State Estimation of Induction Machine using UKF

Ram Milan Verma

Indian Institute of Technology, Bombay

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Contents

Importance of UKF(Unscented Kalman Filter)

- Derivative free sampling based filter.
- Uses deterministic sampling and associates weights.
- Numerical approximations of first and second moments can be found using sample points chosen.
- Sample point generation is called **sigma point generation**.
- Sigma point is generated symmetrically around the mean.
- Hence, works well for signals with **unimodal and symmetric** probability densities.

Unscented Kalman Filter

Augmented Quantities

1. $\chi(k-1) = [\mathbf{X}(k-1)^T, \mathbf{d}(k-1)^T, \mathbf{v}(k-1)^T]^T$
2. It's mean, $\widehat{\chi}(k-1|k-1) = [\widehat{X}(k-1|k-1)^T, \bar{0}^T, \bar{0}^T]^T$
3. Covariance matrix,$\mathbf{P}^a(k-1|k-1) = diag[\mathbf{P}(k-1|k-1), \mathbf{Q}, \mathbf{R}]$

Sigma Point generation

- $\mathbf{M}$=dim(χ); Number of samples, $N_s = 2M + 1$
- $\chi^{(1)}(k-1|k-1) = \widehat{\chi}(k-1|k-1)$
- $\chi^{(i+1)}(k-1|k-1) = \widehat{\chi}(k-1|k-1) + \rho\sqrt{P^a(k-1|k-1)}\zeta^{(i)}$
- $\chi^{(i+M+1)}(k-1|k-1) = \widehat{\chi}(k-1|k-1) - \rho\sqrt{P^a(k-1|k-1)}\zeta^{(i)}$
  where $\zeta^{(i)} = [0, 0, ..., 1, 0, 0]^T$ i=1,...,M; $\rho = \sqrt{M + \kappa}$
  κ (tuning parameter) $= 1$ *or* $(3 - M)$ (A Typical choice)

Unscented Kalman Filter

Weights

- Weights are chosen such that $\sum \omega_i = 1$ and weighted sample average equals $\widehat{\chi}(k-1|k-1)$
- $\omega_1 = \frac{\kappa}{M+\kappa}$
- $\omega_{i+1} = \omega_{i+M+1} = \frac{1}{2(M+\kappa)}$ For i=1,..,M

Propagation using system and measurement models

- state propagation
  - Integrate the model equation for each sample points.
  - Get the weighted sum as sample mean, $\widehat{X}(k|k-1) = \sum \omega_i \widehat{X}^{(i)}(k|k-1)$
- Measurement prediction
  - Get $Y^{(i)}(k|k-1)$ by substituting $\widehat{X}^{(i)}(k|k-1)$
  - $\widehat{Y}(k|k-1) = \sum \omega_i \widehat{Y}^{(i)}(k|k-1)$

Unscented Kalman Filter

Defining error and covariances

- $\varepsilon^{(i)}(k|k-1) = \widehat{X}^{(i)}(k|k-1) - \widehat{X}(k|k-1)$
- $\epsilon^{(i)}(k) = \widehat{Y}^{(i)}(k|k-1) - \widehat{Y}(k|k-1)$
- Covariances
    - $P_{\varepsilon\varepsilon}(k|k-1) \approx \sum \omega_i \varepsilon^{(i)}(k|k-1)[\varepsilon^{(i)}(k|k-1)]^T$
    - $P_{\varepsilon\epsilon}(k) \approx \sum \omega_i \varepsilon^{(i)}(k|k-1)[\epsilon^{(i)}(k)]^T$
    - $P_{\epsilon\epsilon}(k) \approx \sum \omega_i \epsilon^{(i)}(k)[\epsilon^{(i)}(k)]^T$

Kalman Update

- $L(k) = P_{\varepsilon\epsilon}(k)\ P_{\epsilon\epsilon}(k)^{-1}$
- innovation, $e(k) = Y(k) - \widehat{Y}(k|k-1)$
- $\widehat{X}(k|k) = \widehat{X}(k|k-1) + L(k)\ e(k)$
- $P(k|k) = P_{\varepsilon\varepsilon}(k|k-1) - L(k)P_{\epsilon\epsilon}(k)L(k)^T$

Model equations

- State space model of system

$$\frac{dx_1}{dt} = k_1 x_1 + z_1 x_2 + k_2 x_3 + z_2 \tag{1}$$

$$\frac{dx_2}{dt} = z_1 x_1 + k_1 x_2 + k_2 x_4 \tag{2}$$

$$\frac{dx_3}{dt} = k_3 x_1 + k_4 x_3 + (z_1 - x_5) x_4 \tag{3}$$

$$\frac{dx_4}{dt} = k_3 x_2 - (z_1 - x_5) x_3 + k_4 x_4 \tag{4}$$

$$\frac{dx_5}{dt} = k_5 (x_1 x_4 - x_2 x_3) + k_6 z_3 \tag{5}$$

where x_1, x_2 *and* x_3, x_4 are components of stator and rotor flux; x_5 is angular velocity; z_1, z_2, z_3 are inputs indicating frequency, amplitude and load torque

Measurement model and Parameters

- Measurement model equations

$$y_1 = k_7 x_1 + k8 x_3 \tag{6}$$

$$y2 = k_7 x_2 + k_8 x_4 \tag{7}$$

- x(0)=$[0.2, -0.6, -0.4, 0.1, 0.3]^T$; $x(0|0) = 0.9 * x(0)$; $Q = 10^{-4} I_{5x5}$; $R = 10^{-2} I_{2x2}$; $T_s = 0.1\ sec$; $P(0|0) = I_{5x5}$; $N = 500$

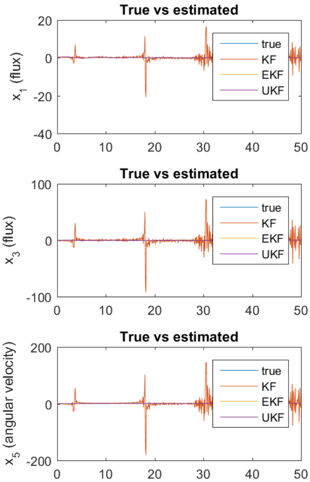
Table: Parameters and inputs

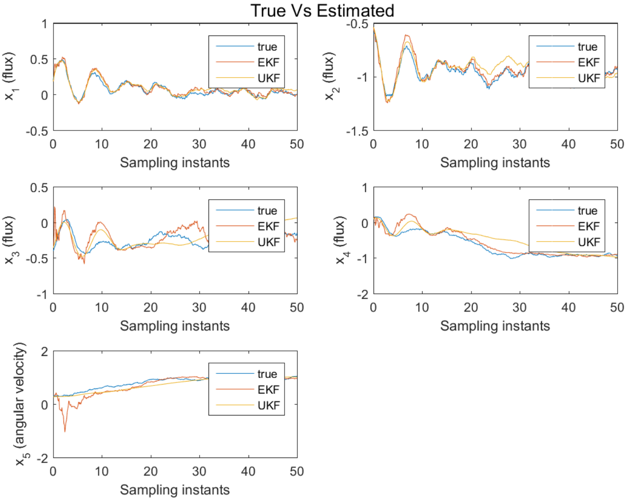
$k1$	k_2	k_3	k_4	k_5	k_6	k_7	k_8	z_1	z_2	z_3
-0.186	0.178	0.225	-0.234	-0.081	4.643	-4.448	1	1	1	0

Linearisation point for KF

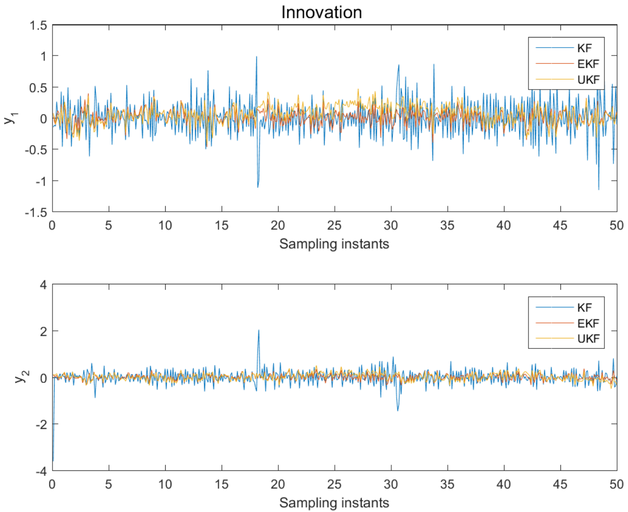
- Used **fsolve** function of MATLAB.
- it takes initial point (x_0) as one argument to perform search.
- Used **feval** to check the correctness of steady state point obtained.
- For x_0=[0 0 0 1 1] or [0 -1 0.5 -1 1]
- $x_{ss} = [0.0148 \; -0.9998 \; 0.0143 \; -0.9613 \; 1.0000]$
- Output of feval: 1.0e-08 *[0.0000 0.0000 0.0481 -0.1631 -0.0023]

Results: True Vs Estimated(All Filter)

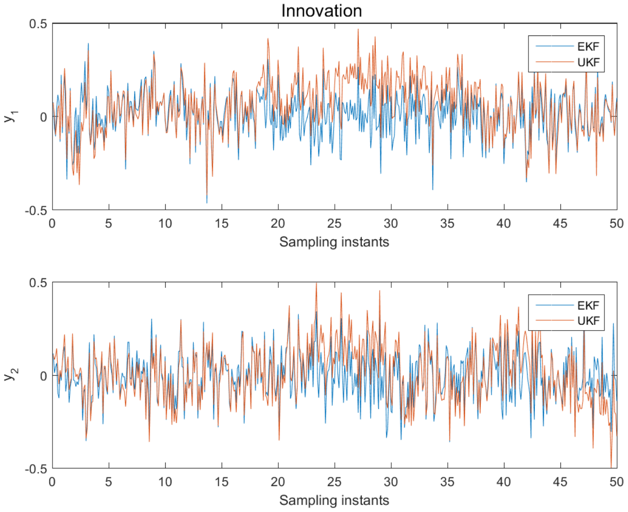



True Vs Estimated
x_1 (flux)
x_2 (flux)
x_3 (flux)
x_4 (flux)
x_5 (angular velocity)
Sampling instants
true
EKF
UKF
1
0.5
0
-0.5
-1
-1.5
-2
2
0
10
20
30
40
50

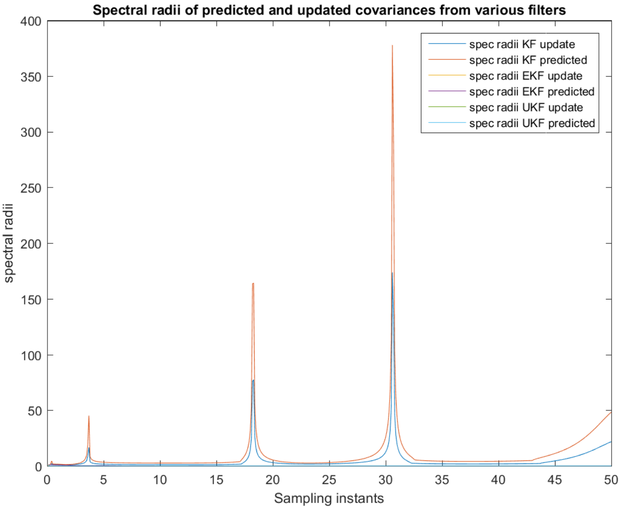
Results: Innovation(All Filter)



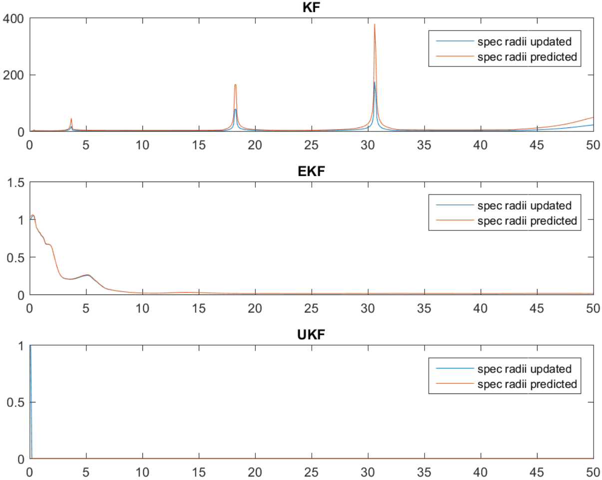
Results: Innovation(EKF and UKF)



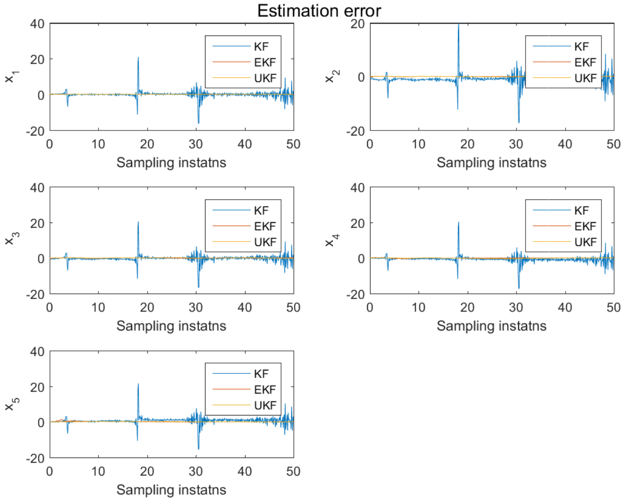
Results: Spectral Radii(All on same plot)



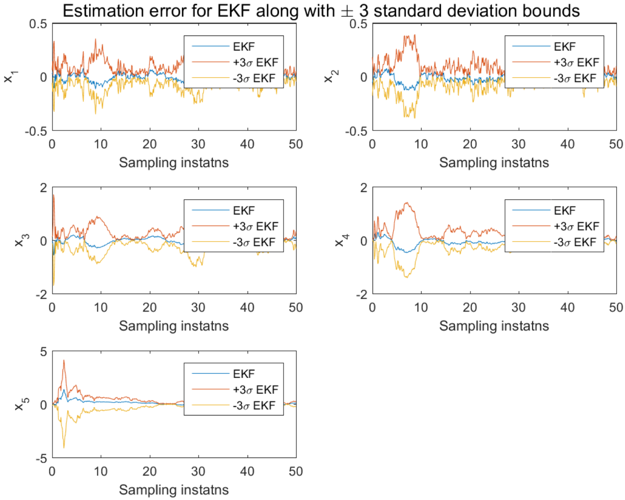
Results: Spectral Radii(subplots)



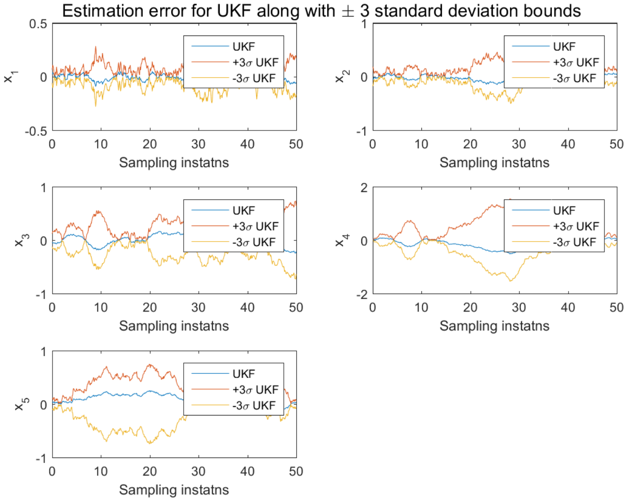
Results: Estimation error(All Filter)



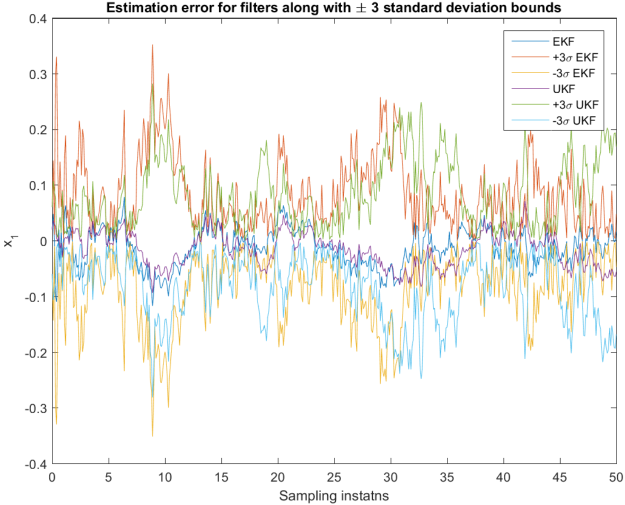
Results: Estimation error for EKF



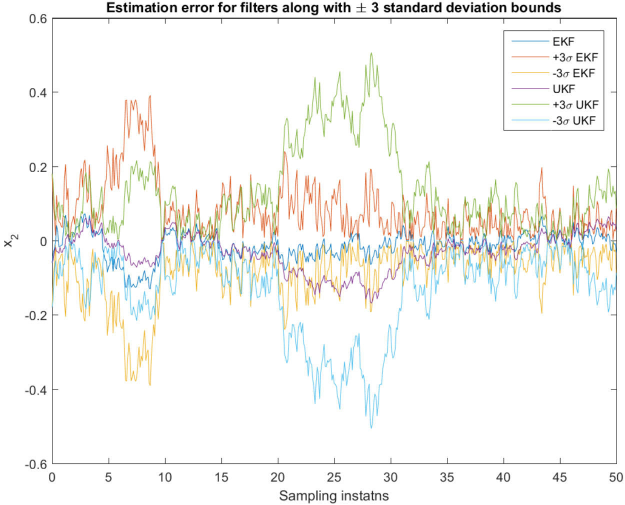
Results: Estimation error for UKF



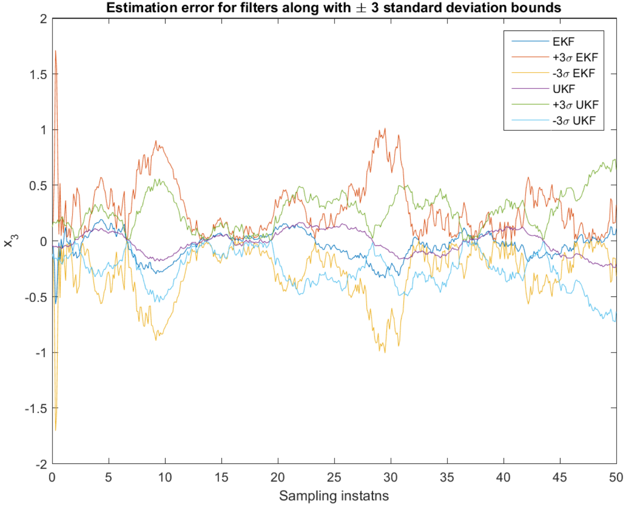
Results: Estimation error for EKF and UKF(State 1)



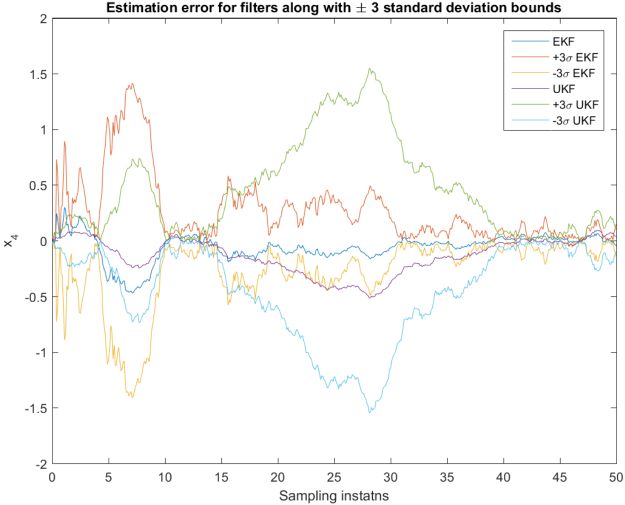
Results: Estimation error for EKF and UKF(State 2)



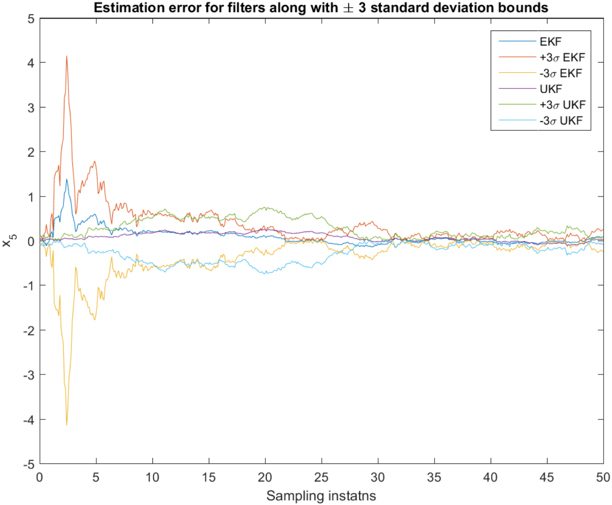
Results: Estimation error for EKF and UKF(State 3)



Results: Estimation error for EKF and UKF(State 4)



Results: Estimation error for EKF and UKF(State 5)



Tables: Mean and Variance of innovation; RMSE of states

Table : Mean and variances across time

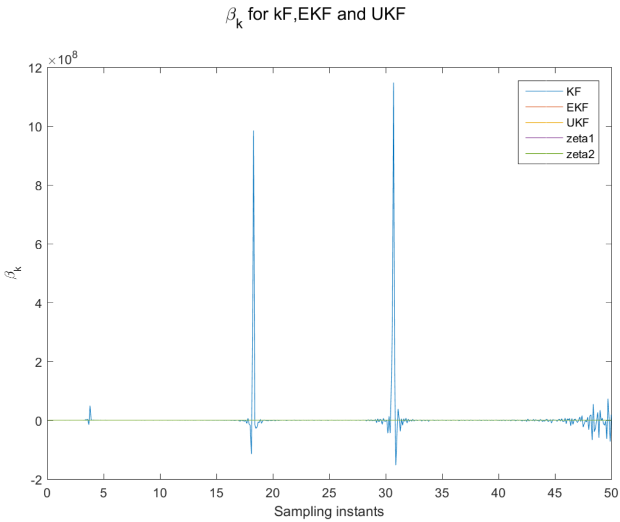
	KF		EKF		UKF	
	y1	y2	y1	y2	y1	y2
Mean	0.0028	-0.0146	0.0096	-0.0058	0.0607	0.0077
Variance	0.0854	0.1210	0.0160	0.0182	0.0205	0.0235

Table: Root Mean Square Error

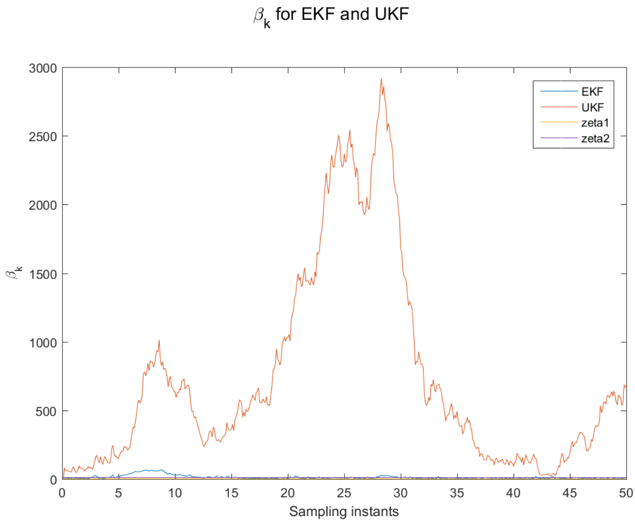
Filter	**State 1**	**State 2**	**State 3**	**State 4**	**State 5**
KF	2.2648	2.9405	10.0561	12.2089	19.8373
EKF	0.0358	0.0387	0.1288	0.1374	0.2158
UKF	0.0343	0.0597	0.1047	0.2104	0.1185

Figure: Mean,Variance and RMSE for filters

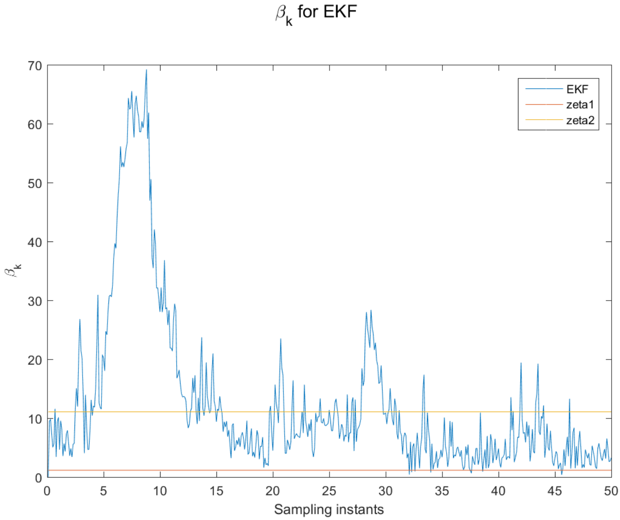
Results:Normalised Estimation Error Squared(NESS)



Results: Normalised Estimation Error Squared(NESS)



Results: Normalised Estimation Error Squared(NESS)



Conclusion

- The fraction of time instants for which β_k exceeded the bounds is 0.368 for EKF and 1 for both KF and UKF.
- No filter is consistent for the initial estimate used.
- Initial estimate should be carefully chosen random guess or far from true state doesn't seemed to work.
- KF didn't work well on this system at chose linearisation point.
- Both EKF and UKF performed decently well but not satisfactory.
- For few states EKF estimated better, for few UKF and for few both did better

References

- Lectrue slides
- Kandepu, R., and Foss, B., and Imsland, L.; Applying the Unscented Kalman Filter for Nonlinear State Estimation, Journal of Process Control, 18, 2008, 753-768.

The End