DC Circuits of Electric Railway Systems

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Abstract

The objective of this research project is to understand the functioning of electric railway systems. This topic is pertinent in the context of Montreal's REM development. Electric railway systems are used for the transportation of electric locomotives in urban settings. With a focus on DC circuits, this paper details on some elements that exist in the circuits of such electric systems and that allow for the railways to function. These elements are traction power substations, braking energy recovery, contact lines, basic electrical calculations, unilateral power supplies, and bilateral power supplies. The methodology used is the reading of various textbooks, research papers, and review papers on the topic, as well as the use of a software to illustrate specific circuits for visualization purposes. This paper contains explanations on theoretical mathematical equations related to electric systems and diagrams of circuits that will serve as examples to explain the theory. Under ideal conditions, bilateral power supplies proved to be four times more optimal than unilateral power supplies. The conclusion yields a better understanding of electric railway systems.

French Translation – Traduction française

L'objectif de ce projet de recherche est de comprendre le fonctionnement des systèmes ferroviaires électriques. Ce sujet se voit pertinent vu les récents développements liés au REM à Montréal. Les systèmes ferroviaires électriques sont utilisés au sein du transport des locomotives électriques en milieux urbains. Cet article détaille sur certains éléments faisant partie des circuits à courant continu (DC) électriques de ce type de système qui permettent leur fonctionnement. Ces éléments sont, entre autres, les sous-stations d'alimentation électrique, la récupération d'énergie de freinage, les lignes aériennes de contact, les calculs fondamentaux de l'utilisation de l'électricité, ainsi que les sources d'alimentation électrique unilatérales et bilatérales. La méthodologie employée consiste de consulter divers manuels, rapports de recherche et documents universitaires, ainsi que de l'utilisation d'un logiciel servant à illustrer certains circuits pour fin de visualisation. Cet article explique la théorie permettant de comprendre des équations mathématiques liées à ces systèmes de circuits électriques and contient des schémas exemplifiant cette théorie. Dans des conditions jugées idéales, les sources d'alimentation électrique bilatérales se trouvent être quatre fois plus optimales que celles unilatérales. En bref, cet article permet d'acquérir une meilleure connaissance des systèmes ferroviaires électriques.

Introduction

Montreal is currently in the middle of developing and building a new, completely automated electric light railway system in order to replace the previous one. This new light rail system is called the *Réseau express métropolitain* (REM) and is scheduled to be fully ready for use by 2024 (REM, 2021). In light of these recent developments, in this paper, we will analyze DC circuits of electric railway systems with the objective of understanding how they function.

Electricity is defined as the flow of electrons within an object that conducts electricity, also called a conductor. It has many properties such as its current, resistance, and voltage. Current (I) is the scalar measurement of the rate at which electrons flow through a surface and its unit is the ampere (A) which is equivalent to one Coulomb (C) of charge per second. As shown in Figure 1, the direction of the current in an electric circuit is always opposite to the direction of the moving electrons. In other words, while electrons move from the anode (-) terminal to the cathode (+) terminal of any source of direct current (DC) such as a battery, the current goes from the positive terminal to the negative terminal, hence flowing from a state of high to low potential energy. This constant potential difference provided by a battery is also called a voltage (V), measured in volts (V).

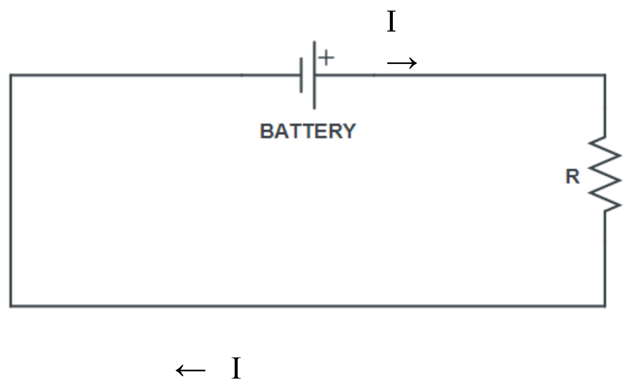

Figure 1 - Simple example of DC circuit.

As shown in Figure 2, inside of a battery itself, external work is required in order to move the electrons from the cathode to the anode, and such an ability is called the electromotive force (it is important to note that despite the name, it is not a force; it is a voltage), also known as emf ($\mathcal{E}$).

$$\mathcal{E} = \frac{W_{ext}}{q} = V$$

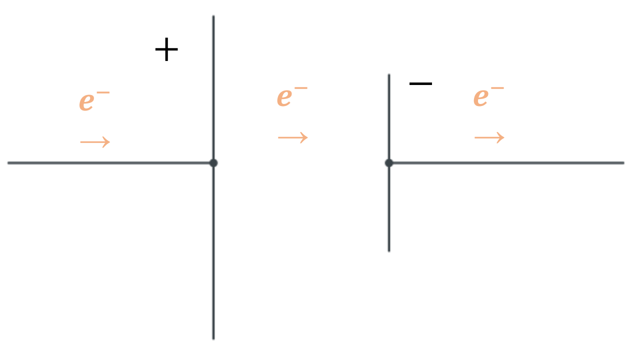

Figure 2 - Electron flow in a battery.

In electric circuits, resistance is provided by resistor devices (R) and carry their own potential when a current flows through them. Resistance is measured in Ohms (Ω) where $1\ \Omega = 1\frac{V}{A}$. Resistance depends on the material and size of an object because each material has its unique resistivity (ρ) and $R = \rho\frac{L}{A}$ where L is the length of the object and A is the area of its cross-section. Ohm's law states that if R is constant, then ΔV is directly proportional to I, and thus

$$\Delta V = RI$$

A resistor is an ohmic device because it follows Ohm's Law and when building a circuit, resistors can be placed in series or in parallel.

Kirchhoff's rules of circuit analysis state the following for closed circuits:

$$\sum I_{in} = \sum I_{out}$$

$$\sum \Delta V = 0$$

The first equation indicates that the sum of all the currents leaving a junction must be equal to the sum of the currents entering that same junction. The second equation indicates that the sum of the changes in potential must be equal to zero. This allows for inference of further rules for series and parallel connections of resistors in a circuit.

For resistors placed in series, such as R1 and R4 in Figure 3, the following rules apply:

$$\Delta V_{total} = \Delta V_1 + \Delta V_4$$

$$I_{eq} = I_1 = I_4$$

$$R_{equivalent} = R_1 + R_4$$

For resistors placed in parallel, such as R_{eq} (which replaces R1 and R4), R2 and R3 in Figure 3, the following rules apply:

$$\Delta V_{total} = \Delta V_{total(1\&4)} = \Delta V_2 = \Delta V_3$$

$$I_{total} = I_{eq} + I_2 + I_3$$

$$R_{equivalent} = \frac{1}{R_{eq(1\&4)} + R_2 + R_3}$$

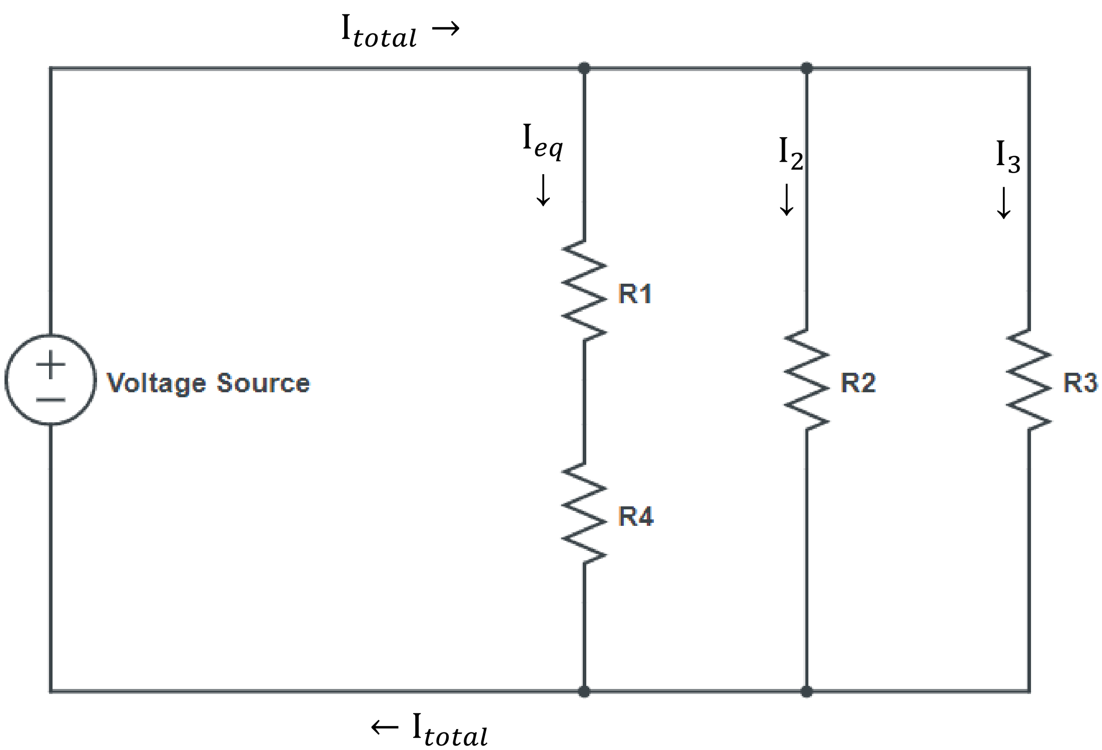

Figure 3 - DC circuit example to show Kirchhoff's rules.

Power (P) is the rate at which the electrical energy is changed, and its unit is the watt (W). It is directly proportional to the current and the potential, such that: $P = I\Delta V$.

Capacitors (C) are devices in circuits that can store charge and electrical energy. They consist of two conductors serving as plates and an insulator. For example, for a parallel-plate capacitor, air can be used as an insulator in-between the two plates. Air is a type of insulator called a dielectric. Other forms of such an insulator are vacuum, glass, and rubber. Each dielectric has its own constant (κ) and for air, $\kappa \cong 1$. Capacitors measure capacitance (C) in farads (F) where $1\ F = 1\ \frac{Coulomb}{Volt}$ and thus, $C = \frac{Q}{\Delta V}$. Capacitance is the ability to store charge or energy and it depends on the shape of the capacitor. According to Farad's law, $C = \kappa\, \varepsilon_0 \frac{A}{d}$ where the capacitance varies according to the area of the plates (A) and the distance that separates them (d) (Openstax, 2016). (ε_0 is a constant called Epsilon Zero and has an approximate value of $8.85 \cdot 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}}$.) Figure 4 illustrates a capacitor in an electric circuit. Capacitors have charging and discharging cycles. To further explain that, suppose the capacitor in Figure 4 were only connected to the battery, it would then be in its charging cycle and accumulate a certain voltage (and if charged to its maximum potential, then its voltage would be the same as the battery's). If this same capacitor were to be disconnected from the battery and then connected to the resistor, then it would enter its discharging cycle which would provide a current in the circuit that would pass by the resistor until the capacitor's voltage drops to zero.

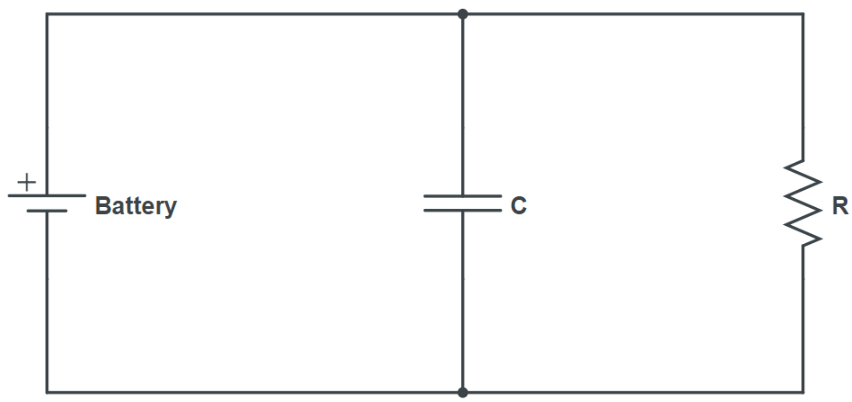

Figure 4 - Circuit with a capacitor.

The software used to illustrate the circuits in this paper is the free online circuit simulation tool CircuitLab (Robbins & Evans, 2021).

Traction Power Substations

In order to represent the entire group of systems, phenomena, and physical equipment required to cause the motion of a vehicle like a locomotive, the term “traction” is used (Brenna, Foiadelli, & Zaninelli, 2018, p. 1). The energy provided to rails and metro lines comes from traction power substations (TPSS).

For systems that serve the purposes that railways do, such as providing means of transportation to living passengers on a nonstop, daily basis, it is important for them to adhere to two fundamental concepts: continuity and quality of service. In other words, in order for the quality of the train service to be maximized, the service must be able to go on even if it faces failures in important parts of its system throughout its lifespan. For example, if a switch were to fail, we must be able to be replace it and therefore, every piece of equipment has to be redundant (p. 104). This particularly applies to substations because they are energy suppliers and an electric system cannot run without energy. A TPSS has the role of converting external sources of electricity into a current that has an appropriate voltage for the railway that will use it, so it serves as a voltage source for the railway (akin to a battery, which is another form of a source of voltage for a circuit). Generally, a TPSS has both an AC and a DC section that each serve different purposes. (AC means alternating current.) Among those, the AC side is provided with an input power through a cable (for example, a high-voltage (HV) one) that can be either high above the ground in the air with supports, or below the ground, depending on the architecture of the area it is located in. All input lines tend to have the same following components: voltage transformers, switches, current transformers, circuit breakers, rotating disconnectors, and surge arresters (p. 105). In order for the system to maintain redundancy, two rows of busbars are usually placed, rather than one (p. 107). A busbar is a conductor that collects power in the form of electricity (common point for incoming and outgoing current) and within a busbar system, there are circuit breakers that make sure the current can be switched off in case there is a problem within the system (Circuit Globe, 2021).

Braking Energy Recovery

Given that electric railways use electric energy in order to function, there is a way to make them more efficient in terms of saving energy (Brenna, Foiadelli, & Zaninelli, 2018, p. 133). Dynamic braking is when kinetic energy is converted into electricity and relies on electric motors doubling their job as generators. The electricity that has been regenerated through braking is either dissipated as heat using rheostats (a type of resistor device) or recycled to be reused within the railway system. The latter is called regenerative braking and it allows to reduce energy consumption. In DC circuits, there exists a certain problem where the regenerated electricity can only be used if the consumption is immediate and simultaneous (Gonzàlez-Gil, Palacin, & Batty, 2013). In other words, if one locomotive is braking, then another locomotive must be accelerating at the same time within the same frame of that electric network. A solution to this problem offered by Brenna et al. (2018) is to store this regenerated energy in supercapacitors and have these devices placed near the tracks.

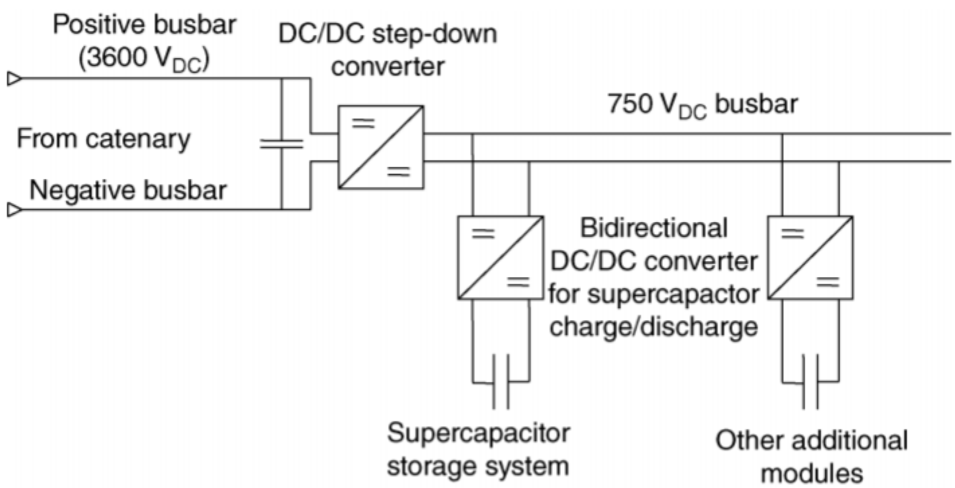

Figure 5 - Connection of the supercapacitor storage system to high-voltage DC lines (Brenna, Foiadelli, & Zaninelli, 2018, p. 134).

The DC/DC step-down converter in Figure 5 is a power converter that lowers the voltage of its input for its output. In Figure 5, the 3600 V coming from the busbars was lowered to 750 V, and the term catenary simply refers to overhead cables.

In the case of subway lines, the positioning of each TPSS is structure in a way that doesn't let the voltage drop more than 33% of the nominal value, meaning the standard value it has been assigned. Ideally, equidistant TPSSs that are considered to be voltage sources (or generators) and each have their own internal resistance would imply that voltage discontinuity is not be present in-between the TPSSs and thus, would not serve as unique sources of power for trains, but rather, all of the TPSSs as a whole, would supply energy to the trains. This would also signify that "each train is predominantly supplied from the nearest TPPSs, while the farthermost ones do not contribute in any significant way" (p. 135). Once again, in order for this ideal system to follow the fundamental concept of continuity, if one of the substations were to malfunction and require to temporarily shut down, then the substation next to it must be able to provide enough power for the entire railway system to keep working.

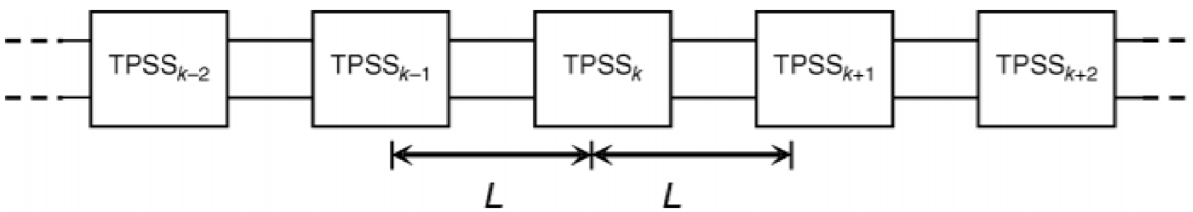

Figure 6 - TPSSs placed along a subway line (Brenna, Foiadelli, & Zaninelli, 2018, p. 135).

In order to recover energy for an entire train line, we must determine the number of TPSSs that need to be installed to figure out how big or how small of a storage is necessary for the entire system. This number depends on length of the line and the "traffic conditions" (p. 136). Using supercapacitors that are "continuously charged and discharged" (p. 139), the following equation computes the ideal energy that needs to be exchanged for every charging-discharging cycle of the supercapacitors ($E_{supercapacitor}$):

$$E_{sc} = \frac{3}{8} \cdot C \cdot V_{sc_n}^2$$

where V_{sc_n} is the nominal voltage of the supercapacitor and C is the capacitance (p. 139).

Contact Lines

An ideal contact line has the following characteristics: keep the height of the contact conductors constant, including at pantographs transit, and the contact conductors symmetrical despite varying road conditions and height of the ground, and remain stable despite strong winds. Catenary suspension is used to make sure these characteristics are followed. Catenary suspension is defined by suspended wires that take on a parabolic shape, as shown in Figure 7, where the top wire is the messenger wire, and the bottom wire is the contact wire (p. 142).

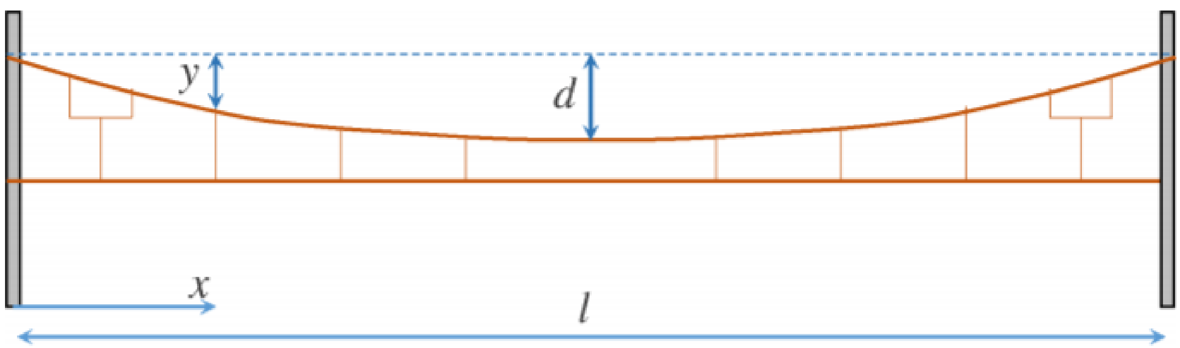

Figure 7 - Catenary suspension (Brenna, Foiadelli, & Zaninelli, 2018, p. 143).

Deflection (y) is the displacement of the bending of an object from its initial undeformed state when a force is exerted on it. It can be calculated using following the equation:

$$y = \frac{m \cdot g \cdot x^2}{2T}$$

where m is the linear mass of the contact wire (in kg/m), g is the gravitational acceleration (9.8 m/s^2), x is the distance (in m) between the two points supported by vertical wires (called droppers), and T is the tension in the wire caused by the pull of the messenger wire (in Newtons). In Figure 7, d represents the point at which there is the maximum deflection of the messenger wire (p. 143).

Electrical Calculations

In the context of electric railway systems, a great majority of the parts are made using copper (p. 144). The resistance of a railway line power supply circuit (r) in $\frac{\Omega}{km}$ is calculated using the sum of the resistance of the catenary (r_c) (which includes the messenger and contact wires) and of the track (r_r):

$$r = r_c + r_r$$

where $r_c = \rho_{Cu} \cdot \frac{1000}{S_c} = \frac{18}{S_c}$ because the resistivity of copper is 0.018, and $r_r = \frac{0.75}{m}$ (S_c is the cross-sectional area of the wire in mm^2). However, due to the erosion of the tracks over time and added resistance from connections of different sections of the railway, the track resistance is approximated to $\frac{0.9}{m}$ (p. 148).

Voltage drop tends to occur because of four fundamental properties: material, wire size, wire length, and current flow (APOGEE INTERACTIVE, 2021). In DC circuits, the only cause for these drops is the resistive component of the power supply line. Supposing the current that the trains absorb is constant, without statistical data on traffic, and using ideal TPSSs that generate a constant voltage, the voltage drops can be approximated (Brenna, Foiadelli, & Zaninelli, 2018, p. 150).

Unilateral Power Supply

If the power supply is unilateral, meaning that there is only one substation that provides power to the train, then the voltage drop (which is a change in electric potential) becomes a linear function of x, the position of the train, with a slope of $r \cdot I$, giving the following equation:

$$\Delta V_{(x)} = r \cdot I \cdot x \qquad \text{(p. 150).}$$

This is an equation that can be used when there is a single train moving along the line and absorbing current, as shown in Figure 8.

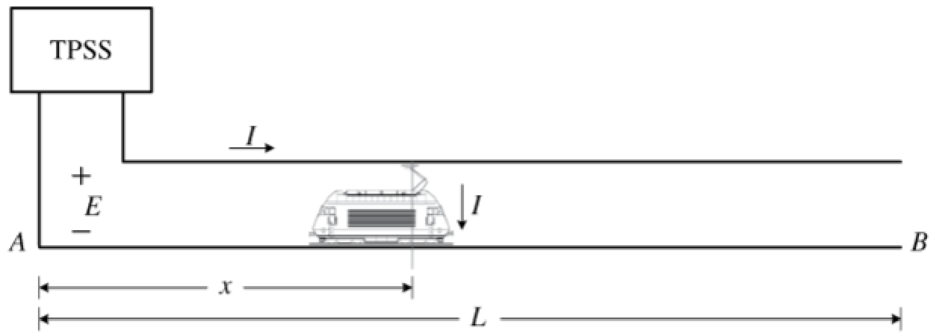

Figure 8 - Single train moving along track from points A to B (Brenna, Foiadelli, & Zaninelli, 2018, p. 150).

When there are more than one trains running on the line, we can replace the number of trains by n and each train absorbs a current I_n, with each train at a distance x_n from the substation at point A. The total voltage drop can be calculated from:

$$\Delta V_n = r\sum_{i=1}^{n} x_i \cdot I_i$$

which represents the summation of every voltage drop caused by each train on the track, where r remains constant throughout the entire line because the contact lines and the rails stay the same all throughout (p. 151).

In real railways, when there is a lot of traffic, the number of trains moving along the same track becomes very large and therefore, we can assume an even distribution of the trains along the track. Thus, $i = \frac{I_s}{L}$, where i is the current absorbed by each train, I_s is the total current supplied by the TPSS, and L is the length of track. This track, if considered only an infinitely small section of the entire line, then $i = \frac{dI}{dx}$, where dI is the infinitely small current supplied by the TPSS for this infinitely small section of L. As illustrated in Figure 9, each infinitely small section of the line (dx) located at a distance x from the TPSS of total current I_s will absorb an infinitely small current ($dI = i \cdot dx$).

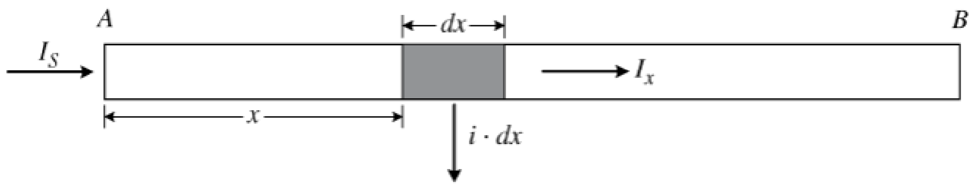

Figure 9 - Uniformly distributed trains with unilateral power supply (Brenna, Foiadelli, & Zaninelli, 2018, p. 152).

From this, to calculate the current at any point x in Figure 9 (I_x), the following equation can be used: $I_x = (L - x) \cdot i$, which actually indicates how much current is left for consumption by trains after some of it has been consumed between point A and x.

The voltage drop experienced at any point x in Figure 9 can be calculated using

$$\Delta V_{(x)} = r \cdot i \cdot x \cdot (L - \frac{x}{2})$$

which comes from replacing I (from the original voltage drop formula) with I_x and integrating the infinitely small $d\Delta V_x$, which is the voltage drop at any small section dx (p. 152).

Bilateral Power Supply

When a train moves along a track that is powered by a TPSS at each end, the power supply is called bilateral. This is illustrated in Figure 10. If the voltage provided by each TPSS is the same, then $E = E_A = E_B$. In the case where there is one single train moving from points A to B, then the voltage at any point x can be calculated using either points, and simply subtracting the voltage drop at x from the initial voltage of TPSS, in other words $V_x = E - V_{(x)\ A\ or\ B}$. It is worth noting that the voltage drop formula used in this case is the one seen for single-train systems in the Unilateral Power Supply section of this paper, not to be confused with the multiple-train equation (p. 153).

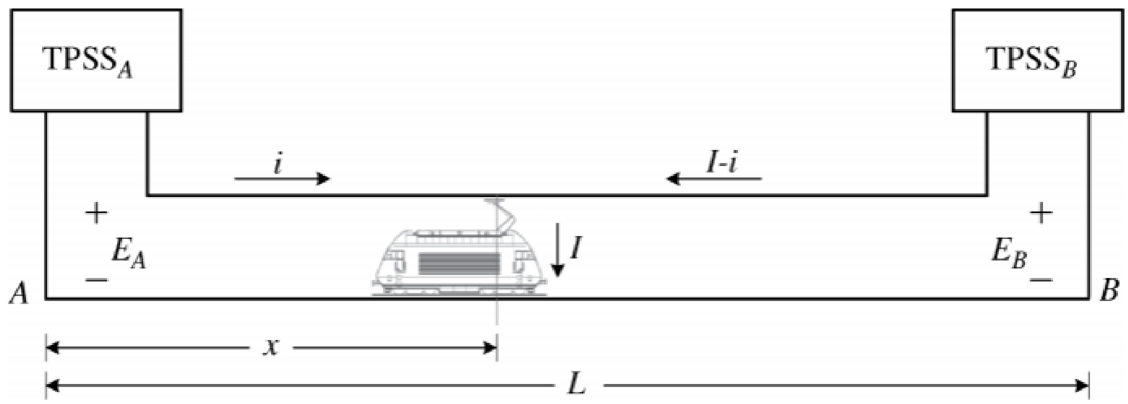

Figure 10 - Single train moving along track from one TPSS to another (Brenna, Foiadelli, & Zaninelli, 2018, p. 153).

The current absorbed by the train (I) can be written as $I_A + I_B$ because the train is powered by both substations. Using this equivalence, the voltage drop at any point x can be derived using the following steps:

$$V_x = E - V_{(x)\,A} = E - V_{(x)\,B}$$

$$V_{(x)\,A} = V_{(x)\,B}$$

$$r \cdot I_A \cdot x_A = r \cdot I_B \cdot x_B \text{ where } x_B = L - x \text{ and } I_B = I - I_A$$

$$r \cdot I_A \cdot x_A = r \cdot (I - I_A) \cdot (L - x) = \Delta V_{(x)}$$

If the train is located at point A, then it will only be supplied by TPSS$_A$, and vice-versa if the train is at point B (p. 154).

Once again, in the case where there is a very high number of trains circulating on a railway line formed by a great number of smaller track sections, the equation to calculate the voltage drop can be derived from considering the following: divide the entire length L of the section into infinitely small segments of length dx at which each current absorbed is of infinitely small value dI. This situation is very similar to multiple-train unilateral systems. The only difference is that the total current supplied throughout the displacement of the trains comes in equal amounts from both TPSSs, given that $I_s = I_A + I_B$. Since we established that an infinite number of trains leads to the assumption that they are equally distributed along the tracks, then we can say that both I_A and I_B are equal to $\frac{I_s}{2}$ (and $\frac{I_s}{2} + \frac{I_s}{2} = I_s$) (p. 155).

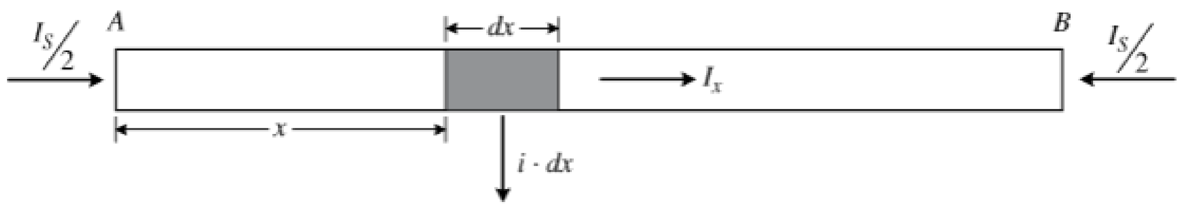

Figure 11 - Uniformly distributed trains with bilateral power supply (Brenna, Foiadelli, & Zaninelli, 2018, p. 155).

From this, by following a similar logic to the previous derivation, the voltage drop at any point x can be calculated using: $\Delta V_{(x)} = r \cdot \frac{x \cdot (L-x)}{2L} \cdot I_s$ (p. 156).

Comparison Between Unilateral and Bilateral Power Supplies

As opposed to a unilateral power supply which has a single train experience a continuous increase in voltage drop until the voltage drop reaches its maximum value (that is, the total voltage supplied by the single TPSS) and the train stops because its power supply has run out, a bilateral power supply allows for the train’s power not to run out as it is constantly being supplied by the second TPSS. For comparison, the graph in Figure 12 shows a curved function that represents the voltage drop for a bilateral power supply, and in red, there is the addition of where a unilateral power supply would stop (once it reached its maximum voltage drop value). It makes sense for the voltage drop of a bilateral power supply to be parabolic because its function is

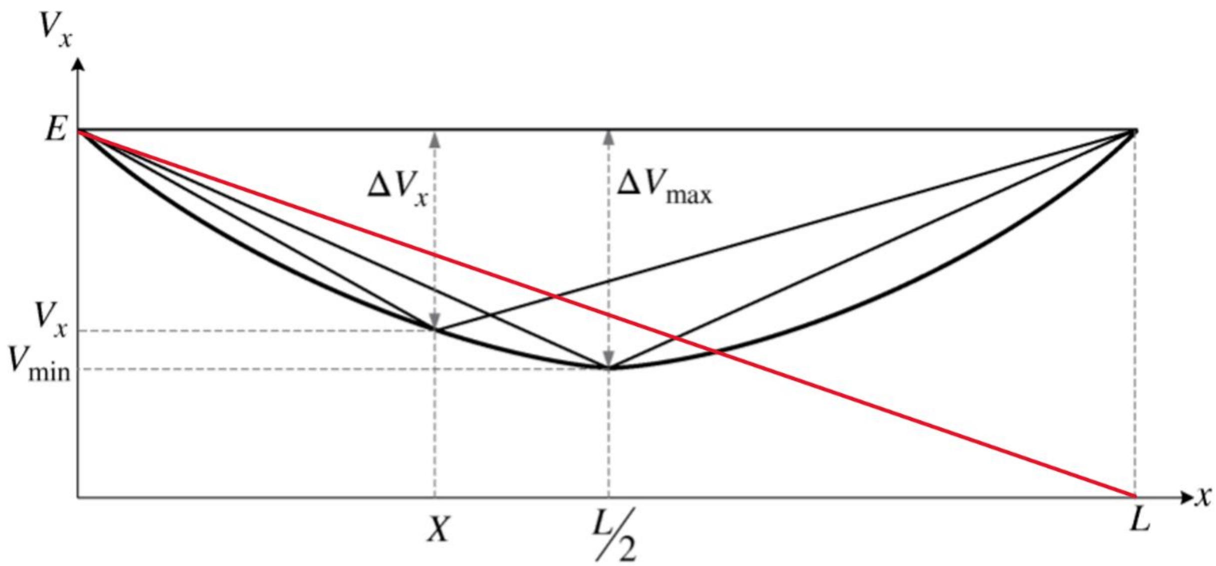

Figure 12 - Voltage vs. Position, unilateral in red, bilateral in black (Brenna, Foiadelli, & Zaninelli, 2018, p. 154).

Given that one aspect of optimizing such electric railway systems is to limit voltage drops, we must compare the maximum voltage drop of a single-train case for unilateral power supplies and bilateral ones.

On one hand, the maximum voltage drop of a unilateral power supply is reached when $x = L$ at point B, so when we replace the variables accordingly, we obtain

$$\Delta V_{\max\ (unilateral)} = r \cdot I \cdot L \qquad \text{(p. 150).}$$

On the other hand, the maximum voltage drop of a bilateral power supply is reached when $x = \frac{L}{2}$, so when we replace the variables accordingly, we obtain

$$\Delta V_{\max\ (bilateral)} = \frac{r \cdot L \cdot I}{4} \qquad \text{(p. 154).}$$

A simple ratio of the two results in the following

$$\frac{\Delta V_{\max\ (unilateral)}}{\Delta V_{\max\ (bilateral)}} = \frac{r \cdot I \cdot L}{\frac{r\ \cdot\ L\ \cdot\ I}{4}} = 4$$

From these observations, a bilateral power supply system proves to be much more effective because its maximum voltage drop is four times smaller than the maximum drop experienced by a train in a unilateral power supply system.

When it comes to comparing unilateral and bilateral power supply systems for multiple trains moving along the tracks, we can compare their maximum voltage drop values as follows. When a lot of trains are involved, for a unilateral power supply system, the voltage drop still reaches its maximum value at point B, so once $x = L$.

$$\Delta V_{\max\ (unilateral)} = r \cdot i \cdot L \cdot \left(L - \frac{L}{2}\right) = r \cdot i \cdot \frac{L^2}{2}$$

This equation can actually be simplified using $i = \frac{I_s}{L}$, and so

$$\Delta V_{\max\ (unilateral)} = r \cdot \left(\frac{I_s}{L}\right) \cdot \frac{L^2}{2} = r \cdot I_s \cdot \frac{L}{2}$$

As for a bilateral power supply, the maximum voltage drop is still achieved half-way through, so at $x = \frac{L}{2}$.

$$\Delta V_{\max\ (bilateral)} = r \cdot \frac{\frac{L}{2} \cdot \left(L - \frac{L}{2}\right)}{2L} \cdot I_s = r \cdot \frac{L}{8} \cdot I_s$$

When comparing their ratio, the following result is obtained

$$\frac{\Delta V_{\max\ (unilateral)}}{\Delta V_{\max\ (bilateral)}} = \frac{r \cdot I_s \cdot \frac{L}{2}}{r \cdot \frac{L}{8} \cdot I_s} = 4$$

This indicates that the maximum voltage drop of a unilateral power supply is four time greater than the maximum voltage drop of a bilateral power supply. From this, we can conclude that bilateral power supply systems are much more preferable in terms of efficiency and saving energy.

Overall, since both ratios give four, we can conclude that no matter how many trains are circulating on a track, whether there is a single train (low traffic) or an infinite number of trains (high traffic), when comparing how to supply power to the trains, a bilateral power supply that uses two TPSSs will always be more effective than a unilateral one because its maximum voltage drop is four times lower.

Conclusion

All in all, there are numerous aspects that contribute to the functioning of electric railway systems. In this paper, we focused on DC circuits. More specifically, we only looked at a limited number of components that go into the operation and optimization of these systems, such as traction power substations, braking energy recovery, contact lines, basic electrical calculations, as well as unilateral and bilateral power supplies. This paper does allow for a better understanding of how electric railways that operate with DC circuits function. There could be further improvement to perform a more in-depth analysis by extending the list of components discussed to a larger one that encompasses all of them. Furthermore, there could be a comparison of DC and AC circuits in the context of electric railways, since AC circuits were only briefly mentioned in this paper to respect the guidelines of the project.

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