Paper Presentation
On
Full Order Observer Design for Nonlinear Systems with Unknown Inputs

Authors: Guannan He, Jing Ji

Presented by: Gautam Sharma, Ram Milan Verma

Introduction

- Observer design problem is a very important problem.
- Applications: Output feedback control, system monitoring, fault detection etc.
- Observer design for nonlinear systems is difficult than for the linear systems.

Objective

- To design an observer for a class of nonlinear systems with unknown inputs.
- State observer estimates the state variables based on the measurements of the output over a period of time.
- Requirement: The system should be “observable” i.e. the observability matrix should be full rank.

Problem Statement

- System dynamics: $$\dot{x} = Ax + Bu + Dv + f(x), \quad y = Cx \tag{1}$$

- Assumption: Function f(x) is assumed to be Lipschitz function.

$$||f(x) - f(\hat{x})|| \leq \gamma ||x - \hat{x}|| \tag{2}$$

- Observer dynamics:

$$\begin{aligned} \dot{z} &= Nz + Gu + Ly + Mf(\hat{x}) \\ \hat{x} &= z - Ey \end{aligned} \tag{3}$$

where $M = I_n + EC$

Error Dynamics

- Estimation error:

$$e(t) = \hat{x} - x = z - x - Ey = z - Mx, \tag{4}$$

- Error dynamics:

$$\begin{aligned} \dot{e}(t) \;=\;& Ne + (NM + LC - MA)x + (G - MB)u \\ & + M(f(\hat{x}) - f(x)) - MDv. \end{aligned} \tag{5}$$

- For eliminating the effect of input and state from error dynamics, to get

$$\dot{e}(t) = Ne + M(f(\hat{x}) - f(x)). \tag{6}$$

- Conditions required

$$\begin{aligned} NM + LC = MA,\; G = MB, \\ M = I_n + EC, MD = 0, \end{aligned} \tag{7}$$

Error Dynamics

- Using
$$N = MA - KC \tag{8}$$
$$K = L + NE. \tag{9}$$
$$L = K(I_n + CE) - MAE. \tag{10}$$

- The observer dynamics equation becomes
$$\dot{z} = (MA - KC)z + Gu + Ly + Mf(\hat{x}) \tag{11}$$

- Hence the problem is reduced to find matrices E & K such that MA - KC is a stability matrix.

Developing LMI

- If there exists E, K, P > 0 such that

$$ECD = -D$$
$$N^T P + PN + \gamma PMM^T P + \gamma I < 0 \qquad (12)$$

- Rewriting error dynamics as $\dot{e}(t) = Ne + M(f(\hat{x}) - f(x)) - MDv. \qquad (13)$
- Defining: $V(t) = e(t)^T Pe(t)$ where P > 0.
- Therefore,

$$\begin{aligned}\dot{V} &= e^T(N^T P + PN)e + 2e^T PM(f(\hat{x}) - f(x)) \\ &\leq e^T(N^T P + PN)e + 2\|e^T PM\|\|(f(\hat{x}) - f(x))\| \\ &\leq e^T(N^T P + PN)e + 2\|e^T PM\|\gamma\|e\| \\ &\leq e^T(N^T P + PN)e + \gamma(\|e^T PM\|^2 + \|e\|^2) \\ &= e^t(N^T P + PN + \gamma PMM^T P + \gamma I)e. \qquad (14)\end{aligned}$$

- Hence asymptotic stable.

Rearranging terms

- Rearranging terms $E = -D(CD)^{+} + Y(I_n - (CD)(CD)^{+})$ (15)
- Defining

$$U = -D(CD)^{+} \qquad\qquad V = I_n - (CD)(CD)^{+}$$

- We get (15) as $E = U + YV.$ (16)
- Substituting (16) in matrix inequality, we get

$$\begin{aligned} &((I+UC)A)^T P + P(I+UC)A \\ &+(VCA)^T Y^T P + PY(VCA) - C^T K^T P - PKC \\ &+\gamma(P(I+UC) + PY(VC)). \\ &(P(I+UC) + PY(VC))^T + \gamma I < 0. \quad (17) \end{aligned}$$

Lemma

The matrix inequality given by (17) has a solution for Y, K and P > 0 if and only if the following LMI has a solution $\bar{Y}, \bar{K}$ and P > 0,

$$\begin{bmatrix} X & X_{12} \\ X_{12}^T & -I \end{bmatrix} < 0 \tag{18}$$

Where X and X_{12} are given by

$$\begin{aligned} X = \quad & ((I+UC)A)^T P + P(I+UC)A \\ & +(VCA)^T \bar{Y}^T + \bar{Y}(VCA) \\ & -C^T \bar{K}^T - \bar{K}C + \gamma I, \\ X_{12} = \quad & \sqrt{\gamma}[P(I+UC) + \bar{Y}(VC)], \end{aligned} \tag{19}$$

with $Y = P^{-1}\bar{Y}$ and $K = P^{-1}\bar{K}$.

Design Algorithm

1. Compute the matrices $U = -D(CD)^{+}$ and $V = I - (CD)(CD)^{+}$ respectively.

2. Solve the LMI for matrices $\bar{Y}, \bar{K}$, and a symmetric matrix P > 0.

3. Solve: $Y = P^{-1}\bar{Y}$ and $K = P^{-1}\bar{K}$.

4. Based on Y & K, the observer gains can be obtained as

   E = U + YV, M = I + EC, N = MA - KC, G = MB, L = K(I + CE) - MAE

Numerical Example

System matrix considered

$$A = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}, \; B = 0, \; D = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix},$$

Comparison of results

M=

-0.0000	-0.0000	-0.0000
-1.0000	0.0000	-1.0000
1.0000	-0.0000	1.0000

$$M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1.358 & 1.569 \\ 0 & 0 & 0 \end{bmatrix}$$

N=

-0.2240	-0.3623	-0.3623
-0.3672	0.4355	-0.5645
-0.3672	-0.5645	0.4355

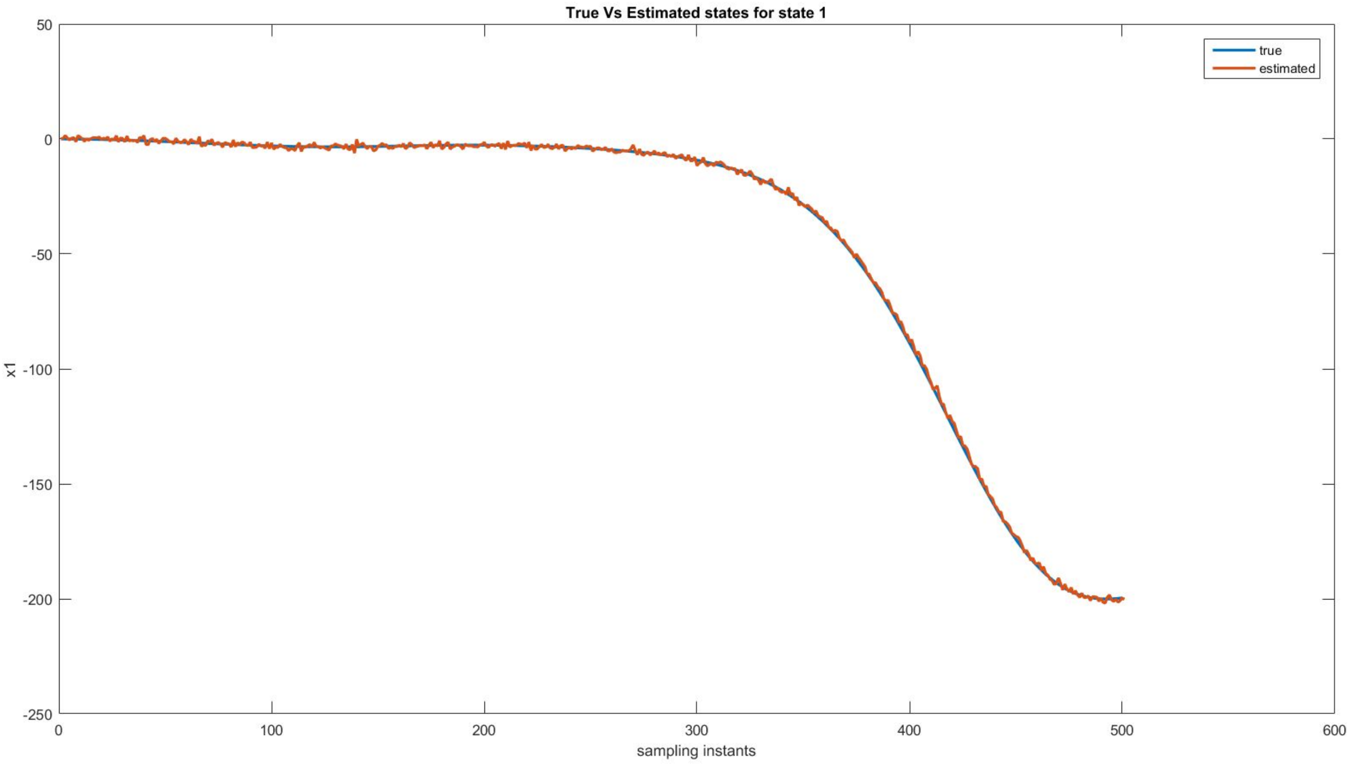
$$N = \begin{bmatrix} -3.453 & 0 & -0.0058 \\ 0 & -1.563 & 0 \\ -0.0058 & 0 & -3.453 \end{bmatrix}$$

L=

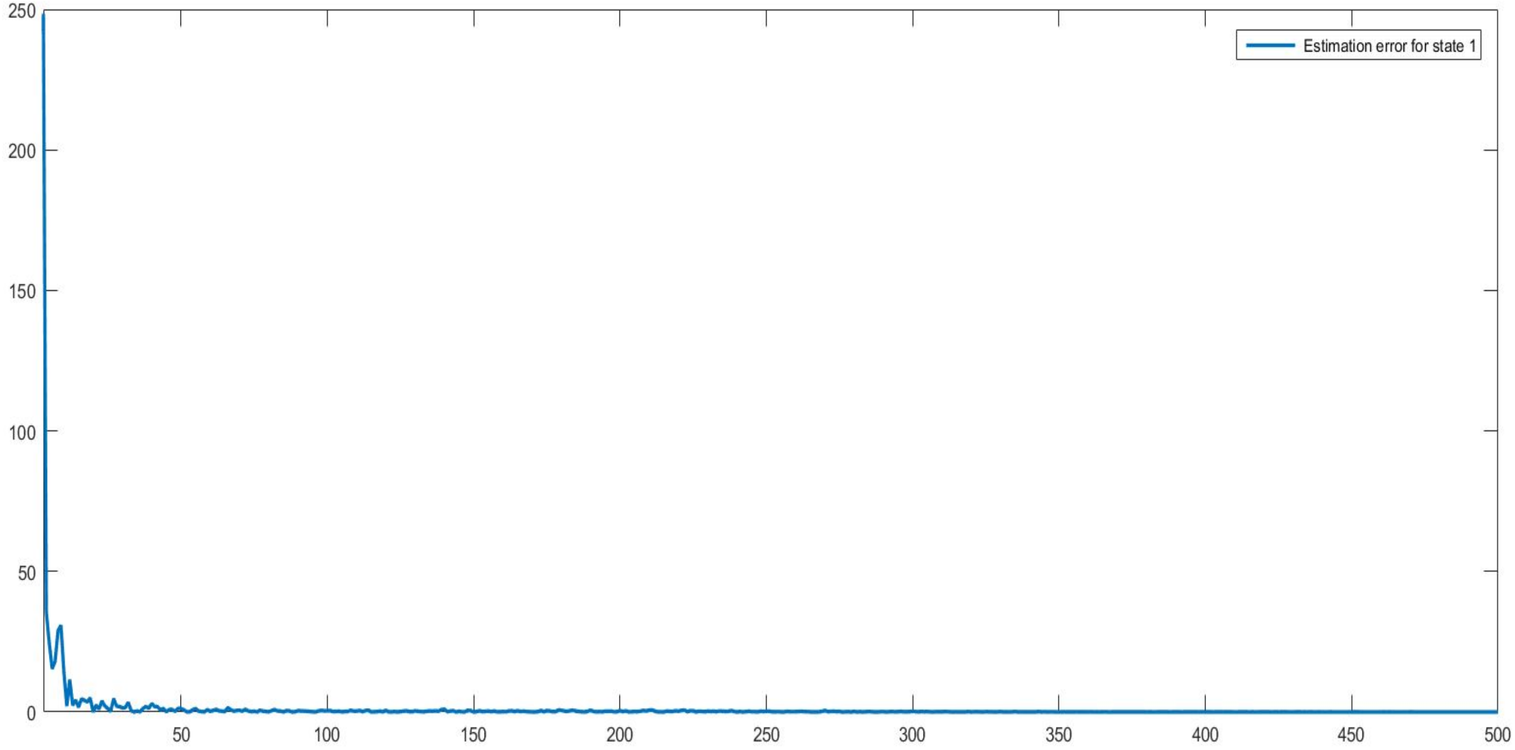
0.0000	0.0000
2.0000	2.0000
-2.0000	-2.0000

$$L = \begin{bmatrix} 0 & 0 \\ -1 & 0.8379 \\ 0 & 0 \end{bmatrix}$$

Estimation using observer



Estimation using observer



Conclusion

- Paper proposed the sufficient conditions for the existence of full order observer for a class of Lipschitz nonlinear systems with unknown input.
- Designed method presented in terms of linear matrix inequalities (LMIs).
- The asymptotic stability of estimated error established by proof is reflected in simulation.

References

1. https://in.mathworks.com/help/robust/ug/specify-lmi-system-at-the-command-line.html
2. https://in.mathworks.com/help/robust/ref/evallmi.html
3. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.613.759&rep=rep1&type=pdf
4. https://in.mathworks.com/help/robust/ug/specify-lmis-with-the-lmi-editor-gui.html