Question 1: Laplace Transforms [10 marks]

The Laplace transform for a function, f(t) is given by:

$$F(s) = \frac{3}{(s+1)^2}$$

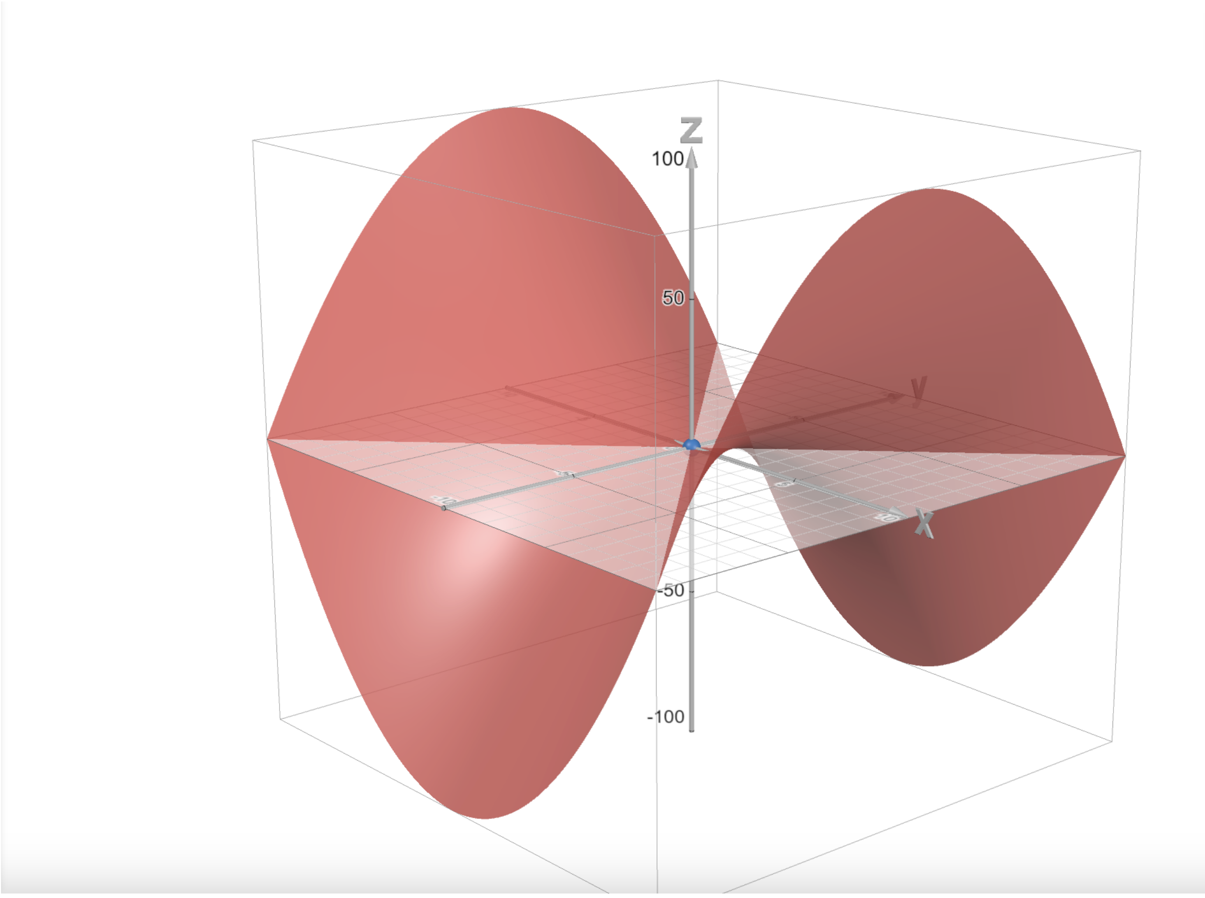
Compute the inverse Laplace transform to find f(t). **[4]**

1. Describe how the function behaves as *t* approaches infinity. Why is this? **[2]**
2. Determine the critical point(s) of the function and classify them. **[2]**
3. Discuss the dampening of this function **[2]**

Question 2: MVC [8 marks]

The function graphed below is:

$$g(x, y) = x^2 - y^2$$



1. What is the nature of the stationary point? Walk me through the steps of verifying this. **[4]**
2. Now, formulate the Jacobian of g(x,y) and evaluate it at the point (1,1). How do we interpret it? **[4]**

Question 3: Fourier Series [8 marks]

1. Can you use Fourier Series with aperiodic functions? **[2]**
2. Discuss Gibb’s ringing phenomena. Where does this occur and what does it have to do with Fourier Series **[3]?**
3. Referring to the graph I have showed you, what would the zeroeth, first, and second order approximations look like?What happens as you sum more terms in a Fourier Series? **[3]**

Question 4: Partial Differential Equations [6 marks]

Temperature of water in the sea varies based on location and depth amongst other things. Imagine two layers of water – a warmer layer sitting on top of a cooler and denser layer – with waves that can travel along the interface. PDEs can be used to represent the displacement of these waves from equilibrium.

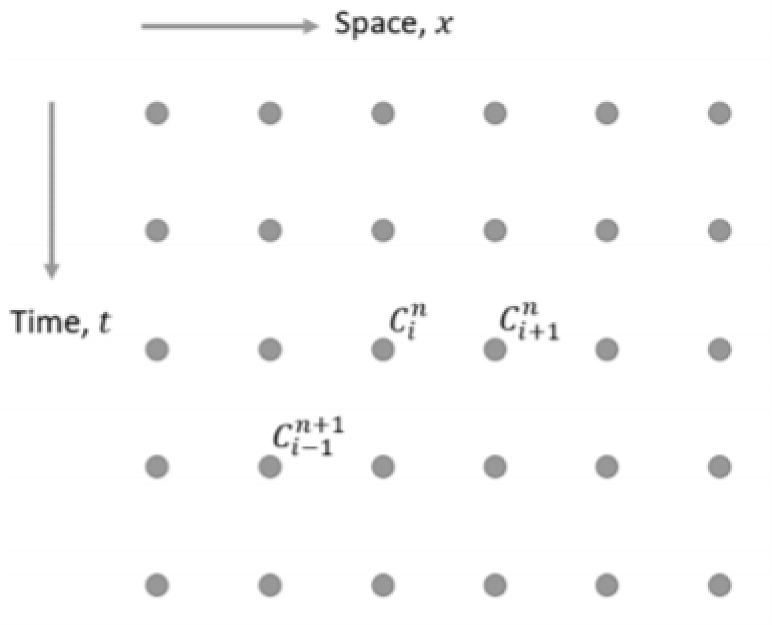
A general solution to these PDEs can be found using separation of variables. Assuming solutions of the form $\psi(x,t) = X(x)T(t)$, we can express the general solution for each layer in terms of complex exponentials, typically representing forward and backward traveling waves:

$$\psi_1(x,t) = A_1 e^{i(kx-\omega t)} + B_1 e^{i(-kx-\omega t)}$$
$$\psi_2(x,t) = A_2 e^{i(kx-\omega t)} + B_2 e^{i(-kx-\omega t)}$$

1. Discuss what a general solution represents and go over each term in the equations for our two waves **[4]**.
2. There are two main PDE equations we focused on in the course. Can you name them and briefly describe each? **[2]**

Question 5: Finite Difference Methods [3 marks]

The following image shows a regularly-spaced grid of nodes representing the distribution of the scalar, *C* in time, *t*, and one-dimensional space, *x*.



The indices *i* and *n* are being used to label discrete points in time and space, separated by Δt and Δx respectively.

1. Write down the expression for the backwards approximation to the time derivative at point: **[1]**

$$C_{i+1}^{n+1}$$

2. Discuss the central approximation technique and compare it to the backwards approximation **[2]**.

Question 6: Probability [4 marks]

When Large Language Models (LLMs) generate text, they select the next token (word or character) based on a probability distribution. Given a current state or context, the model predicts a distribution for the next token. Being trained on massive amounts of text and data, they are effectively learning conditional probability.

1. First, how would you explain a probability distribution? **[1]** If we assume a Gaussian curve and you select a single point underneath, what does that represent in terms of the LLM description above? **[1]**

   Follow-up: LLMs often use mathematical functions to process and normalise their predictions from the last layer of the model into a probability distribution. Something like the soft-max function will exponentiate and normalise before giving the final output. In relation to probability, why is this important? ***[2]***