Lecture 11: Perceptron and Gradient Descent (Mitchell Chapter 4)

CS 167: Machine Learning

Extra Credit Opportunity

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Deep Learning with Big and Small Data

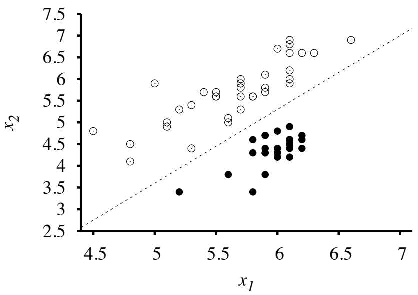
Deep learning has brought tremendous success in many areas with the help of big data and super computing. In this talk, I will present the state of art results of deep learning for image classification. I will also talk about our recent research on how to learn a deep convolutional neural network for fine-grained image classification where big labeled data is difficult to be obtained.

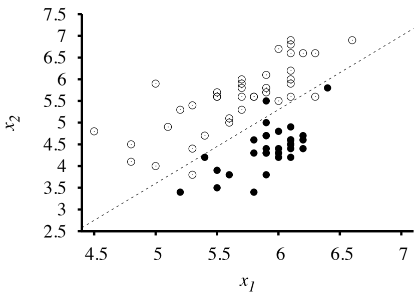
Friday, October 21, in Meredith 106 at 2:00pm.

Earn 3 extra credit (homework) points for

- attend the talk
- write up a paragraph on something you learned

Linear Separation



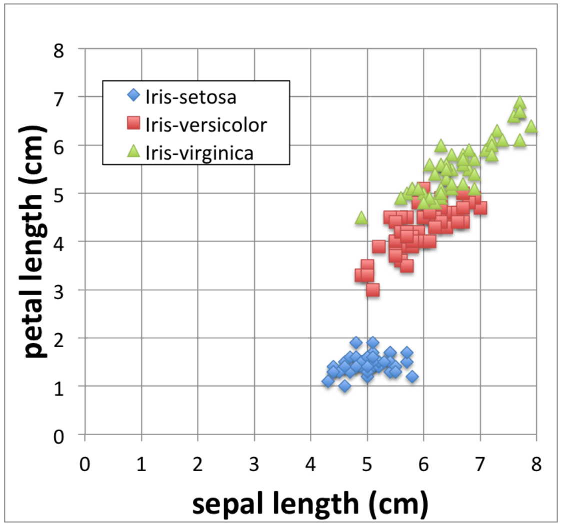


x_1: body wave magnitude, x_2: surface wave magnitude
white: earthquakes, black: underground explosions

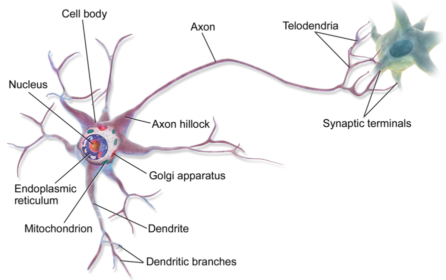
2D data: separate with a line **What do equations of lines look like?**
3D data: separate with a plane **What do equations of planes look like?**
***N*D data**: separate with a hyperplane **What do equations look like?**

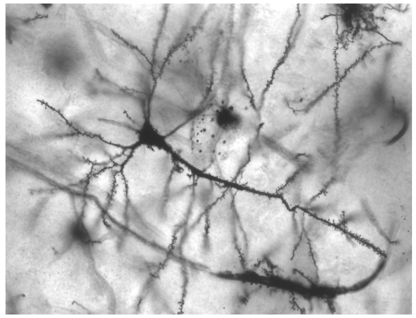
Discussion: Iris Data

Can we use a linear separator as our hypothesis on this data?



Biological Motivation

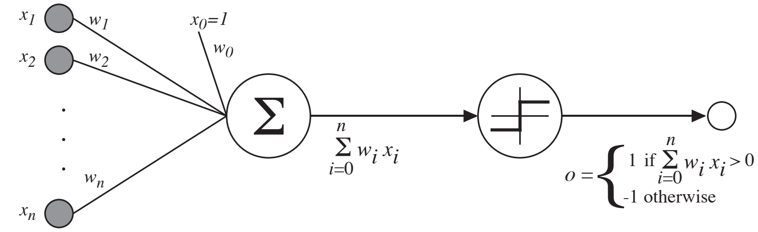




https://commons.wikimedia.org/wiki/Neuron#/media/File:Pyramidal_hippocampal_neuron_40x.jpg
https://en.wikipedia.org/wiki/Neuron#/media/File:Blausen_0657_MultipolarNeuron.png

- dense network, 10^{11} neurons
- each on average connected to 10^4 others
- neuron switching times $<$ .001 seconds - slow
- fast recognition $\Rightarrow$ highly parallel brain
- computer switching times $\approx$ .0000000001 seconds, can't handle complexity of brain

Perceptron



$$o(x_1, \ldots, x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + \cdots + w_n x_n > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Sometimes we'll use simpler vector notation:

$$o(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x} > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Let's train a perceptron

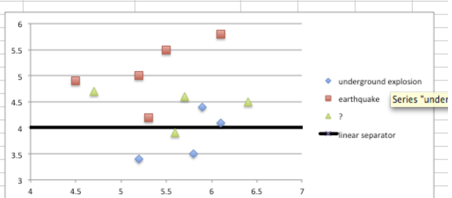
Exercise: *Use the earthquake spreadsheet to come up with the equation of some lines that separate the data.*

Seismometer Data

Body Wave Magnitude	Surface Wave Magnitude	Classification	Result of Weighting Function	After activation function	Prediction (-1 = earthquake, +1 = explosion)
5.2	3.4	underground explosion	0.6	1	underground explosion
5.8	3.5	underground explosion	0.5	1	underground explosion
5.9	4.4	underground explosion	-0.4	-1	earthquake
6.1	4.1	underground explosion	-0.1	-1	earthquake
5.2	5	earthquake	-1	-1	earthquake
4.5	4.9	earthquake	-0.9	-1	earthquake
5.3	4.2	earthquake	-0.2	-1	earthquake
5.5	5.5	earthquake	-1.5	-1	earthquake
6.1	5.8	earthquake	-1.8	-1	earthquake
5.6	3.9	?	0.1	1	underground explosion
5.7	4.6	?	-0.6	-1	earthquake
4.7	4.7	?	-0.7	-1	earthquake
6.4	4.5	?	-0.5	-1	earthquake

w_offset	4	<-- adjust to change offset
w_x	0	<-- adjust to change x weight
w_y	-1	<-- adjust to change y weight

x	y	
0	4	<-- don't change these, updated automatically
10	4	<-- don't change these, updated automatically



Perceptron training rule

Feed a training example into the perceptron, then for each weight w_i

$$w_i \leftarrow w_i + \Delta w_i$$

where

$$\Delta w_i = \eta(t - o)x_i$$

Where:

- $t = c(\vec{x})$ is target value
- o is perceptron output
- η is small constant (e.g., 0.1) called **learning rate**

Exercise: What happens to the weight in each of these cases:

- t and o are both 1
- t and o are both -1
- t is 1, o is -1
- t is -1, o is 1

How to train your perceptron

Do the following over and over and over

1. feed in a training example
2. update weights with weight update rule

Can prove it will converge

- If training data is linearly separable
- and η sufficiently small

Another similar approach

If the data isn't linearly separable, use the **Gradient-Descent** training rule:

Use *unthresholded* output (don't push it to 1 or -1)

$$o = w_0 + w_1 x_1 + \cdots + w_n x_n$$

Sum up Δw_i for all training *examples* before updating w_i

Gradient-Descent Algorithm

$$o = w_0 + w_1 x_1 + \cdots + w_n x_n$$

- Initialize each w_i to some small random value
- Until the termination condition is met, Do
  - Initialize each Δw_i to zero.
  - For each $\langle \vec{x}, t \rangle$ in *training_examples*, Do
    - Input the instance $\vec{x}$ to the unit and compute the output o
    - For each linear unit weight w_i, Do

$$\Delta w_i \leftarrow \Delta w_i + \eta (t - o) x_i$$

  - For each linear unit weight w_i, Do

$$w_i \leftarrow w_i + \Delta w_i$$

Summary

Perceptron training rule guaranteed to succeed if

- Training examples are linearly separable
- Sufficiently small learning rate η

Linear unit training rule uses gradient descent

- Guaranteed to converge to hypothesis with *minimum squared error* (see proof in textbook - uses partial derivatives)
- Given sufficiently small learning rate η
- Even when training data contains noise
- Even when training data not separable by H

And just to make it more confusing, if you update weights one at a time for each training example but use an *unthresholded* unit, that's one way to do it too called **stochastic gradient descent**.

Let's do this with scikit-learn

Add this to the code you were using with the Iris data set last time

```
from sklearn.linear_model import Perceptron

perc = Perceptron()

perc.fit(iris_train[predictors],iris_train['species'])
iris_perc_predictions = perc.predict(iris_test[predictors])
print(metrics.accuracy_score(iris_test['species'],\
                             iris_perc_predictions))
```

Is this the accuracy you expected? Why does it perform this way?

Pandas get_dummies

Let's do a transformation to our original Iris data:

```
iris_dummies = pandas.get_dummies(iris_data)
print(iris_dummies)
```

Discuss: What did that just do?

Exercise: Create Perceptron classifiers for each of viginica, setosa, and versicolor. Evaluate each classifier's performance and explain why they performed as they did.

Review: Regression vs. Classification

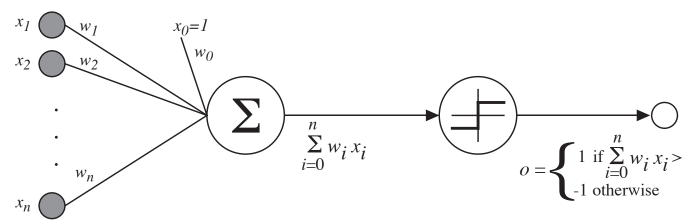
Classification problems: predict which category an examples goes in, examples:

- predict the iris species
- predict whether or not the passenger survived
- predict the acceptability level of a car
- predict creditworthiness
- predict a person's sex from their photo

Regression problems: predict a numerical value, examples:

- predict a car's fuel efficiency in MPG
- predict a person's age from their photo

Group Discussion



Weight update rule:

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

How can we adapt perceptron, gradient descent, and/or stochastic-gradient-descent so that it does regression instead of classification?

Trying out regression in scikit-learn

Exercises:

- read in the Auto MPG data set (note: we will predict the `mpg` column)
- fill in missing values for `horsepower` column
- split into test/train
- try to fit a perceptron learner to the data, **What happens?**, **Is this expected?**
- visualize some of the attributes compared to the `mpg` column:

```
%matplotlib inline
import matplotlib.pyplot as plt
plt.scatter(mpg_data['weight'],mpg_data['mpg'])
plt.show()
```

  Does it seem like we should be able to fit a line to this data?

Regression on MPG Data

Find a regression algorithm based on perceptron, gradient descent, or stochastic gradient descent

http://scikit-learn.org/stable/modules/classes.html

Exercises:

- Use it to make predictions on your test set. **How do these look?**
- Determine what happens when you try to measure the accuracy, explain why it does this

Common Regression Metrics

error(x) on an example x: (absolute value) difference between *predicted* and *actual* value for that example

Mean Absolute Error: find the mean of *error*(x) over all examples x from the test set

Mean Squared Error: find the mean of $(error(x))^2$ over all examples x from the test set

Exercise: *Find the `scikit-learn` functions for measuring these, use them on your earlier predictions*

Improving Performance

Weight update rule:

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

We know there are at least two parameters of this algorithm that can affect performance

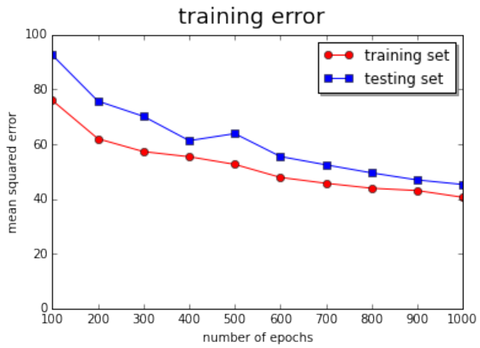
- η, the learning rate
- the number of *epochs*, i.e., the number of times you feed the set of training examples (and update the weights) through the perceptron unit during training

What are these values set to by default?

Exercise: *Find an η and number of epochs that seem to give you decent results.*

Exercise: Make a Plot

Make a plot like this that shows how the performance on the test and training set is affected by the number of epochs



At what point does the MSE stop decreasing?

Exercise: Regression with k-Nearest-Neighbor

Exercise:

- find a way to do regression with Random Forests or k-Nearest-Neighbor in scikit-learn
- comparetheir performance against the Perceptron/GD/SGD method