Exercises for Dynamic Macro

June 6, 2019

Exercises marked with an asterisk are done during class session. All other exercises should be done at home and handed in. For all programming exercises, please hand in MATLAB .m-files. For exercise 1 and 2 please provide texed solutions as pdf files. It is ok to form groups of 3 in handing in solutions. Should the template contain different parameter values than the exercise, please follow the template. Templates will be published block by block, because the templates of the second block contain partly solutions from Block 1 etc.

Block 1

Exercise 1. *Proof the following*
If T is a contraction then $T^n x$ is a Cauchy sequence.

Exercise 2. *Consider the growth model with only capital where productivity z is stochastic and $\ln z$ follows an AR-1 process such that $E(\log z'|z) = \mu(1-\rho) + \rho z$. Assume $\delta = 1$. Show that the value function can be written as $V(k, z) = A + B \ln k + C \ln z$.*

Exercise 3. *(stochastic growth model)*
Extend the cake eating problem, such that the resource constraint is $K' < zK^{\alpha}$ with $\alpha = 0.5$ (stochastic growth model). Consider $z \in \{0.9, 1.1\}$ where the transition probabilities are given by $\begin{bmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{bmatrix}$, i.e., with probability 87.5% the economy remains in one productivity state.

1. *Write a code that solves the model on a grid. Plot consumption as a function of (K, z).*

Exercise 4. ** Use the stochastic growth model with as in Exercise 3.*

1. *Define a 10 point grid for $\log k$.*
2. *Use a spline interpolation to solve for the value function by VFI, starting with $V = 0$ as initial guess, save running times.*
3. *Compare to the true value function and policy function at the grid points and note average absolute difference (on the original grid). Calculate the squared distance evaluating the policy functions at off-grid points for a 640 point refinement of the grid.*
4. *Run a sequence of on-grid value function iterations with $N = 80$, 160, 320, 640 points. Compare average absolute difference to the true model and running times with the off-grid search code.*

Exercise 5. ** Redo Exercise 4.2. but use Broyden's rootfinding procedure instead of VFI - compare running times and numbers of iterations.*

Block 2

Exercise 6. *Solve the stochastic growth model (Exercise 3)using the Policy Function Iteration (off-grid, linear interpolation). Compare running times for a grid of* $N = 40, 80, 160$ *points to VFI.*

Exercise 7. *Solve the stochastic growth model (Exercise 3)using the multi-grid Value Function Iteration (off-grid, linear interpolation) starting with* 20 *grid points. Compare running times for a grid of* $N = 40, 80, 160$ *points to single-grid VFI.*

Exercise 8. *Solve the stochastic growth model (Exercise 3) using the collocation method, using a linear interpolant for the expected future marginal utility. Compare running time and precision to off-grid VFI.*

Exercise 9. *Calculate conditional variances* $E\left(z_{t+1}^2|z_t = z_i\right) - E\left(z_{t+1}|z_t = z_i\right)^2$*, and unconditional kurtosis for the above three state Markov Chain. How do these compare to a normally distributed AR-1 process.*

Exercise 10. *(Savings problem)*
Assume household income can be either 10/9 or 1/9. The probability to move from high income to low income is 4/90 and the probability to move from low income to high income is 2/5. The household can borrow and lend at a 3/90 interest rate and has log utility in consumption. The borrowing constraint is -10/3. Its discount factor is 0.95. Solve the consumption savings model using the endogenous grid method and plot value and policy functions and compare time to computes to off-grid VFI! Use a 100-point log-grid for asset holdings between the borrowing limit and 500*.*

Exercise 11. ** A Bewley Model of Money. Re use the model from exercise 10. Now assume that $r = 0$, the asset they save in bears no interest (money). Further assume the household cannot borrow. Set the maximum asset holdings to 1. Solve the model with EGM, then*

1. *Simulate an agent over T=100,000 periods of time and calculate the average asset holding of the agent in periods $t \geq 10,000$.*

2. *Create a transition matrix from the policy functions. Use this to calculate the ergodic distribution of the model. Calculate the expected asset holdings from that ergodic distribution.*

3. *Compare the histograms of the two distributions.*

Exercise 12. ** Extend the optimal savings problem (Ex. 10) to calculate the equilibrium interest rate. Use a 100-point log-grid for asset holdings between the borrowing limit and a maximum asset holding of 6. Use Brent's method to find the equilibrium, such that aggregate asset demand is zero.*

Exercise 13. ** Same as the last exercise (Ex. 12) but now assume that the asset is physical capital and households have a CRRA utility function with risk aversion 4. The production function is*

$$F(K, N) = K^{\alpha} N^{1-\alpha}.$$

Let $\delta = .1$, $\beta = 0.95$, $\alpha = 0.36$.

Exercise 14. ** Same as above, but introduce a labor income tax $\tau = .2$ to the model. Assume that this tax is rebated lump sum to the households. The lump-sum payments adjust in order to balance the government's budget. How does the equilibrium interest rate change?*

Block 3

Exercise 15. *Solve the stochastic growth model using parameterized expectations.*

Exercise 16. *Solve the stochastic growth model using first order perturbation. For this purpose, first write a function that calculates the Euler equation errors, errors from capital accumulation, and the law of motion for productivity. Define log-consumption as control and log capital and log productivity as states.*

Exercise 17. ** Take an Aiyagari setup: Aggregate output is given by*

$$F(Z, K, N) = ZK^{\alpha}N^{1-\alpha}.$$

Aggregate productivity, Z, can be $.99$ or 1.01, productivity changes with probability $1/8$. Other parameters are: depreciation, $\delta = 1/40$; discount factor, $\beta = 0.99$; capital share, $\alpha = 0.36$; risk aversion, $\gamma = 1$.

Agents can be employed (productivity 10/9) or unemployed. If unemployed they move with 2/5 probability to employment and if employed they loose their job with prob. 4/90. When unemployed, they receive 15% of the employed wage financed by a corresponding labor tax.

1. *Solve for the steady state without aggregate risk.*
2. *Solve for the Krusell and Smith-equilibrium.*
3. *Solve using Reiters's method.*