Path Dependand Exp

For BS model in european call computed earlier, we considered 2 points in time: 0 and T. What if:

- Payoff depends explicitely on the underlying price @ many t?
- We are uncertain of the sampling transi°s i.e. we don't know the exact mvts of the underlying given a time interval.

↳ Solu°: Divide time interval $[0,T]$ into smaller intervals to obtain more accurate estima° to sampling from dist @ time T.

expl. $dS(t) = r\,S(t)\,dt + \sigma(S(t))\,S(t)\,dW(t)$

↳ in most cases, no explicit solu°.

Hence, we partition $[0,T]$ into m subintervals of length $\Delta t = T/m$ and use Eulers discrete approx.

$\forall [t, t+\Delta t]$, $S(t+\Delta t) = S(t) + r\,S(t)\,\Delta t + \sigma(S(t))\,S(t)\sqrt{\Delta t}\,Z$

with $W(t+\Delta) - W(t) \sim \mathcal{N}(0, \sqrt{\Delta t})$

Let's consider Asian opt°:

let $\bar{S} = \frac{1}{m} \sum_{j=1}^{m} S(t_j)$, $0 = t_0 < t_1 < \dots < t_m = T$

to compute $E[e^{-rT}(\bar{S} - K)^+]$ we need to generate samples of $\bar{S}$.

We simulate the paths $S(t_1), \dots, S(t_m)$ then compute the avg s.t.

$$S(t_{j+1}) = S(t_j) \exp\left\{\left(r - \frac{\sigma^2}{2}\right)(t_{j+1} - t_j) + \sigma\sqrt{t_{j+1} - t_j}\, Z_{j+1}\right\}$$

We denote Z_{ij} to illustrate the j^{th} draw from the normal dist along the i^{th} path.

(it's a minus) $\Rightarrow t_{j-1}$

for $i = 1, \dots, n$
  for $j = 1, \dots, m$
    generate Z_{ij}
    set $S_i(t_j) = S_i(t_{j-1}) \exp\left\{\left(r - \frac{\sigma^2}{2}\right)(t_j - t_{j+1}) + \sigma\sqrt{(t_j - t_{j-1})}\, Z_{ij}\right\}$
  set $\bar{S} = (S_i(t_1) + \dots + S_i(t_m))/m$
  set $C_i = e^{-rT^i}(\bar{S} - K)^+$
set $\hat{C}_n = (C_1 + \dots + C_n)/n$