Composite Structures with Partial Interaction

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Based on lectures by Prof. Roberto Crocetti, LTH

Some favorable properties of timber

- Environmentally friendly
- Specific strength and stiffness
- Low weight-Easy to prefabricate, transport and erect

- Easy (and cheap) to shape

- …and much more (aesthetics, low price, availability, workability, etc.)

However, timber has also some unfavorable properties…

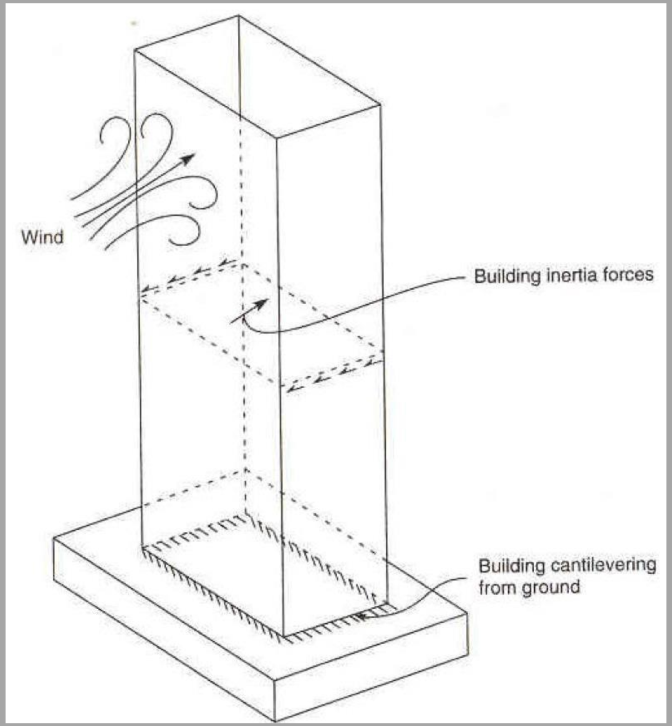
*Some **unfavorable** properties of timber:*

1. Low mass

2. Low Young’s modulus

3. High variability of mechanical properties and

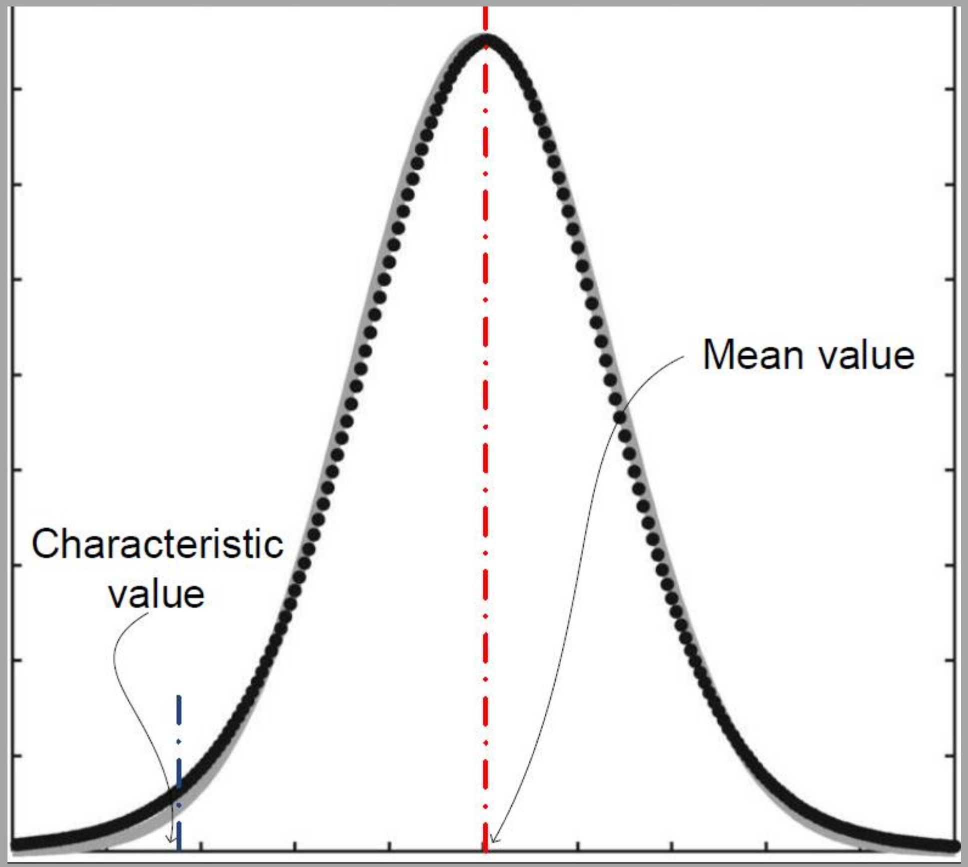
Low mass
Low Young's modulus



- Acoustics and vibrations
- Deep floor structure

- Tilting
- Wind-induced vibration

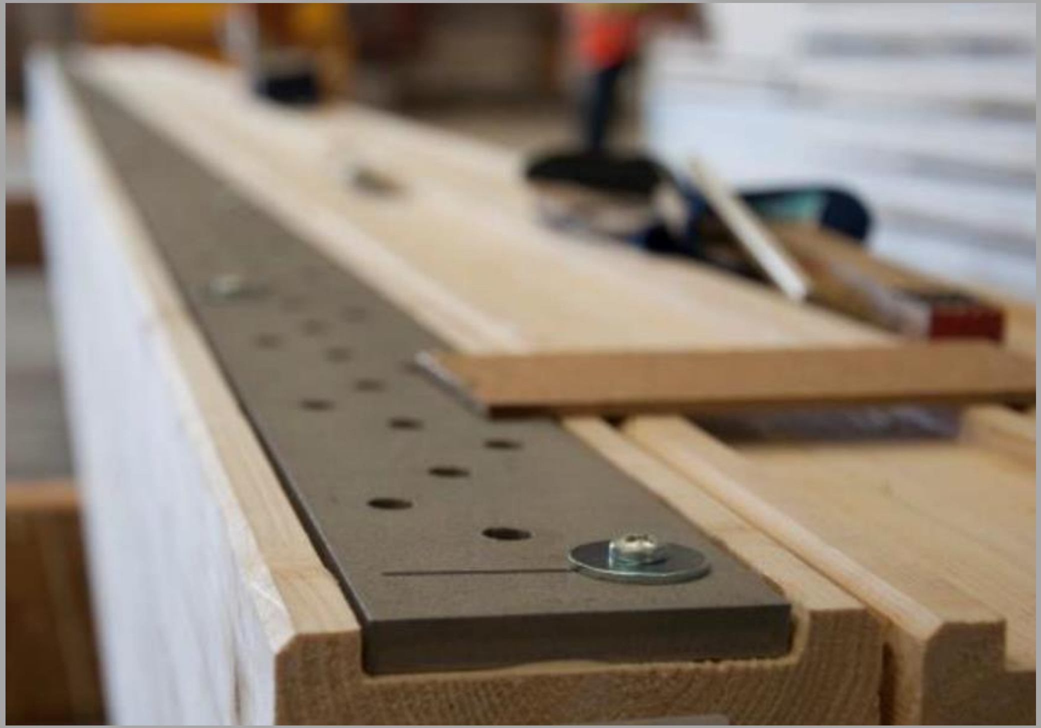
High variability of mechanical properties



Large scatter → low characteristic strength

So, what is the solution?!

Hybrid structures 1: timber + steel or FRP



Glulam beams reinforced by means of steel plate, Lund University, 2014

- Glulam beams 115x270, L=6m, both unreinforced and reinforced with glued steel plate 10x80 mm^2.
- Increase of both strength and stiffness by approx. 80%
- Ductile behaviour if steel plate is located at tension side of the beam.
- Significant reducing in scatter

Hybrid structures 2: prefab floor timber + concrete

Prefabricated concrete floor, span 8 m

- Significant increase of stiffness → reduce problem with vibration
- Increase of mass → better stability against overturning/tilting, and better acoustic performance
- Reduced depth of floor → better economy/ saving space

Question:

Are the conventional composite beam theories valid for analysis of these hybrid structural elements?

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Definitely not!

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Definitely not!

Why? and what is the solution?
(Lets think & discuss…)

*For hybrid structural elements having **Connections**:*

- **By glue:**

  Slip between the parts may not be much of a trouble!

  But, slip must be considered when quality of the glue-line cannot be relied upon, or having thick and soft core layer (e.g. foam core, in sandwich elements…

- **By mechanical connectors:**

  slipping between member parts must be considered!

*For hybrid structural elements having **Connections**:*

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  Slip between the parts may not be much of a trouble!

  But, slip must be considered when quality of the glue-line cannot be relied upon, or having thick and soft layer of glue/core layer, like sandwich elements with foam core…

- **By mechanical connectors:**

  slipping between member parts must be considered!

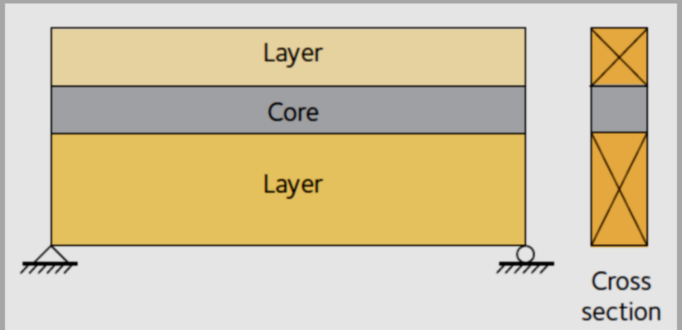
*Then, we employ **Partial Composite Theory** for these cases*

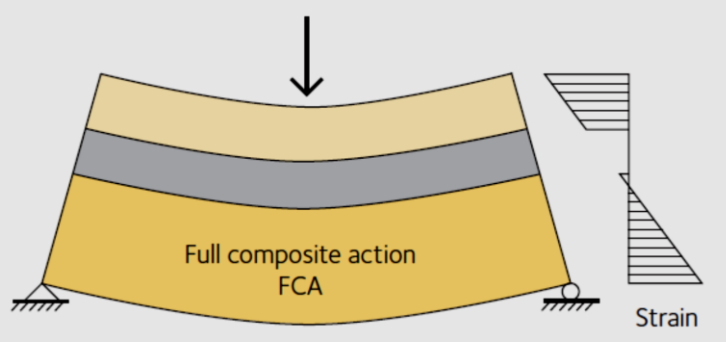
Partial Composite Theory

Generalized for both ***soft core*** and ***mechanical connectors***

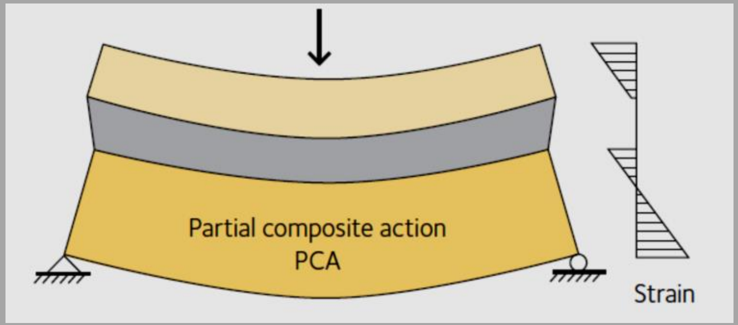
Partial Composite Interaction:

meant that the shear deformation between separate parts is **non-negligible**!

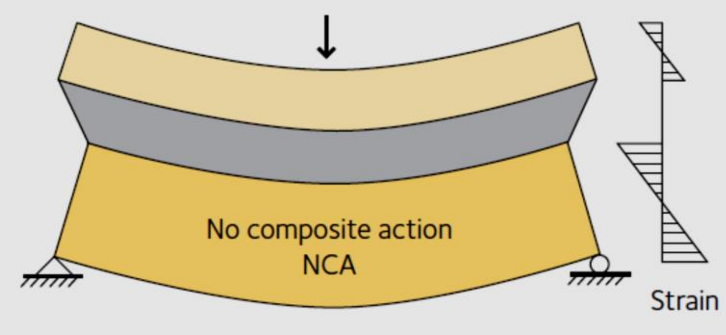




full composite action (FCA)
(if the shear connection is *infinitely stiff*)



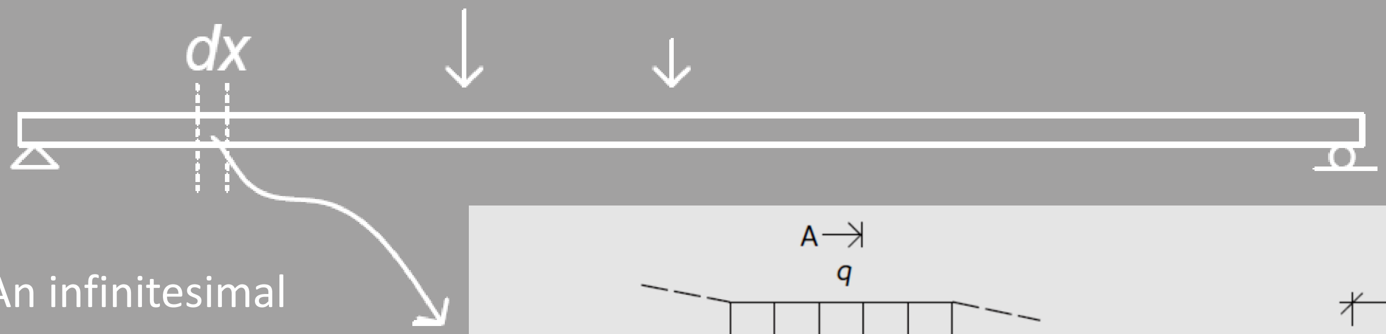
partial composite action (PCA)
if the shear connection has a finite stiffness, "true" response!



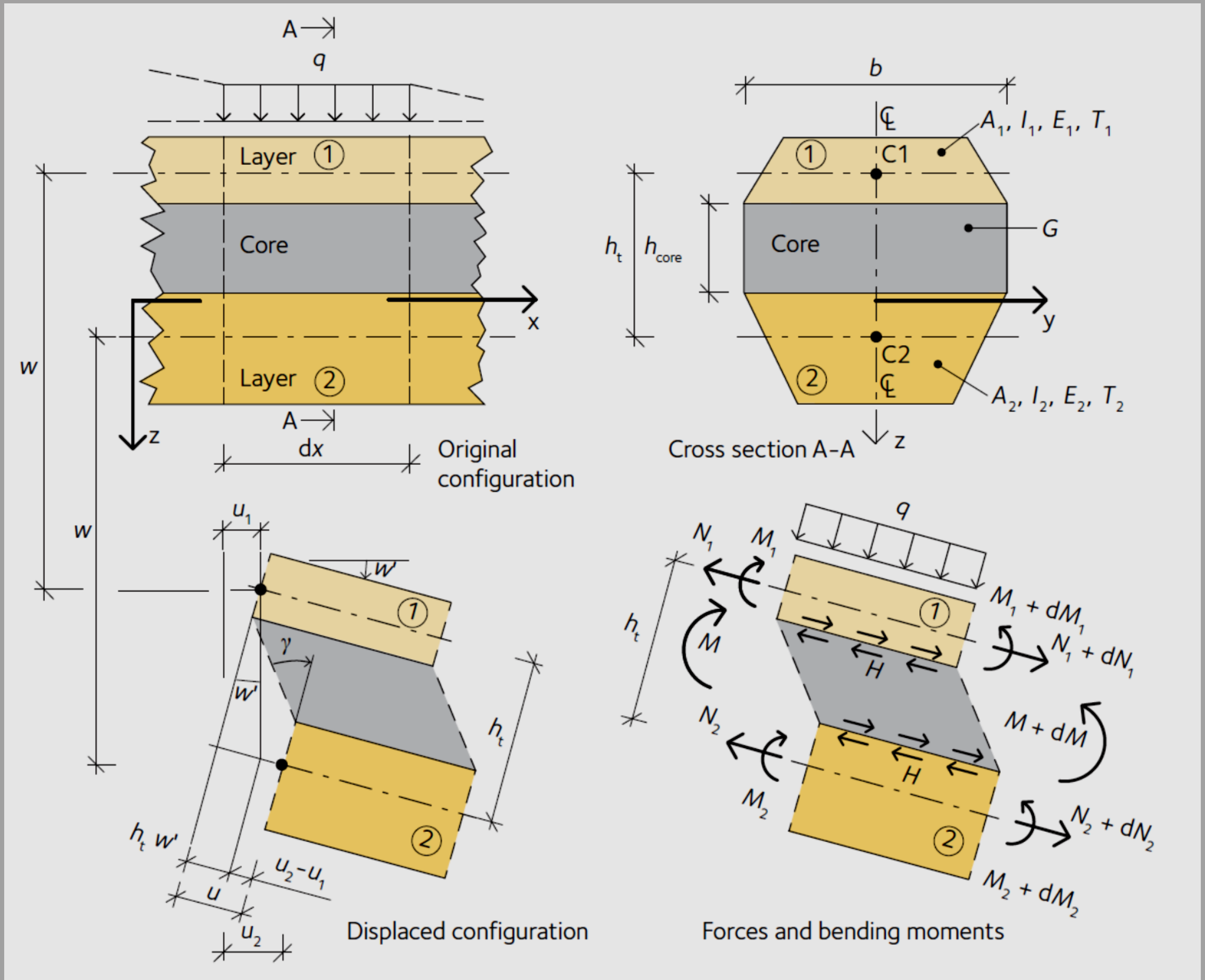
Non composite action (NCA)
(if the shear connection has *no stiffness at all*)
Independent-layers: no shear forces are transferred along the joint line

Assumptions of PC theory

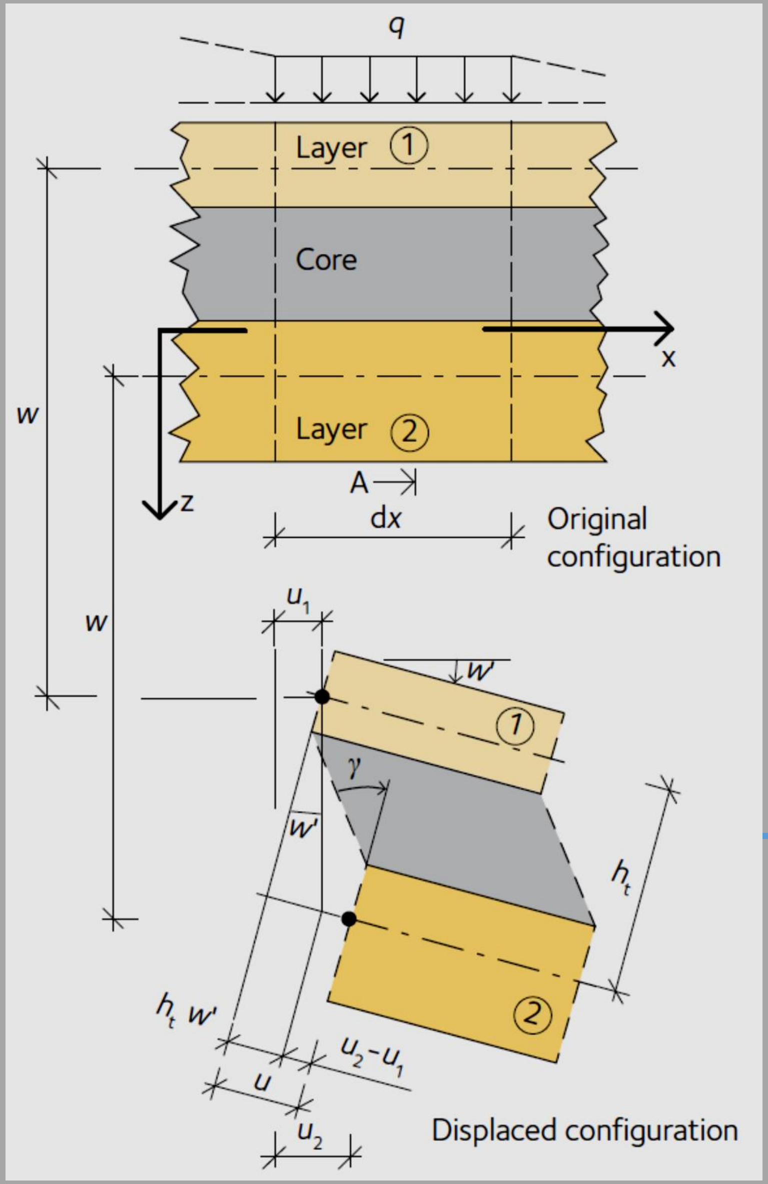
- All material remains linear elastic.
- The layers must have equal deflections, i.e. no separation, no tensile or compressive stresses in thickness direction.
- All layers have equal radius of curvature => the thickness of the element is much less than the radius of curvature.
- No shear deformation within layers is considered, only the shear deformation of core/interface is accounted for.
- Any influence of straining perpendicular to the longitudinal axis is neglected. This implies that plane cross-sections remain plain, holds for each layer.
- The core only have shear stiffness and its only purpose is to act as a shear connection between the two layers.
- In addition to the strain difference the only allowed load is a bending moment caused by some external load acting in the transverse direction. External axial loads are not accounted for.
- The model can only account for deflections within vertical plane through longitudinal direction and the cross-section must, therefore, be symmetric about the vertical axis.



An infinitesimal segment
cut out along the
longitudinal axis



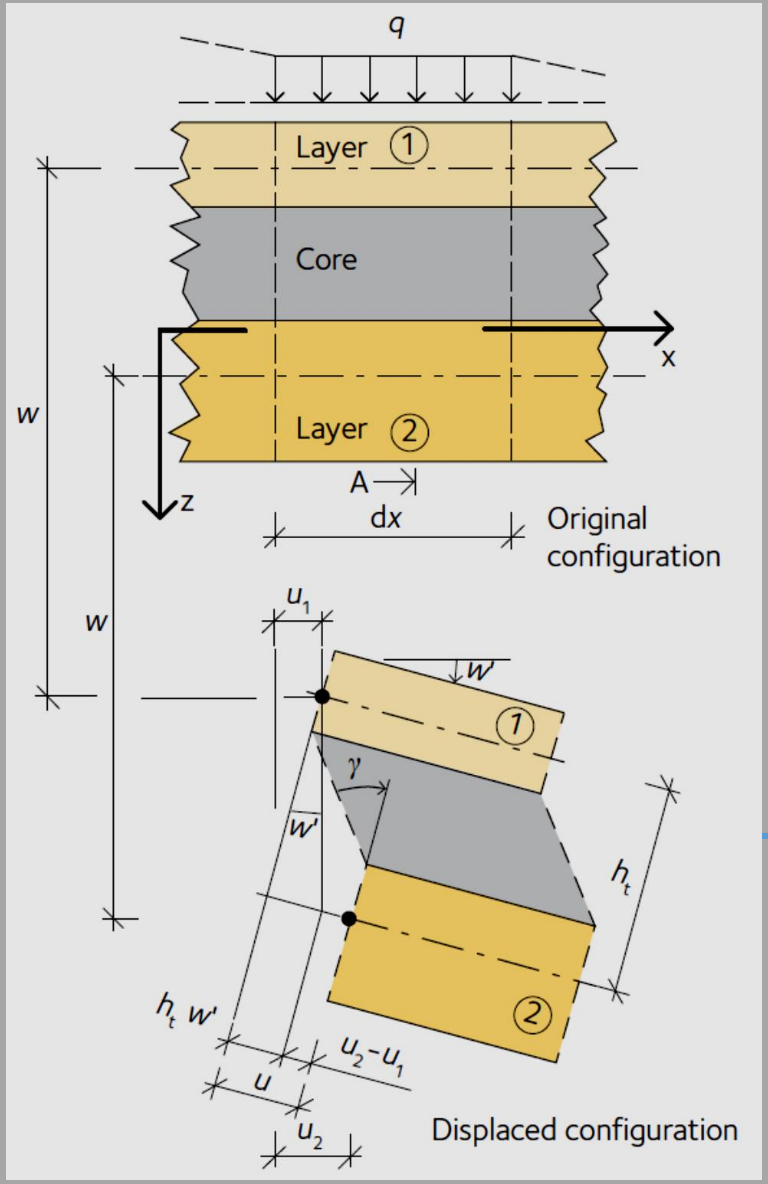
Horizontal displacements of the parts



Relative displacement between the layers (slip):

$$u = u_2 - u_1 + h_t w'$$

Horizontal displacements of the parts

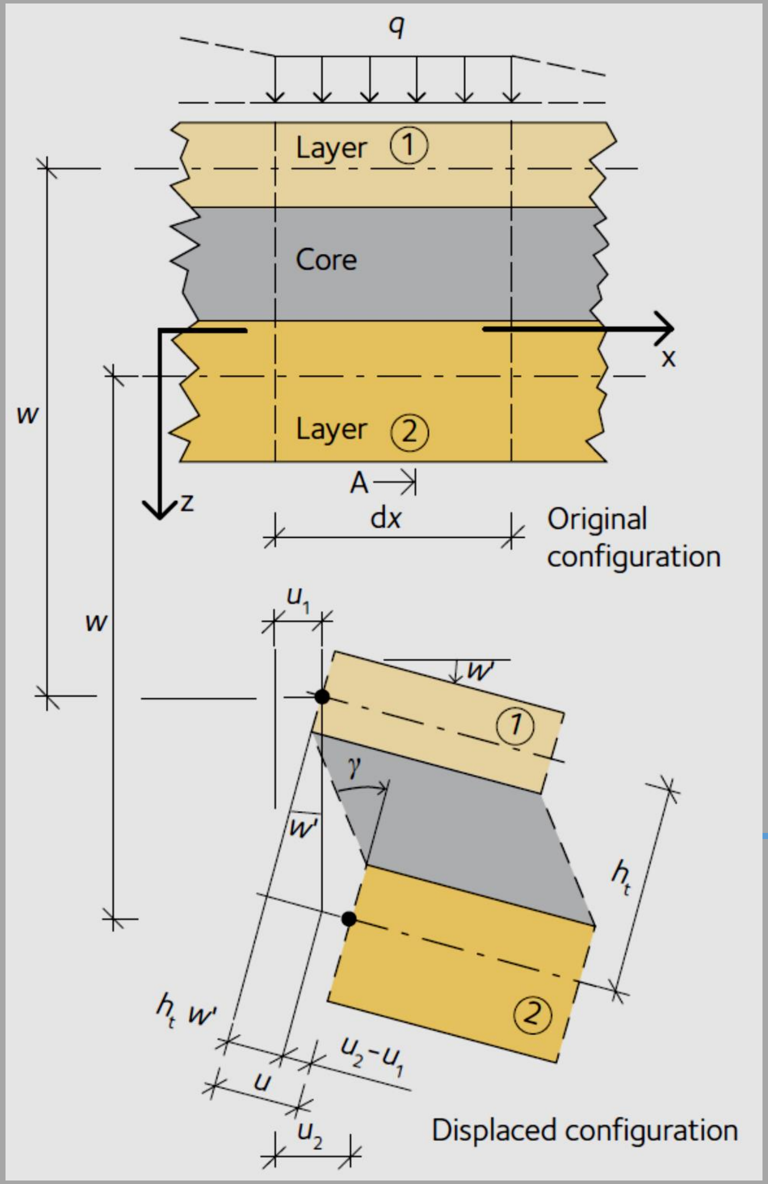


Relative displacement between the layers (slip):

$$u = u_2 - u_1 + h_t w'$$

$$u' = u_2' - u_1' + h_t w'' = \varepsilon_2 - \varepsilon_1 + h_t w''$$

Horizontal displacements of the parts



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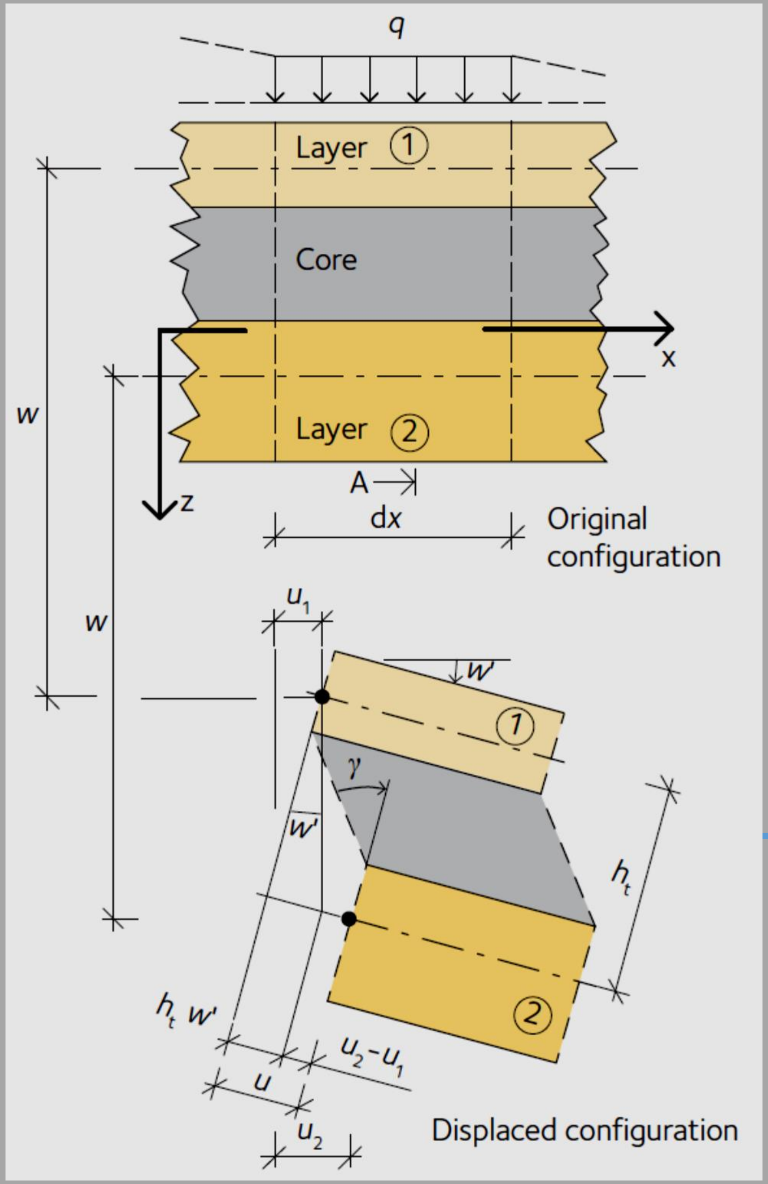
$$u' = u_2' - u_1' + h_t w'' = \varepsilon_2 - \varepsilon_1 + h_t w''$$

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \varepsilon \quad (strain)$$

Horizontal displacements of the parts



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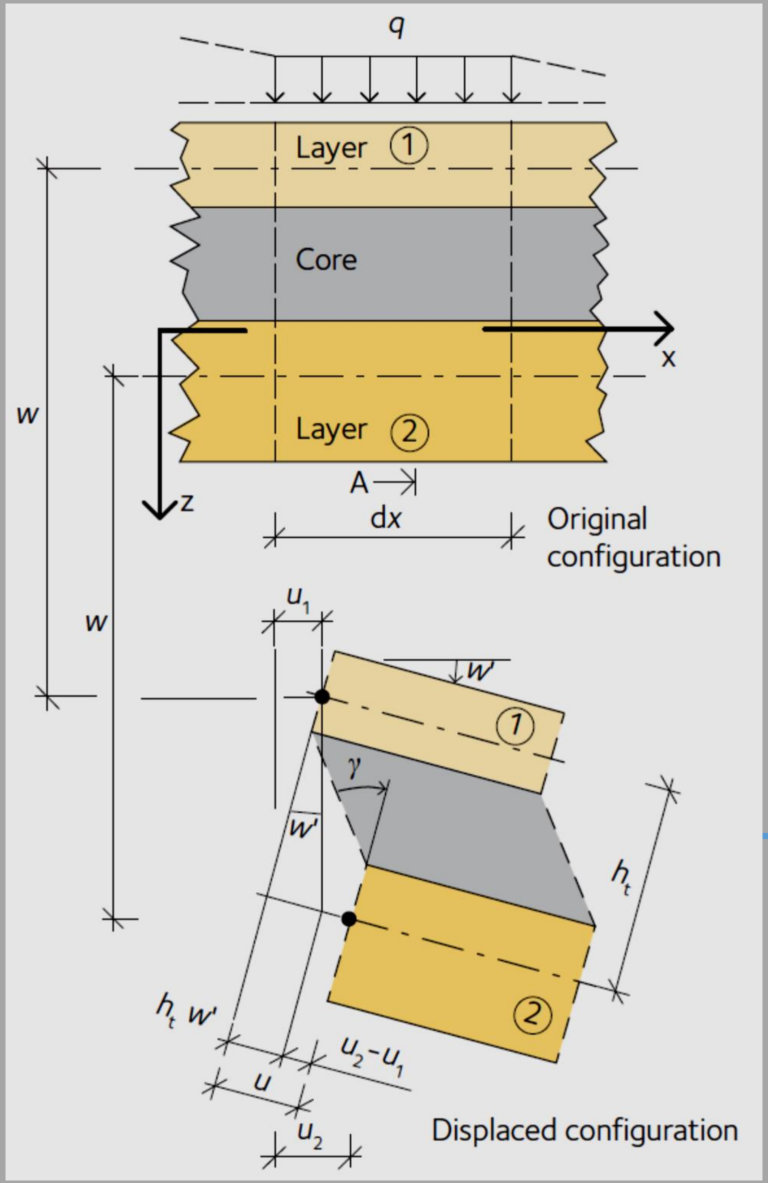
classical bar theory:

$$\varepsilon_1 = \frac{N_1}{EA_1}$$

$$\varepsilon_2 = \frac{N_2}{EA_2}$$

Horizontal displacements of the parts



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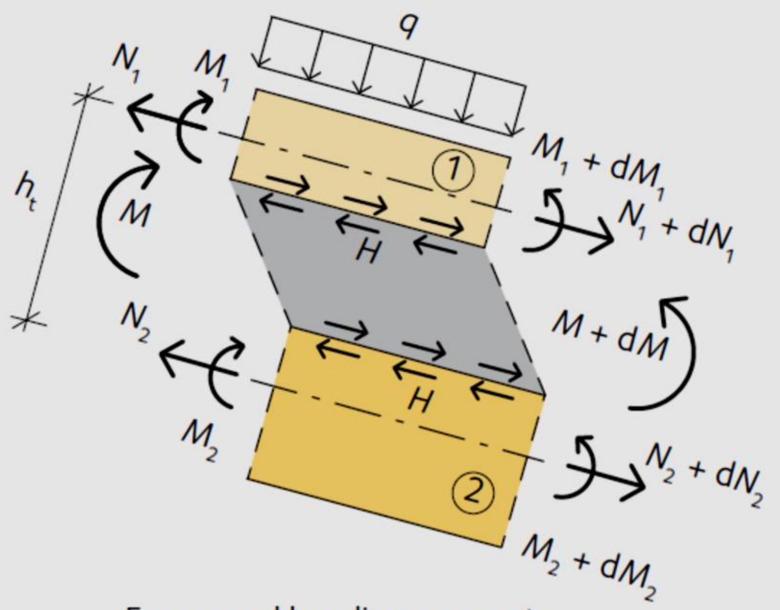
classical bar theory:

$$\varepsilon_1 = \frac{N_1}{E A_1}$$

$$\varepsilon_2 = \frac{N_2}{E A_2}$$

$$\boxed{u' = h_t w'' - \frac{N_1}{E A_1} + \frac{N_2}{E A_2}}$$

Axial force equilibrium: Zero external axial force



Forces and bending moments

Pure bending due to transverse loading; No external axial force:
N1 +N2 =0

$$N_2 = -N_1$$

$$u' = h_t w'' - \frac{N_1}{EA_1} + \frac{N_2}{EA_2}$$

$$\boxed{u' = h_t w'' - \left(\frac{1}{EA_1} + \frac{1}{EA_2}\right) N_1}$$

Now, let's assume that the interface has a shear stiffness k_{core}:

$$H = k_{core} \cdot u$$

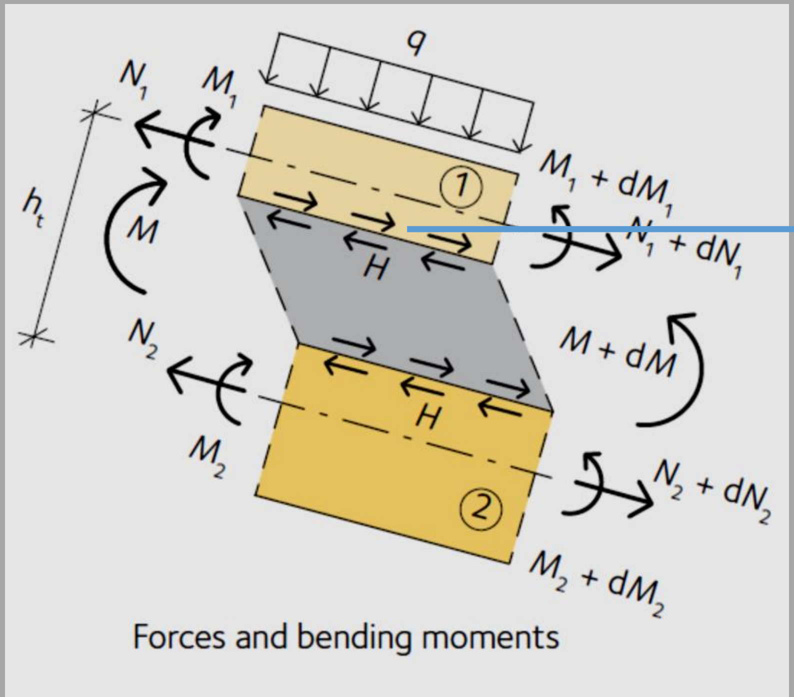
shear flow

slip modulus

slip between layers

This shear stiffness may be due to mechanical connector, thick soft core layer or both, and will be defined later…

Force equilibrium for layer 1:



H: shear flow

$$H = -N_1'$$

From previous slide:

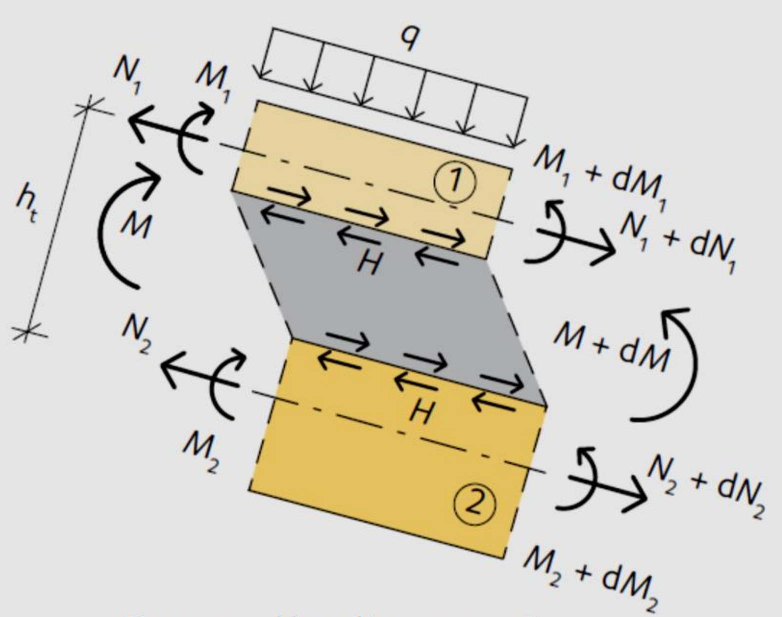
$$H = k_{core}\, u$$

$$u' = h_t w'' - \left(\frac{1}{EA_1} + \frac{1}{EA_2}\right) N_1$$

Equation I (in terms of):
w: deflection, and
N1: Axial load in layer 1

$$\boxed{\frac{-N_1''}{k_{core}} = h_t w'' - \left(\frac{1}{EA_1} + \frac{1}{EA_2}\right) N_1}$$

External moment M equilibrium:

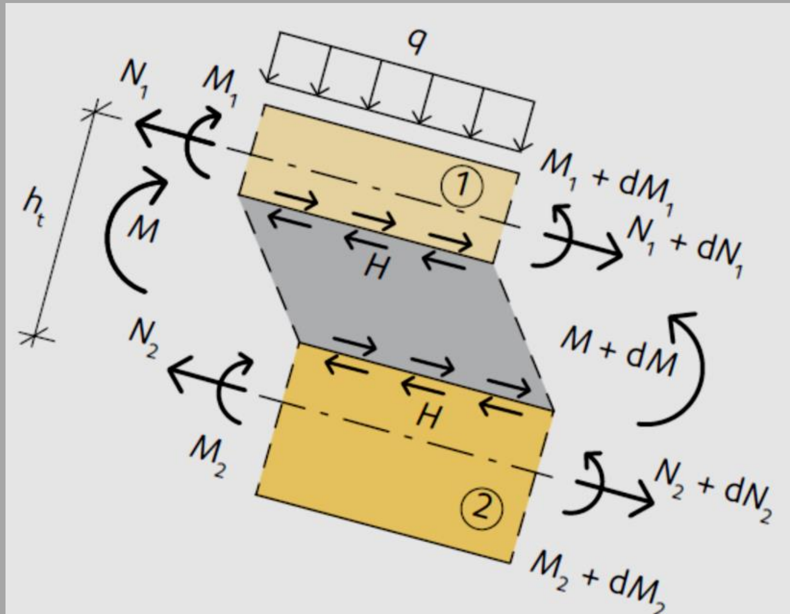


Forces and bending moments

From equilibrium about the center of layer 2:

$$M = M_1 + M_2 - N_1 h_t$$

External moment M equilibrium:



Forces and bending moments

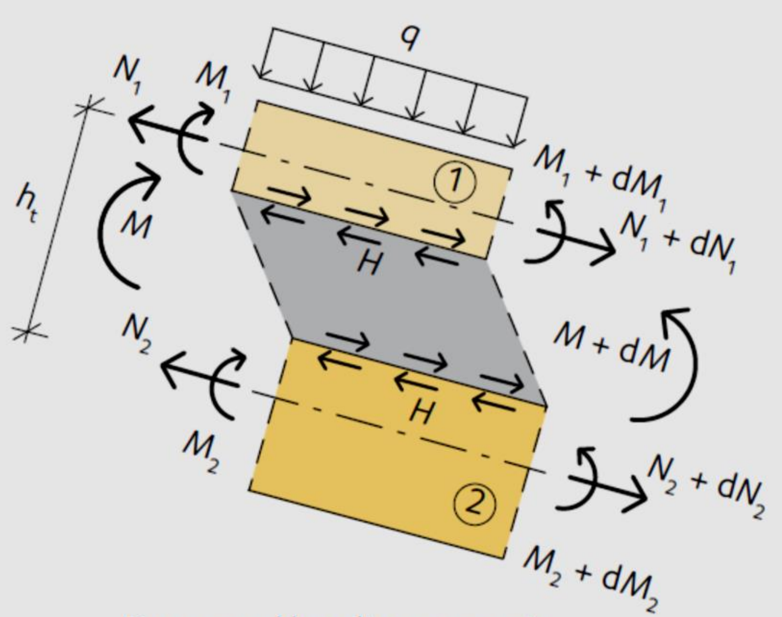
From equilibrium about the center of layer 2:

$$M = M_1 + M_2 - N_1 h_t$$

Assuming that both layers have equal radius of curvature (Thin layers as Euler Beams + ***no separation***):

$$w'' = -\frac{M_1}{EI_1} = -\frac{M_2}{EI_2}$$

External moment M equilibrium:



Forces and bending moments

From equilibrium about the center of layer 2:

$$M = M_1 + M_2 - N_1 h_t$$

$$\boxed{M = -(EI_1 + EI_2)w'' - N_1 h_t}$$

EquationII (in terms of):
w: deflection, and
N_1: Axial load in layer 1

Assuming that both layers have equal radius of curvature
(Thin layers as Euler Beams + ***no separation***):

$$w'' = -\frac{M_1}{EI_1} = -\frac{M_2}{EI_2}$$

and after some manipulations….

(elimination of N_1 from Eqs. I & II)

Governing differential equations:

$$w'''' - \omega^2 w'' = C_{\mathrm{Mb}}(C_{\mathrm{M}} M - M'')$$

where:

$$C_{\mathrm{Mb}} = \frac{1}{EI_1 + EI_2}$$

$$C_{\mathrm{M}} = k_{\mathrm{core}}\left(\frac{1}{EA_1} + \frac{1}{EA_2}\right)$$

$$\omega = \sqrt{k_{\mathrm{core}}\left(\frac{1}{EA_1} + \frac{1}{EA_2} + \frac{h_t^{\,2}}{EI_1 + EI_2}\right)}$$

w: Deflection of the partial composite beam
M: Total bending moment from transverse load
EA_i: Axial stiffness of i-th layer of composite beam
EI_i: Flexural stiffness of i-th layer of composite beam
h_t: Distance between the the neutral axis of each layer of composite beam
k_{core}: slip modulus of the core/interface

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Let's think and compare the PC theory with Euler beam theory! If e.g. slip modulus = 0…

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where:

$$C_{\mathrm{Mb}} = \frac{1}{EI_1 + EI_2}$$

$$C_{\mathrm{M}} = \cancel{k_{\mathrm{core}}}\left(\frac{1}{EA_1} + \frac{1}{EA_2}\right)$$

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k_{core}: slip modulus of the core/interface

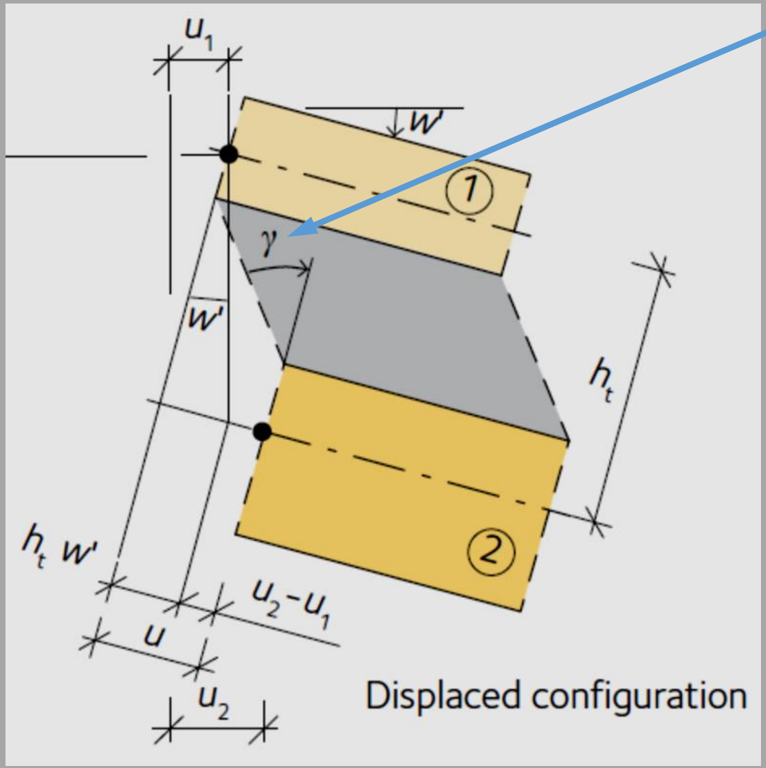
Let's think and compare the PC theory with Euler beam theory! If e.g. slip modulus = 0…

$$w'' = \frac{M}{EI}$$

Classical Euler Beam

But, how to calculate slip modulus?!

Slip modulus for thick/soft core:



Core material is displaced an angle:

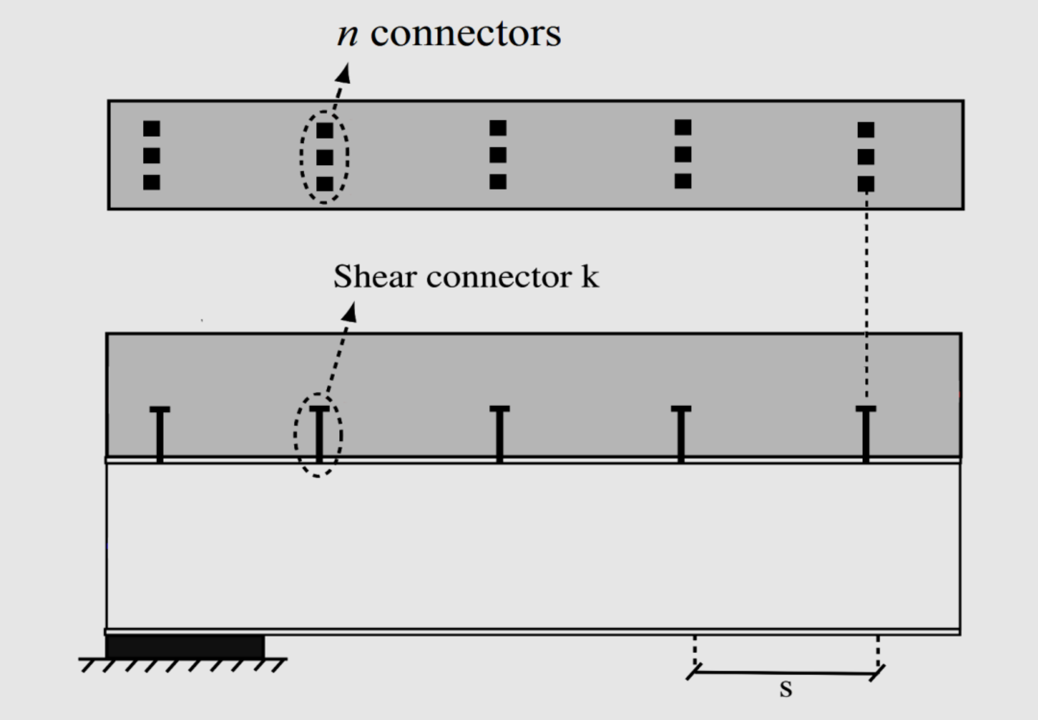
$$\gamma = \frac{\text{Shear stress}}{\text{Shear modulus}} = \frac{H}{Gb}$$

$$u = \gamma\, h_{core} = \frac{H\, h_{core}}{Gb} = \frac{H}{k_{core}}$$

$$\boxed{k_{core} = \frac{Gb}{h_{core}}}$$

Slip modulus of mechanical connectors:

$$k_{core} = \frac{k\,n}{s}$$



- k is the slip modulus of a single connector
  (for instance K_{ser} or K_u for a timber structure designed according to EC5)

- s is the constant spacing between the connectors in longitudinal direction

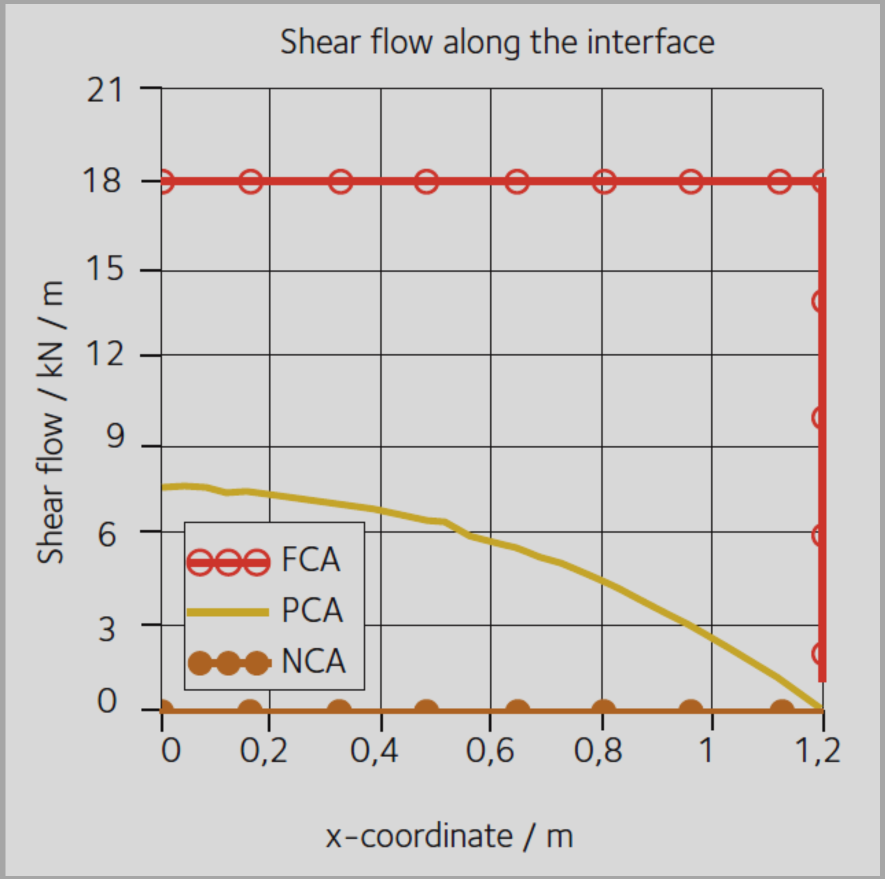
- n is the number of connectors placed on a line perpendicular to longitudinal direction (along width)

In case of **both** ***mechanical connectors*** and ***soft core***:

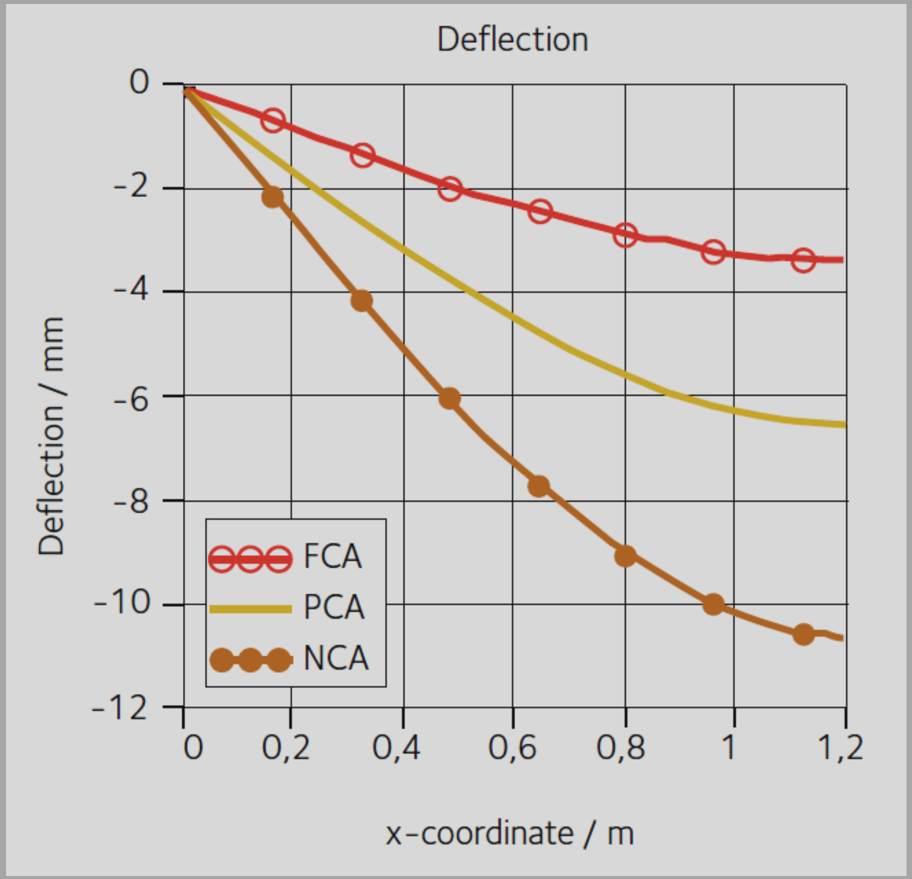
Parallel springs: $$k_{\text{core}} = \frac{k\,n}{s} + \frac{G\,b}{h_{\text{core}}}$$

Now, we proceed with solution for governing equations of PC theory!

Partial Composite Action (PCA) in comparison with the upper and lower limits



Distribution of shear flow (shear stress*beam width) at the interface of the composite beam layers

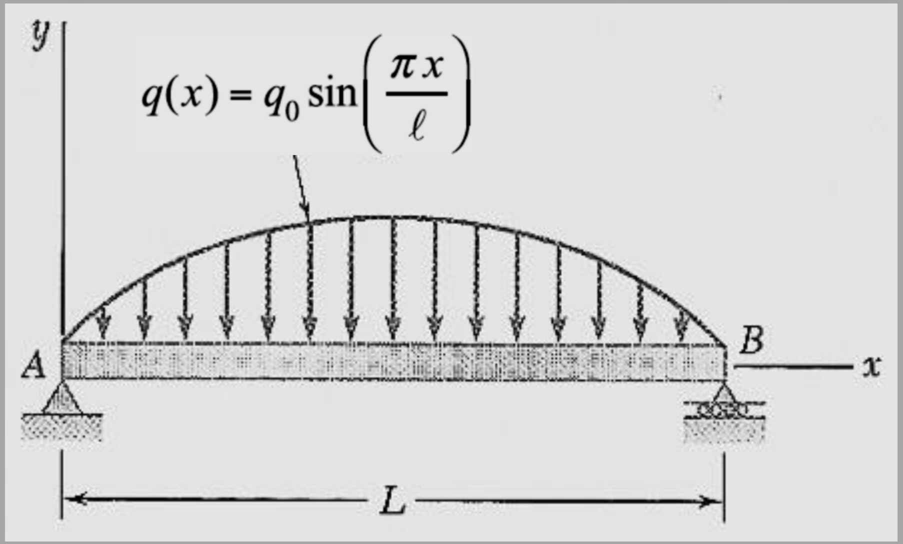


Deflection in a simply supported composite beam (half-span) any approximate engineering solution?!

Gamma method for analysis of partial composite members

Approximate solutions of the governing equations:

- ✓ the approximate solution is based on the exact solution to an elementary case in which a simply supported beam is loaded by a ***sinusoidally*** distributed load, that is half a sinewave.



$$M'' = -q$$

$$M(x) = q_0\left(\frac{\ell}{\pi}\right)^2 \sin\left(\frac{\pi x}{\ell}\right)$$

✓ We substitute the bending moment into the governing differential equation of *Partial Composite* beams and solve it to obtain the beam deflection:

$$w(x) = \frac{q_0 \ell^4}{\pi^4 E I_{\mathrm{fca}}} \frac{1 + \frac{1}{C_{\mathrm{M}}}\left(\frac{\pi}{\ell}\right)^2}{1 + \left(\frac{\pi}{\omega \ell}\right)^2} \sin\left(\frac{\pi x}{\ell}\right)$$

For the PCA beam

- ✓ We substitute the bending moment into the governing differential equation of *Partial Composite* beams and solve it to obtain the beam deflection:

$$w(x) = \frac{q_0\,\ell^4}{\pi^4\,E\,I_{\text{fca}}}\underbrace{\frac{1+\dfrac{1}{C_{\text{M}}}\left(\dfrac{\pi}{\ell}\right)^2}{1+\left(\dfrac{\pi}{\omega\,\ell}\right)^2}}_{\gamma\ \textit{factor}}\sin\left(\frac{\pi\,x}{\ell}\right) \xrightarrow{}$$

For the PCA beam

$$= \frac{q_0\,\ell^4}{\pi^4\,E\,I_{\text{fca}}}\gamma_{\text{dmf}}\sin\left(\frac{\pi\,x}{\ell}\right)$$

<u>We compare this solution with that for a classical full composite beam</u>

✓ We substitute the bending moment into the governing differential equation of *Partial Composite* beams and solve it to obtain the beam deflection:

$$w(x) = \frac{q_0\,\ell^4}{\pi^4\,E\,I_{\mathrm{fca}}}\underbrace{\frac{1+\dfrac{1}{C_{\mathrm{M}}}\left(\dfrac{\pi}{\ell}\right)^2}{1+\left(\dfrac{\pi}{\omega\,\ell}\right)^2}}_{\gamma\ \text{factor}}\sin\left(\frac{\pi\,x}{\ell}\right)$$

For the PCA beam

$$= \frac{q_0\,\ell^4}{\pi^4\,E\,I_{\mathrm{fca}}}\gamma_{\mathrm{dmf}}\sin\left(\frac{\pi\,x}{\ell}\right)$$

For the FCA beam

$$w(x) = \frac{q_0\,\ell^4}{\pi^4\,E\,I_{\mathrm{fca}}}\sin\left(\frac{\pi\,x}{\ell}\right)$$

Clearly, γ_{dmf} represents a ***displacement magnification factor***, which multiplied by the *FCA* displacement, gives the "correct" displacement, accounting for interlayer slip.

✓ In other words, the effective stiffness is reduced for a PCA beam as: ($1/\gamma_{dmf}$ acts as a ***stiffness reduction factor***):

$$E\,I_{ef} = E\,I_{fca}\,\frac{1}{\gamma_{dmf}}$$

where:

$$\gamma_{dmf} = \frac{1 + \dfrac{1}{C_M}\left(\dfrac{\pi}{\ell}\right)^2}{1 + \left(\dfrac{\pi}{\omega\,\ell}\right)^2}$$

The method for designing built-up beams and columns, characterized by PCA, in ***Eurocode 5*** depends entirely ***on this simplified solution***. In Eurocode 5 the effective bending stiffness in Equation 5.81 is expressed differently, but is really the same thing!

Gamma method based on EC5:

Gamma method based on the latest version of EC5:

Assumptions

- ✓ The design method is based on the theory of linear elasticity and the following assumptions:
- ✓ The beams are simply supported with a span *L*. For continuous beams the expressions may be used with *L* equal to 0,8 of the relevant span and for cantilevered beams with *L* equal to twice the cantilever length.
- ✓ The individual parts are connected to each other by mechanical fasteners with a slip modulus K.
- ✓ The spacing *s* between the fasteners is constant or varies uniformly according to the shear force between s_{min} and s_{max}, with $s_{max} < 4\, s_{min}$
- ✓ The load is acting in the z-direction giving a moment $M = M(x)$ varying sinusoidally or parabolically.

Spacings

- ✓ Where a flange consists of two parts jointed to a web or where a web consists of two parts (as in a box beam), the spacing s_i is determined by the sum of the fasteners per unit length in the two jointing planes.

Effective bending stiffness:

The effective bending stiffness should be taken as:

$$(EI)_{ef} = \sum_{i=1}^{3} (E_i I_i + \gamma_i E_i A_i a_i^2)$$

using mean values of E and where:

$$A_i = b_i h_i$$

$$I_i = \frac{b_i h_i^3}{12}$$

$$\gamma_2 = 1$$

$$\gamma_i = \left[1 + \pi^2 E_i A_i s_i / (K_i l^2)\right]^{-1} \quad \text{for } i = 1 \text{ and } i = 3$$

$$a_2 = \frac{\gamma_1 E_1 A_1 (h_1 + h_2) - \gamma_3 E_3 A_3 (h_2 + h_3)}{2 \sum_{i=1}^{3} \gamma_i E_i A_i}$$

For T-sections $h_3 = 0$

$K_i = K_{ser,i}$ for the serviceability limit state calculations;
$K_i = K_{u,i}$ for the ultimate limit state calculations.

Normal stress:

The normal stress should be taken as

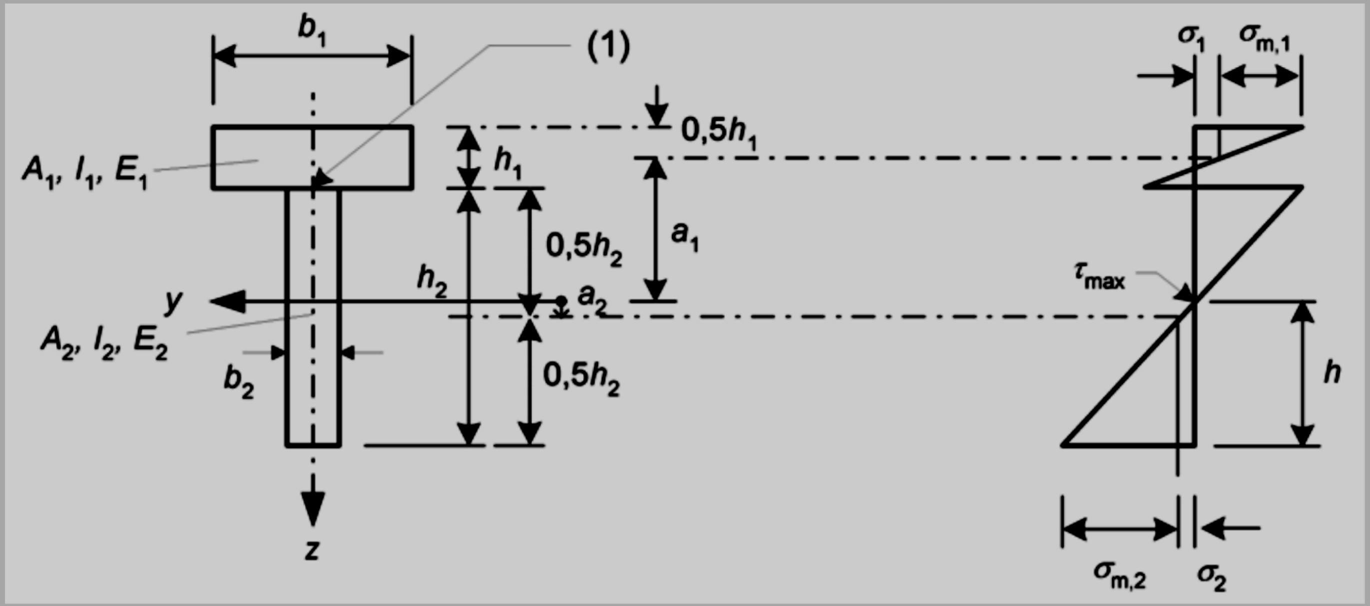
$$\sigma_{\mathrm{i}} = \frac{\gamma_{\mathrm{i}}\, E_{\mathrm{i}}\, a_{\mathrm{i}}\, M}{(E\,I)_{\mathrm{ef}}}$$

Maximum shear stress

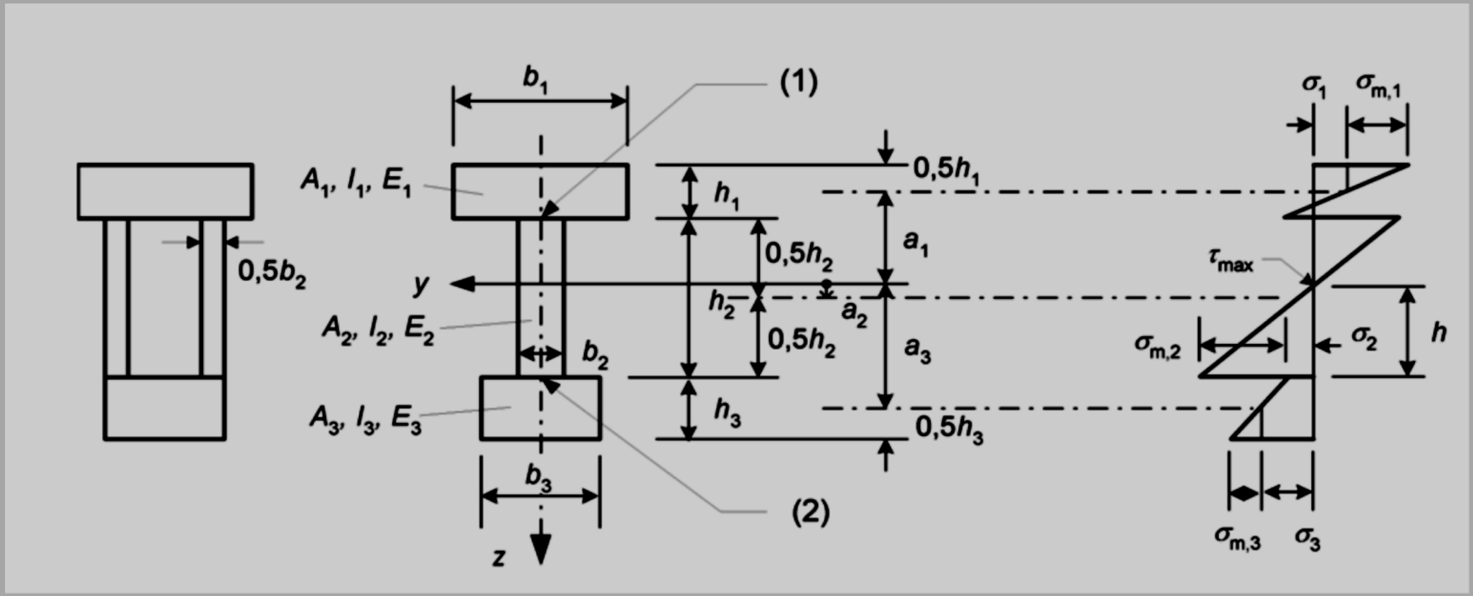
The maximum shear stresses occur where the normal stresses are zero. The maximum shear stresses in the web member (part 2) should be taken as:

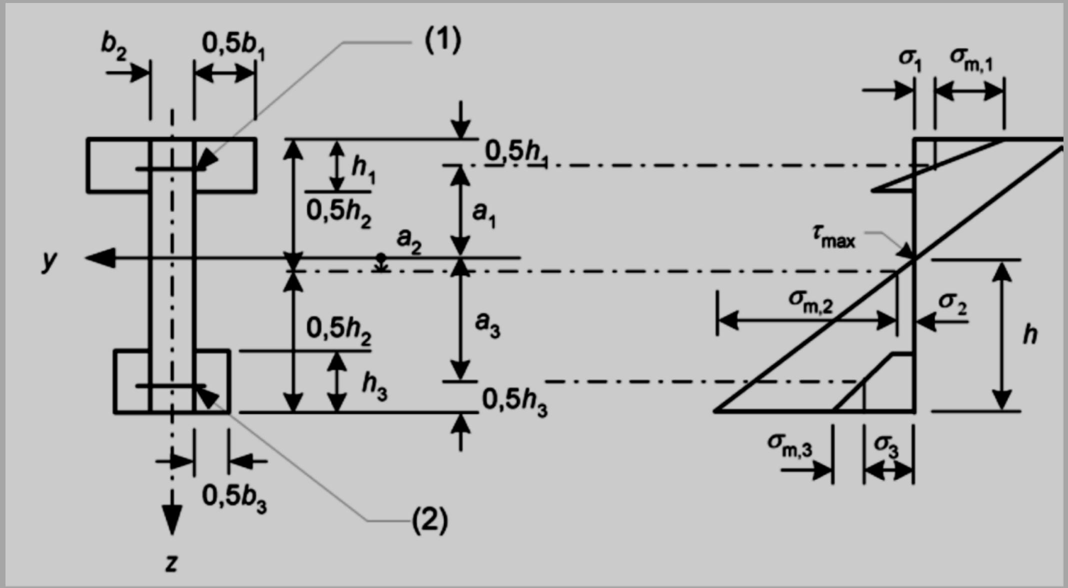
$$\tau_{2,\mathrm{max}} = \frac{\gamma_3\, E_3\, A_3\, a_3 + 0{,}5\, E_2\, b_2\, h_2^2}{b_2 (E\,I)_{\mathrm{ef}}} V$$

2-layer composite beams:



3-layer composite beams:

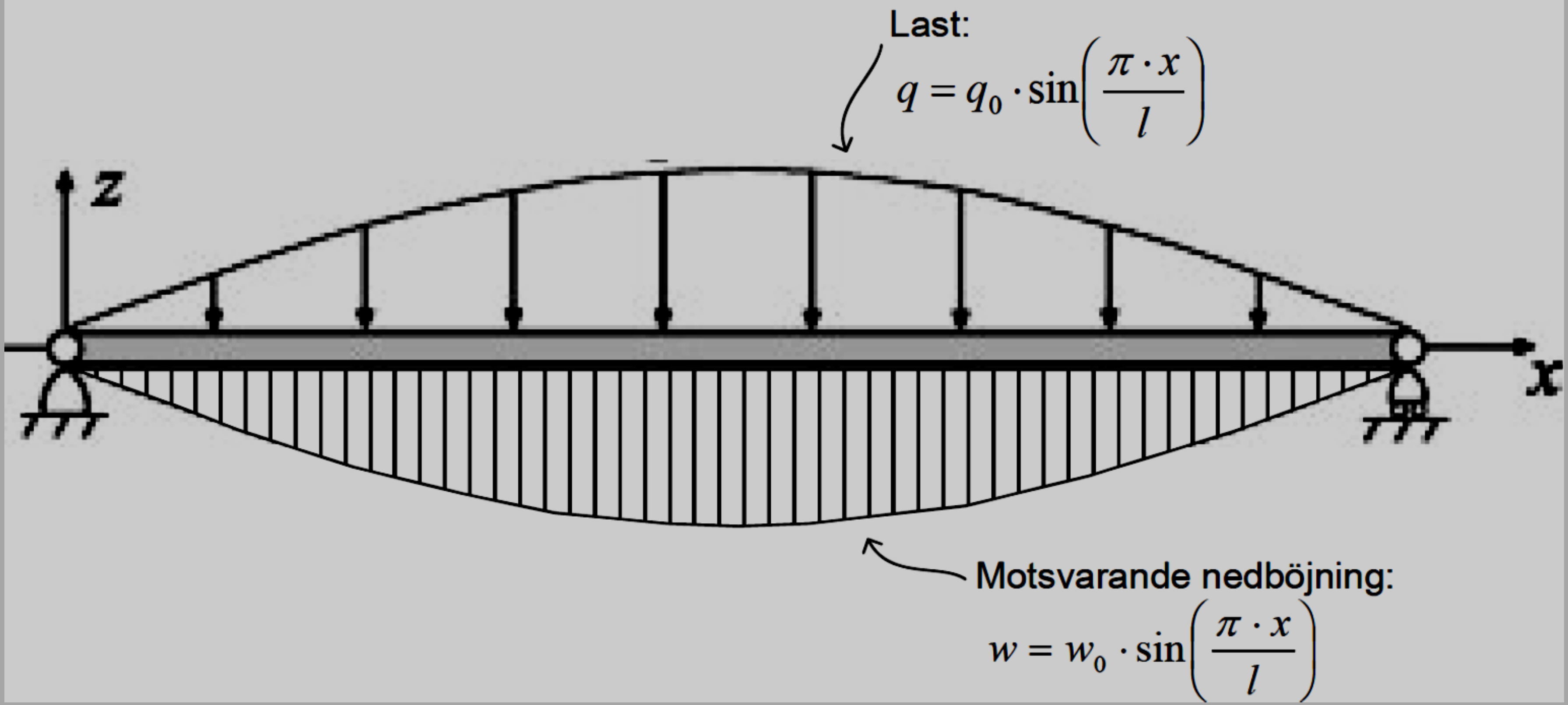


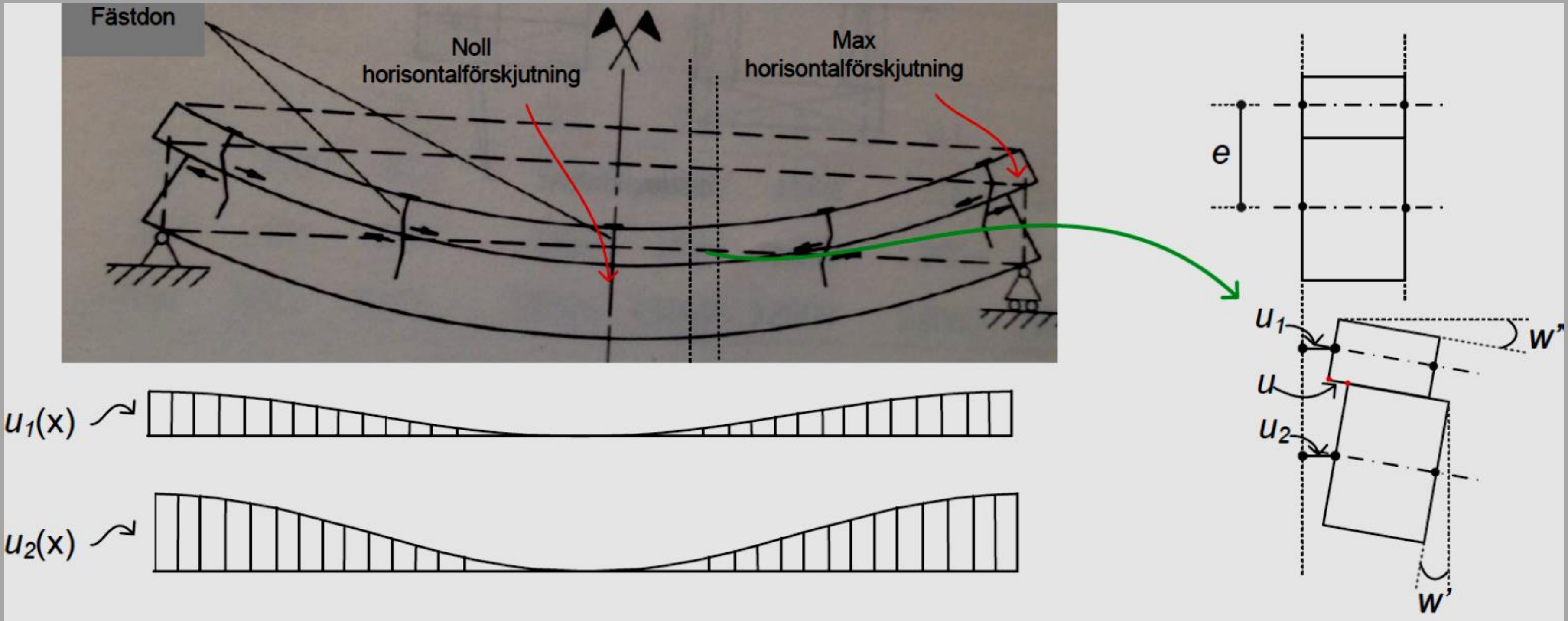


Rehearsal question

1. Derive a formula for the slip modulus of a two layer beam with a thick soft core and investigate how the thickness of the core and the shear stiffness of the core material affect the slip.
2. Derive a simple formula for the displacement magnification factor (gamma) of a pinned-pinned partial composite beam under a sinusoidal half-wave load.
3. Consider a two layer beam composed of two identical layers of glulam with square cross section shape of dimension a (so the total height of the beam is $2a$), connected by nails with spacing $s = L/10$. So, the beam shows partial composite behavior under loading. The beam is of length L and is simply supported at its both ends and is subjected to a half wave sine loading. Obtain the maximum deflection of the beam, one time based on the method in EC5, and next based on Eq. 5.80 in “Design of Timber Structures” book, and compare them. Is there any difference between them?

Efficient use of mechanical connectors…


Last:
$q = q_0 \cdot \sin\left(\frac{\pi \cdot x}{l}\right)$
z
x
Motsvarande nedböjning:
$w = w_0 \cdot \sin\left(\frac{\pi \cdot x}{l}\right)$



$u = u_2 - u_1 + e \cdot w' \quad \Rightarrow$ *u* skall ha samma form som *w'*, d.v.s. cosinusform

$$u_1 = u_{10} \cdot \cos\left(\frac{\pi \cdot x}{l}\right) \qquad u_2 = u_{20} \cdot \cos\left(\frac{\pi \cdot x}{l}\right)$$

Slip / shear stress at composite members

Increasing shear flow / slip toward the edges.
Minimum slip around the center of composite element (zero for symmetric case!)

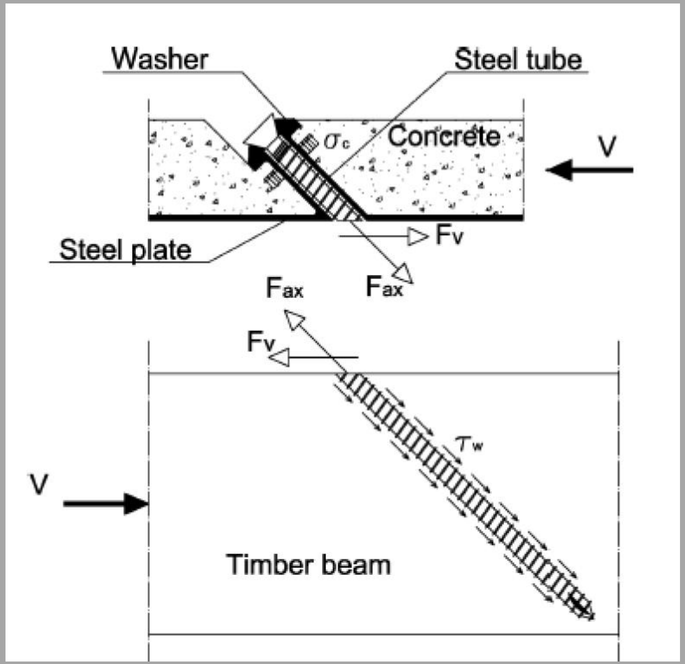
So, we need to higher number of mechanical connectors at end-zones;

Slip / shear stress at composite members

Increasing shear flow / slip toward the edges.
Minimum slip around the center of composite element (zero for symmetric case!)

So, we need to a higher number of mechanical connectors at end-zones;

or efficiently using them…



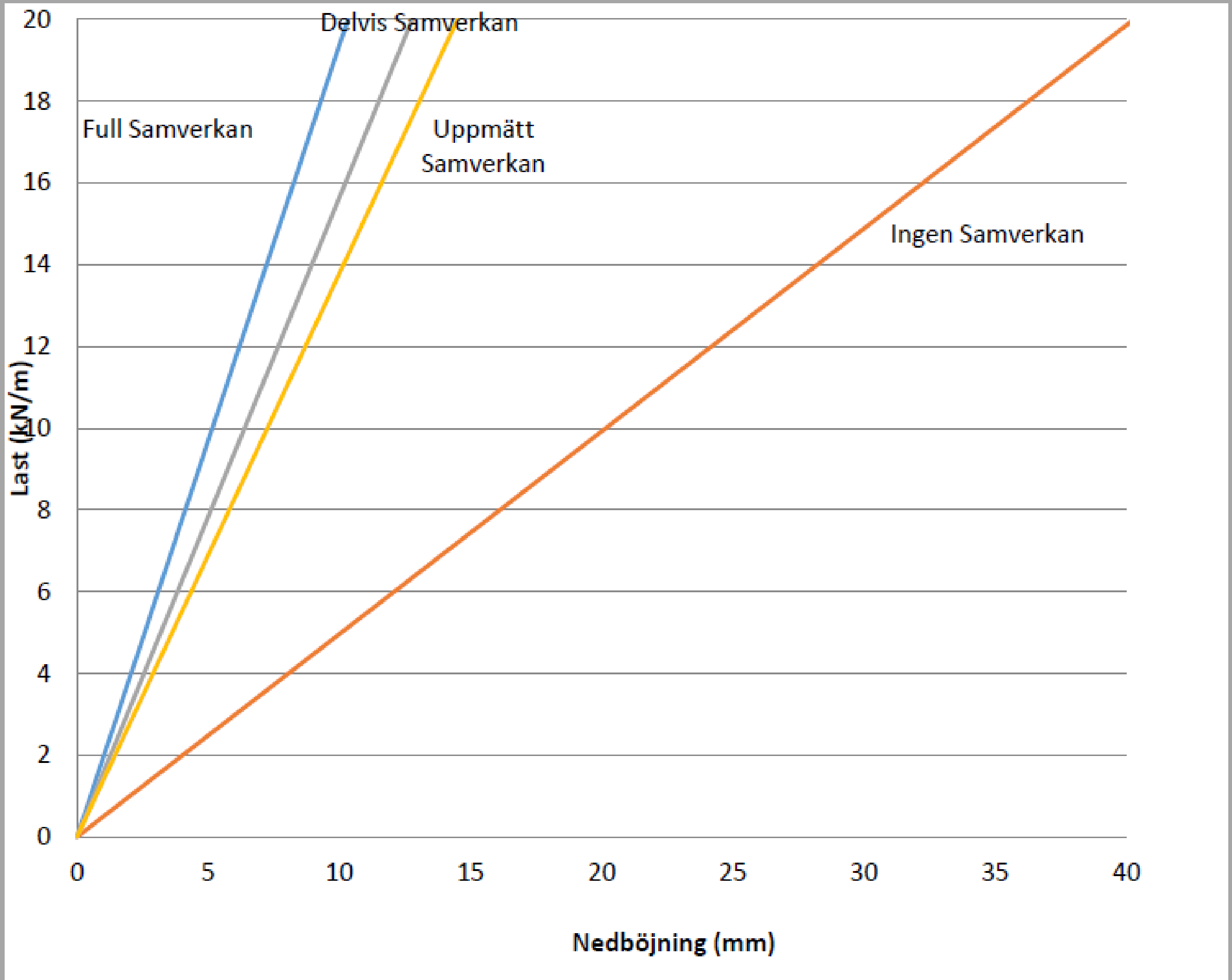
Inclined screws transmits shear load on them to tensile…

Insert screws

2: Twist upside down and cast concrete

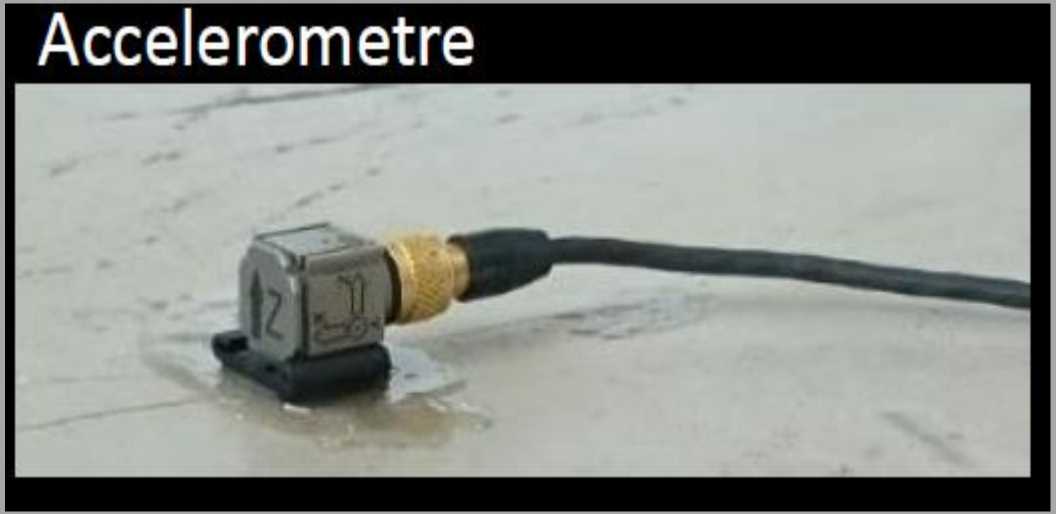
3: Let cure and twist upside down again

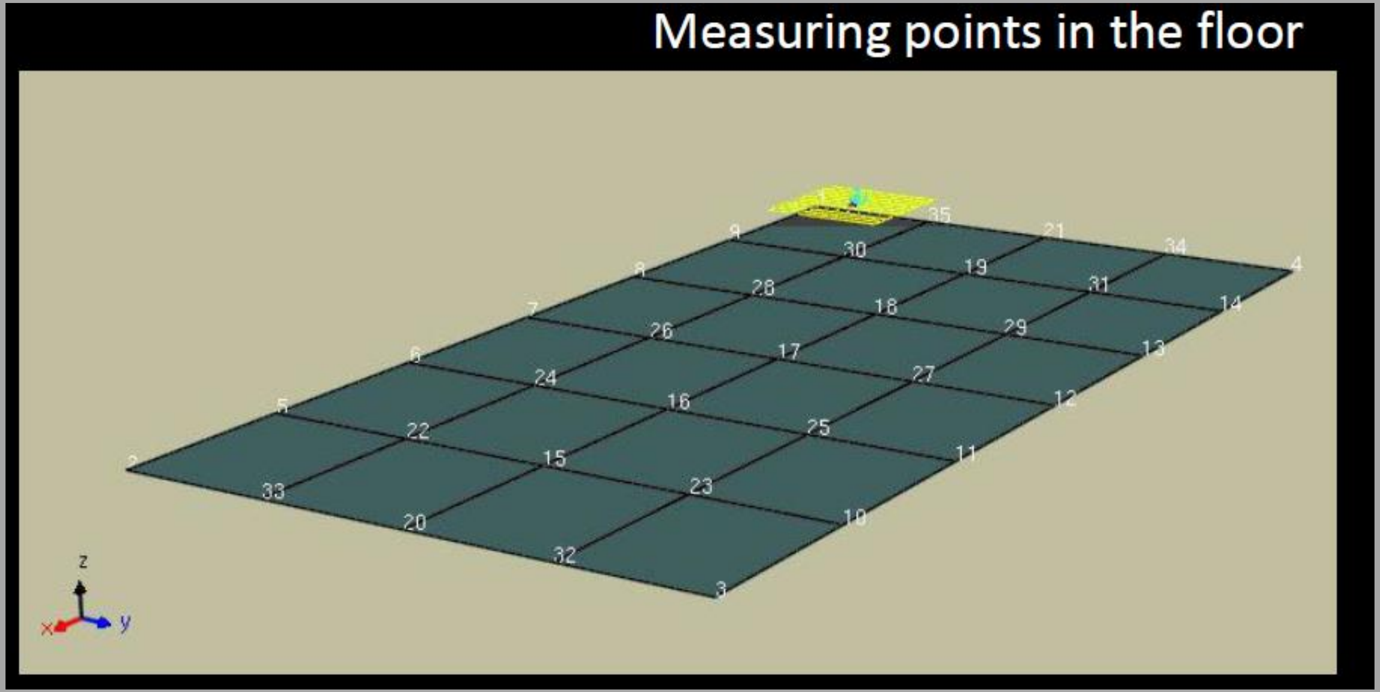
Tests



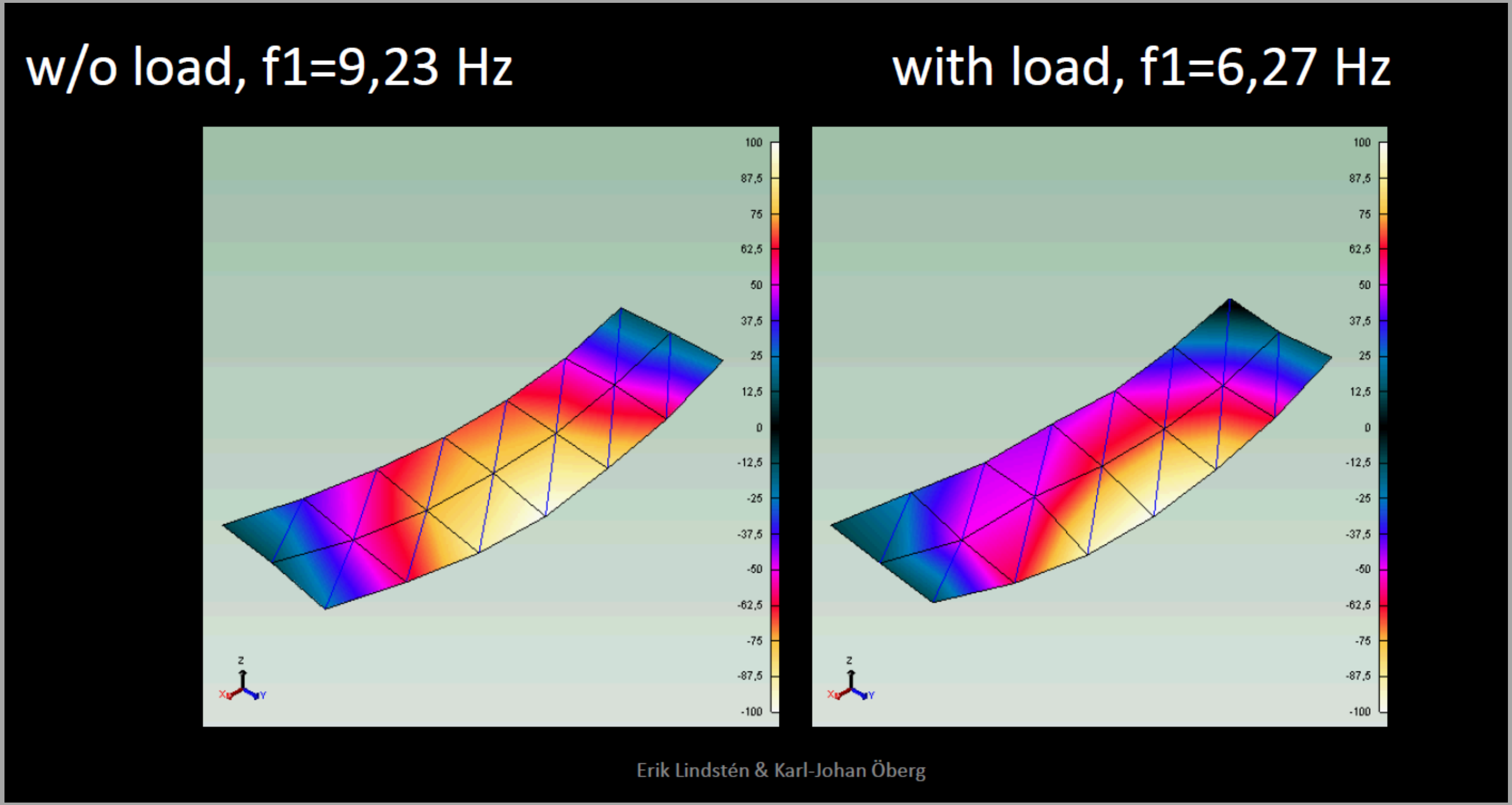
Delvis Samverkan
Full Samverkan
Uppmätt Samverkan
Ingen Samverkan
Last (kN/m)
Nedböjning (mm)
0
2
4
6
8
10
12
14
16
18
20
0
5
10
15
20
25
30
35
40

Dynamic tests / Modal analysis





Results


w/o load, f1=9,23 Hz
100
87,5
75
62,5
50
37,5
25
12,5
0
-12,5
-25
-37,5
-50
-62,5
-75
-87,5
-100
with load, f1=6,27 Hz
100
87,5
75
62,5
50
37,5
25
12,5
0
-12,5
-25
-37,5
-50
-62,5
-75
-87,5
-100
Erik Lindstén & Karl-Johan Öberg

Thanks for listening!