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TIMBER ENGINEERING - VSM196

LECTURE 3 –

DESIGN OF BEAMS IN SHEAR, NOTCHED BEAMS AND HOLES

SPRING 2020

Topic

- Design of beams in shear, notched beams and holes

Content

- Design of beams in shear and with loading at the angle to the grain
- Various types and causes of fracture perpendicular to the grain
- Design of notched beams and beams with holes

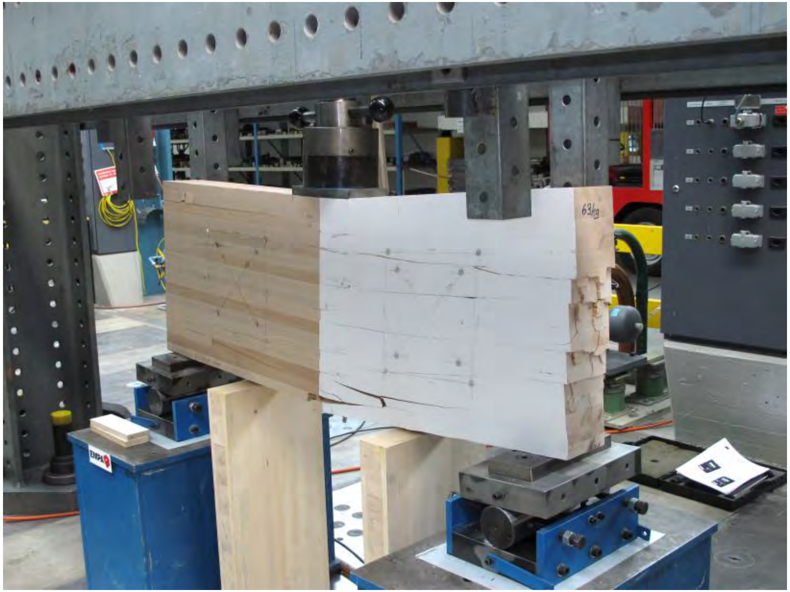
- Exercises E1 & E2

Intended Learning Outcomes of this lecture

- You understand the different types of shear failure in timber
- You can describe the problem of stress singularities at notches
- You can distinguish the impact of shear force and moment contribution on the stresses at holes in beams
- You know the benefits of reinforcement for notches and holes

Shear failure of timber members

Examples of shear failure of beams



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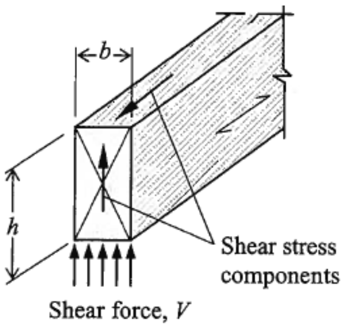
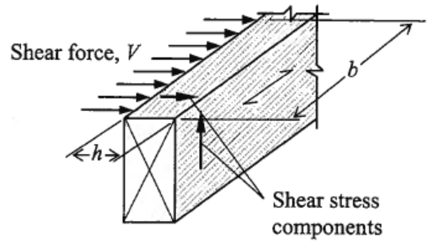
Shear in timber

- Parallel shear



- Rolling shear



Shear strength values

- Glued laminated timber (EN 14080:2013)
  - Homogeneous glulam

Table 5 — Characteristic strength and stiffness properties an N/mm² and densities in kg/m³ for homogeneous glulam

Property	Symbol	Glulam strength class						
		GL 20h	GL 22h	GL 24h	GL 26h	GL 28h	GL 30h	GL 32h
Bending strength	$f_{m,g,k}$	20	22	24	26	28	30	32
Tensile strength	$f_{t,0,g,k}$	16	17,6	19,2	20,8	22,3	24	25,6
	$f_{t,90,g,k}$	0,5						
Compression strength	$f_{c,0,g,k}$	20	22	24	26	28	30	32
	$f_{c,90,g,k}$	2,5						
Shear strength (shear and torsion)	$f_{v,g,k}$	3,5						
Rolling shear strength	$f_{r,g,k}$	1,2						
Modulus of elasticity	$E_{0,g,mean}$	8 400	10 500	11 500	12 100	12 600	13 600	14 200
	$E_{0,g,05}$	7 000	8 800	9 600	10 100	10 500	11 300	11 800
	$E_{90,g,mean}$	300						
	$E_{90,g,05}$	250						
Shear modulus	$G_{g,mean}$	650						
	$G_{g,05}$	540						
Rolling shear modulus	$G_{r,g,mean}$	65						
	$G_{r,g,05}$	54						
Density	$\rho_{g,k}$	340	370	385	405	425	430	440
	$\rho_{g,mean}$	370	410	420	445	460	480	490

Shear strength values

- Solid timber (EN 338:2016)
  - Softwood

Table 1 — Strength classes for softwood based on edgewise bending tests – strength, stiffness and density values

	Class	C14	**C16**	**C18**	C20	C22	**C24**	C27	**C30**	C35	C40	C45	C50
Strength properties in N/mm²													
Bending	$f_{m,k}$	14	**16**	**18**	20	22	**24**	27	**30**	35	40	45	50
Tension parallel	$f_{t,0,k}$	7,2	**8,5**	**10**	11,5	13	**14,5**	16,5	**19**	22,5	26	30	33,5
Tension perpendicular	$f_{t,90,k}$	0,4	**0,4**	**0,4**	0,4	0,4	**0,4**	0,4	**0,4**	0,4	0,4	0,4	0,4
Compression parallel	$f_{c,0,k}$	16	**17**	**18**	19	20	**21**	22	**24**	25	27	29	30
Compression perpendicular	$f_{c,90,k}$	2,0	**2,2**	**2,2**	2,3	2,4	**2,5**	2,5	**2,7**	2,7	2,8	2,9	3,0
Shear	$f_{v,k}$	3,0	**3,2**	**3,4**	3,6	3,8	**4,0**	4,0	**4,0**	4,0	4,0	4,0	4,0
Stiffness properties in kN/mm²													
Mean modulus of elasticity parallel bending	$E_{m,0,mean}$	7,0	**8,0**	**9,0**	9,5	10,0	**11,0**	11,5	**12,0**	13,0	14,0	15,0	16,0
5 percentile modulus of elasticity parallel bending	$E_{m,0,k}$	4,7	**5,4**	**6,0**	6,4	6,7	**7,4**	7,7	**8,0**	8,7	9,4	10,1	10,7
Mean modulus of elasticity perpendicular	$E_{m,90,mean}$	0,23	**0,27**	**0,30**	0,32	0,33	**0,37**	0,38	**0,40**	0,43	0,47	0,50	0,53
Mean shear modulus	G_{mean}	0,44	**0,50**	**0,56**	0,59	0,63	**0,69**	0,72	**0,75**	0,81	0,88	0,94	1,00
Density in kg/m³													
5 percentile density	ρ_k	290	**310**	**320**	330	340	**350**	360	**380**	390	400	410	430
Mean density	ρ_{mean}	350	**370**	**380**	400	410	**420**	430	**460**	470	480	490	520

NOTE 1 Values given above for tension strength, compression strength, shear strength, char. modulus of elasticity in bending, mean modulus of elasticity perpendicular to grain and mean shear modulus have been calculated using the equations given in EN 384.

NOTE 2 The tension strength values are conservatively estimated since grading is done for bending strength.

NOTE 3 The tabulated properties are compatible with timber at moisture content consistent with a temperature of 20 °C and a relative humidity of 65 %, which corresponds to a moisture content of 12 % for most species.

NOTE 4 Characteristic values for shear strength are given for timber without fissures, according to EN 408.

NOTE 5 These classes may also be used for hardwoods with similar strength and density profiles such as e.g. poplar or chestnut.

NOTE 6 The edgewise bending strength may also be used in the case of flatwise bending.

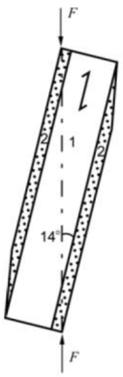
Volume effect for shear strength

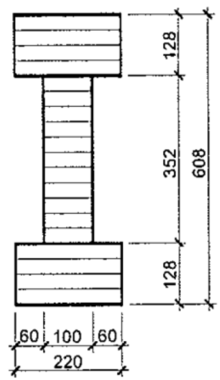
- Comparison of tests with specimens with two different sizes
  - Test specimen according to EN 408

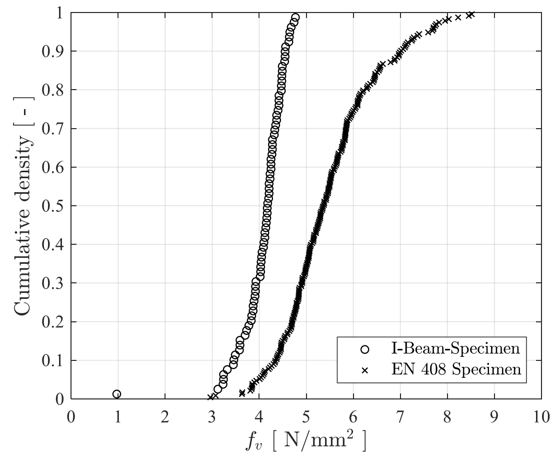
    $V = 588cm^3$

  - Full size I-beams

    $V = 44'700cm^3$







⇨ Timber element with larger volumens under shear stress have lower strength!

Shear verification

- Shear stress shall satisfy: $\tau_d \leq f_{v,d}$
  - Rectangular cross-section:

$$\tau_d = \frac{3V_d}{2A_{ef}} = \frac{1.5V_d}{b_{ef}h}$$

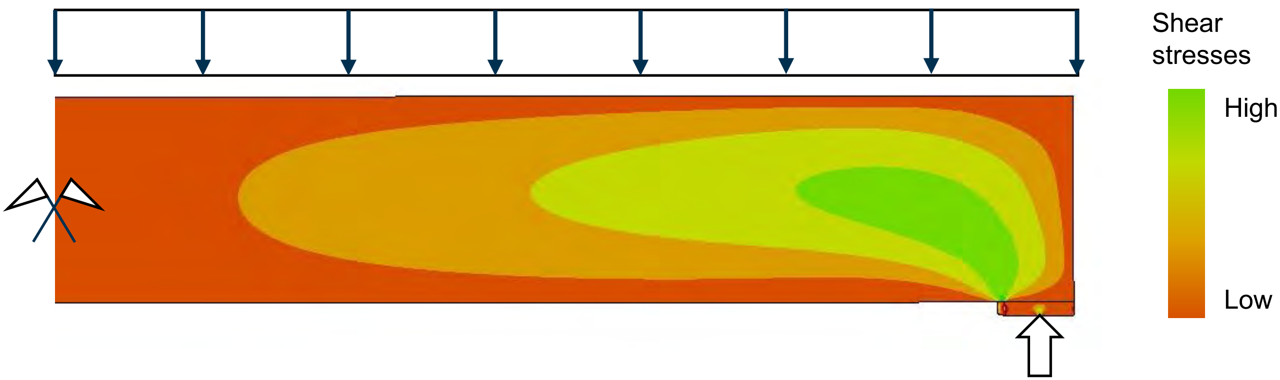
  - For the verification of shear resistance of members in bending, the influence of cracks should be taken using an effective width of the member given as:

$$b_{ef} = k_{cr} \cdot b$$

Where

- b is the width of the section of the member
- $k_{cr} = 0{,}67$ for solid timber and glulam (not LVL)

Shear stress distribution in beams



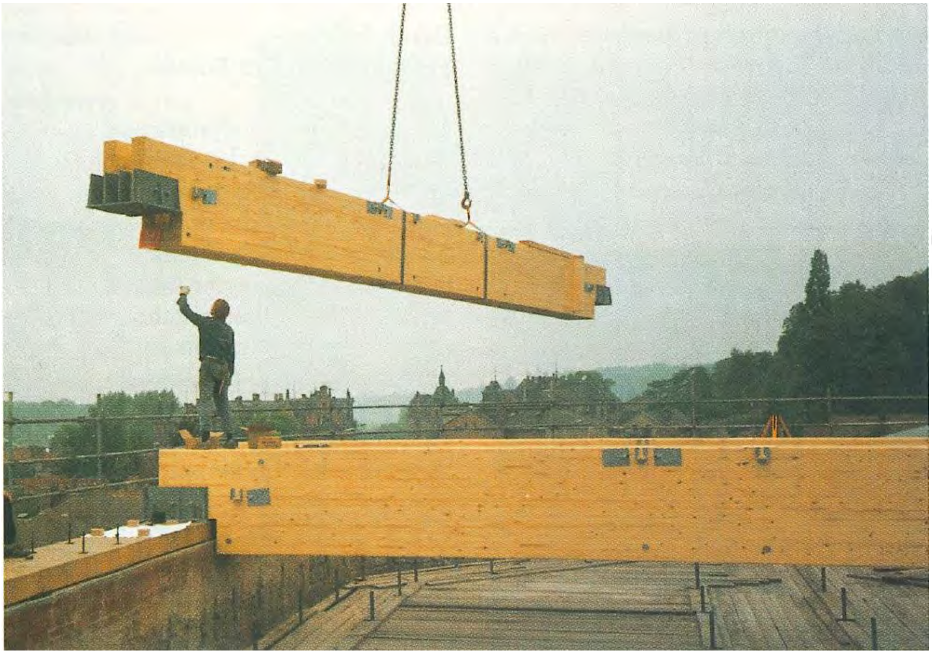
Beams with notches at the support

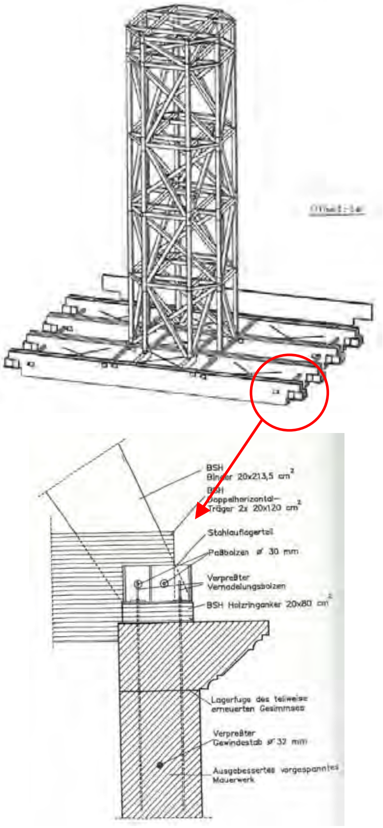
Examples of end-notched beams




Examples of end-notched beams

- Roof structure of a historic church



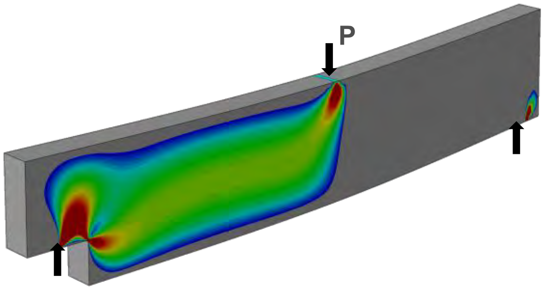


Schröter/Hüttemann (1993)

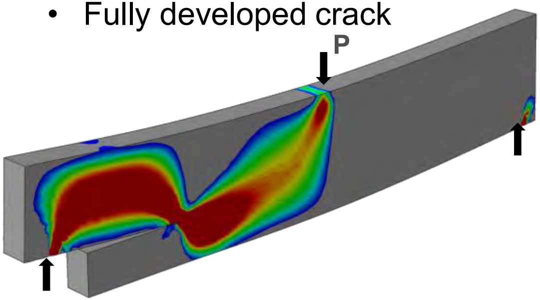
Stresses at the notch

Stress level:
high low

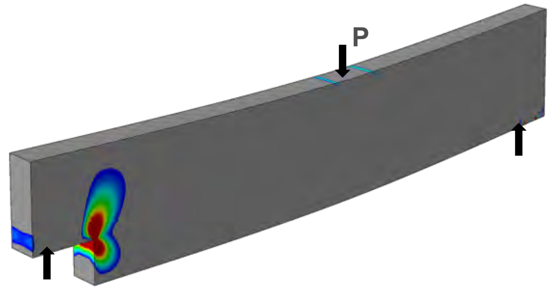
- Shear
  - Crack initiation



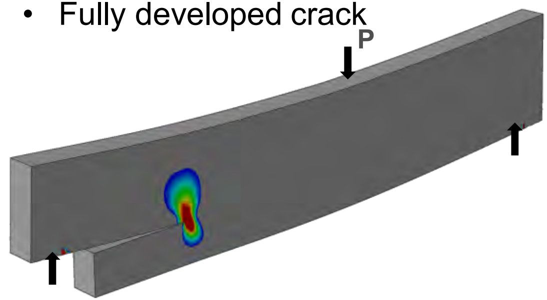
  - Fully developed crack



- Tension perpendicular to the grain
  - Crack initiation

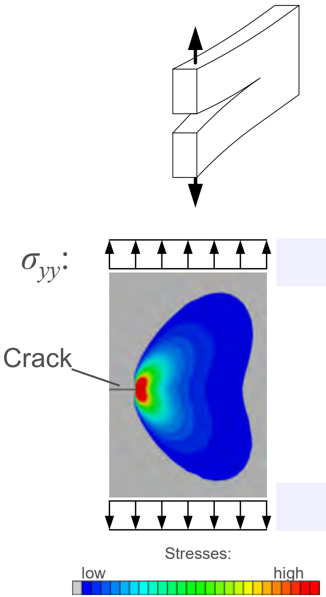


  - Fully developed crack

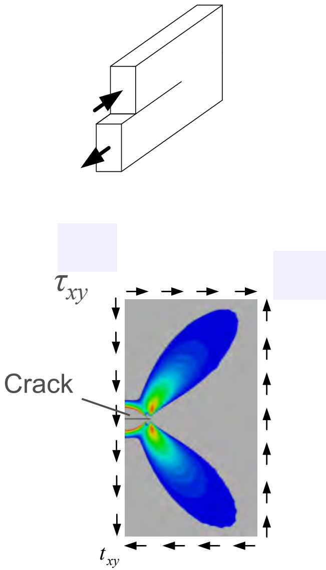


Stresses at a crack

- Tension perpendicular to the grain (Mode 1)



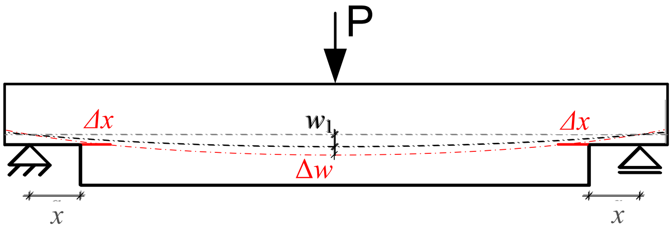
- Shear along the grain (Mode 2)

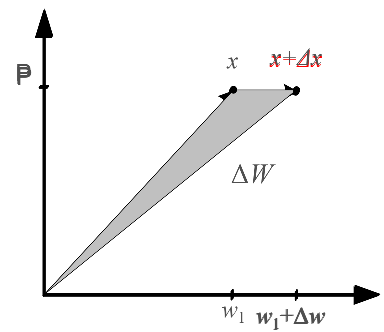


Fracture energy approach

- Energy release rate per unit crack growth $\Delta A = b\Delta x$

$$G = \frac{-\partial W}{\partial A}$$





$$\Delta W = -\frac{1}{2}P\Delta w = -G_c b\Delta x$$

Where	G_c	Fracture energy [N/mm]
	A	Crack surface [mm²]
	P	Force [N]
	w	Deflection in beam center [mm]
	x	Crack length [mm]
	b	Beam width [mm]

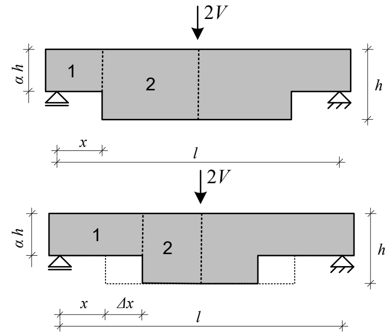
Notched beams

Per Johan Gustafsson
†2020

- Gustafsson approach for notched beams

$$\frac{1}{2}V\Delta w = G_c b \Delta x \quad \Rightarrow \quad \frac{V}{b\alpha h} = \sqrt{\frac{2G_c}{b\alpha h} \Big/ \frac{\delta w}{\delta x}}$$

$$\Delta w \begin{cases} \text{Before crack growth} \\ w = w_1 + w_2 \\ \\ \text{After crack growth} \\ w = w_1 + w_2 \end{cases}$$

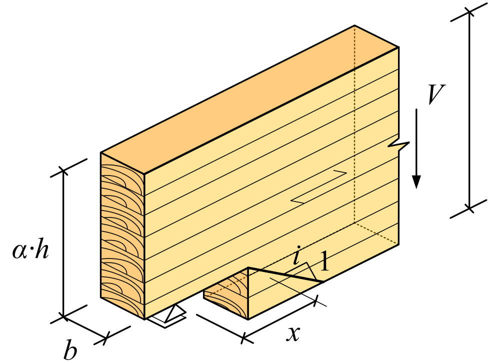


$$\Rightarrow \quad \frac{V}{b \cdot \alpha \cdot h} = \frac{\sqrt{G_c}}{\sqrt{h} \cdot \left(\sqrt{0.6(\alpha - \alpha^2)\frac{1}{G_0}} + \beta\sqrt{6\left(\frac{1}{\alpha} - \alpha^2\right)\frac{1}{E_0}}\right)}$$

Notched beams in EC5

- Shear verification of the notched beam

$$\tau_d = 1.5 \frac{V_d}{b_{ef} \alpha h} \leq k_v f_{v,d}$$



Where

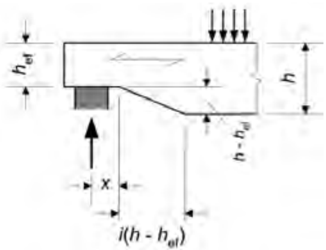
- Reduction factor for the notch

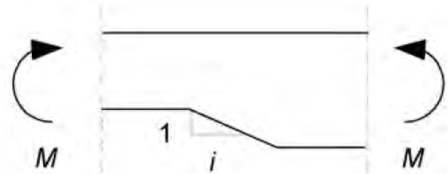
$$k_v = \min \begin{cases} 1 \\ \dfrac{k_n \left(1 + \dfrac{1.1 i^{1.5}}{\sqrt{h}}\right)}{\sqrt{h}\left(\sqrt{\alpha - \alpha^2} + 0.8 \dfrac{x}{h} \sqrt{\dfrac{1}{\alpha} - \alpha^2}\right)} \end{cases}$$

$$k_n = \begin{cases} 4{,}5 & \text{for LVL} \\ 5{,}0 & \text{for solid timber} \\ 6{,}5 & \text{for glued laminated timber} \end{cases}$$

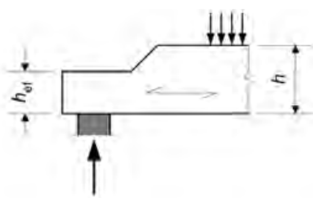
Notched beams in EC5

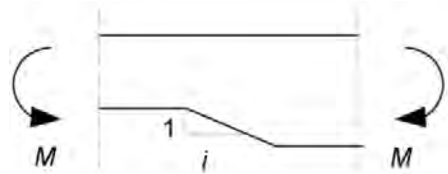
- Notch with tensile stresses at the notch $\left(h_{ef} = \alpha \cdot h\right)$
  - $k_v = \min\left\{1{,}0;\ \dfrac{k_n\left(1 + \dfrac{1.1 i^{1.5}}{\sqrt{h}}\right)}{\sqrt{h}\left(\sqrt{\alpha - \alpha^2} + 0.8\beta\sqrt{\dfrac{1}{\alpha} - \alpha^2}\right)}\right.$





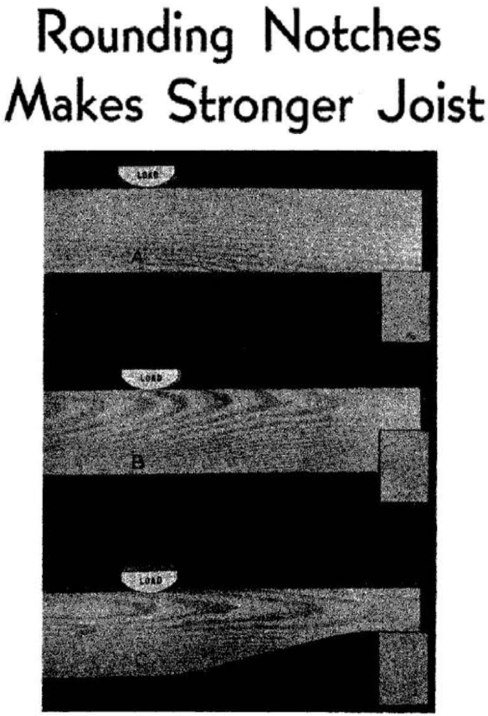
- Notch with compressive stresses at the notch
  - $k_v = 1{,}0$

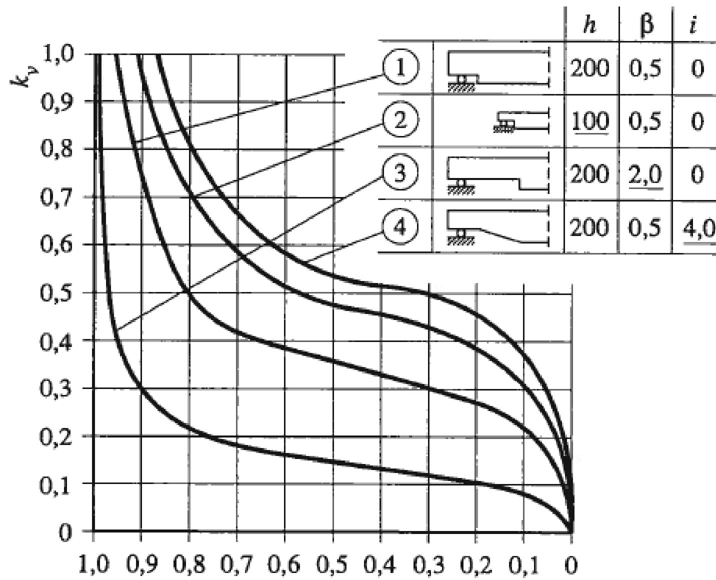




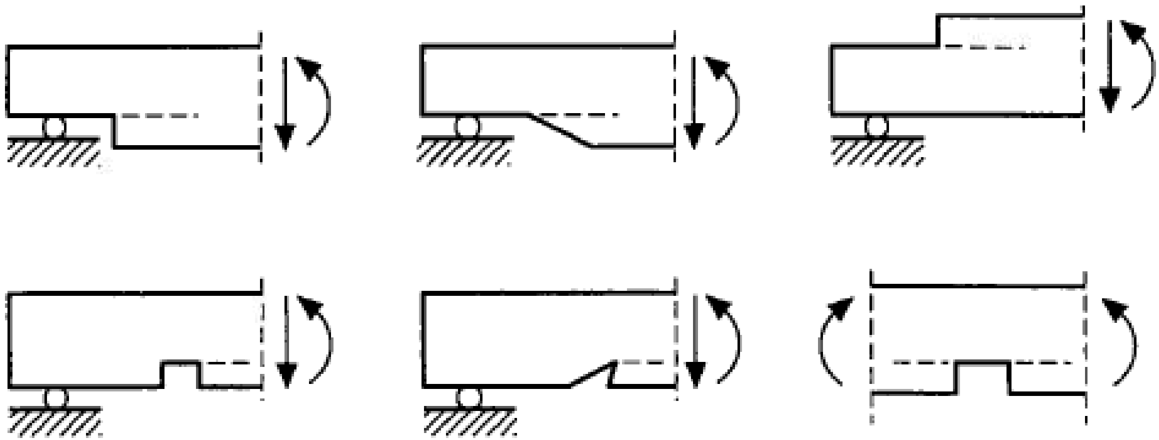
Influences on the notch capacity

- Scholten (1935)





Varieties of notched beams



- Broken line indicates probable crack propagation path

Principles for notches

The following three main principles should be considered:

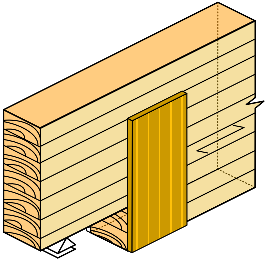
1. Avoid notches!
2. Reinforce notches!
3. Use unreinforced notches only if really necessary.
   Consider an adequate level of safety and robustness (e.g. regarding moisture variations)!

Interventions – reinforcement

- Notched beams
  - Internal, dowel-type reinforcement:
    - fully threaded screws according to EN 14592 or European Technical Assessment
    - screwed-in threaded rods with wood screw thread according to European Technical Assessment
    - glued-in threaded or ribbed steel rods

Interventions – reinforcement

- Notched beams
  - Plane, external reinforcement may be applied:
    - glued-on plywood according to EN 13986
    - glued-on structural laminated veneer lumber according to EN 14374
    - glued-on laminations made of either structural solid timber according to EN 14081-1 or plywood according to EN 13986 or structural laminated veneer lumber according to EN 14374
    - pressed-in punched metal plate fasteners



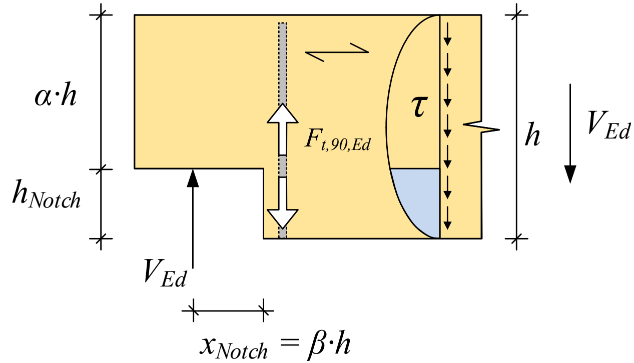
Interventions – reinforcement

- Notched beams

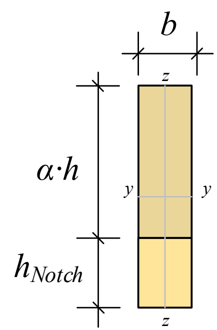
  Force on reinforcement:

  - Beam theory

$$\tau_d = \frac{V_{Ed} \cdot S_y}{b \cdot I_y} = \frac{V_{Ed} \cdot \int_z^{h/2} z \cdot dA}{b \cdot I_y} = \frac{V_{Ed} \cdot \frac{b}{2}\left(\frac{h^2}{4} - z^2\right)}{b \cdot I_y}$$



$$F_{t,90,Ed} = \iint \tau_d(z) \cdot dA = \frac{V_{Ed} \cdot b/2}{b \cdot bh^3/12} \cdot \int_{h/2-h_{Notch}}^{h/2} (h^2/4 - z^2)\, bdz$$



$$\Rightarrow \quad F_{t,90,Ed} = \left[3 \cdot (1 - \alpha h)^2 - 2 \cdot (1 - \alpha h)^3 \right] \cdot V_{Ed}$$

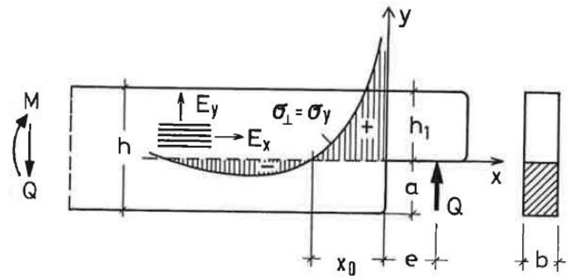
Interventions – reinforcement

- Notched beams

  Force on reinforcement:

  - Plate-theory by Henrici (1984, 1990)

$$F_{t,90,d} = k_\beta k_s k_\alpha \left[3(1-\alpha)^2 - 2(1-\alpha)^3\right] V_{Ed}$$

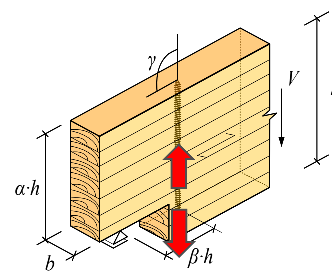


$$k_\beta = 1 + 2\beta$$

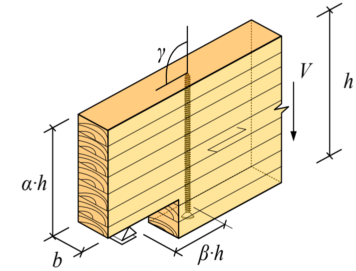
$$k_s = 1 - (s_0 - 1)[1.44(1-\alpha)(1-2\alpha) + 0.10]$$

$$k_\alpha = 1.5(1 - 0.7(1-\alpha))$$

$$s_0 = \sqrt[4]{E_0 / E_{90}}$$



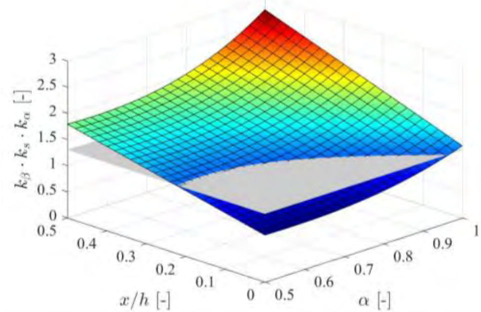
Interventions – reinforcement



- Notched beams

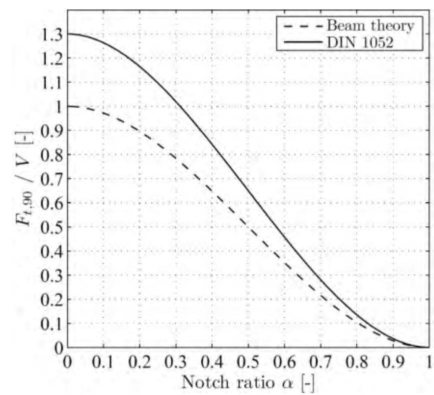
  Force on reinforcement:

  - Plate-theory by Henrici (1984, 1990)
  - product of parameters ($k_\beta \cdot k_\alpha \cdot k_s$) in dependency of relative notch height α and notch length x/h.



  - For softwood:
    Henrici summarized the parameters to a constant value of 1.3 as a conservative estimate for
    $\alpha < 0.8$ and $\beta < 0.2$

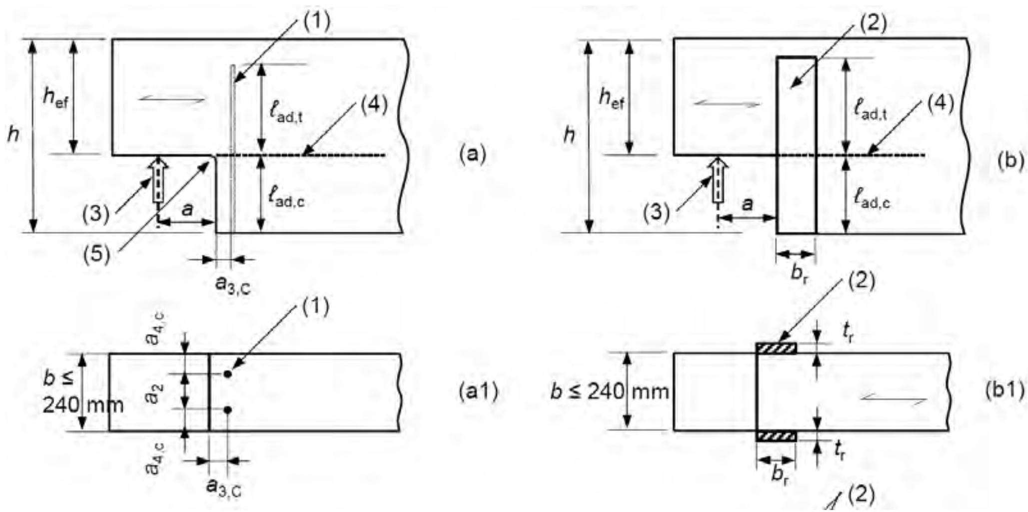
$$F_{t,90,d} = 1.3\left[3(1-\alpha)^2 - 2(1-\alpha)^3\right]V_{Ed}$$



Interventions – reinforcement

- Notched beams
  - Force on reinforcement:

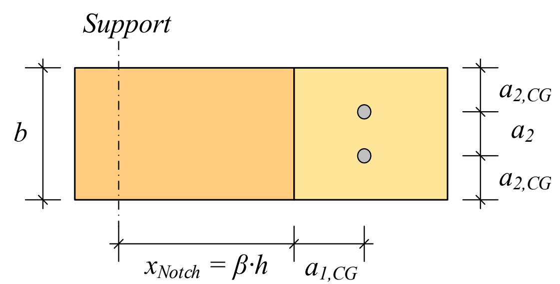
$$F_{t,90,Ed} = 1.3\ [3(1-\alpha)^2 + 2(1-\alpha)^3]V_{Ed}$$



Interventions – reinforcement

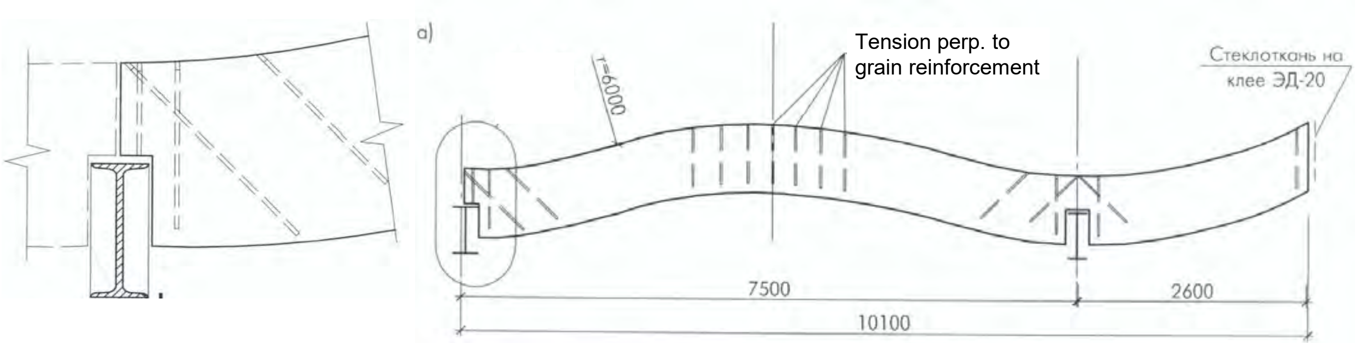
- Notched beams
  - Minimum spacings and end and edge distances for reinforcement by means of self-tapping fully threaded screws according to EN 1995-1-1

Minimum screw spacing in a plane parallel to the grain a_1	Minimum screw spacing perpendicular to a plane parallel to the Grain a_2	Minimum end distance of the screw in the member $a_{1,CG}$	Minimum edge distance of the screw in the member $a_{2,CG}$
$7d$	$5d$	$10d$	$4d$



Reinforced notched beam

Turksovsky (2013)

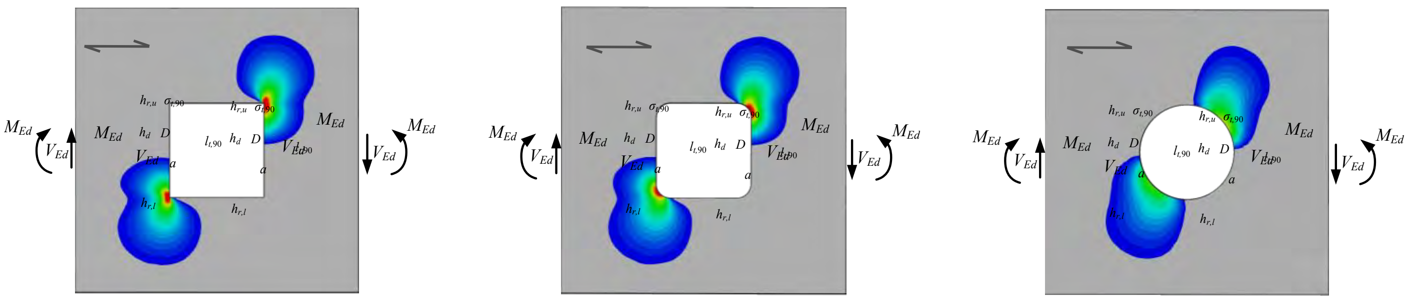


Holes in beams

Examples of beams with holes

Problem

- Tension perpendicular to grain stresses around holes in beams loaded in bending and shear

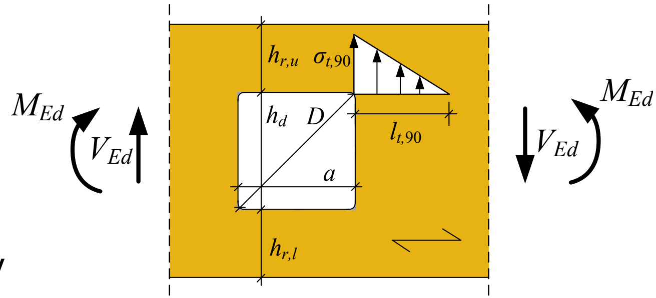


Tensile stresses perpendicular to grain:
high low

Interventions – reinforcement

- Holes in beams
  - Tension perpendicular to grain stresses around holes in beams

$$\sigma_{t,90,d} = \frac{F_{t,90,d}}{0.5 \cdot l_{t,90} \cdot b} \leq k_{t,90} \cdot f_{t,90,d}$$

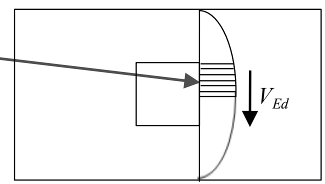


  - Force perp. to grain

$$F_{t,90,d} = F_{t,V,d} + F_{t,M,d}$$

  - $F_{t,V,d}$ derived from beam theory

$$F_{t,V,d} = \frac{V_d \cdot h_d}{4 \cdot h}\left[3 - \left(\frac{h_d}{h}\right)^2\right]$$



  - $F_{t,M,d}$ derived from FE-models

$$F_{t,M,d} = 0.008\frac{M_d}{h_r}$$

where $h_r = \begin{cases} \min\{h_{ru}; h_{rl}\} & \text{for rectangular holes} \\ \min\{h_{ru} + 0.15h_d; h_{rl} + 0.15h_d\} & \text{for round holes} \end{cases}$

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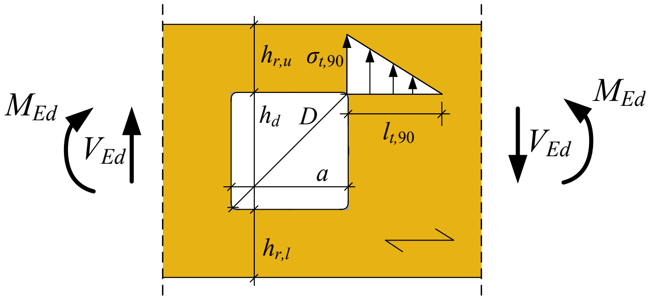
Interventions – reinforcement

- Holes in beams
  - Tension perpendicular to grain stresses around holes in beams

$$\sigma_{t,90,d} = \frac{F_{t,90,d}}{0.5 \cdot l_{t,90} \cdot b} \leq k_{t,90} \cdot f_{t,90,d}$$



  - Force perp. to grain

$$F_{t,90,d} = F_{t,V,d} + F_{t,M,d}$$

  - Size effect factor

$$k_{t,90} = \min\left\{1\,;\,(450/h)^{0.5}\right\}$$

  - Effective length of tensile area

$$l_{t,90} = \begin{cases} 0.5(h_d + h) \\ 0.353 h_d + 0.5h \end{cases}$$

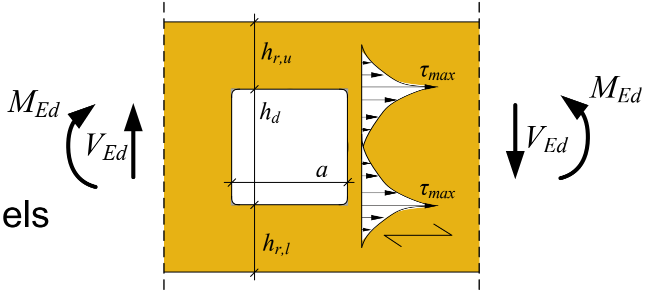
  - Reduced cross section

$$h_r = \begin{cases} \min\{h_{ru} ; h_{rl}\} \\ \min\{h_{ru} + 0.15 h_d ; h_{rl} + 0.15 h_d\} \end{cases}$$

Interventions – reinforcement

- Holes in beams
  - Shear stresses at holes in beams (Blaß, Bejtka 2003)

$$\tau_{\max,d} = \kappa_{\max} \frac{1.5 V_{Ed}}{b\left(h - h_d\right)}$$



  - Parameter derived from FE Models

$$\kappa_{\max} = 1.84\left[1 + \frac{a}{h}\right]\left(\frac{h_d}{h}\right)^{0.2}$$

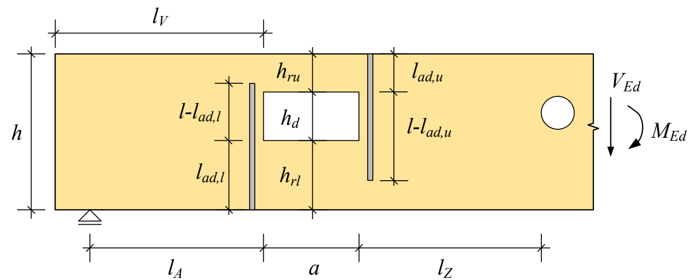
Holtes in beams

- Reinforcement:

Brunauer (2015)

Holes in beams

- Reinforcement
  - Force perp. to grain acting on reinforcement at holes in beams



$$F_{t,90,d} = F_{t,V,d} + F_{t,M,d}$$

- $F_{t,V,d}$ derived from beam theory

$$F_{t,V,d} = \frac{V_d \cdot h_d}{4 \cdot h}\left[3 - \left(\frac{h_d}{h}\right)^2\right]$$

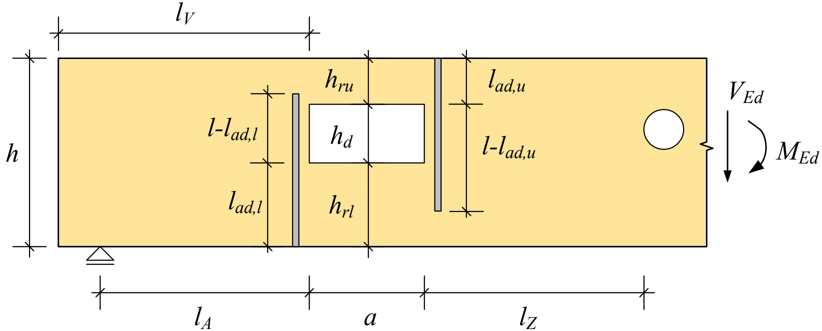
- $F_{t,M,d}$ derived from FE-models

$$F_{t,M,d} = 0.008\frac{M_d}{h_r}$$

where $h_r = \begin{cases} \min\{h_{ru}; h_{rl}\} & \text{for rectangular holes} \\ \min\{h_{ru} + 0.15h_d; h_{rl} + 0.15h_d\} & \text{for round holes} \end{cases}$

Interventions – reinforcement

- Holes in beams
  - Requirements on reinforcement
    - End distance $l_v \geq h$
    - Spacing between holes $l_z \geq h$ and not less than 300 mm
    - Distance to support $l_A \geq h/2$
    - Reduced beam height $h_{rl(ru)} \geq 0.25\ h$
    - Length of the holes $a \leq h$
    - Dimensions of the hole $a/h_d \leq 2.5$
    - Height of the hole $h_d \leq 0.3\ h$



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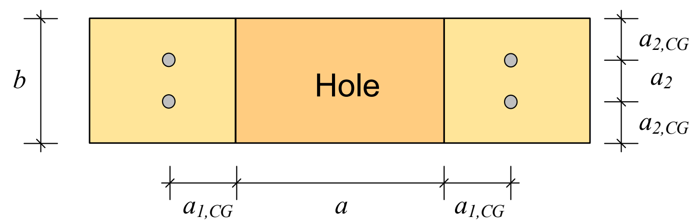
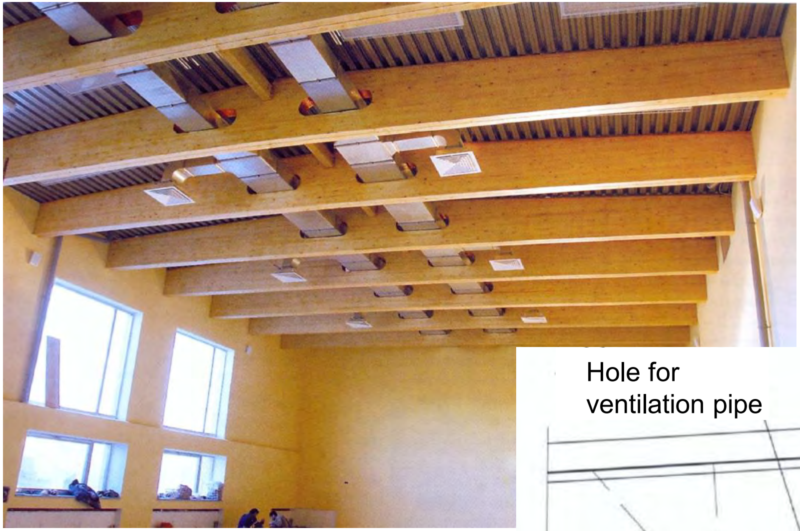
Interventions – reinforcement

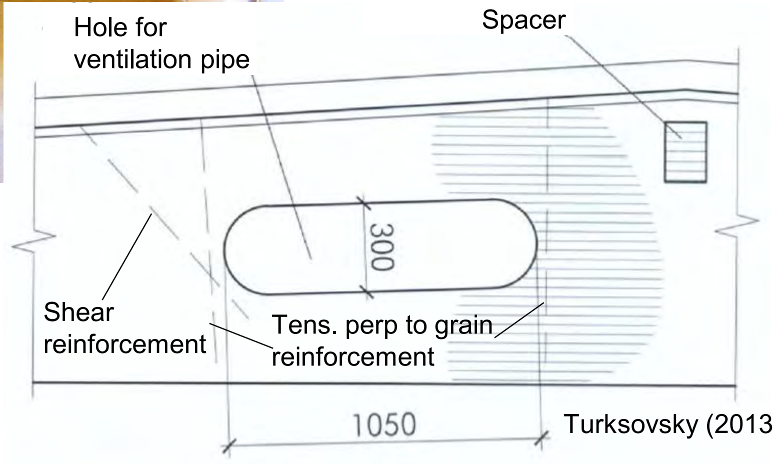
- Holes in beams
  - Minimum spacings and end and edge distances for reinforcement by means of self-tapping fully threaded screws according to EN 1995-1-1

Minimum screw spacing in a plane parallel to the grain a_1	Minimum screw spacing perpendicular to a plane parallel to the Grain a_2	Minimum end distance of the screw in the member $a_{1,CG}$	Minimum edge distance of the screw in the member $a_{2,CG}$
$7d$	$5d$	$10d$	$4d$



Examples: Beam with reinforced holes





Turksovsky (2013)

General - moisture

- Notched members, holes in beams and pitched cambered beams should be reinforced for tensile stresses perpendicular to the grain. Where the design tensile stresses perpendicular to the grain exceed 60 % of the design tensile strength perpendicular to the grain of curved and double tapered beams, these should be reinforced.

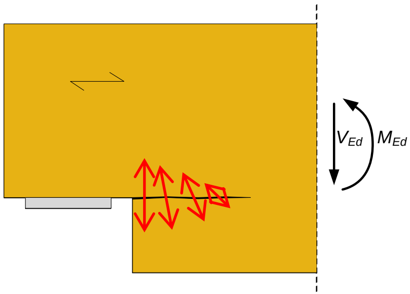
- Notched members, holes in beams, double tapered, curved and pitched cambered beams, assigned to Service Class 3 should be reinforced.

- The reinforcement should be approved for the Service Class of the reinforced timber element.

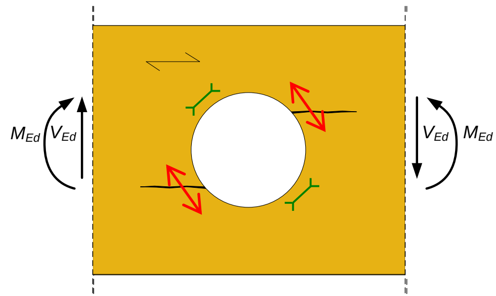
- Corrosion protection of steel reinforcement should be considered.

General – attention in situations with tension perp. to grain!

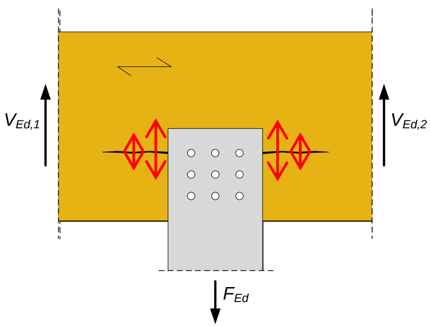
- Notch



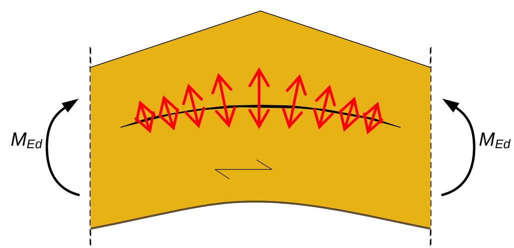
- Hole



- Connection perp. to grain



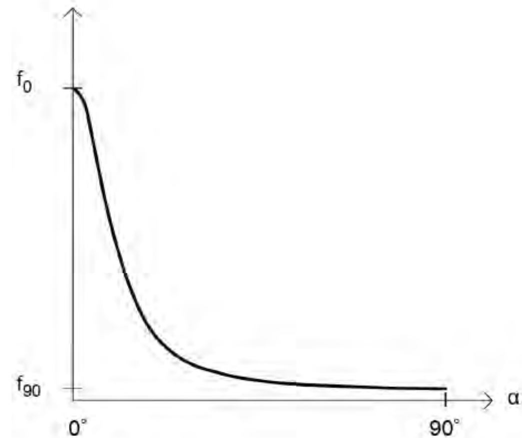
- Curved beams



General – attention in situations with tension perp. to grain!

- Tensile strength at an angle to the grain

$$f_\alpha = \frac{f_0 \cdot f_{90}}{f_0 \sin^2 \alpha + f_{90} \cos^2 \alpha}$$



Eq. 2.3 Design of timber sructures

Example: E1

End-notched glulam beam

Example: E2

End-notched beam

References

- Blaß, H.J., Bejtka, I., (2004), «Reinforcements perpendicular to the grain using self-tapping screws», Proceedings of the 8th world conference on timber engineering. Vol. 1.
- Henrici D. (1984): Beitrag zur Spannungsermittlung in ausgeklinkten Biegeträgern aus Holz. PhD thesis, Technische Universität München, Munich, Germany
- Henrici D. (1990): Beitrag zur Bemessung ausgeklinkter Brettschichtholzträger. Bauen mit Holz 11:806–811
- Aicher, S., Höfflin, L. (2001), «Round holes in glulam members. Part 1: Analysis (in German)», Bautechnik, Vol 78, pp. 706-715,
- Aicher, S., Höfflin, L. (2009), «Glulam Beams with Holes Reinforced by Steel Bars», Proc. of the CIB-W18 Meeting 42, Paper 42-12-1, Zürich.
- Aicher, S. (2011), «Glulam Beams with Internally and Externally Reinforced Holes - Test, Detailing and Design», Proc. of the CIB-W18 Meeting 44, Paper 44-12-4, Alghero.
- Blaß, H.J.; Bejtka, I. (2003), «Querzugverstärkungen in gefährdeten Bereichen mit selbstbohrenden Holzschrauben», Report, Versuchsanstalt für Stahl, Holz und Steine, Universität Karlsruhe, Germany.
- Kolb, H., Epple, A. (1985), «Verstärkungen von durchbrochenen Brettschichtholzbindern», Schlussbericht zum Forschungsvorhaben I.4 – 34810, Forschungs- und Materialprüfungsanstalt Baden-Württemberg, Stuttgart, Germany.
- Scholten. “Rounding Notches Makes Stronger Joists.” *American Lumberman*, 1935.

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