Berth Allocation

The problem considers a port terminal quay of total length $L \in \mathbb{N}$, as shown in Figure 1. A set V of vessels are scheduled to arrive at the terminal on a weekly basis. Each vessel $i \in V$ has scheduled arrival and departure times $start_i, end_i \in \mathbb{N}$, respectively, which correspond to exact hours within the week. For example, a vessel i with $start_i = 10$ and $end_i = 30$ must be moored at the quay from Monday 10am to Tuesday 6am. Each vessel is also characterised by its length $\ell_i \in \mathbb{N}$, in meters. Each vessel must be assigned to a berthing position in the quay, where it remains moored for its entire stay. The assignment of a vessel $i \in V$ to a position $k \in [0, L]$ is represented by a **Berth Allocation** (BA) $A_i = k$, which indicates that i covers part $[k, k+\ell_i]$ of the quay, as shown for Vessel 5 in Figure 1. Vessels with overlapping stays must not only moor at non-overlapping berthing positions, but must also ensure they respect a minimum safety distance $d \in \mathbb{N}$.

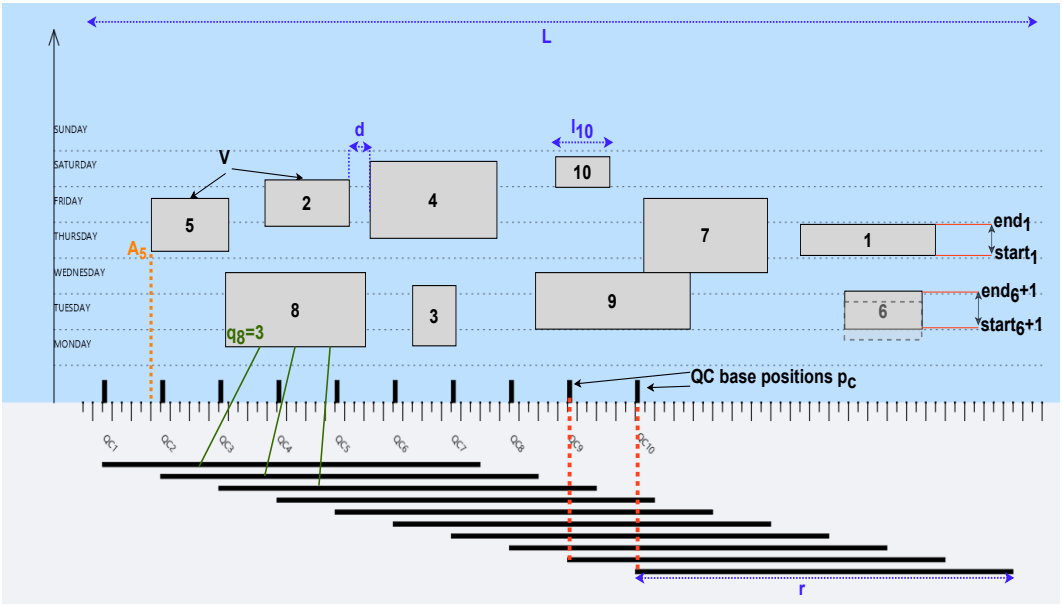

Figure 1: Setting configuration. Vessels are represented by rectangles. Vertical axis = time and horizontal axis = space.

Note that the arrival schedule is cyclic and is repeated every week. In this manner when $start_i > end_i$ for a vessel i, it is assumed that its arrival

time is before and its departure time after Sunday 23.59, which should be considered for time-space overlaps as shown for Vessels 1 and 4 in Figure 2.

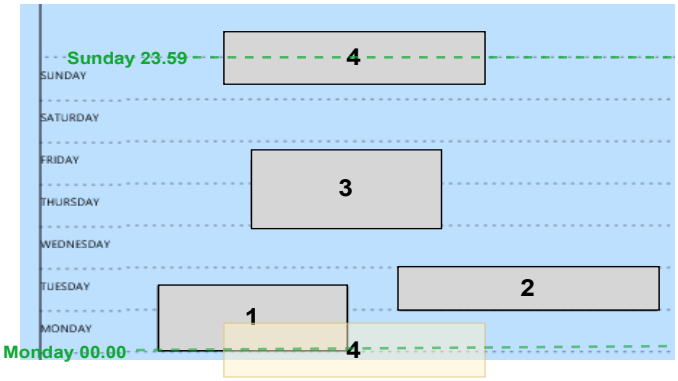

Figure 2: Cyclic schedule representation. Vessel 4 arrives on Sunday and departs on Monday. The yellow, transparent rectangle visualises its stay for the previous calendar week.

Flexible berthing windows

Vessels are also allowed to shift their berthing time window for a maximum time of s hours later, as long as their stay **duration** remains unchanged and that they arrive at an exact hour. For example, given $s = 3$, a vessel i with $start_i = 10$ and $end_i = 30$ can be moored at the quay within one of the windows $[10, 30]$, $[11, 31]$, $[12, 32]$ and $[13, 33]$. Vessel 6's stay in Figure 1 is shifted 1 hour later than its scheduled berthing time window, which is depicted by the transparent rectangle.

Quay Crane assignments

A set of Quay Cranes (QCs) C are located uniformly along the quay. Each QC $c \in C$ has a base position p_c and may operate within a limited range of fixed length r, serving part $[p_c, p_c + r]$ of the quay as shown in Figure 1. Each crane may serve only one vessel at any time and must be assigned to it for its entire stay. Each vessel i is finally characterized by a number q_i which indicates the minimum number of QCs which must be serving it during its stay. In order for a QC c to be compatible with a vessel i assigned at a position k, it must hold that $p_c \leq k < k + \ell_i \leq p_c + r$, as it holds for Vessel 8 and QCs 1, 2 and 3 in Figure 1.