STUDY PLAN WITH SAMPLE TEST QUESTIONS FOR REVISION

The course slides and tutorial notes represent the core reference material. The **core textbook** provides support for the material discussed in class. I have added optional detailed textbook references for those of you who would like to study some topic a bit further or see it presented from a more technical angle.

Please note that although answers can always be supported by mathematical formulation, I would also like you to try to articulate your answers in a simple, yet precise narrative. This is very helpful for open-ended exam questions and extremely important for job interviews!

Part 1 - PRICING/HEDGING DERIVATIVES WITH TREES

- How do we model the evolution of a security price with a tree? What are the basic no-arbitrage restrictions we would have to impose on a binomial tree? Can you devise an arbitrage opportunity in case they are violated? (slides 1-7) [Shreve I, pp. 1-3]
- What is the idea of pricing a derivative via replication? Can you obtain the cash and stock position of a replicating portfolio in a simple one period binomial model? (slides 8-9, 12) [Shreve I, pp. 4-6]
- What is the idea of pricing via risk neutral valuation? How is it related to replication and perfect hedging? What are risk neutral probabilities in a one-period binomial model, and how are they related to real world probabilities? (slides 10-11, 12) [Shreve I, pp. 6-8]
- What is the difference between a complete and an incomplete market? Is pricing by no-arbitrage possible in both markets? If so, what is the main difference in no-arbitrage prices? How does such difference translate in terms of risk neutral probabilities? Is perfect hedging possible in an incomplete market? In case it is not, does our approach to hedging (or our preferences about hedging precision) matter for pricing? Provide an example of a tree model resulting in an incomplete market. (slides 13-18) [Shreve I, p. 14; Shreve II, pp. 223, 231-234]
- How do we model the evolution of a security price with a two-period tree? What are the basic no-arbitrage restrictions we would have to impose on the tree? Can you obtain the structure of the dynamic replicating portfolio in a two-period binomial model? How is it related to risk-neutral valuation? What are risk-neutral probabilities in a two-period binomial? (slides 19-24; key is slide 22) [Shreve I, pp. 8-12]
- How do you price an American option in a two-period binomial tree? Is the value of an American option always strictly greater than the one of a European option? (slides 16-29) [Shreve I, pp. 89-95; Shreve II, pp. 340-345]
- How do you extend a two-period binomial tree to the case of a continuous dividend yield? Is the replicating portfolio of a call option more or less invested in the underlying stock in the case of a strictly positive dividend yield? What is the intuition? How do risk-neutral probabilities look like in the case of a strictly positive continuous dividend yield? (slides 30-33)

Part 2 - FROM TREES TO BLACK & SCHOLES

- How do you model the evolution of a security price with a multi period tree? What are the basic no-arbitrage restrictions you would have to impose on the tree? Can you obtain the structure of the dynamic replicating portfolio in a multi period binomial model? Write down the (real world) probability of a stock to reach a terminal value corresponding to 10 upward moves and 5 downward moves in a 15-period, recombining binomial tree. How is this probability related to risk neutral valuation, if at all? How does risk neutral valuation work for a multiperiod binomial model? (slides 1-9; slides 22 of part 1 is still key) [Shreve I, pp. 12-20]

- Outline how you could obtain the Black & Scholes formula from a multiperiod binomial model as you let the discretization step become smaller and smaller [outline only, no derivation]. (slides 10-13) [tutorial**]**
- Consider the trees discussed in the previous point. How are stock prices distributed in the limit? (slide 14) [tutorial; Shreve II, pp. 91-93]
- What are the main Greeks and how are they defined within the Black & Scholes framework [no explicit derivation required; no memorization of explicit solutions required]? Can you relate the Greeks obtained for the Black & Scholes model to key quantities used in derivative pricing via trees? (slides 15-19) [Shreve I, p. 6; Shreve II, pp. 159-162]
- Consider any of the north-western charts for the option positions considered in slides 20-21. Make sure you understand what the Profit & Loss diagram represents, and how it differs from the mark to market (MTM) value of the position before maturity. For each position, make sure you can understand the sign of Delta and Gamma by using their definition and using the top left chart. Please also make sure you can discuss how the MTM value of the position would change in response to changes in volatility and time to maturity (i.e., Theta and Vega). (slides 20-21) [You can ignore Rho]
- Consider the strategies of slide 24: how do they differ, and in what sense to the allow investors to bet on volatility? Consider the backspread of slide 23: if the underlying trades at 1645 and there are only two days from maturity, is the strategy essentially equivalent to a short maturity long straddle, and hence to a long bet on volatility? Explain. (slides 22-26) [you are not required to remember the names of options trading strategies, with the exception of straddles]

Part 3 - GREEKS: REFINEMENTS

- You have a complex derivative portfolio with a Delta of USD 1m. How is this value related to the Deltas of each individual component of the portfolio? Can you estimate its change in value over the next small time step by using the Delta approach? What are the key limitations of your approximation? (slides 5-8) How do the Delta-Gamma and Delta-Gamma-Theta can help you overcome some of these limitations? (slides 9-12) [Shreve II, pp. 159-162]
- Take the perspective of a market maker (MM) operating in an options market. How would could you explain the Black and Scholes formula in light of a MM hedging its positions frequently enough? How could you determine the dynamics of the MTM value of any of the positions discussed in part 2 (from slide 20 onwards) by using Ito's formula? Would you be able to determine the sign of the integrands appearing in Ito's formula? (slides 13-17; slide 17 is key) [Shreve II, pp. 193-196]
- Make sure you would be able to compute Delta, Gamma, and Theta in the context of a binomial tree. (slides 18-20)

Part 4 - INTEREST RATE DERIVATIVES

- Make sure you can price zero coupon bonds as well as coupon bearing bonds via risk neutral valuation. Define spot-LIBOR rates, the short rate, and the yield/zero coupon curve. (slides 1-10) [Brigo and Mercurio, pp. 4-9, 13]
- Define Forward Rate Agreements (FRAs) and illustrate the transaction flow on a diagram. How would you price a FRA via risk neutral valuation? What do market participants mean by a forward rate, and how is it defined in light of your pricing expression for a FRA? Are forward rates good predictors of future interest rates? (slides 11-17) [Brigo and Mercurio, pp. 11-12]
- Define Interest Rate Swaps (IRSs) and illustrate the transaction flow on a diagram. What could be the rationale for a hedger to enter a receiver IRS? And for someone entering a payer IRS? Write down a no arbitrage pricing formula for an IRS? What do market participants mean when they quote a swap rate? How can you mark to market an IRS you entered into some some time ago? Two years ago you entered a 10-year receiver IRS at a fixed rate of 1%: how would you mark-to-market your position today? If you were to close a swap at time t, how would you determine whether you would make money or have to incur a cost to close it? (mainly slides 18-21 and 24) [Brigo and Mercurio, pp. 13-16]
- What are the payoffs of caplets, floorlets, caps, and floors? If caplets and floorlets are liquidly quoted, is it straightforward to price caps and floors? Explain. Define swaptions: why are they harder to price than caps and floors? (slides 25-30) [Brigo and Mercurio, pp. 16-20]

Part 5 - SOME SHORT RATE MODELS

- What are common models of the short rate referred to as endogenous models? Why are they called in this way? Which ones allow you to model positive rates? Discuss the Vasicek and the CIR models, providing an explanation of their key parameters and explaining how they affect the dynamic behavior of the short rate. (slides 9-14) [Shreve II, pp. 273-276] [Brigo et al., pp. 72-77] [Brigo and Mercurio, pp. 51-58]
- Consider the Vasicek model. Can you determine an explicit expression for the short rate? What does it tell you about the average conditional short rate at different future points in time? And how does the short rate behave on average in the long run? (slides 15-16) [derivation of the conditional variance is not required] [Shreve II, pp. 273-276] [Brigo and Mercurio, pp. 58-60]
- Both the Vasicek and the CIR model are called affine, as their drift and instantaneous variance are affine in the state variable r. What does it mean for zero coupon bond prices? Are they available in closed form, and if so, what is their functional form? (slide 17) [you are not required to derive or memorize the explicit expressions of functions such as A and B appearing in slide 17] [Shreve II, pp. 405-423] [Brigo and Mercurio, pp. 64-70]
- What are the limitations of endogenous models, and how are they addressed by exogenous models? Provide some examples of exogenous models for the short rate. Which exogenous models allow you to model positive rates? Consider the Hull-White models and the shifted Vasicek or CIR models: how do you interpret their main parameters and how do they affect the dynamic behavior of the short rate? (slides 18-22) [Shreve II, pp. 265-266, 273-276] [Brigo et al., pp. 72-77] [Brigo and Mercurio, pp. 71-74, 80-84, 95-103, 110-112]

Part 6 - CREDIT RISK

- What are the main differences between IRSs and OISs? What do market participants mean by multicurve modeling? How would you interpret the behavior of the spread depicted in the chart of slide 9? (slides 6-12) [Brigo et al., pp. 242]
- How would you price a defaultable zero coupon bond with zero recovery? How would you price a defaultable zero coupon bond with deterministic recovery R(t)? How would you price a defaultable zero coupon bond with random R(τ)? (slides 14-17) [use also the material discussed in the tutorials] [Brigo and Mercurio, pp. 723-724]
- Explain how Credit Default Swaps work and how they can be used by a hedger to mitigate the impact of default risk on a portfolio of defaultable assets. What do market participants mean when they quote a CDS rate? Explain under what assumptions you could develop a model-independent formula that could be used to recover the term structure of default probabilities supporting CDS quotes observed in the market. In the CDS stripping procedure, why do we start from shorter maturities and then use longer maturity CDSs? (slides 19-33) [Brigo and Mercurio, pp. 724-734]
- Introduce a modeling framework that could be used to model randomness in default events, as well as time varying, deterministic default probabilities. Extend the model to capture randomness in both default probabilities and default events. Explain how you would simulate a default occurrence under the two modeling frameworks introduced before. How would the pricing of a defaultable zero coupon bond (see second bullet of this section) simplify in the case of each of the two frameworks? Consider the case of zero, deterministic, and random recovery. (slides 35-41) [Brigo and Mercurio, pp. 757-764]

Part 7 - CREDIT RISK: FURTHER DETAILS

- Discuss a simple approach to CDS stripping based on the assumption that the default time coincides with the first jump of a Poisson process with parameter $\lambda>0$. Show that, under the assumption of a continuous CDS premium rate payment, such premium rate is naturally equal to the instantaneous risk-neutral expected default losses (i.e., "instantaneous risk-neutral default probability times Loss Given Default (LGD)"). Familiarize yourself with the basic computations of slide 9. (slides 6-10) [Brigo and Mercurio., pp. 735-737] Brigo et al., pp. 70-71]
- Explain how CDS stripping could be implemented under the assumption of a piecewise constant default intensity. Use the case study of slides 13-16 to familiarize yourself with common shapes of the structure of implied default intensities in the case of names under financial distress. (slides 11-16) [Brigo and Mercurio, pp. 764-776] [Brigo et al., pp. 62-64, 71]
- Introduce a setup that could allow one to model the stochastic evolution of credit spreads. Provide examples of stochastic processes that could be used to model such spreads. Introduce and discuss at least one model that could be useful in reproducing the shape of the term structure of quoted CDS rates. (slides 18-20) [Brigo et al., pp. 785-788] [Brigo et al., pp. 72-78]

Part 8 - COUNTERPARTY RISK

- Consider a bilateral transaction in which only one of the two counterparties can default: write down the transaction's payoffs from the point of view of the default-free counterparty; write down the corresponding valuation formula; explain the meaning of each term appearing in the valuation formula; explain how the UCVA term changes as the credit quality of the defaultable party improves vs. deteriorates; explain why counterparty risk introduces optionality in otherwise plain vanilla derivatives (consider an IRS as an example and provide a link with swaptions valuation). (slides 5-9, 12) [Brigo et al., pp. 89-96]
- Repeat the analysis carried out in the previous point from the point of view of the defaultable counterparty. (slides 13-14) [Brigo et al., pp. 248-251]
- Consider a bilateral transaction in which both counterparties can default. Write down the payoffs from the point of view of a counterparty of your choice, as well as the corresponding valuation formula. Explain how the BVA term is shaped by DVA and CVA. Explain how the valuation formula written from the point of view of the chosen counterparty would change in response to changes in the relative credit quality of the other counterparty. Explain why some financial institutions may want to monetize DVA? Explain what the term "DVA hedging" means and how such hedging may be implemented in practice? (slides 16-22) [Brigo et al., pp. 251-255]

REFERENCES

- Brigo and Mercurio (2006). *Interest rate models: Theory and practice*, Springer.
- Brigo, Morini and Pallavicini (2013). *Counterparty credit risk, collateral, funding*, Wiley Finance.
- Shreve (2004). *Stochastic calculus for finance*, volume I, Springer.
- Shreve (2004). *Stochastic calculus for finance*, volume II, Springer.