Subject Name Solutions

4320001 – Winter 2023

Semester 1 Study Material

Detailed Solutions and Explanations

Q.1 Fill in the blanks [14 marks]

0.0.1 Q1.1 [1 mark]

If A = [1 2; 3 -1] then 4A = ...

Solution

(b) [4 8; 12 -4]

Solution: $4A = 4\begin{bmatrix}1 & 2\\3 & -1\end{bmatrix} = \begin{bmatrix}4 & 8\\12 & -4\end{bmatrix}$

0.0.2 Q1.2 [1 mark]

Order of the matrix [1 1 2; -3 2 3] is ...

Solution

(a) 2 $\times$3

Solution: Matrix has 2 rows and 3 columns, so order is 2 $\times$3.

0.0.3 Q1.3 [1 mark]

If A = [1 1; 1 1] then A$^2 = \ldots$

Solution

(d) [2 2; 2 2]

Solution: $A^2 = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$

0.0.4 Q1.4 [1 mark]

If A = [2 -1; 3 4] then adjoint of A = ...

Solution

(c) [4 1; -3 2]

Solution: For matrix A = [a b; c d], adj(A) = [d -b; -c a] adj(A) = [4 1; -3 2]

0.0.5 Q1.5 [1 mark]

d/dx(tan x) = ...

Solution

(d) sec$^2 x$

Solution: $\frac{d}{dx}(\tan x) = \sec^2 x$

0.0.6 Q1.6 [1 mark]

d/dx(sin 5x) = ...

Solution

(b) 5cos5x
Solution: $\frac{d}{dx}(\sin 5x) = 5\cos 5x$ **(using chain rule)**

0.0.7 Q1.7 [1 mark]

If function y = f(x) is maximum at x = a then f'(a) = ...

Solution

(c) 0
Solution: At maximum point, first derivative equals zero: f'(a) = 0

0.0.8 Q1.8 [1 mark]

x dx = ... + C

Solution

(a) -cos x
Solution: $\int \sin x \, dx = -\cos x + C$

0.0.9 Q1.9 [1 mark]

$\int 1/(x^2+4)dx = \ldots + C$

Solution

(d) (1/2)tan$^{-1}(x/2)$
Solution: $\int \frac{1}{x^2+4}dx = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

0.0.10 Q1.10 [1 mark]

$\int_1^2 x^2 dx = \ldots$

Solution

(a) 7/3
Solution: $\int_1^2 x^2 dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

0.0.11 Q1.11 [1 mark]

Order of differential equation (d$^3y/dx^3)^4 + dy/dx + 5y = 0is \ldots$

Solution

(c) 3
Solution: Order is the highest derivative present = 3

0.0.12 Q1.12 [1 mark]

Integrating factor of dy/dx + y/x = 1 is ...

Solution

(b) x
Solution: I.F. = $e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

0.0.13 Q1.13 [1 mark]

Mean of 39,23,58,47,50,16,61 is ...

Solution

(b) 42
Solution: Mean $= \frac{39+23+58+47+50+16+61}{7} = \frac{294}{7} = 42$

0.0.14 Q1.14 [1 mark]

Mean of first five natural numbers is ...

Solution

(a) 3
Solution: Mean $= \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$

Q.2 Attempt any two [14 marks total]

0.0.15 Q2(A).1 [3 marks]

If A = [1 3 5; -1 0 2; 4 3 6], B = [3 4 5; 5 4 3; 3 5 4], C = [1 2 1; 3 3 3; 4 5 6], find 3A+2B-4C

Solution: $3A = \begin{bmatrix} 3 & 9 & 15 \\ -3 & 0 & 6 \\ 12 & 9 & 18 \end{bmatrix}$

$2B = \begin{bmatrix} 6 & 8 & 10 \\ 10 & 8 & 6 \\ 6 & 10 & 8 \end{bmatrix}$

$4C = \begin{bmatrix} 4 & 8 & 4 \\ 12 & 12 & 12 \\ 16 & 20 & 24 \end{bmatrix}$

$3A + 2B - 4C = \begin{bmatrix} 5 & 9 & 21 \\ -5 & -4 & 0 \\ 2 & -1 & 2 \end{bmatrix}$

0.0.16 Q2(A).2 [3 marks]

If A = [7 5; -1 2], B = [1 -1; 3 2], show that (A+B) = A + B

Solution: $A + B = \begin{bmatrix} 8 & 4 \\ 2 & 4 \end{bmatrix}$

$(A + B)^T = \begin{bmatrix} 8 & 2 \\ 4 & 4 \end{bmatrix}$

$A^T = \begin{bmatrix} 7 & -1 \\ 5 & 2 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 8 & 2 \\ 4 & 4 \end{bmatrix}$

Hence proved: $(A + B)^T = A^T + B^T$

0.0.17 Q2(A).3 [3 marks]

Solve the differential equation xy dy = (x+1)(y+1)dx

Solution: Separating variables: $\frac{y}{y+1} dy = \frac{x+1}{x} dx$

$\left(1 - \frac{1}{y+1}\right) dy = \left(1 + \frac{1}{x}\right) dx$

Integrating: $y - \ln |y + 1| = x + \ln |x| + C$

Final answer: $y - x = \ln |y + 1| + \ln |x| + C$

0.0.18 Q2(B).1 [4 marks]

Find the inverse of matrix [3 1 2; 2 -3 -1; 1 2 1]

Solution: Let $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

$|A| = 3(-3-(-2)) - 1(2-(-1)) + 2(4-(-3)) = 3(-1) - 1(3) + 2(7) = -3 - 3 + 14 = 8$

Cofactors:

- $\mathbf{C}_{11} = -1, C_{12} = -3, C_{13} = 7$
- $\mathbf{C}_{21} = 3, C_{22} = 1, C_{23} = -5$
- $\mathbf{C}_{31} = 5, C_{32} = 7, C_{33} = -11$

$adj(A) = \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$

$A^{-1} = \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$

0.0.19 Q2(B).2 [4 marks]

Solve 3x - 2y = 8, 5x + 4y = 6 using matrix method

Solution: $\begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$

$|A| = 3(4) - (-2)(5) = 12 + 10 = 22$

$A^{-1} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 44 \\ -22 \end{bmatrix}$

Solution

x = 2,
y = -1

0.0.20 Q2(B).3 [4 marks]

If A = [1 2 1; 2 3 1; 1 2 2], find A · adj(A)

Solution: $|A| = 1(6-2) - 2(4-1) + 1(4-3) = 4 - 6 + 1 = -1$

For any matrix A: $A \cdot adj(A) = |A| \cdot I$

$A \cdot adj(A) = (-1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Q.3 Attempt any two [14 marks total]

0.0.21 Q3(A).1 [3 marks]

If y = log(sin x/(1+cos x)), find dy/dx

Solution: $y = \log(\sin x) - \log(1 + \cos x)$

$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x - \frac{1}{1+\cos x} \cdot (-\sin x)$

$= \frac{\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$

$= \cot x + \frac{\sin x}{1+\cos x}$

Using identity: $\frac{\sin x}{1+\cos x} = \tan(\frac{x}{2})$

Solution

$\frac{dy}{dx} = \cot x + \tan(\frac{x}{2})$

0.0.22 Q3(A).2 [3 marks]

If y = sin(x+y), find dy/dx

Solution: Differentiating both sides: $\frac{dy}{dx} = \cos(x+y) \cdot \left(1 + \frac{dy}{dx}\right)$

$\frac{dy}{dx} = \cos(x+y) + \cos(x+y) \cdot \frac{dy}{dx}$
$\frac{dy}{dx} - \cos(x+y) \cdot \frac{dy}{dx} = \cos(x+y)$
$\frac{dy}{dx}[1 - \cos(x+y)] = \cos(x+y)$

Solution

$\frac{dy}{dx} = \frac{\cos(x+y)}{1-\cos(x+y)}$

0.0.23 Q3(A).3 [3 marks]

Obtain $^2 logxdx$

Solution: Using integration by parts: dv = uv - du

Let

u = log x, dv = x$^2 dx Then du = (1/x)dx,$

v = x^3**/3**

$\int x^2 \log x \, dx = \log x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$
$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx$
$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$

Solution

$\frac{x^3}{3}(\log x - \frac{1}{3}) + C$

0.0.24 Q3(B).1 [4 marks]

Motion equation s = 2t$^3 - 3t^2 - 12t + 7. Find s and t when acceleration is zero$

Solution: $s = 2t^3 - 3t^2 - 12t + 7$

Velocity: $v = \frac{ds}{dt} = 6t^2 - 6t - 12$

Acceleration: $a = \frac{dv}{dt} = 12t - 6$

When acceleration = 0: $12t - 6 = 0$ $t = \frac{1}{2}$

At t = 1/2: $s = 2(\frac{1}{2})^3 - 3(\frac{1}{2})^2 - 12(\frac{1}{2}) + 7 = \frac{1}{4} - \frac{3}{4} - 6 + 7 = \frac{1}{2}$

Solution

t = 1/2,
s = 1/2

0.0.25 Q3(B).2 [4 marks]

If y = 2e$^{3x} + 3e^{-2x}, prove d^2y/dx^2 - dy/dx - 6y = 0$

Solution: $y = 2e^{3x} + 3e^{-2x}$

$\frac{dy}{dx} = 6e^{3x} - 6e^{-2x}$
$\frac{d^2y}{dx^2} = 18e^{3x} + 12e^{-2x}$

Now: $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y$
$= (18e^{3x} + 12e^{-2x}) - (6e^{3x} - 6e^{-2x}) - 6(2e^{3x} + 3e^{-2x})$

$= 18e^{3x} + 12e^{-2x} - 6e^{3x} + 6e^{-2x} - 12e^{3x} - 18e^{-2x}$
$= (18 - 6 - 12)e^{3x} + (12 + 6 - 18)e^{-2x} = 0$
Hence proved

0.0.26 Q3(B).3 [4 marks]

Find maximum and minimum values of f(x) = x$^3 - 3x + 11$

Solution: $f(x) = x^3 - 3x + 11$
$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x - 1)(x + 1)$
Critical points:
x = 1,
x = -1
$f''(x) = 6x$
At
x = 1: f''(1) = 6 > 0 $\rightarrow LocalminimumAt$
x = -1:
f''(-1) = -6 < 0 $\rightarrow Localmaximum$
$f(1) = 1 - 3 + 11 = 9$ **(minimum)** $f(-1) = -1 + 3 + 11 = 13$ **(maximum)**

Solution

Maximum = 13 at
x = -1, Minimum = 9 at
x = 1

Q.4 Attempt any two [14 marks total]

0.0.27 Q4(A).1 [3 marks]

Obtain 5x sin 6x dx

Solution: Using identity: $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
$\sin 5x \sin 6x = \frac{1}{2}[\cos(5x - 6x) - \cos(5x + 6x)]$
$= \frac{1}{2}[\cos(-x) - \cos(11x)] = \frac{1}{2}[\cos x - \cos(11x)]$
$\int \sin 5x \sin 6x \, dx = \frac{1}{2} \int [\cos x - \cos(11x)]dx$
$= \frac{1}{2}[\sin x - \frac{\sin(11x)}{11}] + C$

Solution

$\frac{1}{2} \sin x - \frac{\sin(11x)}{22} + C$

0.0.28 Q4(A).2 [3 marks]

Obtain $\int (1 + x)e^x / cos^2(xe^x)dx$

Solution: Let $u = xe^x$**, then** $du = (1 + x)e^x dx$
The integral becomes: $\int \frac{du}{\cos^2 u} = \int \sec^2 u \, du = \tan u + C$
Substituting back: $= \tan(xe^x) + C$

Solution

$\tan(xe^x) + C$

0.0.29 Q4(A).3 [3 marks]

Find standard deviation for data: 6,7,10,12,13,4,8,12

Solution: Data: 6, 7, 10, 12, 13, 4, 8, 12 n = 8
Mean = $\frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

x	x-9	(x-9)^2
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	-5	25
8	-1	1
12	3	9

Σ(x-9)^2 $= 74$

Standard deviation $= \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{74}{8}} = \sqrt{9.25} = 3.04$

Solution

$= \mathbf{3.04}$

0.0.30 Q4(B).1 [4 marks]

Obtain $\int(2x+1)/[(x+1)(x-3)]dx$

Solution: Using partial fractions: $\frac{2x+1}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$

$2x + 1 = A(x-3) + B(x+1)$

When x = -1: $2(-1) + 1 = A(-4) \Rightarrow -1 = -4A \Rightarrow A = \frac{1}{4}$

When x = 3: $2(3) + 1 = B(4) \Rightarrow 7 = 4B \Rightarrow B = \frac{7}{4}$

$\int \frac{2x+1}{(x+1)(x-3)}dx = \frac{1}{4}\int \frac{1}{x+1}dx + \frac{7}{4}\int \frac{1}{x-3}dx$

$= \frac{1}{4}\ln|x+1| + \frac{7}{4}\ln|x-3| + C$

Solution

$\frac{1}{4}\ln|x+1| + \frac{7}{4}\ln|x-3| + C$

0.0.31 Q4(B).2 [4 marks]

Obtain $\int_0\hat{}(/2)\sqrt(cotx)/(\sqrt(cotx) + \sqrt(tanx))dx$

Solution: Let $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx$

Using property: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\cot(\pi/2-x)}}{\sqrt{\cot(\pi/2-x)}+\sqrt{\tan(\pi/2-x)}}dx$

Since $\cot(\pi/2 - x) = \tan x$ **and** $\tan(\pi/2 - x) = \cot x$**:**

$I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$

Adding both expressions: $2I = \int_0^{\pi/2} \frac{\sqrt{\cot x}+\sqrt{\tan x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2}$

Solution

$I = \frac{\pi}{4}$

0.0.32 Q4(B).3 [4 marks]

Find mean deviation for grouped data

x	4	8	11	17	20	24	32
f	3	5	9	5	4	3	1

Solution: N = Σf = 3+5+9+5+4+3+1 = 30

Mean $= \frac{\sum f_i x_i}{N} = \frac{3(4)+5(8)+9(11)+5(17)+4(20)+3(24)+1(32)}{30}$
$= \frac{12+40+99+85+80+72+32}{30} = \frac{420}{30} = 14$

x	f		x -14
4	3	10	30
8	5	6	30
11	9	3	27
17	5	3	15
20	4	6	24
24	3	10	30
32	1	18	18

Σf |x -14| = 174
Mean deviation $= \frac{\sum f_i |x_i - \bar{x}|}{N} = \frac{174}{30} = 5.8$

Solution

Mean deviation = 5.8

Q.5 Attempt any two [14 marks total]

0.0.33 Q5(A).1 [3 marks]

Find mean deviation for grouped data

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Freq	3	7	12	15	8	3	2

Solution:

Class	Mid-value	f	f x
30-40	35	3	105
40-50	45	7	315
50-60	55	12	660
60-70	65	15	975
70-80	75	8	600
80-90	85	3	255
90-100	95	2	190

N = 50, Σf x = 3100
Mean = 3100/50 = 62

Class	x	f		x -62
30-40	35	3	27	81
40-50	45	7	17	119
50-60	55	12	7	84
60-70	65	15	3	45
70-80	75	8	13	104
80-90	85	3	23	69
90-100	95	2	33	66

Mean deviation = 568/50 = 11.36

Solution

Mean deviation = 11.36

0.0.34 Q5(A).2 [3 marks]

Find standard deviation for given data

Class	60	61	62	63	64	65	66	67	68
Freq	2	1	12	29	25	12	10	4	5

Solution: N = 100, Mean = $(2 \times 60 + 1 \times 61 + \ldots + 5 \times 68)/100 = 6380/100 = 63.8$

x	f	(x -63.8)	$(x - 63.8)^2$	$f(x - 63.8)^2$
60	2	-3.8	14.44	28.88
61	1	-2.8	7.84	7.84
62	12	-1.8	3.24	38.88
63	29	-0.8	0.64	18.56
64	25	0.2	0.04	1.00
65	12	1.2	1.44	17.28
66	10	2.2	4.84	48.40
67	4	3.2	10.24	40.96
68	5	4.2	17.64	88.20

$\Sigma f(x - \bar{x})^2 = 290$

Standard deviation = $\sqrt{(}290/100) = \sqrt{2}.9 = 1.70$

Solution

= **1.70**

0.0.35 Q5(A).3 [3 marks]

Find mean for grouped data

Class	0-20	20-40	40-60	60-80	80-100	100-120
Freq	26	31	35	42	82	71

Solution:

Class	Mid-value	f	f x
0-20	10	26	260
20-40	30	31	930
40-60	50	35	1750
60-80	70	42	2940
80-100	90	82	7380
100-120	110	71	7810

N = 287, Σf x = 21070

Mean = $\frac{\sum f_i x_i}{N} = \frac{21070}{287} = 73.42$

Solution

Mean = 73.42

0.0.36 Q5(B).1 [4 marks]

Solve differential equation (x + y + 1)$^2 dy/dx = 1$

Solution: Let z = x + y + 1, then dz/dx = 1 + dy/dx So dy/dx = dz/dx - 1

Substituting: $z^2(dz/dx - 1) = 1$ $z^2 dz/dx - z^2 = 1$ $z^2 dz/dx = 1 + z^2$ $\frac{z^2}{1+z^2} dz = dx$

Integrating: $\int \frac{z^2}{1+z^2} dz = \int dx$

$\int \left(1 - \frac{1}{1+z^2}\right) dz = x + C$

$z - \tan^{-1} z = x + C$

Substituting back z = x + y + 1: $(x + y + 1) - \tan^{-1}(x + y + 1) = x + C$

Solution

$y + 1 = \tan^{-1}(x + y + 1) + C$

0.0.37 Q5(B).2 [4 marks]

Solve dy/dx + y/x = e$^x, y(0) = 2$

Solution: This is a linear differential equation of the form dy/dx + P(x)y = Q(x)

Here P(x) = 1/x, Q(x) = e^x

Integrating factor: $I.F. = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = |x| = x$ **(for x > 0)**

Multiplying the equation by x: $x\frac{dy}{dx} + y = xe^x$

$\frac{d}{dx}(xy) = xe^x$

Integrating both sides: $xy = \int xe^x dx$

Using integration by parts for $^x dx : Let$

u = x, dv = e$^x dx Then du =$

dx, v = e^x

$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x = e^x(x - 1)$

So: $xy = e^x(x - 1) + C$ $y = \frac{e^x(x-1)+C}{x}$

Using initial condition y(0) = 2: As x $\to 0, we need to use L'Hpital's rule or series expansion.$

From the original equation at

x = 0: dy/dx = e$^x - y/x This suggests we$

need to be more careful with the initial condition.

Alternative approach: Since the equation has a singularity at x = 0, we solve in the neighborhood where x $\neq 0$.

Solution

$y = \frac{e^x(x-1)+C}{x}$ **where C is determined by boundary conditions.**

0.0.38 Q5(B).3 [4 marks]

Solve y dy/dx = $\sqrt{(1 + x^2 + y^2 + x^2y^2)}$

Solution: $y\frac{dy}{dx} = \sqrt{1 + x^2 + y^2 + x^2y^2}$

$y\frac{dy}{dx} = \sqrt{(1 + x^2)(1 + y^2)}$

$\frac{ydy}{\sqrt{1+y^2}} = \sqrt{1 + x^2}dx$

Integrating both sides: $\int \frac{ydy}{\sqrt{1+y^2}} = \int \sqrt{1 + x^2}dx$

For the left side, let

u = 1 + y$^2, then du = 2ydy :$

$\int \frac{ydy}{\sqrt{1+y^2}} = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \sqrt{u} = \sqrt{1 + y^2}$

For the right side: $\int \sqrt{1 + x^2}dx = \frac{x\sqrt{1+x^2}}{2} + \frac{1}{2}\ln|x + \sqrt{1 + x^2}| + C$

Therefore: $\sqrt{1 + y^2} = \frac{x\sqrt{1+x^2}}{2} + \frac{1}{2}\ln|x + \sqrt{1 + x^2}| + C$

Solution

$\sqrt{1 + y^2} = \frac{x\sqrt{1+x^2}}{2} + \frac{1}{2}\ln|x + \sqrt{1 + x^2}| + C$

Formula Cheat Sheet

0.0.39 Matrix Operations

- $(A+B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$
- $A \cdot adj(A) = |A| \cdot I$
- **For $2{\times}2 matrix[a\ b; c\ d]$:** $adj = [d\ \ -b; -c\ a]$

0.0.40 Differentiation Formulas

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$
- $\frac{d}{dx}(e^x) = e^x$
- **Chain rule:** $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

0.0.41 Integration Formulas

- $\int \sin x\, dx = -\cos x + C$
- $\int \cos x\, dx = \sin x + C$
- $\int \sec^2 x\, dx = \tan x + C$
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}(\frac{x}{a}) + C$

0.0.42 Differential Equations

- **Linear DE:** $\frac{dy}{dx} + P(x)y = Q(x)$
- **Integrating Factor:** $I.F. = e^{\int P(x)dx}$
- **Variable Separable:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \frac{dy}{g(y)} = f(x)dx$

0.0.43 Statistics

- **Mean:** $\bar{x} = \frac{\sum x_i}{n}$ **(ungrouped),** $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ **(grouped)**
- **Mean Deviation:** $M.D. = \frac{\sum |x_i - \bar{x}|}{n}$
- **Standard Deviation:** $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

Problem-Solving Strategies

0.0.44 Matrix Problems

1. **Always check dimensions before operations**
2. **For inverse: Calculate determinant first, then adjoint**
3. **For system of equations: Use $X = A^{-1}B$ where $AX = B$**

0.0.45 Differentiation Problems

1. **Identify the type: Chain rule, product rule, quotient rule**
2. **For implicit differentiation: Differentiate both sides, collect dy/dx terms**
3. **For parametric: Use $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$**

0.0.46 Integration Problems

1. **Try substitution if you see function and its derivative**

2. Use integration by parts for products (LIATE rule)
3. For definite integrals: Check for symmetry properties

0.0.47 Differential Equations

1. Identify type: Separable, linear, exact
2. For linear DE: Find integrating factor first
3. Always verify your solution by substitution

0.0.48 Statistics Problems

1. Find mean first for deviation calculations
2. Use grouped data formulas when data is in classes
3. Create frequency table to organize calculations

Common Mistakes to Avoid

1. Matrix multiplication: Order matters (AB $\neq BAgenerally$)

1. Chain rule: Don't forget to multiply by derivative of inner function
2. Integration by parts: Choose u and dv carefully using LIATE
3. Differential equations: Don't forget the constant of integration
4. Statistics: Use correct formula for grouped vs ungrouped data

Exam Tips

1. Read questions carefully - especially for OR questions
2. Show all steps - partial marks are awarded
3. Check units and signs in your final answers
4. Verify solutions when possible by substitution
5. Manage time wisely - attempt questions you're confident about first
6. Use standard formulas - memorize the formula sheet content
7. For fill-in-blanks: Eliminate obviously wrong options first