Challenge Problem

Probability of rolling a 5 before 7 when rolling a pair of dice

$$P(\text{rolling a 5 on first roll}) = \frac{4}{36} = \frac{1}{9}$$

$$P(\text{rolling a 5 a second roll}) = P(\text{not a 5 or 7 on first}) \cdot P(\text{5 on second})$$

$$= \left(1 - \frac{10}{36}\right) \cdot \frac{1}{9} = \frac{1}{9} \cdot \frac{13}{18}$$

$$P(\text{rolling a 5 on third roll}) = P(\text{not a 5 or 7 on first}) \cdot P(\text{not a 5 or 7 on second}) \cdot P(\text{5 on third})$$

$$= \frac{13}{18} \cdot \frac{13}{18} \cdot \frac{1}{9}$$

So the pattern is:

$$\frac{1}{9} + \frac{1}{9} \cdot \frac{13}{18} + \frac{1}{9}\left(\frac{13}{18}\right)^2 + \frac{1}{9}\left(\frac{13}{18}\right)^3 + \cdots$$

$$= \frac{1}{9}\left(1 + \frac{13}{18} + \left(\frac{13}{18}\right)^2 + \left(\frac{13}{18}\right)^3 + \cdots\right)$$

$$= \frac{1}{9} \sum_{n=0}^{\infty} \left(\frac{13}{18}\right)^n$$

geometric series with $a = 1$, $r = \frac{13}{18}$

$$= \frac{1}{1 - \frac{13}{18}} = \frac{1}{\frac{5}{18}} = \frac{18}{5}$$

$$= \frac{1}{9} \cdot \frac{18^2}{5} = \boxed{\frac{2}{5}}$$