{r_{it}^{1j*}}{q_{jt}^{i*}s}(j=1)$ are equal to 0.261 in period 1 and equal to 0.306 in period 5 while $\frac{r_{it}^{2j*}}{q_{jt}^{i*}s}(j=2)$ are equal to 0.237 in period 1 and equal to 0.232 in period 5. For another instance, $\frac{r_{jt}^{2k*}}{q_{kt}^{j*}s}(j=1)$ are equal to 0.261 in period 2 and equal to 0.306 in period 5 while $\frac{r_{jt}^{2k*}}{q_{kt}^{j*}s}(j=2)$ are equal to 0.237 in period 2 and equal to 0.232 in period 5. However, the the average over all risks related to manufacturer 1 are consistently lower than those related to manufacturer 2.

Finally, in Case 3 we can see that the profit of manufacturer 1 increases from 2.848 in period 1 to 3.962 in period 5 while the profit of manufacturer 2 decreases from 2.343 to 2.231.

In summary, our main finding in Case 3 is that the manufacturer with more reliable productions (manufacturer 1) will have better relationships with supply chain partners, higher transaction quantities, higher expected profit, and lower average opportunism risks and total risks. However, the manufacturer with more reliable productions will have higher supply disruption risks.

4. Conclusions and insights

In this paper, we describes the effects of relationships in a multiperiod supply chain network consisting of suppliers, manufacturers, and retailers. Decision-makers in a given tier of the network can decide on the relationship levels that they want to achieve with decision-makers associated with the other tiers of the network in other to maximize profit and minimize risks (opportunism and disruption risks). Establishing/maintaining a certain relationship level induces some costs, but may also lower the risk. We explicitly describe the role of relationships in influencing risk. Both the risk functions and the relationship cost functions are allowed to depend on the relationship levels present and past.

We address the following questions and capture the timing and location of investments in social relationships, and their impact on product flows, price, risk and profit: (1) how do the evolving relationships affect the profitability and risks of supply chain firms as well as the prices and demands of the product in the market? Our analysis summarized Case 1, indicate that as the relationships between the firms improve over time, the opportunism risks and the total risks in the supply chain which will reduce. The evolving relationship will increase the product flows, and the total supply to consumers, and will reduce the price of the product in for consumers. The improving relationships will also increase the profits for the supply chain members (suppliers, manufacturers,and retailers). (2) how do the relationships with the upstream supply chain firms affect the relationships with the downstream firms, and how these relationships influence the profitability and risks of the supply chain firms? The answer to this questions is summarized in Case 2. Our results indicate that better relationships with upstream supply chain firms will induce better relationships with downstream supply chain partners. Moreover, the manufacturer with better relationships will also have higher profits than its competitors. (3) how do the supply disruption risks interact with the opportunism risks through supply chain relationships, and how these risks influence the profitability of the firms? Our main finding in Case 3 shows that the manufacturer with more reliable productions (manufacturer 1) will have better relationships with supply chain partners, higher transaction quantities, higher expected profit, and lower average opportunism risks and total risks. However, the manufacturer with more reliable productions will have higher likelihood of supply disruption risks due to the increase in transactions.

Results of this paper highlight the importance of considering the time dimension in analyzing the impact of relationship levels in a supply chain context. When considering risk issues it is important to the firm to create a portfolio of relationships. Strong supply chain relationships enable firms to react to changes in the market, improve forecasting and create supply chain visibility, which lead to improve profit margins (Flint, 2004). Moreover, multiple relationships can help companies deal with the negative consequences related to dependence on supply chain partners. Thus, supply chain vulnerability can be minimized as an exposure to serious disturbance is reduced. Companies will be able to deal with disturbance arising from risks within the supply chain as well as risks external to the supply chain when there is an optimal interaction and cooperation between the entities along the chain. (Spekman and Davis, 2004). These results are also supported by our economic model where we demonstrated that a higher level of relationship lowers supply chain risk and uncertainty while reducing supply chain prices and increasing supply chain transactions.

Finally, it is important for companies to analyze the impact of upfront investment in their supply chain social relationship. Our results indicate that early investments in relationships would make the supply chain more profitable. The longest the relationship is, the highest is it impact to the reduction of future cost and risk in a competitive multiperiod supply chain environment. Risk management in supply chain should be based on clear performance requirements, optimal time and level of investment in supply chain relationships, and on process alignment and cooperation within and between the entities in the supply chain. Therefore, the time and location of investments in a portfolio of relationship will determine how risky is supply chain network.

Future research will include the extension of this framework to the international arena and the incorporation of other criteria. Future research may also focus on the study of type of relationship investments are required in each time period and stages of the supply chain network. It is important to know to what extent do the relationships between elements in supply chain differ depending on the stage of the alliance and on aspects of the costs and benefits of the relationship. How do the combination of these elements play in each time period and stage of the supply chain? These questions await future study. We feel we have only scratched the surface and look forward to future studies that will help researchers better conceptualize and theorize specific aspect of supply chain relationship, vulnerability and risk management.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2011.03.044.

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