International Economics (Master) - Problem Set 2b

<u>General remarks:</u>
There are two different sets of questions: review questions and exercises. Review questions are for self-study at home. They serve to highlight what you (should) already have learned in the lecture. The review questions will, therefore, not be discussed in class. The exercises, instead, will. Please try and solve the exercises on your own. And ask questions if there are any.

Submit your solutions on ecampus by May 27, 6 pm.

There is example code online (for the exercises with two states). Modify this according to your needs. When we discuss the problem set, please have your code ready to show. You could be asked to explain it.

Review questions - slides Ch02

- Be able to define the terms "trade balance" and "current account."

- Be able to define the term "permanent income."

- In the intertemporal model of the current account, what drives the trade balance and the current account?

Exercises

The exercise is meant to help you

- practice setting up the Bellman equation

- practice setting up the Lagrangian

- understand what a *state-contingent* choice is, and

- see why the expectations terms appear in the first-order conditions on the slides in the way they appear.

1. The intertemporal model of the current account (pencil and paper part)

Consider the following model of an open economy. The economy is populated by a representative household. The household lives forever. Letting $t = 0, 1, 2, ...$ denote time, the household has an exogenous stream of income. In each period, $y_t \in \{y_L, y_H\}$, with $0 < y_L < y_H$. Let $prob(y_{t+1}|y_t)$ denote the probability, that income will be y_{t+1} tomorrow if is y_t today. Let y_t follow a first-order Markov process (so that the transition probabilities for income tomorrow depend only on today's income state and on nothing else).

The household has access to international financial markets. Let b_t denote savings by the household (in a bond). Let $r > 0$ be the fixed real rate of interest. Suppose that there is a borrowing constraint, such that $b_t \geq \underline{b}$ in all periods t.

The household maximizes expected lifetime utility

$$\max_{\{c_t, b_t\}_{t=0}^{\infty}} \quad E_0\left\{ \sum_{t=0}^{\infty} \beta^t \left[c_t^{1-\sigma}/(1-\sigma) \right] \right\}, \ \sigma > 0, \beta \in (0, 1/(1+r))$$

subject to the period budget constraint

$$c_t + b_t = y_t + b_{t-1}(1 + r).$$

1. Explain what we mean by a "representative household."

2. What is the lowest-possible value for constraint $\underline{b}$? That is, give a formula for the "natural borrowing constraint."

3. What is/are the state variables of the household's problem?

    — in the following, adopt the precise notation that we used in class —

4. Using the notation applied in class, what is s_t, what is s^t?

5. Set up the Bellman equation.

6. Set up the Lagrangian for the household problem in precise notation.

7. Derive the first-order conditions.

8. Suppose that the borrowing constraint does not bind ever. Derive the consumption Euler equation.

9. Define the terms "trade balance" and "current account," both verbally and in terms of equations.

The purpose of the next exercise is to solve the intertemporal model of the current account of exercise 1 on a computer using one method. The exercise will help you

- practice value function iteration

- see how the income process shapes consumption and saving behavior.

2. The intertemporal model of the current account solved on a computer (I).
Consider the same model as in Exercise 1. Let $r = 0.02$, $\beta = 0.975$, $\sigma = 4$. Let $y_L = 0.85$ and $y_H = 1.15$. Let $prob(y'_L|y_L) = prob(y'_H|y_H) = 0.95$. Let $\underline{b} = -2$.

1. What are $prob(y'_H|y_L)$ and $prob(y'_L|y_H)$?

2. Choices on a grid. Specify a grid for assets such that $b \in [\underline{b}, 20]$. Choose an evenly-spaced grid with 2000 grid points.

3. Assume that current assets and future assets have to be on the grid. Do value function iteration to solve for the asset policies and consumption policies.

4. Plot the asset policy function and consumption policy function.

5. How do the policies depend on the amplitude of fluctuations (y_L and y_H)?

6. How do the policies depend on the persistence of fluctuations ($prob(y'_L|y_L)$ and $prob(y'_H|y_H)$)?

The purpose of the next exercise is to solve the intertemporal model of the current account of exercise 1 on a computer using another method. You will

- solve for policy functions by iterating on the Euler equation.

- using the endogenous grid method.

- simulate the model.

- see how the income process shapes the correlation between the trade balance and income.

3. The intertemporal model of the current account solved on a computer (II).
Consider the same configuration as in exercise 2. Solve the model using policy function iteration and the endogenous grid method. Specify a grid for assets such that $b \in [\underline{b}, 20]$. Choose an evenly-spaced grid with 300 grid points.

1. Use the endogenous grid method to solve the model, iterating on the Euler equation.

2. How do your results for the consumption and savings policy compare to the policies derived in Exercise 2?

3. Simulate 1,000 draws from the model.

4. How does the correlation of the trade balance with income depend on the persistence of the income process? Document your findings and explain the economics.

5. Assume now that there are three income states, y_L, y_M, y_H. Solve the model using the endogenous grid method. Assume that $y_L = 0.85$, $y_M = 1$, $y_H = 1.15$. Assume further that $prob(y'_L|y_L) = prob(y'_M|y_M) = prob(y'_H|y_H) = 0.95$. In addition, assume that $prob(y'_M|y_L) = prob(y'_M|y_H) = 0.05$ and $prob(y'_L|y_M) = prob(y'_H|y_M) = 0.025$.

6. Same as above (item 5), but now assume that there is a saving constraint as well. That is, the household may not *save* more than a certain amount. Assume that $b_t \leq \bar{b}$ and put $\bar{b} = 5$. Solve the model using the endogenous grid method.