GCLC Prover Output for conjecture "konstrukcija-trougla"

Groebner bases method used

April 28, 2022

1 Construction and prover internal state

Construction commands:

- Point O
- Point M_b
- Point M_c
- Line m_b: O M_b
- Line m_c: O M_c
- Perpendicular, b: M_b m_b
- Perpendicular, c: M_c m_c
- Intersection of lines, A: c b
- Towards C A A M_b 2
- Towards B M_c M_c A -1
- Line c': A B
- Line b': A C
- Midpoint, M'_b: A C
- Midpoint, M'_c: A B
- Perpendicular, m'_c: M'_c c
- Perpendicular, m'_b: M'_b b
- Intersection of lines, O': m'_b m'_c

Coordinates assigned to the points:

- $O = (0, 0)$
- $M_b = (u_1, 0)$
- $M_c = (u_2, u_3)$
- $A = (u_1, x_1)$
- $C = (u_1, x_3)$
- $B = (x_6, x_5)$
- $M_b' = (u_1, x_7)$
- $M_c' = (x_{10}, x_9)$
- $O' = (x_{12}, x_7)$

Conjecture(s):

1. Given conjecture

   - **GCLC code:**

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     identical M_{c} M_{c}'
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   - **Expression:**
     $P_{M_c M_c' M_c} = 0$

2 Resolving constructed lines

- $m_b \ni O, M_b$; line is horizontal (i.e., $y(O) = y(M_b)$)
- $m_c \ni O, M_c$
- $b \ni M_b, A, C, M_b'$; line is vertical (i.e., $x(M_b) = x(A)$)
- $c \ni M_c, A, B, M_c'$
- $c' \ni A, B$
- $b' \ni A, C$
- $m_c' \ni M_c', O'$
- $m_b' \ni M_b', O'$; line is horizontal (i.e., $y(M_b') = y(O')$)

3 Creating polynomials from hypotheses

- Point O
  no condition
- Point M_b
  no condition
- Point M_c
  no condition
- Line m_b: O M_b
  - point O is on the line (O, M_b)
    no condition
  - point M_b is on the line (O, M_b)
    no condition
- Line m_c: O M_c
  - point O is on the line (O, M_c)
    no condition
  - point M_c is on the line (O, M_c)
    no condition
- Perpendicular, b: M_b m_b
  - Line (M_b, A) perpendicular with line (O, M_b)
    — true by the construction
- Perpendicular, c: M_c m_c
  - Line (M_c, A) perpendicular with line (O, M_c)

$$p_1 = u_3 x_1 + (-u_3^2 - u_2^2 + u_2 u_1)$$

- Intersection of lines, A: c b
  - point A is on the line (M_c, A)
    no condition
  - point A is on the line (M_b, A)
    no condition
- Towards C A A M_b 2
  - Line (A, C) parallel with line (A, M_b)
    — true by the construction
  - Ratio of projections of the segments (A, C) and (A, M_b) on axis y equal to 2.000

$$p_2 = -x_3 - x_1$$

- Towards B M_c M_c A -1
  - Line (M_c, B) parallel with line (M_c, A)

$$p_3 = x_6x_1 - u_3x_6 + (u_2 - u_1)x_5 - u_2x_1 + u_3u_1$$

  - Ratio of projections of the segments (M_c, B) and (M_c, A) on axis x equal to -1.000

$$p_4 = -x_6 + (2u_2 - u_1)$$

- Line c': A B
  - point A is on the line (A, B)
    no condition
  - point B is on the line (A, B)
    no condition
- Line b': A C
  - point A is on the line (A, C)
    no condition
  - point C is on the line (A, C)
    no condition
- Midpoint, M_b': A C
  - point M_b' is on the line (A, C) — true by the construction
    no condition
  - Segment [M_b', A] equal size as segment [M_b', C]

$$p_5 = 2x_7x_3 - 2x_7x_1 - x_3^2 + x_1^2$$

- Midpoint, M_c': A B
  - point M_c' is on the line (A, B)

$$p_6 = -x_{10}x_5 + x_{10}x_1 + x_9x_6 - u_1x_9 - x_6x_1 + u_1x_5$$

  - Segment [M_c', A] equal size as segment [M_c', B]

$$p_7 = 2x_{10}x_6 - 2u_1x_{10} + 2x_9x_5 - 2x_9x_1 - x_6^2 - x_5^2 + x_1^2 + u_1^2$$

- Perpendicular, m_c': M_c' c
  - Line (M_c', O') perpendicular with line (M_c, A)

$$p_8 = (-u_2 + u_1)x_{12} + (u_2 - u_1)x_{10} - x_9x_1 + u_3x_9 + x_7x_1 - u_3x_7$$

- Perpendicular, m_b': M_b' b

- Line (M'_b, O') perpendicular with line (M_b, A)
  — true by the construction

- Intersection of lines, O': m'_b m'_c
  - point O' is on the line (M'_b, O')
    no condition
  - point O' is on the line (M'_c, O')
    no condition

4 Creating polynomial from the conjecture

- Processing given conjecture(s).

Conjecture 1:

$$p_9 = 2x_{10}^2 - 4u_2x_{10} + 2x_9^2 - 4u_3x_9 + (2u_3^2 + 2u_2^2)$$

5 Invoking the theorem prover

The used proving method is Buchberger's method.
Input polynomial system is:

$$\begin{aligned}
p_0 &= u_3x_1 + (-u_3^2 - u_2^2 + u_2u_1) \\
p_1 &= -x_3 - x_1 \\
p_2 &= x_6x_1 - u_3x_6 + (u_2 - u_1)x_5 - u_2x_1 + u_3u_1 \\
p_3 &= -x_6 + (2u_2 - u_1) \\
p_4 &= 2x_7x_3 - 2x_7x_1 - x_3^2 + x_1^2 \\
p_5 &= -x_{10}x_5 + x_{10}x_1 + x_9x_6 - u_1x_9 - x_6x_1 + u_1x_5 \\
p_6 &= 2x_{10}x_6 - 2u_1x_{10} + 2x_9x_5 - 2x_9x_1 - x_6^2 - x_5^2 + x_1^2 + u_1^2 \\
p_7 &= (-u_2 + u_1)x_{12} + (u_2 - u_1)x_{10} - x_9x_1 + u_3x_9 + x_7x_1 - u_3x_7
\end{aligned}$$

5.1 Iteration 1

Current set is $S_1 =$

$$\begin{aligned}
p_0 &= u_3x_1 + (-u_3^2 - u_2^2 + u_2u_1) \\
p_1 &= -x_3 - x_1 \\
p_2 &= x_6x_1 - u_3x_6 + (u_2 - u_1)x_5 - u_2x_1 + u_3u_1 \\
p_3 &= -x_6 + (2u_2 - u_1) \\
p_4 &= 2x_7x_3 - 2x_7x_1 - x_3^2 + x_1^2 \\
p_5 &= -x_{10}x_5 + x_{10}x_1 + x_9x_6 - u_1x_9 - x_6x_1 + u_1x_5 \\
p_6 &= 2x_{10}x_6 - 2u_1x_{10} + 2x_9x_5 - 2x_9x_1 - x_6^2 - x_5^2 + x_1^2 + u_1^2 \\
p_7 &= (-u_2 + u_1)x_{12} + (u_2 - u_1)x_{10} - x_9x_1 + u_3x_9 + x_7x_1 - u_3x_7
\end{aligned}$$

1. Creating S-polynomial from the pair (p_0, p_1).
   Skipping pair p_0 and p_1 because gcd of their leading monoms is zero.
2. Creating S-polynomial from the pair (p_0, p_2).
   Forming S-pol of p_0 and p_2:
   $$p_{26} = (-u_2^2 + u_2u_1)x_6 + (-u_3u_2 + u_3u_1)x_5 + u_3u_2x_1 - u_3^2u_1$$
   S-pol added.
3. Creating S-polynomial from the pair (p_0, p_3).
   Skipping pair p_0 and p_3 because gcd of their leading monoms is zero.
4. Creating S-polynomial from the pair (p_0, p_4).
   Skipping pair p_0 and p_4 because gcd of their leading monoms is zero.
5. Creating S-polynomial from the pair (p_0, p_5).
   Skipping pair p_0 and p_5 because gcd of their leading monoms is zero.
6. Creating S-polynomial from the pair (p_0, p_6).
   Skipping pair p_0 and p_6 because gcd of their leading monoms is zero.
7. Creating S-polynomial from the pair (p_0, p_7).
   Skipping pair p_0 and p_7 because gcd of their leading monoms is zero.
8. Creating S-polynomial from the pair (p_1, p_2).
   Skipping pair p_1 and p_2 because gcd of their leading monoms is zero.
9. Creating S-polynomial from the pair (p_1, p_3).
   Skipping pair p_1 and p_3 because gcd of their leading monoms is zero.
10. Creating S-polynomial from the pair (p_1, p_4).
    Forming S-pol of p_1 and p_4:
    $$p_{27} = -4x_7x_1 - x_3^2 + x_1^2$$
    S-pol added.
11. Creating S-polynomial from the pair (p_1, p_5).
    Skipping pair p_1 and p_5 because gcd of their leading monoms is zero.
12. Creating S-polynomial from the pair (p_1, p_6).
    Skipping pair p_1 and p_6 because gcd of their leading monoms is zero.
13. Creating S-polynomial from the pair (p_1, p_7).
    Skipping pair p_1 and p_7 because gcd of their leading monoms is zero.
14. Creating S-polynomial from the pair (p_2, p_3).
    Forming S-pol of p_2 and p_3:
    $$p_{28} = u_3x_6 + (-u_2 + u_1)x_5 + (-u_2 + u_1)x_1 - u_3u_1$$
    S-pol added.

15. Creating S-polynomial from the pair (p_2, p_4).
    Skipping pair p_2 and p_4 because gcd of their leading monoms is zero.

16. Creating S-polynomial from the pair (p_2, p_5).
    Skipping pair p_2 and p_5 because gcd of their leading monoms is zero.

17. Creating S-polynomial from the pair (p_2, p_6).
    Forming S-pol of p_2 and p_6:

$$p_{29} = -2u_3x_{10}x_6 + (2u_2 - 2u_1)x_{10}x_5 +$$
$$(-2u_2 + 2u_1)x_{10}x_1 + 2u_3u_1x_{10} - 2x_9x_5x_1 + 2x_9x_1^2 + x_6^2x_1 + x_5^2x_1 -$$
$$x_1^3 - u_1^2x_1$$

    S-pol added.

18. Creating S-polynomial from the pair (p_2, p_7).
    Skipping pair p_2 and p_7 because gcd of their leading monoms is zero.

19. Creating S-polynomial from the pair (p_3, p_4).
    Skipping pair p_3 and p_4 because gcd of their leading monoms is zero.

20. Creating S-polynomial from the pair (p_3, p_5).
    Skipping pair p_3 and p_5 because gcd of their leading monoms is zero.

21. Creating S-polynomial from the pair (p_3, p_6).
    Forming S-pol of p_3 and p_6:

$$p_{30} = (4u_2 - 4u_1)x_{10} + 2x_9x_5 - 2x_9x_1 - x_6^2 - x_5^2 + x_1^2 + u_1^2$$

    S-pol added.

22. Creating S-polynomial from the pair (p_3, p_7).
    Skipping pair p_3 and p_7 because gcd of their leading monoms is zero.

23. Creating S-polynomial from the pair (p_4, p_5).
    Skipping pair p_4 and p_5 because gcd of their leading monoms is zero.

24. Creating S-polynomial from the pair (p_4, p_6).
    Skipping pair p_4 and p_6 because gcd of their leading monoms is zero.

25. Creating S-polynomial from the pair (p_4, p_7).
    Skipping pair p_4 and p_7 because gcd of their leading monoms is zero.

26. Creating S-polynomial from the pair (p_5, p_6).
    Forming S-pol of p_5 and p_6:

$$p_{31} = 2x_{10}x_6x_1 - 2u_1x_{10}x_5 + 2x_9x_6^2 - 2u_1x_9x_6 + 2x_9x_5^2 - 2x_9x_5x_1 -$$
$$x_6^2x_5 - 2x_6^2x_1 + 2u_1x_6x_5 - x_5^3 + x_5x_1^2 + u_1^2x_5$$

    S-pol added.

27. Creating S-polynomial from the pair (p_5, p_7).
    Skipping pair p_5 and p_7 because gcd of their leading monoms is zero.

28. Creating S-polynomial from the pair (p_6, p_7).
    Skipping pair p_6 and p_7 because gcd of their leading monoms is zero.

5.2 Iteration 2

Current set is $S_2 =$

$$
\begin{aligned}
p_0 &= u_3x_1 + (-u_3^2 - u_2^2 + u_2u_1)\\
p_1 &= -x_3 - x_1\\
p_2 &= x_6x_1 - u_3x_6 + (u_2 - u_1)x_5 - u_2x_1 + u_3u_1\\
p_3 &= -x_6 + (2u_2 - u_1)\\
p_4 &= 2x_7x_3 - 2x_7x_1 - x_3^2 + x_1^2\\
p_5 &= -x_{10}x_5 + x_{10}x_1 + x_9x_6 - u_1x_9 - x_6x_1 + u_1x_5\\
p_6 &= 2x_{10}x_6 - 2u_1x_{10} + 2x_9x_5 - 2x_9x_1 - x_6^2 - x_5^2 + x_1^2 + u_1^2\\
p_7 &= (-u_2 + u_1)x_{12} + (u_2 - u_1)x_{10} - x_9x_1 + u_3x_9 + x_7x_1 - u_3x_7\\
p_8 &= (u_3^2u_2 - u_3^2u_1)x_5 +\\
&\quad (-u_3^3u_2 + u_3^3u_1 + u_3u_2^3 - 2u_3u_2^2u_1 + u_3u_2u_1^2)\\
p_9 &= (-4u_3^4 - 4u_3^2u_2^2 + 4u_3^2u_2u_1)x_7\\
p_{10} &= (u_3u_2 - u_3u_1)x_5 + (-u_3^2u_2 + u_3^2u_1 + u_2^3 - 2u_2^2u_1 + u_2u_1^2)\\
p_{11} &= (4u_3^{10}u_2 - 4u_3^{10}u_1)x_{10} + (2u_3^{10} + 2u_3^8u_2^2 - 2u_3^8u_2u_1)x_9x_5 +\\
&\quad (-2u_3^{11} - 8u_3^9u_2^2 + 12u_3^9u_2u_1 - 4u_3^9u_1^2 - 2u_3^7u_2^4 + 4u_3^7u_2^3u_1 -\\
&\quad 2u_3^7u_2^2u_1^2)x_9 + (-u_3^{10} - u_3^8u_2^2 + u_3^8u_2u_1)x_5^2 +\\
&\quad (-2u_3^9u_2u_1 + 2u_3^9u_1^2)x_5 +\\
&\quad (u_3^{12} + 3u_3^{10}u_2^2 - 5u_3^{10}u_2u_1 + 2u_3^{10}u_1^2 + 3u_3^8u_2^4 - 8u_3^8u_2^3u_1 +\\
&\quad 7u_3^8u_2^2u_1^2 - 2u_3^8u_2u_1^3 + u_3^6u_2^6 - 3u_3^6u_2^5u_1 +\\
&\quad 3u_3^6u_2^4u_1^2 - u_3^6u_2^3u_1^3)\\
p_{12} &= (4u_3^3u_2 - 4u_3^3u_1)x_{10} + 2u_3^3x_9x_5 +\\
&\quad (-2u_3^4 - 2u_3^2u_2^2 + 2u_3^2u_2u_1)x_9 - u_3^3x_5^2 +\\
&\quad (u_3^5 - 2u_3^3u_2^2 + 2u_3^3u_2u_1 + u_3u_2^4 - 2u_3u_2^3u_1 + u_3u_2^2u_1^2)\\
p_{13} &= (\\
&\quad 4u_3^8u_2 - 4u_3^8u_1 + 4u_3^6u_2^3 - 8u_3^6u_2^2u_1 + 4u_3^6u_2u_1^2)x_{10} +\\
&\quad 2u_3^7x_9x_5^2 + (-2u_3^8 - 2u_3^6u_2^2 + 2u_3^6u_2u_1)x_9x_5 +\\
&\quad (8u_3^7u_2^2 - 16u_3^7u_2u_1 + 8u_3^7u_1^2)x_9 - u_3^7x_5^3 +\\
&\quad (u_3^9 - 2u_3^7u_2^2 + 6u_3^7u_2u_1 - 4u_3^7u_1^2 + u_3^5u_2^4 - 2u_3^5u_2^3u_1 +\\
&\quad u_3^5u_2^2u_1^2)x_5 +\\
&\quad (-8u_3^8u_2^2 + 12u_3^8u_2u_1 - 4u_3^8u_1^2 - 8u_3^6u_2^4 + 20u_3^6u_2^3u_1 -\\
&\quad 16u_3^6u_2^2u_1^2 + 4u_3^6u_2u_1^3)
\end{aligned}
$$

1. Creating S-polynomial from the pair (p_0, p_8).

Skipping pair p_0 and p_8 because gcd of their leading monoms is zero.

2. Creating S-polynomial from the pair (p_0, p_9).
   Skipping pair p_0 and p_9 because gcd of their leading monoms is zero.
3. Creating S-polynomial from the pair (p_0, p_{10}).
   Skipping pair p_0 and p_{10} because gcd of their leading monoms is zero.
4. Creating S-polynomial from the pair (p_0, p_{11}).
   Skipping pair p_0 and p_{11} because gcd of their leading monoms is zero.
5. Creating S-polynomial from the pair (p_0, p_{12}).
   Skipping pair p_0 and p_{12} because gcd of their leading monoms is zero.
6. Creating S-polynomial from the pair (p_0, p_{13}).
   Skipping pair p_0 and p_{13} because gcd of their leading monoms is zero.
7. Creating S-polynomial from the pair (p_1, p_8).
   Skipping pair p_1 and p_8 because gcd of their leading monoms is zero.
8. Creating S-polynomial from the pair (p_1, p_9).
   Skipping pair p_1 and p_9 because gcd of their leading monoms is zero.
9. Creating S-polynomial from the pair (p_1, p_{10}).
   Skipping pair p_1 and p_{10} because gcd of their leading monoms is zero.
10. Creating S-polynomial from the pair (p_1, p_{11}).
    Skipping pair p_1 and p_{11} because gcd of their leading monoms is zero.
11. Creating S-polynomial from the pair (p_1, p_{12}).
    Skipping pair p_1 and p_{12} because gcd of their leading monoms is zero.
12. Creating S-polynomial from the pair (p_1, p_{13}).
    Skipping pair p_1 and p_{13} because gcd of their leading monoms is zero.
13. Creating S-polynomial from the pair (p_2, p_8).
    Skipping pair p_2 and p_8 because gcd of their leading monoms is zero.
14. Creating S-polynomial from the pair (p_2, p_9).
    Skipping pair p_2 and p_9 because gcd of their leading monoms is zero.
15. Creating S-polynomial from the pair (p_2, p_{10}).
    Skipping pair p_2 and p_{10} because gcd of their leading monoms is zero.
16. Creating S-polynomial from the pair (p_2, p_{11}).
    Skipping pair p_2 and p_{11} because gcd of their leading monoms is zero.
17. Creating S-polynomial from the pair (p_2, p_{12}).
    Skipping pair p_2 and p_{12} because gcd of their leading monoms is zero.

18. Creating S-polynomial from the pair (p_2, p_{13}).
    Skipping pair p_2 and p_{13} because gcd of their leading monoms is zero.

19. Creating S-polynomial from the pair (p_3, p_8).
    Skipping pair p_3 and p_8 because gcd of their leading monoms is zero.

20. Creating S-polynomial from the pair (p_3, p_9).
    Skipping pair p_3 and p_9 because gcd of their leading monoms is zero.

21. Creating S-polynomial from the pair (p_3, p_{10}).
    Skipping pair p_3 and p_{10} because gcd of their leading monoms is zero.

22. Creating S-polynomial from the pair (p_3, p_{11}).
    Skipping pair p_3 and p_{11} because gcd of their leading monoms is zero.

23. Creating S-polynomial from the pair (p_3, p_{12}).
    Skipping pair p_3 and p_{12} because gcd of their leading monoms is zero.

24. Creating S-polynomial from the pair (p_3, p_{13}).
    Skipping pair p_3 and p_{13} because gcd of their leading monoms is zero.

25. Creating S-polynomial from the pair (p_4, p_8).
    Skipping pair p_4 and p_8 because gcd of their leading monoms is zero.

26. Creating S-polynomial from the pair (p_4, p_9).
    Forming S-pol of p_4 and p_9:

$$p_{46} = (8u_3^4 + 8u_3^2u_2^2 - 8u_3^2u_2u_1)x_7x_1 + \\ (4u_3^4 + 4u_3^2u_2^2 - 4u_3^2u_2u_1)x_3^2 + \\ (-4u_3^4 - 4u_3^2u_2^2 + 4u_3^2u_2u_1)x_1^2$$

    Reduced to zero.

27. Creating S-polynomial from the pair (p_4, p_{10}).
    Skipping pair p_4 and p_{10} because gcd of their leading monoms is zero.

28. Creating S-polynomial from the pair (p_4, p_{11}).
    Skipping pair p_4 and p_{11} because gcd of their leading monoms is zero.

29. Creating S-polynomial from the pair (p_4, p_{12}).
    Skipping pair p_4 and p_{12} because gcd of their leading monoms is zero.

30. Creating S-polynomial from the pair (p_4, p_{13}).
    Skipping pair p_4 and p_{13} because gcd of their leading monoms is zero.

31. Creating S-polynomial from the pair (p_5, p_8).
Forming S-pol of p_5 and p_8:

$$p_{47} = (u_3^2u_2 - u_3^2u_1)x_{10}x_1 + (-u_3^3u_2 + u_3^3u_1 + u_3u_2^3 - 2u_3u_2^2u_1 + u_3u_2u_1^2)x_{10} + (u_3^2u_2 - u_3^2u_1)x_9x_6 + (-u_3^2u_2u_1 + u_3^2u_1^2)x_9 + (-u_3^2u_2 + u_3^2u_1)x_6x_1 + (u_3^2u_2u_1 - u_3^2u_1^2)x_5$$

S-pol added.

32. Creating S-polynomial from the pair (p_5, p_9).
Skipping pair p_5 and p_9 because gcd of their leading monoms is zero.

33. Creating S-polynomial from the pair (p_5, p_{10}).
Forming S-pol of p_5 and p_{10}:

$$p_{48} = (u_3u_2 - u_3u_1)x_{10}x_1 + (-u_3^2u_2 + u_3^2u_1 + u_2^3 - 2u_2^2u_1 + u_2u_1^2)x_{10} + (u_3u_2 - u_3u_1)x_9x_6 + (-u_3u_2u_1 + u_3u_1^2)x_9 + (-u_3u_2 + u_3u_1)x_6x_1 + (u_3u_2u_1 - u_3u_1^2)x_5$$

S-pol added.

34. Creating S-polynomial from the pair (p_5, p_{11}).
Forming S-pol of p_5 and p_{11}:

$$p_{49} = (4u_3^{10}u_2 - 4u_3^{10}u_1)x_{10}x_1 + (4u_3^{10}u_2 - 4u_3^{10}u_1)x_9x_6 + (2u_3^{10} + 2u_3^8u_2^2 - 2u_3^8u_2u_1)x_9x_5^2 + (-2u_3^{11} - 8u_3^9u_2^2 + 12u_3^9u_2u_1 - 4u_3^9u_1^2 - 2u_3^7u_2^4 + 4u_3^7u_2^3u_1 - 2u_3^7u_2^2u_1^2)x_9x_5 + (-4u_3^{10}u_2u_1 + 4u_3^{10}u_1^2)x_9 + (-4u_3^{10}u_2 + 4u_3^{10}u_1)x_6x_1 + (-u_3^{10} - u_3^8u_2^2 + u_3^8u_2u_1)x_5^3 + (-2u_3^9u_2u_1 + 2u_3^9u_1^2)x_5^2 + (u_3^{12} + 3u_3^{10}u_2^2 - u_3^{10}u_2u_1 - 2u_3^{10}u_1^2 + 3u_3^8u_2^4 - 8u_3^8u_2^3u_1 + 7u_3^8u_2^2u_1^2 - 2u_3^8u_2u_1^3 + u_3^6u_2^6 - 3u_3^6u_2^5u_1 + 3u_3^6u_2^4u_1^2 - u_3^6u_2^3u_1^3)x_5$$

S-pol added.

35. Creating S-polynomial from the pair (p_5, p_{12}).
Forming S-pol of p_5 and p_{12}:

$$p_{50} = (4u_3^3u_2 - 4u_3^3u_1)x_{10}x_1 + (4u_3^3u_2 - 4u_3^3u_1)x_9x_6 + 2u_3^3x_9x_5^2 + (-2u_3^4 - 2u_3^2u_2^2 + 2u_3^2u_2u_1)x_9x_5 + (-4u_3^3u_2u_1 + 4u_3^3u_1^2)x_9 + (-4u_3^3u_2 + 4u_3^3u_1)x_6x_1 - u_3^3x_5^3 + (u_3^5 - 2u_3^3u_2^2 + 6u_3^3u_2u_1 - 4u_3^3u_1^2 + u_3u_2^4 - 2u_3u_2^3u_1 + u_3u_2^2u_1^2)x_5$$

S-pol added.

36. Creating S-polynomial from the pair (p_5, p_{13}).

Forming S-pol of p_5 and p_{13}:

$$p_{51} = (4u_3^8u_2 - 4u_3^8u_1 + 4u_3^6u_2^3 - 8u_3^6u_2^2u_1 + 4u_3^6u_2u_1^2)x_{10}x_1 +$$
$$(4u_3^8u_2 - 4u_3^8u_1 + 4u_3^6u_2^3 - 8u_3^6u_2^2u_1 + 4u_3^6u_2u_1^2)x_9x_6 +$$
$$2u_3^7x_9x_5^3 + (-2u_3^8 - 2u_3^6u_2^2 + 2u_3^6u_2u_1)x_9x_5^2 +$$
$$(8u_3^7u_2^2 - 16u_3^7u_2u_1 + 8u_3^7u_1^2)x_9x_5 +$$
$$(-4u_3^8u_2u_1 + 4u_3^8u_1^2 - 4u_3^6u_2^3u_1 + 8u_3^6u_2^2u_1^2 -$$
$$4u_3^6u_2u_1^3)x_9 +$$
$$(-4u_3^8u_2 + 4u_3^8u_1 - 4u_3^6u_2^3 + 8u_3^6u_2^2u_1 - 4u_3^6u_2u_1^2)x_6x_1 -$$
$$u_3^7x_5^4 +$$
$$(u_3^9 - 2u_3^7u_2^2 + 6u_3^7u_2u_1 - 4u_3^7u_1^2 + u_3^5u_2^4 - 2u_3^5u_2^3u_1 +$$
$$u_3^5u_2^2u_1^2)x_5^2 +$$
$$(-8u_3^8u_2^2 + 16u_3^8u_2u_1 - 8u_3^8u_1^2 - 8u_3^6u_2^4 + 24u_3^6u_2^3u_1 -$$
$$24u_3^6u_2^2u_1^2 + 8u_3^6u_2u_1^3)x_5$$

S-pol added.

37. Creating S-polynomial from the pair (p_6, p_8).

Skipping pair p_6 and p_8 because gcd of their leading monoms is zero.

38. Creating S-polynomial from the pair (p_6, p_9).

Skipping pair p_6 and p_9 because gcd of their leading monoms is zero.

39. Creating S-polynomial from the pair (p_6, p_{10}).

Skipping pair p_6 and p_{10} because gcd of their leading monoms is zero.

40. Creating S-polynomial from the pair (p_6, p_{11}).

Forming S-pol of p_6 and p_{11}:

$$p_{52} = (-8u_3^{10}u_2u_1 + 8u_3^{10}u_1^2)x_{10} +$$
$$(-4u_3^{10} - 4u_3^8u_2^2 + 4u_3^8u_2u_1)x_9x_6x_5 +$$
$$(4u_3^{11} + 16u_3^9u_2^2 - 24u_3^9u_2u_1 + 8u_3^9u_1^2 + 4u_3^7u_2^4 - 8u_3^7u_2^3u_1 +$$
$$4u_3^7u_2^2u_1^2)x_9x_6 + (8u_3^{10}u_2 - 8u_3^{10}u_1)x_9x_5 +$$
$$(-8u_3^{10}u_2 + 8u_3^{10}u_1)x_9x_1 + (-4u_3^{10}u_2 + 4u_3^{10}u_1)x_6^2 +$$
$$(2u_3^{10} + 2u_3^8u_2^2 - 2u_3^8u_2u_1)x_6x_5^2 +$$
$$(4u_3^9u_2u_1 - 4u_3^9u_1^2)x_6x_5 +$$
$$(-2u_3^{12} - 6u_3^{10}u_2^2 + 10u_3^{10}u_2u_1 - 4u_3^{10}u_1^2 - 6u_3^8u_2^4 +$$
$$16u_3^8u_2^3u_1 - 14u_3^8u_2^2u_1^2 + 4u_3^8u_2u_1^3 - 2u_3^6u_2^6 +$$
$$6u_3^6u_2^5u_1 - 6u_3^6u_2^4u_1^2 + 2u_3^6u_2^3u_1^3)x_6 +$$
$$(-4u_3^{10}u_2 + 4u_3^{10}u_1)x_5^2 + (4u_3^{10}u_2 - 4u_3^{10}u_1)x_1^2 +$$
$$(4u_3^{10}u_2u_1^2 - 4u_3^{10}u_1^3)$$

S-pol added.

41. Creating S-polynomial from the pair (p_6, p_{12}).

    Forming S-pol of p_6 and p_{12}:

$$p_{53} = (-8u_3^3u_2u_1 + 8u_3^3u_1^2)x_{10} - 4u_3^3x_9x_6x_5 +$$
$$(4u_3^4 + 4u_3^2u_2^2 - 4u_3^2u_2u_1)x_9x_6 + (8u_3^3u_2 - 8u_3^3u_1)x_9x_5 +$$
$$(-8u_3^3u_2 + 8u_3^3u_1)x_9x_1 + (-4u_3^3u_2 + 4u_3^3u_1)x_6^2 + 2u_3^3x_6x_5^2 +$$
$$(-2u_3^5 + 4u_3^3u_2^2 - 4u_3^3u_2u_1 - 2u_3u_2^4 + 4u_3u_2^3u_1 - 2u_3u_2^2u_1^2)x_6 +$$
$$(-4u_3^3u_2 + 4u_3^3u_1)x_5^2 + (4u_3^3u_2 - 4u_3^3u_1)x_1^2 +$$
$$(4u_3^3u_2u_1^2 - 4u_3^3u_1^3)$$

    S-pol added.

42. Creating S-polynomial from the pair (p_6, p_{13}).

    Forming S-pol of p_6 and p_{13}: Polynomial too big for output (text size is 1364 characters, number of terms is 13)

    S-pol added.

43. Creating S-polynomial from the pair (p_7, p_8).

    Skipping pair p_7 and p_8 because gcd of their leading monoms is zero.

44. Creating S-polynomial from the pair (p_7, p_9).

    Skipping pair p_7 and p_9 because gcd of their leading monoms is zero.

45. Creating S-polynomial from the pair (p_7, p_{10}).

    Skipping pair p_7 and p_{10} because gcd of their leading monoms is zero.

46. Creating S-polynomial from the pair (p_7, p_{11}).

    Skipping pair p_7 and p_{11} because gcd of their leading monoms is zero.

47. Creating S-polynomial from the pair (p_7, p_{12}).

    Skipping pair p_7 and p_{12} because gcd of their leading monoms is zero.

48. Creating S-polynomial from the pair (p_7, p_{13}).

    Skipping pair p_7 and p_{13} because gcd of their leading monoms is zero.

49. Creating S-polynomial from the pair (p_8, p_9).

    Skipping pair p_8 and p_9 because gcd of their leading monoms is zero.

50. Creating S-polynomial from the pair (p_8, p_{10}).

    Forming S-pol of p_8 and p_{10}:

$$p_{54} = 0$$

    Reduced to zero.

51. Creating S-polynomial from the pair (p_8, p_{11}).

    Skipping pair p_8 and p_{11} because gcd of their leading monoms is zero.

52. Creating S-polynomial from the pair (p_8, p_{12}).
    Skipping pair p_8 and p_{12} because gcd of their leading monoms is zero.

53. Creating S-polynomial from the pair (p_8, p_{13}).
    Skipping pair p_8 and p_{13} because gcd of their leading monoms is zero.

54. Creating S-polynomial from the pair (p_9, p_{10}).
    Skipping pair p_9 and p_{10} because gcd of their leading monoms is zero.

55. Creating S-polynomial from the pair (p_9, p_{11}).
    Skipping pair p_9 and p_{11} because gcd of their leading monoms is zero.

56. Creating S-polynomial from the pair (p_9, p_{12}).
    Skipping pair p_9 and p_{12} because gcd of their leading monoms is zero.

57. Creating S-polynomial from the pair (p_9, p_{13}).
    Skipping pair p_9 and p_{13} because gcd of their leading monoms is zero.

58. Creating S-polynomial from the pair (p_{10}, p_{11}).
    Skipping pair p_{10} and p_{11} because gcd of their leading monoms is zero.

59. Creating S-polynomial from the pair (p_{10}, p_{12}).
    Skipping pair p_{10} and p_{12} because gcd of their leading monoms is zero.

60. Creating S-polynomial from the pair (p_{10}, p_{13}).
    Skipping pair p_{10} and p_{13} because gcd of their leading monoms is zero.

61. Creating S-polynomial from the pair (p_{11}, p_{12}).
    Forming S-pol of p_{11} and p_{12}:

$$p_{55} = (8u_3^{11}u_2^3 - 16u_3^{11}u_2^2u_1 + 8u_3^{11}u_2u_1^2)x_9x_5 +$$
$$(-24u_3^{12}u_2^3 + 64u_3^{12}u_2^2u_1 - 56u_3^{12}u_2u_1^2 + 16u_3^{12}u_1^3 - 8u_3^{10}u_2^5 +$$
$$24u_3^{10}u_2^4u_1 - 24u_3^{10}u_2^3u_1^2 + 8u_3^{10}u_2^2u_1^3)x_9 +$$
$$(-4u_3^{11}u_2^3 + 8u_3^{11}u_2^2u_1 - 4u_3^{11}u_2u_1^2)x_5^2 +$$
$$(-8u_3^{12}u_2^2u_1 + 16u_3^{12}u_2u_1^2 - 8u_3^{12}u_1^3)x_5 +$$
$$(20u_3^{13}u_2^3 - 48u_3^{13}u_2^2u_1 + 36u_3^{13}u_2u_1^2 - 8u_3^{13}u_1^3 + 8u_3^{11}u_2^5 -$$
$$32u_3^{11}u_2^4u_1 + 48u_3^{11}u_2^3u_1^2 - 32u_3^{11}u_2^2u_1^3 + 8u_3^{11}u_2u_1^4 +$$
$$4u_3^9u_2^7 - 16u_3^9u_2^6u_1 + 24u_3^9u_2^5u_1^2 - 16u_3^9u_2^4u_1^3 +$$
$$4u_3^9u_2^3u_1^4)$$

    S-pol added.

62. Creating S-polynomial from the pair (p_{11}, p_{13}).
    Forming S-pol of p_{11} and p_{13}: Polynomial too big for output (text size is 1874 characters, number of terms is 7)

    S-pol added.

63. Creating S-polynomial from the pair (p_{12}, p_{13}).

Forming S-pol of p_{12} and p_{13}: Polynomial too big for output (text size is 1227 characters, number of terms is 7)

S-pol added.

5.3 Iteration 3

Current set is $S_3 =$

$$
\begin{aligned}
p_0 &= u_3x_1 + (-u_3^2 - u_2^2 + u_2u_1)\\
p_1 &= -x_3 - x_1\\
p_2 &= x_6x_1 - u_3x_6 + (u_2 - u_1)x_5 - u_2x_1 + u_3u_1\\
p_3 &= -x_6 + (2u_2 - u_1)\\
p_4 &= 2x_7x_3 - 2x_7x_1 - x_3^2 + x_1^2\\
p_5 &= -x_{10}x_5 + x_{10}x_1 + x_9x_6 - u_1x_9 - x_6x_1 + u_1x_5\\
p_6 &= 2x_{10}x_6 - 2u_1x_{10} + 2x_9x_5 - 2x_9x_1 - x_6^2 - x_5^2 + x_1^2 + u_1^2\\
p_7 &= (-u_2 + u_1)x_{12} + (u_2 - u_1)x_{10} - x_9x_1 + u_3x_9 + x_7x_1 - u_3x_7\\
p_8 &= (u_3^2u_2 - u_3^2u_1)x_5 +\\
&\quad (-u_3^3u_2 + u_3^3u_1 + u_3u_2^3 - 2u_3u_2^2u_1 + u_3u_2u_1^2)\\
p_9 &= (-4u_3^4 - 4u_3^2u_2^2 + 4u_3^2u_2u_1)x_7\\
p_{10} &= (u_3u_2 - u_3u_1)x_5 + (-u_3^2u_2 + u_3^2u_1 + u_2^3 - 2u_2^2u_1 + u_2u_1^2)\\
p_{11} &= (4u_3^{10}u_2 - 4u_3^{10}u_1)x_{10} + (2u_3^{10} + 2u_3^8u_2^2 - 2u_3^8u_2u_1)x_9x_5 +\\
&\quad (-2u_3^{11} - 8u_3^9u_2^2 + 12u_3^9u_2u_1 - 4u_3^9u_1^2 - 2u_3^7u_2^4 + 4u_3^7u_2^3u_1 -\\
&\quad 2u_3^7u_2^2u_1^2)x_9 + (-u_3^{10} - u_3^8u_2^2 + u_3^8u_2u_1)x_5^2 +\\
&\quad (-2u_3^9u_2u_1 + 2u_3^9u_1^2)x_5 +\\
&\quad (u_3^{12} + 3u_3^{10}u_2^2 - 5u_3^{10}u_2u_1 + 2u_3^{10}u_1^2 + 3u_3^8u_2^4 - 8u_3^8u_2^3u_1 +\\
&\quad 7u_3^8u_2^2u_1^2 - 2u_3^8u_2u_1^3 + u_3^6u_2^6 - 3u_3^6u_2^5u_1 +\\
&\quad 3u_3^6u_2^4u_1^2 - u_3^6u_2^3u_1^3)\\
p_{12} &= (4u_3^3u_2 - 4u_3^3u_1)x_{10} + 2u_3^3x_9x_5 +\\
&\quad (-2u_3^4 - 2u_3^2u_2^2 + 2u_3^2u_2u_1)x_9 - u_3^3x_5^2 +\\
&\quad (u_3^5 - 2u_3^3u_2^2 + 2u_3^3u_2u_1 + u_3u_2^4 - 2u_3u_2^3u_1 + u_3u_2^2u_1^2)\\
p_{13} &= (\\
&\quad 4u_3^8u_2 - 4u_3^8u_1 + 4u_3^6u_2^3 - 8u_3^6u_2^2u_1 + 4u_3^6u_2u_1^2)x_{10} +\\
&\quad 2u_3^7x_9x_5^2 + (-2u_3^8 - 2u_3^6u_2^2 + 2u_3^6u_2u_1)x_9x_5 +\\
&\quad (8u_3^7u_2^2 - 16u_3^7u_2u_1 + 8u_3^7u_1^2)x_9 - u_3^7x_5^3 +\\
&\quad (u_3^9 - 2u_3^7u_2^2 + 6u_3^7u_2u_1 - 4u_3^7u_1^2 + u_3^5u_2^4 - 2u_3^5u_2^3u_1 +\\
&\quad u_3^5u_2^2u_1^2)x_5 +\\
&\quad (-8u_3^8u_2^2 + 12u_3^8u_2u_1 - 4u_3^8u_1^2 - 8u_3^6u_2^4 + 20u_3^6u_2^3u_1 -\\
&\quad 16u_3^6u_2^2u_1^2 + 4u_3^6u_2u_1^3)\\
p_{14} &= \ldots\\
p_{15} &= \ldots
\end{aligned}
$$

$$
\begin{aligned}
p_{16} &= \dots \\
p_{17} &= \dots \\
p_{18} &= \dots \\
p_{19} &= (\\
&\quad 64u_3^{28}u_2^7 - 448u_3^{28}u_2^6u_1 + 1344u_3^{28}u_2^5u_1^2 - 2240u_3^{28}u_2^4u_1^3 + \\
&\quad 2240u_3^{28}u_2^3u_1^4 - 1344u_3^{28}u_2^2u_1^5 + 448u_3^{28}u_2u_1^6 - 64u_3^{28}u_1^7 + \\
&\quad 64u_3^{26}u_2^9 - 448u_3^{26}u_2^8u_1 + 1344u_3^{26}u_2^7u_1^2 - 2240u_3^{26}u_2^6u_1^3 + \\
&\quad 2240u_3^{26}u_2^5u_1^4 - 1344u_3^{26}u_2^4u_1^5 + 448u_3^{26}u_2^3u_1^6 - \\
&\quad 64u_3^{26}u_2^2u_1^7)x_9 + \\
&\quad (-64u_3^{29}u_2^7 + 448u_3^{29}u_2^6u_1 - 1344u_3^{29}u_2^5u_1^2 + 2240u_3^{29}u_2^4u_1^3 - \\
&\quad 2240u_3^{29}u_2^3u_1^4 + 1344u_3^{29}u_2^2u_1^5 - 448u_3^{29}u_2u_1^6 + 64u_3^{29}u_1^7 - \\
&\quad 64u_3^{27}u_2^9 + 448u_3^{27}u_2^8u_1 - 1344u_3^{27}u_2^7u_1^2 + 2240u_3^{27}u_2^6u_1^3 - \\
&\quad 2240u_3^{27}u_2^5u_1^4 + 1344u_3^{27}u_2^4u_1^5 - 448u_3^{27}u_2^3u_1^6 + \\
&\quad 64u_3^{27}u_2^2u_1^7) \\
p_{20} &= (\\
&\quad 32u_3^{21}u_2^6u_1 - 192u_3^{21}u_2^5u_1^2 + 480u_3^{21}u_2^4u_1^3 - \\
&\quad 640u_3^{21}u_2^3u_1^4 + 480u_3^{21}u_2^2u_1^5 - 192u_3^{21}u_2u_1^6 + 32u_3^{21}u_1^7 + \\
&\quad 32u_3^{19}u_2^8u_1 - 192u_3^{19}u_2^7u_1^2 + 480u_3^{19}u_2^6u_1^3 - \\
&\quad 640u_3^{19}u_2^5u_1^4 + 480u_3^{19}u_2^4u_1^5 - 192u_3^{19}u_2^3u_1^6 + \\
&\quad 32u_3^{19}u_2^2u_1^7)x_9 + \\
&\quad (-32u_3^{22}u_2^6u_1 + 192u_3^{22}u_2^5u_1^2 - 480u_3^{22}u_2^4u_1^3 + \\
&\quad 640u_3^{22}u_2^3u_1^4 - 480u_3^{22}u_2^2u_1^5 + 192u_3^{22}u_2u_1^6 - 32u_3^{22}u_1^7 - \\
&\quad 32u_3^{20}u_2^8u_1 + 192u_3^{20}u_2^7u_1^2 - 480u_3^{20}u_2^6u_1^3 + \\
&\quad 640u_3^{20}u_2^5u_1^4 - 480u_3^{20}u_2^4u_1^5 + 192u_3^{20}u_2^3u_1^6 - \\
&\quad 32u_3^{20}u_2^2u_1^7) \\
p_{21} &= \dots \\
p_{22} &= (\\
&\quad -16u_3^{18}u_2^6 + 96u_3^{18}u_2^5u_1 - 240u_3^{18}u_2^4u_1^2 + 320u_3^{18}u_2^3u_1^3 - \\
&\quad 240u_3^{18}u_2^2u_1^4 + 96u_3^{18}u_2u_1^5 - 16u_3^{18}u_1^6 - 16u_3^{16}u_2^8 + \\
&\quad 96u_3^{16}u_2^7u_1 - 240u_3^{16}u_2^6u_1^2 + 320u_3^{16}u_2^5u_1^3 - \\
&\quad 240u_3^{16}u_2^4u_1^4 + 96u_3^{16}u_2^3u_1^5 - 16u_3^{16}u_2^2u_1^6)x_9 + \\
&\quad (16u_3^{19}u_2^6 - 96u_3^{19}u_2^5u_1 + 240u_3^{19}u_2^4u_1^2 - 320u_3^{19}u_2^3u_1^3 + \\
&\quad 240u_3^{19}u_2^2u_1^4 - 96u_3^{19}u_2u_1^5 + 16u_3^{19}u_1^6 + 16u_3^{17}u_2^8 - \\
&\quad 96u_3^{17}u_2^7u_1 + 240u_3^{17}u_2^6u_1^2 - 320u_3^{17}u_2^5u_1^3 + \\
&\quad 240u_3^{17}u_2^4u_1^4 - 96u_3^{17}u_2^3u_1^5 + 16u_3^{17}u_2^2u_1^6) \\
p_{23} &= \dots \\
p_{24} &= \dots
\end{aligned}
$$

1. Creating S-polynomial from the pair (p_0, p_{14}).

   Skipping pair p_0 and p_{14} because gcd of their leading monoms is zero.

2. Creating S-polynomial from the pair (p_0, p_{15}).
   Skipping pair p_0 and p_{15} because gcd of their leading monoms is zero.
3. Creating S-polynomial from the pair (p_0, p_{16}).
   Skipping pair p_0 and p_{16} because gcd of their leading monoms is zero.
4. Creating S-polynomial from the pair (p_0, p_{17}).
   Skipping pair p_0 and p_{17} because gcd of their leading monoms is zero.
5. Creating S-polynomial from the pair (p_0, p_{18}).
   Skipping pair p_0 and p_{18} because gcd of their leading monoms is zero.
6. Creating S-polynomial from the pair (p_0, p_{19}).
   Skipping pair p_0 and p_{19} because gcd of their leading monoms is zero.
7. Creating S-polynomial from the pair (p_0, p_{20}).
   Skipping pair p_0 and p_{20} because gcd of their leading monoms is zero.
8. Creating S-polynomial from the pair (p_0, p_{21}).
   Skipping pair p_0 and p_{21} because gcd of their leading monoms is zero.
9. Creating S-polynomial from the pair (p_0, p_{22}).
   Skipping pair p_0 and p_{22} because gcd of their leading monoms is zero.
10. Creating S-polynomial from the pair (p_0, p_{23}).
    Skipping pair p_0 and p_{23} because gcd of their leading monoms is zero.
11. Creating S-polynomial from the pair (p_0, p_{24}).
    Skipping pair p_0 and p_{24} because gcd of their leading monoms is zero.
12. Creating S-polynomial from the pair (p_1, p_{14}).
    Skipping pair p_1 and p_{14} because gcd of their leading monoms is zero.
13. Creating S-polynomial from the pair (p_1, p_{15}).
    Skipping pair p_1 and p_{15} because gcd of their leading monoms is zero.
14. Creating S-polynomial from the pair (p_1, p_{16}).
    Skipping pair p_1 and p_{16} because gcd of their leading monoms is zero.
15. Creating S-polynomial from the pair (p_1, p_{17}).
    Skipping pair p_1 and p_{17} because gcd of their leading monoms is zero.
16. Creating S-polynomial from the pair (p_1, p_{18}).
    Skipping pair p_1 and p_{18} because gcd of their leading monoms is zero.
17. Creating S-polynomial from the pair (p_1, p_{19}).
    Skipping pair p_1 and p_{19} because gcd of their leading monoms is zero.
18. Creating S-polynomial from the pair (p_1, p_{20}).
    Skipping pair p_1 and p_{20} because gcd of their leading monoms is zero.

19. Creating S-polynomial from the pair (p_1, p_{21}).
    Skipping pair p_1 and p_{21} because gcd of their leading monoms is zero.
20. Creating S-polynomial from the pair (p_1, p_{22}).
    Skipping pair p_1 and p_{22} because gcd of their leading monoms is zero.
21. Creating S-polynomial from the pair (p_1, p_{23}).
    Skipping pair p_1 and p_{23} because gcd of their leading monoms is zero.
22. Creating S-polynomial from the pair (p_1, p_{24}).
    Skipping pair p_1 and p_{24} because gcd of their leading monoms is zero.
23. Creating S-polynomial from the pair (p_2, p_{14}).
    Skipping pair p_2 and p_{14} because gcd of their leading monoms is zero.
24. Creating S-polynomial from the pair (p_2, p_{15}).
    Skipping pair p_2 and p_{15} because gcd of their leading monoms is zero.
25. Creating S-polynomial from the pair (p_2, p_{16}).
    Skipping pair p_2 and p_{16} because gcd of their leading monoms is zero.
26. Creating S-polynomial from the pair (p_2, p_{17}).
    Skipping pair p_2 and p_{17} because gcd of their leading monoms is zero.
27. Creating S-polynomial from the pair (p_2, p_{18}).
    Skipping pair p_2 and p_{18} because gcd of their leading monoms is zero.
28. Creating S-polynomial from the pair (p_2, p_{19}).
    Skipping pair p_2 and p_{19} because gcd of their leading monoms is zero.
29. Creating S-polynomial from the pair (p_2, p_{20}).
    Skipping pair p_2 and p_{20} because gcd of their leading monoms is zero.
30. Creating S-polynomial from the pair (p_2, p_{21}).
    Skipping pair p_2 and p_{21} because gcd of their leading monoms is zero.
31. Creating S-polynomial from the pair (p_2, p_{22}).
    Skipping pair p_2 and p_{22} because gcd of their leading monoms is zero.
32. Creating S-polynomial from the pair (p_2, p_{23}).
    Skipping pair p_2 and p_{23} because gcd of their leading monoms is zero.
33. Creating S-polynomial from the pair (p_2, p_{24}).
    Skipping pair p_2 and p_{24} because gcd of their leading monoms is zero.
34. Creating S-polynomial from the pair (p_3, p_{14}).
    Skipping pair p_3 and p_{14} because gcd of their leading monoms is zero.
35. Creating S-polynomial from the pair (p_3, p_{15}).
    Skipping pair p_3 and p_{15} because gcd of their leading monoms is zero.

36. Creating S-polynomial from the pair (p_3, p_{16}).
    Skipping pair p_3 and p_{16} because gcd of their leading monoms is zero.
37. Creating S-polynomial from the pair (p_3, p_{17}).
    Skipping pair p_3 and p_{17} because gcd of their leading monoms is zero.
38. Creating S-polynomial from the pair (p_3, p_{18}).
    Skipping pair p_3 and p_{18} because gcd of their leading monoms is zero.
39. Creating S-polynomial from the pair (p_3, p_{19}).
    Skipping pair p_3 and p_{19} because gcd of their leading monoms is zero.
40. Creating S-polynomial from the pair (p_3, p_{20}).
    Skipping pair p_3 and p_{20} because gcd of their leading monoms is zero.
41. Creating S-polynomial from the pair (p_3, p_{21}).
    Skipping pair p_3 and p_{21} because gcd of their leading monoms is zero.
42. Creating S-polynomial from the pair (p_3, p_{22}).
    Skipping pair p_3 and p_{22} because gcd of their leading monoms is zero.
43. Creating S-polynomial from the pair (p_3, p_{23}).
    Skipping pair p_3 and p_{23} because gcd of their leading monoms is zero.
44. Creating S-polynomial from the pair (p_3, p_{24}).
    Skipping pair p_3 and p_{24} because gcd of their leading monoms is zero.
45. Creating S-polynomial from the pair (p_4, p_{14}).
    Skipping pair p_4 and p_{14} because gcd of their leading monoms is zero.
46. Creating S-polynomial from the pair (p_4, p_{15}).
    Skipping pair p_4 and p_{15} because gcd of their leading monoms is zero.
47. Creating S-polynomial from the pair (p_4, p_{16}).
    Skipping pair p_4 and p_{16} because gcd of their leading monoms is zero.
48. Creating S-polynomial from the pair (p_4, p_{17}).
    Skipping pair p_4 and p_{17} because gcd of their leading monoms is zero.
49. Creating S-polynomial from the pair (p_4, p_{18}).
    Skipping pair p_4 and p_{18} because gcd of their leading monoms is zero.
50. Creating S-polynomial from the pair (p_4, p_{19}).
    Skipping pair p_4 and p_{19} because gcd of their leading monoms is zero.
51. Creating S-polynomial from the pair (p_4, p_{20}).
    Skipping pair p_4 and p_{20} because gcd of their leading monoms is zero.
52. Creating S-polynomial from the pair (p_4, p_{21}).
    Skipping pair p_4 and p_{21} because gcd of their leading monoms is zero.

53. Creating S-polynomial from the pair (p_4, p_{22}).
    Skipping pair p_4 and p_{22} because gcd of their leading monoms is zero.
54. Creating S-polynomial from the pair (p_4, p_{23}).
    Skipping pair p_4 and p_{23} because gcd of their leading monoms is zero.
55. Creating S-polynomial from the pair (p_4, p_{24}).
    Skipping pair p_4 and p_{24} because gcd of their leading monoms is zero.
56. Creating S-polynomial from the pair (p_5, p_{14}).
    Skipping pair p_5 and p_{14} because gcd of their leading monoms is zero.
57. Creating S-polynomial from the pair (p_5, p_{15}).
    Skipping pair p_5 and p_{15} because gcd of their leading monoms is zero.
58. Creating S-polynomial from the pair (p_5, p_{16}).
    Skipping pair p_5 and p_{16} because gcd of their leading monoms is zero.
59. Creating S-polynomial from the pair (p_5, p_{17}).
    Skipping pair p_5 and p_{17} because gcd of their leading monoms is zero.
60. Creating S-polynomial from the pair (p_5, p_{18}).
    Skipping pair p_5 and p_{18} because gcd of their leading monoms is zero.
61. Creating S-polynomial from the pair (p_5, p_{19}).
    Skipping pair p_5 and p_{19} because gcd of their leading monoms is zero.
62. Creating S-polynomial from the pair (p_5, p_{20}).
    Skipping pair p_5 and p_{20} because gcd of their leading monoms is zero.
63. Creating S-polynomial from the pair (p_5, p_{21}).
    Skipping pair p_5 and p_{21} because gcd of their leading monoms is zero.
64. Creating S-polynomial from the pair (p_5, p_{22}).
    Skipping pair p_5 and p_{22} because gcd of their leading monoms is zero.
65. Creating S-polynomial from the pair (p_5, p_{23}).
    Skipping pair p_5 and p_{23} because gcd of their leading monoms is zero.
66. Creating S-polynomial from the pair (p_5, p_{24}).
    Skipping pair p_5 and p_{24} because gcd of their leading monoms is zero.
67. Creating S-polynomial from the pair (p_6, p_{14}).
    Skipping pair p_6 and p_{14} because gcd of their leading monoms is zero.
68. Creating S-polynomial from the pair (p_6, p_{15}).
    Skipping pair p_6 and p_{15} because gcd of their leading monoms is zero.
69. Creating S-polynomial from the pair (p_6, p_{16}).
    Skipping pair p_6 and p_{16} because gcd of their leading monoms is zero.

70. Creating S-polynomial from the pair (p_6, p_{17}).
    Skipping pair p_6 and p_{17} because gcd of their leading monoms is zero.
71. Creating S-polynomial from the pair (p_6, p_{18}).
    Skipping pair p_6 and p_{18} because gcd of their leading monoms is zero.
72. Creating S-polynomial from the pair (p_6, p_{19}).
    Skipping pair p_6 and p_{19} because gcd of their leading monoms is zero.
73. Creating S-polynomial from the pair (p_6, p_{20}).
    Skipping pair p_6 and p_{20} because gcd of their leading monoms is zero.
74. Creating S-polynomial from the pair (p_6, p_{21}).
    Skipping pair p_6 and p_{21} because gcd of their leading monoms is zero.
75. Creating S-polynomial from the pair (p_6, p_{22}).
    Skipping pair p_6 and p_{22} because gcd of their leading monoms is zero.
76. Creating S-polynomial from the pair (p_6, p_{23}).
    Skipping pair p_6 and p_{23} because gcd of their leading monoms is zero.
77. Creating S-polynomial from the pair (p_6, p_{24}).
    Skipping pair p_6 and p_{24} because gcd of their leading monoms is zero.
78. Creating S-polynomial from the pair (p_7, p_{14}).
    Skipping pair p_7 and p_{14} because gcd of their leading monoms is zero.
79. Creating S-polynomial from the pair (p_7, p_{15}).
    Skipping pair p_7 and p_{15} because gcd of their leading monoms is zero.
80. Creating S-polynomial from the pair (p_7, p_{16}).
    Skipping pair p_7 and p_{16} because gcd of their leading monoms is zero.
81. Creating S-polynomial from the pair (p_7, p_{17}).
    Skipping pair p_7 and p_{17} because gcd of their leading monoms is zero.
82. Creating S-polynomial from the pair (p_7, p_{18}).
    Skipping pair p_7 and p_{18} because gcd of their leading monoms is zero.
83. Creating S-polynomial from the pair (p_7, p_{19}).
    Skipping pair p_7 and p_{19} because gcd of their leading monoms is zero.
84. Creating S-polynomial from the pair (p_7, p_{20}).
    Skipping pair p_7 and p_{20} because gcd of their leading monoms is zero.
85. Creating S-polynomial from the pair (p_7, p_{21}).
    Skipping pair p_7 and p_{21} because gcd of their leading monoms is zero.
86. Creating S-polynomial from the pair (p_7, p_{22}).
    Skipping pair p_7 and p_{22} because gcd of their leading monoms is zero.

87. Creating S-polynomial from the pair (p_7, p_{23}).
    Skipping pair p_7 and p_{23} because gcd of their leading monoms is zero.
88. Creating S-polynomial from the pair (p_7, p_{24}).
    Skipping pair p_7 and p_{24} because gcd of their leading monoms is zero.
89. Creating S-polynomial from the pair (p_8, p_{14}).
    Skipping pair p_8 and p_{14} because gcd of their leading monoms is zero.
90. Creating S-polynomial from the pair (p_8, p_{15}).
    Skipping pair p_8 and p_{15} because gcd of their leading monoms is zero.
91. Creating S-polynomial from the pair (p_8, p_{16}).
    Skipping pair p_8 and p_{16} because gcd of their leading monoms is zero.
92. Creating S-polynomial from the pair (p_8, p_{17}).
    Skipping pair p_8 and p_{17} because gcd of their leading monoms is zero.
93. Creating S-polynomial from the pair (p_8, p_{18}).
    Skipping pair p_8 and p_{18} because gcd of their leading monoms is zero.
94. Creating S-polynomial from the pair (p_8, p_{19}).
    Skipping pair p_8 and p_{19} because gcd of their leading monoms is zero.
95. Creating S-polynomial from the pair (p_8, p_{20}).
    Skipping pair p_8 and p_{20} because gcd of their leading monoms is zero.
96. Creating S-polynomial from the pair (p_8, p_{21}).
    Skipping pair p_8 and p_{21} because gcd of their leading monoms is zero.
97. Creating S-polynomial from the pair (p_8, p_{22}).
    Skipping pair p_8 and p_{22} because gcd of their leading monoms is zero.
98. Creating S-polynomial from the pair (p_8, p_{23}).
    Skipping pair p_8 and p_{23} because gcd of their leading monoms is zero.
99. Creating S-polynomial from the pair (p_8, p_{24}).
    Skipping pair p_8 and p_{24} because gcd of their leading monoms is zero.
100. Creating S-polynomial from the pair (p_9, p_{14}).
    Skipping pair p_9 and p_{14} because gcd of their leading monoms is zero.
101. Creating S-polynomial from the pair (p_9, p_{15}).
    Skipping pair p_9 and p_{15} because gcd of their leading monoms is zero.
102. Creating S-polynomial from the pair (p_9, p_{16}).
    Skipping pair p_9 and p_{16} because gcd of their leading monoms is zero.
103. Creating S-polynomial from the pair (p_9, p_{17}).
    Skipping pair p_9 and p_{17} because gcd of their leading monoms is zero.

104. Creating S-polynomial from the pair (p_9, p_{18}).
Skipping pair p_9 and p_{18} because gcd of their leading monoms is zero.

105. Creating S-polynomial from the pair (p_9, p_{19}).
Skipping pair p_9 and p_{19} because gcd of their leading monoms is zero.

106. Creating S-polynomial from the pair (p_9, p_{20}).
Skipping pair p_9 and p_{20} because gcd of their leading monoms is zero.

107. Creating S-polynomial from the pair (p_9, p_{21}).
Skipping pair p_9 and p_{21} because gcd of their leading monoms is zero.

108. Creating S-polynomial from the pair (p_9, p_{22}).
Skipping pair p_9 and p_{22} because gcd of their leading monoms is zero.

109. Creating S-polynomial from the pair (p_9, p_{23}).
Skipping pair p_9 and p_{23} because gcd of their leading monoms is zero.

110. Creating S-polynomial from the pair (p_9, p_{24}).
Skipping pair p_9 and p_{24} because gcd of their leading monoms is zero.

111. Creating S-polynomial from the pair (p_{10}, p_{14}).
Skipping pair p_{10} and p_{14} because gcd of their leading monoms is zero.

112. Creating S-polynomial from the pair (p_{10}, p_{15}).
Skipping pair p_{10} and p_{15} because gcd of their leading monoms is zero.

113. Creating S-polynomial from the pair (p_{10}, p_{16}).
Skipping pair p_{10} and p_{16} because gcd of their leading monoms is zero.

114. Creating S-polynomial from the pair (p_{10}, p_{17}).
Skipping pair p_{10} and p_{17} because gcd of their leading monoms is zero.

115. Creating S-polynomial from the pair (p_{10}, p_{18}).
Skipping pair p_{10} and p_{18} because gcd of their leading monoms is zero.

116. Creating S-polynomial from the pair (p_{10}, p_{19}).
Skipping pair p_{10} and p_{19} because gcd of their leading monoms is zero.

117. Creating S-polynomial from the pair (p_{10}, p_{20}).
Skipping pair p_{10} and p_{20} because gcd of their leading monoms is zero.

118. Creating S-polynomial from the pair (p_{10}, p_{21}).
Skipping pair p_{10} and p_{21} because gcd of their leading monoms is zero.

119. Creating S-polynomial from the pair (p_{10}, p_{22}).
Skipping pair p_{10} and p_{22} because gcd of their leading monoms is zero.

120. Creating S-polynomial from the pair (p_{10}, p_{23}).
Skipping pair p_{10} and p_{23} because gcd of their leading monoms is zero.

121. Creating S-polynomial from the pair (p_{10}, p_{24}).
Skipping pair p_{10} and p_{24} because gcd of their leading monoms is zero.

122. Creating S-polynomial from the pair (p_{11}, p_{14}).
Skipping pair p_{11} and p_{14} because gcd of their leading monoms is zero.

123. Creating S-polynomial from the pair (p_{11}, p_{15}).
Skipping pair p_{11} and p_{15} because gcd of their leading monoms is zero.

124. Creating S-polynomial from the pair (p_{11}, p_{16}).
Skipping pair p_{11} and p_{16} because gcd of their leading monoms is zero.

125. Creating S-polynomial from the pair (p_{11}, p_{17}).
Skipping pair p_{11} and p_{17} because gcd of their leading monoms is zero.

126. Creating S-polynomial from the pair (p_{11}, p_{18}).
Skipping pair p_{11} and p_{18} because gcd of their leading monoms is zero.

127. Creating S-polynomial from the pair (p_{11}, p_{19}).
Skipping pair p_{11} and p_{19} because gcd of their leading monoms is zero.

128. Creating S-polynomial from the pair (p_{11}, p_{20}).
Skipping pair p_{11} and p_{20} because gcd of their leading monoms is zero.

129. Creating S-polynomial from the pair (p_{11}, p_{21}).
Skipping pair p_{11} and p_{21} because gcd of their leading monoms is zero.

130. Creating S-polynomial from the pair (p_{11}, p_{22}).
Skipping pair p_{11} and p_{22} because gcd of their leading monoms is zero.

131. Creating S-polynomial from the pair (p_{11}, p_{23}).
Skipping pair p_{11} and p_{23} because gcd of their leading monoms is zero.

132. Creating S-polynomial from the pair (p_{11}, p_{24}).
Skipping pair p_{11} and p_{24} because gcd of their leading monoms is zero.

133. Creating S-polynomial from the pair (p_{12}, p_{14}).
Skipping pair p_{12} and p_{14} because gcd of their leading monoms is zero.

134. Creating S-polynomial from the pair (p_{12}, p_{15}).
Skipping pair p_{12} and p_{15} because gcd of their leading monoms is zero.

135. Creating S-polynomial from the pair (p_{12}, p_{16}).
Skipping pair p_{12} and p_{16} because gcd of their leading monoms is zero.

136. Creating S-polynomial from the pair (p_{12}, p_{17}).
Skipping pair p_{12} and p_{17} because gcd of their leading monoms is zero.

137. Creating S-polynomial from the pair (p_{12}, p_{18}).
Skipping pair p_{12} and p_{18} because gcd of their leading monoms is zero.

138. Creating S-polynomial from the pair (p_{12}, p_{19}).
Skipping pair p_{12} and p_{19} because gcd of their leading monoms is zero.

139. Creating S-polynomial from the pair (p_{12}, p_{20}).
Skipping pair p_{12} and p_{20} because gcd of their leading monoms is zero.

140. Creating S-polynomial from the pair (p_{12}, p_{21}).
Skipping pair p_{12} and p_{21} because gcd of their leading monoms is zero.

141. Creating S-polynomial from the pair (p_{12}, p_{22}).
Skipping pair p_{12} and p_{22} because gcd of their leading monoms is zero.

142. Creating S-polynomial from the pair (p_{12}, p_{23}).
Skipping pair p_{12} and p_{23} because gcd of their leading monoms is zero.

143. Creating S-polynomial from the pair (p_{12}, p_{24}).
Skipping pair p_{12} and p_{24} because gcd of their leading monoms is zero.

144. Creating S-polynomial from the pair (p_{13}, p_{14}).
Skipping pair p_{13} and p_{14} because gcd of their leading monoms is zero.

145. Creating S-polynomial from the pair (p_{13}, p_{15}).
Skipping pair p_{13} and p_{15} because gcd of their leading monoms is zero.

146. Creating S-polynomial from the pair (p_{13}, p_{16}).
Skipping pair p_{13} and p_{16} because gcd of their leading monoms is zero.

147. Creating S-polynomial from the pair (p_{13}, p_{17}).
Skipping pair p_{13} and p_{17} because gcd of their leading monoms is zero.

148. Creating S-polynomial from the pair (p_{13}, p_{18}).
Skipping pair p_{13} and p_{18} because gcd of their leading monoms is zero.

149. Creating S-polynomial from the pair (p_{13}, p_{19}).
Skipping pair p_{13} and p_{19} because gcd of their leading monoms is zero.

150. Creating S-polynomial from the pair (p_{13}, p_{20}).
Skipping pair p_{13} and p_{20} because gcd of their leading monoms is zero.

151. Creating S-polynomial from the pair (p_{13}, p_{21}).
Skipping pair p_{13} and p_{21} because gcd of their leading monoms is zero.

152. Creating S-polynomial from the pair (p_{13}, p_{22}).
Skipping pair p_{13} and p_{22} because gcd of their leading monoms is zero.

153. Creating S-polynomial from the pair (p_{13}, p_{23}).
Skipping pair p_{13} and p_{23} because gcd of their leading monoms is zero.

154. Creating S-polynomial from the pair (p_{13}, p_{24}).
Skipping pair p_{13} and p_{24} because gcd of their leading monoms is zero.

155. Creating S-polynomial from the pair (p_{14}, p_{15}).
Forming S-pol of p_{14} and p_{15}:

$$p_{73} = 0$$

Reduced to zero.

156. Creating S-polynomial from the pair (p_{14}, p_{16}).
Forming S-pol of p_{14} and p_{16}:

$$p_{74} = 0$$

Reduced to zero.

157. Creating S-polynomial from the pair (p_{14}, p_{17}).
Forming S-pol of p_{14} and p_{17}:

$$p_{75} = 0$$

Reduced to zero.

158. Creating S-polynomial from the pair (p_{14}, p_{18}).
Forming S-pol of p_{14} and p_{18}:

$$p_{76} = 0$$

Reduced to zero.

159. Creating S-polynomial from the pair (p_{14}, p_{19}).
Forming S-pol of p_{14} and p_{19}:

$$p_{77} = 0$$

Reduced to zero.

160. Creating S-polynomial from the pair (p_{14}, p_{20}).
Forming S-pol of p_{14} and p_{20}:

$$p_{78} = 0$$

Reduced to zero.

161. Creating S-polynomial from the pair (p_{14}, p_{21}).
Forming S-pol of p_{14} and p_{21}:

$$p_{79} = 0$$

Reduced to zero.

162. Creating S-polynomial from the pair (p_{14}, p_{22}).
Forming S-pol of p_{14} and p_{22}:

$$p_{80} = 0$$

Reduced to zero.

163. Creating S-polynomial from the pair (p_{14}, p_{23}).
Forming S-pol of p_{14} and p_{23}:

$$p_{81} = 0$$

Reduced to zero.

164. Creating S-polynomial from the pair (p_{14}, p_{24}).
Forming S-pol of p_{14} and p_{24}:

$$p_{82} = 0$$

Reduced to zero.

165. Creating S-polynomial from the pair (p_{15}, p_{16}).
Forming S-pol of p_{15} and p_{16}:

$$p_{83} = 0$$

Reduced to zero.

166. Creating S-polynomial from the pair (p_{15}, p_{17}).
Forming S-pol of p_{15} and p_{17}:

$$p_{84} = 0$$

Reduced to zero.

167. Creating S-polynomial from the pair (p_{15}, p_{18}).
Forming S-pol of p_{15} and p_{18}:

$$p_{85} = 0$$

Reduced to zero.

168. Creating S-polynomial from the pair (p_{15}, p_{19}).
Forming S-pol of p_{15} and p_{19}:

$$p_{86} = 0$$

Reduced to zero.

169. Creating S-polynomial from the pair (p_{15}, p_{20}).
Forming S-pol of p_{15} and p_{20}:

$$p_{87} = 0$$

Reduced to zero.

170. Creating S-polynomial from the pair (p_{15}, p_{21}).
Forming S-pol of p_{15} and p_{21}:

$$p_{88} = 0$$

Reduced to zero.

171. Creating S-polynomial from the pair (p_{15}, p_{22}).
Forming S-pol of p_{15} and p_{22}:

$$p_{89} = 0$$

Reduced to zero.

172. Creating S-polynomial from the pair (p_{15}, p_{23}).
Forming S-pol of p_{15} and p_{23}:

$$p_{90} = 0$$

Reduced to zero.

173. Creating S-polynomial from the pair (p_{15}, p_{24}).
Forming S-pol of p_{15} and p_{24}:

$$p_{91} = 0$$

Reduced to zero.

174. Creating S-polynomial from the pair (p_{16}, p_{17}).
Forming S-pol of p_{16} and p_{17}:

$$p_{92} = 0$$

Reduced to zero.

175. Creating S-polynomial from the pair (p_{16}, p_{18}).
Forming S-pol of p_{16} and p_{18}:

$$p_{93} = 0$$

Reduced to zero.

176. Creating S-polynomial from the pair (p_{16}, p_{19}).
Forming S-pol of p_{16} and p_{19}:

$$p_{94} = 0$$

Reduced to zero.

177. Creating S-polynomial from the pair (p_{16}, p_{20}).
Forming S-pol of p_{16} and p_{20}:

$$p_{95} = 0$$

Reduced to zero.

178. Creating S-polynomial from the pair (p_{16}, p_{21}).
Forming S-pol of p_{16} and p_{21}:

$$p_{96} = 0$$

Reduced to zero.

179. Creating S-polynomial from the pair (p_{16}, p_{22}).
Forming S-pol of p_{16} and p_{22}:

$$p_{97} = 0$$

Reduced to zero.

180. Creating S-polynomial from the pair (p_{16}, p_{23}).
Forming S-pol of p_{16} and p_{23}:

$$p_{98} = 0$$

Reduced to zero.

181. Creating S-polynomial from the pair (p_{16}, p_{24}).
Forming S-pol of p_{16} and p_{24}:

$$p_{99} = 0$$

Reduced to zero.

182. Creating S-polynomial from the pair (p_{17}, p_{18}).
Forming S-pol of p_{17} and p_{18}:

$$p_{100} = 0$$

Reduced to zero.

183. Creating S-polynomial from the pair (p_{17}, p_{19}).
Forming S-pol of p_{17} and p_{19}:

$$p_{101} = 0$$

Reduced to zero.

184. Creating S-polynomial from the pair (p_{17}, p_{20}).
Forming S-pol of p_{17} and p_{20}:

$$p_{102} = 0$$

Reduced to zero.

185. Creating S-polynomial from the pair (p_{17}, p_{21}).
Forming S-pol of p_{17} and p_{21}:

$$p_{103} = 0$$

Reduced to zero.

186. Creating S-polynomial from the pair (p_{17}, p_{22}).
Forming S-pol of p_{17} and p_{22}:

$$p_{104} = 0$$

Reduced to zero.

187. Creating S-polynomial from the pair (p_{17}, p_{23}).
Forming S-pol of p_{17} and p_{23}:

$$p_{105} = 0$$

Reduced to zero.

188. Creating S-polynomial from the pair (p_{17}, p_{24}).
Forming S-pol of p_{17} and p_{24}:

$$p_{106} = 0$$

Reduced to zero.

189. Creating S-polynomial from the pair (p_{18}, p_{19}).
Forming S-pol of p_{18} and p_{19}:

$$p_{107} = 0$$

Reduced to zero.

190. Creating S-polynomial from the pair (p_{18}, p_{20}).
Forming S-pol of p_{18} and p_{20}:

$$p_{108} = 0$$

Reduced to zero.

191. Creating S-polynomial from the pair (p_{18}, p_{21}).
Forming S-pol of p_{18} and p_{21}:

$$p_{109} = 0$$

Reduced to zero.

192. Creating S-polynomial from the pair (p_{18}, p_{22}).
Forming S-pol of p_{18} and p_{22}:

$$p_{110} = 0$$

Reduced to zero.

193. Creating S-polynomial from the pair (p_{18}, p_{23}).
Forming S-pol of p_{18} and p_{23}:

$$p_{111} = 0$$

Reduced to zero.

194. Creating S-polynomial from the pair (p_{18}, p_{24}).
Forming S-pol of p_{18} and p_{24}:

$$p_{112} = 0$$

Reduced to zero.

195. Creating S-polynomial from the pair (p_{19}, p_{20}).
Forming S-pol of p_{19} and p_{20}:

$$p_{113} = 0$$

Reduced to zero.

196. Creating S-polynomial from the pair (p_{19}, p_{21}).
Forming S-pol of p_{19} and p_{21}:

$$p_{114} = 0$$

Reduced to zero.

197. Creating S-polynomial from the pair (p_{19}, p_{22}).
Forming S-pol of p_{19} and p_{22}:

$$p_{115} = 0$$

Reduced to zero.

198. Creating S-polynomial from the pair (p_{19}, p_{23}).
Forming S-pol of p_{19} and p_{23}:

$$p_{116} = 0$$

Reduced to zero.

199. Creating S-polynomial from the pair (p_{19}, p_{24}).
Forming S-pol of p_{19} and p_{24}:

$$p_{117} = 0$$

Reduced to zero.

200. Creating S-polynomial from the pair (p_{20}, p_{21}).
Forming S-pol of p_{20} and p_{21}:

$$p_{118} = 0$$

Reduced to zero.

201. Creating S-polynomial from the pair (p_{20}, p_{22}).
Forming S-pol of p_{20} and p_{22}:

$$p_{119} = 0$$

Reduced to zero.

202. Creating S-polynomial from the pair (p_{20}, p_{23}).
Forming S-pol of p_{20} and p_{23}:

$$p_{120} = 0$$

Reduced to zero.

203. Creating S-polynomial from the pair (p_{20}, p_{24}).
Forming S-pol of p_{20} and p_{24}:

$$p_{121} = 0$$

Reduced to zero.

204. Creating S-polynomial from the pair (p_{21}, p_{22}).
Forming S-pol of p_{21} and p_{22}:

$$p_{122} = 0$$

Reduced to zero.

205. Creating S-polynomial from the pair (p_{21}, p_{23}).
Forming S-pol of p_{21} and p_{23}:

$$p_{123} = 0$$

Reduced to zero.

206. Creating S-polynomial from the pair (p_{21}, p_{24}).
Forming S-pol of p_{21} and p_{24}:

$$p_{124} = 0$$

Reduced to zero.

207. Creating S-polynomial from the pair (p_{22}, p_{23}).
Forming S-pol of p_{22} and p_{23}:

$$p_{125} = 0$$

Reduced to zero.

208. Creating S-polynomial from the pair (p_{22}, p_{24}).
Forming S-pol of p_{22} and p_{24}:

$$p_{126} = 0$$

Reduced to zero.

209. Creating S-polynomial from the pair (p_{23}, p_{24}).
Forming S-pol of p_{23} and p_{24}:

$$p_{127} = 0$$

Reduced to zero.

5.4 Groebner Basis

Groebner basis has 25 polynomials:

$$
\begin{aligned}
p_0 &= u_3x_1 + (-u_3^2 - u_2^2 + u_2u_1)\\
p_1 &= -x_3 - x_1\\
p_2 &= x_6x_1 - u_3x_6 + (u_2 - u_1)x_5 - u_2x_1 + u_3u_1\\
p_3 &= -x_6 + (2u_2 - u_1)\\
p_4 &= 2x_7x_3 - 2x_7x_1 - x_3^2 + x_1^2\\
p_5 &= -x_{10}x_5 + x_{10}x_1 + x_9x_6 - u_1x_9 - x_6x_1 + u_1x_5\\
p_6 &= 2x_{10}x_6 - 2u_1x_{10} + 2x_9x_5 - 2x_9x_1 - x_6^2 - x_5^2 + x_1^2 + u_1^2\\
p_7 &= (-u_2 + u_1)x_{12} + (u_2 - u_1)x_{10} - x_9x_1 + u_3x_9 + x_7x_1 - u_3x_7\\
p_8 &= (u_3^2u_2 - u_3^2u_1)x_5 +\\
&\quad (-u_3^3u_2 + u_3^3u_1 + u_3u_2^3 - 2u_3u_2^2u_1 + u_3u_2u_1^2)\\
p_9 &= (-4u_3^4 - 4u_3^2u_2^2 + 4u_3^2u_2u_1)x_7\\
p_{10} &= (u_3u_2 - u_3u_1)x_5 + (-u_3^2u_2 + u_3^2u_1 + u_2^3 - 2u_2^2u_1 + u_2u_1^2)\\
p_{11} &= (4u_3^{10}u_2 - 4u_3^{10}u_1)x_{10} + (2u_3^{10} + 2u_3^8u_2^2 - 2u_3^8u_2u_1)x_9x_5 +\\
&\quad (-2u_3^{11} - 8u_3^9u_2^2 + 12u_3^9u_2u_1 - 4u_3^9u_1^2 - 2u_3^7u_2^4 + 4u_3^7u_2^3u_1 -\\
&\quad 2u_3^7u_2^2u_1^2)x_9 + (-u_3^{10} - u_3^8u_2^2 + u_3^8u_2u_1)x_5^2 +\\
&\quad (-2u_3^9u_2u_1 + 2u_3^9u_1^2)x_5 +\\
&\quad (u_3^{12} + 3u_3^{10}u_2^2 - 5u_3^{10}u_2u_1 + 2u_3^{10}u_1^2 + 3u_3^8u_2^4 - 8u_3^8u_2^3u_1 +\\
&\quad 7u_3^8u_2^2u_1^2 - 2u_3^8u_2u_1^3 + u_3^6u_2^6 - 3u_3^6u_2^5u_1 +\\
&\quad 3u_3^6u_2^4u_1^2 - u_3^6u_2^3u_1^3)\\
p_{12} &= (4u_3^3u_2 - 4u_3^3u_1)x_{10} + 2u_3^3x_9x_5 +\\
&\quad (-2u_3^4 - 2u_3^2u_2^2 + 2u_3^2u_2u_1)x_9 - u_3^3x_5^2 +\\
&\quad (u_3^5 - 2u_3^3u_2^2 + 2u_3^3u_2u_1 + u_3u_2^4 - 2u_3u_2^3u_1 + u_3u_2^2u_1^2)\\
p_{13} &= (\\
&\quad 4u_3^8u_2 - 4u_3^8u_1 + 4u_3^6u_2^3 - 8u_3^6u_2^2u_1 + 4u_3^6u_2u_1^2)x_{10} +\\
&\quad 2u_3^7x_9x_5^2 + (-2u_3^8 - 2u_3^6u_2^2 + 2u_3^6u_2u_1)x_9x_5 +\\
&\quad (8u_3^7u_2^2 - 16u_3^7u_2u_1 + 8u_3^7u_1^2)x_9 - u_3^7x_5^3 +\\
&\quad (u_3^9 - 2u_3^7u_2^2 + 6u_3^7u_2u_1 - 4u_3^7u_1^2 + u_3^5u_2^4 - 2u_3^5u_2^3u_1 +\\
&\quad u_3^5u_2^2u_1^2)x_5 +\\
&\quad (-8u_3^8u_2^2 + 12u_3^8u_2u_1 - 4u_3^8u_1^2 - 8u_3^6u_2^4 + 20u_3^6u_2^3u_1 -\\
&\quad 16u_3^6u_2^2u_1^2 + 4u_3^6u_2u_1^3)\\
p_{14} &= \ldots\\
p_{15} &= \ldots\\
p_{16} &= \ldots\\
p_{17} &= \ldots\\
p_{18} &= \ldots\\
p_{19} &= (\\
&\quad 64u_3^{28}u_2^7 - 448u_3^{28}u_2^6u_1 + 1344u_3^{28}u_2^5u_1^2 - 2240u_3^{28}u_2^4u_1^3 +
\end{aligned}
$$

$$
\begin{aligned}
& 2240u_3^{28}u_2^3u_1^4 - 1344u_3^{28}u_2^2u_1^5 + 448u_3^{28}u_2u_1^6 - 64u_3^{28}u_1^7 + \\
& 64u_3^{26}u_2^9 - 448u_3^{26}u_2^8u_1 + 1344u_3^{26}u_2^7u_1^2 - 2240u_3^{26}u_2^6u_1^3 + \\
& 2240u_3^{26}u_2^5u_1^4 - 1344u_3^{26}u_2^4u_1^5 + 448u_3^{26}u_2^3u_1^6 - \\
& 64u_3^{26}u_2^2u_1^7)x_9 + \\
& (-64u_3^{29}u_2^7 + 448u_3^{29}u_2^6u_1 - 1344u_3^{29}u_2^5u_1^2 + 2240u_3^{29}u_2^4u_1^3 - \\
& 2240u_3^{29}u_2^3u_1^4 + 1344u_3^{29}u_2^2u_1^5 - 448u_3^{29}u_2u_1^6 + 64u_3^{29}u_1^7 - \\
& 64u_3^{27}u_2^9 + 448u_3^{27}u_2^8u_1 - 1344u_3^{27}u_2^7u_1^2 + 2240u_3^{27}u_2^6u_1^3 - \\
& 2240u_3^{27}u_2^5u_1^4 + 1344u_3^{27}u_2^4u_1^5 - 448u_3^{27}u_2^3u_1^6 + \\
& 64u_3^{27}u_2^2u_1^7)
\end{aligned}
$$

$$
\begin{aligned}
p_{20} = \; & (\\
& 32u_3^{21}u_2^6u_1 - 192u_3^{21}u_2^5u_1^2 + 480u_3^{21}u_2^4u_1^3 - \\
& 640u_3^{21}u_2^3u_1^4 + 480u_3^{21}u_2^2u_1^5 - 192u_3^{21}u_2u_1^6 + 32u_3^{21}u_1^7 + \\
& 32u_3^{19}u_2^8u_1 - 192u_3^{19}u_2^7u_1^2 + 480u_3^{19}u_2^6u_1^3 - \\
& 640u_3^{19}u_2^5u_1^4 + 480u_3^{19}u_2^4u_1^5 - 192u_3^{19}u_2^3u_1^6 + \\
& 32u_3^{19}u_2^2u_1^7)x_9 + \\
& (-32u_3^{22}u_2^6u_1 + 192u_3^{22}u_2^5u_1^2 - 480u_3^{22}u_2^4u_1^3 + \\
& 640u_3^{22}u_2^3u_1^4 - 480u_3^{22}u_2^2u_1^5 + 192u_3^{22}u_2u_1^6 - 32u_3^{22}u_1^7 - \\
& 32u_3^{20}u_2^8u_1 + 192u_3^{20}u_2^7u_1^2 - 480u_3^{20}u_2^6u_1^3 + \\
& 640u_3^{20}u_2^5u_1^4 - 480u_3^{20}u_2^4u_1^5 + 192u_3^{20}u_2^3u_1^6 - \\
& 32u_3^{20}u_2^2u_1^7)
\end{aligned}
$$

$$
\begin{aligned}
p_{21} = \; & \ldots \\
p_{22} = \; & (\\
& -16u_3^{18}u_2^6 + 96u_3^{18}u_2^5u_1 - 240u_3^{18}u_2^4u_1^2 + 320u_3^{18}u_2^3u_1^3 - \\
& 240u_3^{18}u_2^2u_1^4 + 96u_3^{18}u_2u_1^5 - 16u_3^{18}u_1^6 - 16u_3^{16}u_2^8 + \\
& 96u_3^{16}u_2^7u_1 - 240u_3^{16}u_2^6u_1^2 + 320u_3^{16}u_2^5u_1^3 - \\
& 240u_3^{16}u_2^4u_1^4 + 96u_3^{16}u_2^3u_1^5 - 16u_3^{16}u_2^2u_1^6)x_9 + \\
& (16u_3^{19}u_2^6 - 96u_3^{19}u_2^5u_1 + 240u_3^{19}u_2^4u_1^2 - 320u_3^{19}u_2^3u_1^3 + \\
& 240u_3^{19}u_2^2u_1^4 - 96u_3^{19}u_2u_1^5 + 16u_3^{19}u_1^6 + 16u_3^{17}u_2^8 - \\
& 96u_3^{17}u_2^7u_1 + 240u_3^{17}u_2^6u_1^2 - 320u_3^{17}u_2^5u_1^3 + \\
& 240u_3^{17}u_2^4u_1^4 - 96u_3^{17}u_2^3u_1^5 + 16u_3^{17}u_2^2u_1^6)
\end{aligned}
$$

$$
\begin{aligned}
p_{23} = \; & \ldots \\
p_{24} = \; & \ldots
\end{aligned}
$$

Groebner basis succesfully computed.

6 Reducing Polynomial Conjecture

Reducing with polynomial p_{11}, the result is:

$$p_{145} = (-4u_3^{10} - 4u_3^8u_2^2 + 4u_3^8u_2u_1)x_{10}x_9x_5 +$$

$$(4u_3^{11} + 16u_3^9u_2^2 - 24u_3^9u_2u_1 + 8u_3^9u_1^2 + 4u_3^7u_2^4 - 8u_3^7u_2^3u_1 +$$
$$4u_3^7u_2^2u_1^2)x_{10}x_9 + (2u_3^{10} + 2u_3^8u_2^2 - 2u_3^8u_2u_1)x_{10}x_5^2 +$$
$$(4u_3^9u_2u_1 - 4u_3^9u_1^2)x_{10}x_5 +$$
$$(-2u_3^{12} - 22u_3^{10}u_2^2 + 26u_3^{10}u_2u_1 - 4u_3^{10}u_1^2 - 6u_3^8u_2^4 +$$
$$16u_3^8u_2^3u_1 - 14u_3^8u_2^2u_1^2 + 4u_3^8u_2u_1^3 - 2u_3^6u_2^6 +$$
$$6u_3^6u_2^5u_1 - 6u_3^6u_2^4u_1^2 + 2u_3^6u_2^3u_1^3)x_{10} +$$
$$(8u_3^{10}u_2 - 8u_3^{10}u_1)x_9^2 + (-16u_3^{11}u_2 + 16u_3^{11}u_1)x_9 +$$
$$(8u_3^{12}u_2 - 8u_3^{12}u_1 + 8u_3^{10}u_2^3 - 8u_3^{10}u_2^2u_1)$$

Reducing with polynomial p_5, the result is:

Polynomial too big for output (text size is 1091 characters, number of terms is 11)

Reducing with polynomial p_0, the result is:

$$p_{146} = (-8u_3^{10}u_2^2 + 16u_3^{10}u_2u_1 - 8u_3^{10}u_1^2)x_{10}x_9 +$$
$$(-2u_3^{11} - 2u_3^9u_2^2 + 2u_3^9u_2u_1)x_{10}x_5^2 +$$
$$(-4u_3^{10}u_2u_1 + 4u_3^{10}u_1^2)x_{10}x_5 +$$
$$(2u_3^{13} + 22u_3^{11}u_2^2 - 26u_3^{11}u_2u_1 + 4u_3^{11}u_1^2 + 6u_3^9u_2^4 -$$
$$16u_3^9u_2^3u_1 + 14u_3^9u_2^2u_1^2 - 4u_3^9u_2u_1^3 + 2u_3^7u_2^6 -$$
$$6u_3^7u_2^5u_1 + 6u_3^7u_2^4u_1^2 - 2u_3^7u_2^3u_1^3)x_{10} +$$
$$(4u_3^{11} + 4u_3^9u_2^2 - 4u_3^9u_2u_1)x_9^2x_6 +$$
$$(-8u_3^{11}u_2 + 4u_3^{11}u_1 - 4u_3^9u_2^2u_1 + 4u_3^9u_2u_1^2)x_9^2 +$$
$$(-4u_3^{11} - 4u_3^9u_2^2 + 4u_3^9u_2u_1)x_9x_6x_1 +$$
$$(4u_3^{11}u_1 + 4u_3^9u_2^2u_1 - 4u_3^9u_2u_1^2)x_9x_5 +$$
$$(16u_3^{12}u_2 - 16u_3^{12}u_1)x_9 +$$
$$(-8u_3^{13}u_2 + 8u_3^{13}u_1 - 8u_3^{11}u_2^3 + 8u_3^{11}u_2^2u_1)$$

Reducing with polynomial p_{11}, the result is:

Polynomial too big for output (text size is 2459 characters, number of terms is 11)

Reducing with polynomial p_5, the result is:

Polynomial too big for output (text size is 2842 characters, number of terms is 14)

Reducing with polynomial p_0, the result is:

Polynomial too big for output (text size is 2881 characters, number of terms is 13)

Reducing with polynomial p_5, the result is:

Polynomial too big for output (text size is 3665 characters, number of terms is 16)

Reducing with polynomial p_0, the result is:
Polynomial too big for output (text size is 3213 characters, number of terms is 15)

Reducing with polynomial p_{11}, the result is:
Polynomial too big for output (text size is 5289 characters, number of terms is 14)

Reducing with polynomial p_3, the result is:
Polynomial too big for output (text size is 5090 characters, number of terms is 13)

Reducing with polynomial p_8, the result is:
Polynomial too big for output (text size is 5877 characters, number of terms is 12)

Reducing with polynomial p_{14}, the result is:
Polynomial too big for output (number of terms is 11) Reducing with polynomial p_3, the result is:
Polynomial too big for output (number of terms is 10) Reducing with polynomial p_0, the result is:
Polynomial too big for output (number of terms is 9) Reducing with polynomial p_3, the result is:
Polynomial too big for output (number of terms is 8) Reducing with polynomial p_8, the result is:
Polynomial too big for output (number of terms is 7) Reducing with polynomial p_8, the result is:
Polynomial too big for output (number of terms is 6) Reducing with polynomial p_{14}, the result is:
Polynomial too big for output (number of terms is 5) Reducing with polynomial p_0, the result is:
Polynomial too big for output (number of terms is 5) Reducing with polynomial p_3, the result is:
Polynomial too big for output (number of terms is 4) Reducing with polynomial p_0, the result is:
Polynomial too big for output (number of terms is 4) Reducing with polynomial p_3, the result is:
Polynomial too big for output (number of terms is 3) Reducing with polynomial p_8, the result is:
Polynomial too big for output (number of terms is 2) Reducing with polynomial p_8, the result is:

$$p_{147} = 0$$

Conclusion is reduced to zero.

7 Prover report

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 797 terms.

Time Complexity: Time spent by the prover: 0.955 seconds. There are no ndg conditions.