

Mathematics-I Solutions

DI01000021 – Summer 2025

Semester 1 Study Material

Detailed Solutions and Explanations

Q.1 [14 marks]

Fill in the blanks/MCQs using appropriate choice from the given options

0.0.1 Q1.1 [1 mark]

** $\log_3 1 = ?$ **

Solution

d. 0

Solution: For any base $a > 0, a \neq 1$: $\log_a 1 = 0$ Therefore: $\log_3 1 = 0$

0.0.2 Q1.2 [1 mark]

If $f(x) = e^{x-1}$ then $f(1) = ?$

Solution

c. 1

Solution: $f(x) = e^{x-1}$ $f(1) = e^{1-1} = e^0 = 1$

0.0.3 Q1.3 [1 mark]

** $\log_5 125 = ?$ **

Solution

b. 3

Solution: $\log_5 125 = \log_5 5^3 = 3$ Since $5^3 = 125$

0.0.4 Q1.4 [1 mark]

If $f(x) = x^3 - 7$ then $f(-2) = ?$

Solution

c. -15

Solution: $f(x) = x^3 - 7$ $f(-2) = (-2)^3 - 7 = -8 - 7 = -15$

0.0.5 Q1.5 [1 mark]

Principal period of $\cos x$ is _____

Solution

c. 2π

Solution: The cosine function repeats every 2π radians, so its principal period is 2π .

0.0.6 Q1.6 [1 mark]

$150^\circ = ?$

Solution**a.** $\frac{5\pi}{6}$ **Solution:** Converting degrees to radians: $150^\circ = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$ **0.0.7 Q1.7 [1 mark]** $\sin^{-1}(-1)x + \cos^{-1}(-1)x = \$$ _____**Solution****a.** $\frac{\pi}{2}$ **Solution:** This is a standard identity: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for $x \in [-1, 1]$ **0.0.8 Q1.8 [1 mark]** $(1,0,0) \times (1,0,0) =$ _____**Solution****d.** (0,0,0)**Solution:** Cross product of any vector with itself is zero vector: $(1,0,0) \times (1,0,0) = (0,0,0)$ **0.0.9 Q1.9 [1 mark]**If $\vec{a} = 4\hat{i} - 3\hat{j}$ then $||\vec{a}|| =$ \$ _____**Solution****b.** 5**Solution:** $|\vec{a}| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ **0.0.10 Q1.10 [1 mark]**If a line makes an angle 45° with positive x-axis then slope of the line is _____**Solution****c.** 1**Solution:** Slope $m = \tan(45^\circ) = 1$ **0.0.11 Q1.11 [1 mark]**Radius of the circle $x^2 + y^2 = 4$ is _____**Solution****d.** 2**Solution:** Standard form: $x^2 + y^2 = r^2$ Comparing: $r^2 = 4$, so $r = 2$ **0.0.12 Q1.12 [1 mark]**

**\$lim

0.0.12 Q1.13 [1 mark]

**\$lim

0.0.12 Q1.14 [1 mark]

**\$lim

Q.2 (A) [6 marks]

Attempt any two

0.0.13 Q2(A).1 [3 marks]

Find value: $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

Solution

0

Solution: $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(5 \times 9 - 6 \times 8) - 2(4 \times 9 - 6 \times 7) + 3(4 \times 8 - 5 \times 7)$
 $= 1(45 - 48) - 2(36 - 42) + 3(32 - 35) = 1(-3) - 2(-6) + 3(-3) = -3 + 12 - 9 = 0$

0.0.14 Q2(A).2 [3 marks]

Prove that: $\log\left(\frac{x^p}{x^q}\right) + \log\left(\frac{x^q}{x^r}\right) + \log\left(\frac{x^r}{x^p}\right) = 0$

Solution: LHS $= \log\left(\frac{x^p}{x^q}\right) + \log\left(\frac{x^q}{x^r}\right) + \log\left(\frac{x^r}{x^p}\right)$

Using logarithm properties: $= \log(x^p) - \log(x^q) + \log(x^q) - \log(x^r) + \log(x^r) - \log(x^p) = p \log x - q \log x + q \log x - r \log x + r \log x - p \log x = 0 = \text{RHS}$

0.0.15 Q2(A).3 [3 marks]

Find value: $\tan(75^\circ)$

Solution

$$2 + \sqrt{3}$$

Solution: $\tan(75^\circ) = \tan(45^\circ + 30^\circ)$

Using $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$:

$$\begin{aligned} \tan(75^\circ) &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \\ &= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2 + \sqrt{3} \end{aligned}$$

Q.2 (B) [8 marks]

Attempt any two

0.0.16 Q2(B).1 [4 marks]

Prove that: $\frac{1}{\log_{12} 120} + \frac{1}{\log_2 120} + \frac{1}{\log_5 120} = 1$

Solution: Using change of base formula: $\frac{1}{\log_a b} = \log_b a$

$$\text{LHS} = \log_{120} 12 + \log_{120} 2 + \log_{120} 5$$

Using logarithm properties: $= \log_{120}(12 \times 2 \times 5) = \log_{120} 120 = 1 = \text{RHS}$

0.0.17 Q2(B).2 [4 marks]

Solve: $\begin{vmatrix} x & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3$

Solution: Expanding along third row: $\begin{vmatrix} x & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3 \begin{vmatrix} x & 1 \\ 1 & 2 \end{vmatrix}$

$= 3(2x - 1) = 6x - 3$

Given: $6x - 3 = 3$ $6x = 6$ $x = 1$

0.0.18 Q2(B).3 [4 marks]

If $f(x) = \frac{1-x}{1+x}$ prove that: (i) $f(x) + f\left(\frac{1}{x}\right) = 0$ (ii) $f(x) \times f(-x) = 1$

Solution: Given: $f(x) = \frac{1-x}{1+x}$

(i) $f\left(\frac{1}{x}\right) = \frac{1-\frac{1}{x}}{1+\frac{1}{x}} = \frac{\frac{x-1}{x}}{\frac{x+1}{x}} = \frac{x-1}{x+1} = -\frac{1-x}{1+x} = -f(x)$

Therefore: $f(x) + f\left(\frac{1}{x}\right) = f(x) + (-f(x)) = 0$

(ii) $f(-x) = \frac{1-(-x)}{1+(-x)} = \frac{1+x}{1-x}$

$f(x) \times f(-x) = \frac{1-x}{1+x} \times \frac{1+x}{1-x} = 1$

Q.3 (A) [6 marks]

Attempt any two

0.0.19 Q3(A).1 [3 marks]

Prove that: $\frac{\sin(180^\circ - x) + (180^\circ - x) + \tan(180^\circ + x)}{\cos(90^\circ + x) + \sec(90^\circ + x) + \cot(90^\circ + x)} = -3$

Solution: Using trigonometric identities:

- $\sin(180^\circ - x) = \sin x$
- $(180^\circ - x) = x$
- $\tan(180^\circ + x) = \tan x$
- $\cos(90^\circ + x) = -\sin x$
- $\sec(90^\circ + x) = -x$
- $\cot(90^\circ + x) = -\tan x$

Numerator $= \sin x + x + \tan x$ Denominator $= -\sin x - x - \tan x = -(\sin x + x + \tan x)$

Therefore: $\frac{\sin x + x + \tan x}{-(\sin x + x + \tan x)} = -1 \neq -3$

Note: There appears to be an error in the problem statement or expected answer.

0.0.20 Q3(A).2 [3 marks]

Prove that: $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) = 45^\circ$

Solution: Using $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$:

$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \times \frac{1}{2}}\right)$

$= \tan^{-1}\left(\frac{\frac{5}{6}}{1 - \frac{1}{6}}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) = 45^\circ$

0.0.21 Q3(A).3 [3 marks]

Find out equation of the line whose X-intercept is 3 and Y-intercept is 2.

Solution: Using intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

Where $a = 3$ (x-intercept) and $b = 2$ (y-intercept)

$\frac{x}{3} + \frac{y}{2} = 1$

Multiplying by 6: $2x + 3y = 6$

Q.3 (B) [8 marks]

Attempt any two

0.0.22 Q3(B).1 [4 marks]

Prove that: $\tan(70^\circ) = \frac{\cos(25^\circ) + \sin(25^\circ)}{\cos(25^\circ) - \sin(25^\circ)}$

Solution: RHS = $\frac{\cos(25^\circ) + \sin(25^\circ)}{\cos(25^\circ) - \sin(25^\circ)}$

Dividing numerator and denominator by $\cos(25^\circ)$:

$$= \frac{1 + \tan(25^\circ)}{1 - \tan(25^\circ)}$$

Using $\tan(45^\circ +) = \frac{1 + \tan}{1 - \tan}$:

$$= \tan(45^\circ + 25^\circ) = \tan(70^\circ) = \text{LHS}$$

0.0.23 Q3(B).2 [4 marks]

Prove that: $\frac{\sin + \sin 2 + \sin 3}{\cos + \cos 2 + \cos 3} = \tan 2$

Solution: Using sum-to-product formulas:

$$\text{Numerator: } \sin + \sin 3 + \sin 2 = 2 \sin 2 \cos + \sin 2 = \sin 2(2 \cos + 1)$$

$$\text{Denominator: } \cos + \cos 3 + \cos 2 = 2 \cos 2 \cos + \cos 2 = \cos 2(2 \cos + 1)$$

$$\text{Therefore: } \frac{\sin 2(2 \cos + 1)}{\cos 2(2 \cos + 1)} = \frac{\sin 2}{\cos 2} = \tan 2$$

0.0.24 Q3(B).3 [4 marks]

If $\vec{a} = (1, 2, 3)$, $\vec{b} = (4, 0, 0)$ and $\vec{c} = (2, 0, 1)$ find $2\vec{a} + 3\vec{b} - 5\vec{c}$

Solution: $2\vec{a} = 2(1, 2, 3) = (2, 4, 6)$ $3\vec{b} = 3(4, 0, 0) = (12, 0, 0)$ $5\vec{c} = 5(2, 0, 1) = (10, 0, 5)$

$$2\vec{a} + 3\vec{b} - 5\vec{c} = (2, 4, 6) + (12, 0, 0) - (10, 0, 5) = (2 + 12 - 10, 4 + 0 - 0, 6 + 0 - 5) = (4, 4, 1)$$

Q.4 (A) [6 marks]

Attempt any two

0.0.25 Q4(A).1 [3 marks]

If the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + m\hat{j} - 4\hat{k}$ are perpendicular, find m .

Solution: For perpendicular vectors: $\vec{a} \cdot \vec{b} = 0$

$$\vec{a} \cdot \vec{b} = (1)(2) + (-2)(m) + (3)(-4) = 2 - 2m - 12 = -10 - 2m$$

$$\text{Setting equal to zero: } -10 - 2m = 0 \quad 2m = -10 \quad m = -5$$

0.0.26 Q4(A).2 [3 marks]

Find the direction cosines and direction angles of the vector $\vec{a} = 5\hat{i} - 12\hat{k}$

Solution: $\vec{a} = 5\hat{i} + 0\hat{j} - 12\hat{k}$

$$\text{Magnitude: } |\vec{a}| = \sqrt{5^2 + 0^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

Direction cosines:

$$\begin{aligned} \bullet \quad l &= \frac{5}{13} \\ \bullet \quad m &= \frac{0}{13} = 0 \end{aligned}$$

$$\bullet \quad n = \frac{-12}{13}$$

Direction angles:

$$\begin{aligned} \bullet \quad &= \cos^{-1}\left(\frac{5}{13}\right) \\ \bullet \quad &= \cos^{-1}(0) = 90^\circ \\ \bullet \quad &= \cos^{-1}\left(\frac{-12}{13}\right) \end{aligned}$$

0.0.27 Q4(A).3 [3 marks]

Find out equation of the circle having center at $(2, -3)$ and radius 3.

Solution: Standard form: $(x - h)^2 + (y - k)^2 = r^2$

Where $(h, k) = (2, -3)$ and $r = 3$

$$(x - 2)^2 + (y + 3)^2 = 9$$

$$\text{Expanding: } x^2 - 4x + 4 + y^2 + 6y + 9 = 9 \quad x^2 + y^2 - 4x + 6y + 4 = 0$$

Q.4 (B) [8 marks]

Attempt any two

0.0.28 Q4(B).1 [4 marks]

Show that the angle between vectors $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + 3\hat{k}$ is $\sin^{-1} \sqrt{\frac{46}{55}}$

Solution: $\vec{a} \cdot \vec{b} = (1)(1) + (2)(1) + (0)(3) = 1 + 2 = 3$

$$|\vec{a}| = \sqrt{1^2 + 2^2} = \sqrt{5} \quad |\vec{b}| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}$$

$$\cos = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{3}{\sqrt{5}\sqrt{11}} = \frac{3}{\sqrt{55}}$$

$$\sin^2 = 1 - \cos^2 = 1 - \frac{9}{55} = \frac{46}{55}$$

$$\text{Therefore: } = \sin^{-1} \sqrt{\frac{46}{55}}$$

0.0.29 Q4(B).2 [4 marks]

Under effect of the forces $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$ a particle moves from the point $(1, 2, -3)$ to the point $(5, 3, 7)$. Find out work done.

Solution: Net force: $\vec{F} = (2\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 3\hat{j} - \hat{k}) = 3\hat{i} + 4\hat{j}$

Displacement: $\vec{s} = (5, 3, 7) - (1, 2, -3) = (4, 1, 10)$

$$\text{Work done: } W = \vec{F} \cdot \vec{s} = (3)(4) + (4)(1) + (0)(10) = 12 + 4 = 16 \text{ units}$$

0.0.30 Q4(B).3 [4 marks]

Evaluate: $\lim_{x \rightarrow 0} \frac{2^x - 5^x}{x}$

Solution: Using L'Hôpital's rule or the derivative definition:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2^x - 5^x}{x} &= \lim_{x \rightarrow 0} \frac{2^x \ln 2 - 5^x \ln 5}{1} \\ &= 2^0 \ln 2 - 5^0 \ln 5 = \ln 2 - \ln 5 = \ln \left(\frac{2}{5} \right) \end{aligned}$$

Q.5 (A) [6 marks]

Attempt any two

0.0.31 Q5(A).1 [3 marks]

Evaluate: $\lim_{x \rightarrow 0} \left(1 + \frac{3x}{7}\right)^{\frac{1}{x}}$

Solution: Let $y = \left(1 + \frac{3x}{7}\right)^{\frac{1}{x}}$

Taking natural log: $\ln y = \frac{1}{x} \ln \left(1 + \frac{3x}{7}\right)$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \frac{\ln \left(1 + \frac{3x}{7}\right)}{x}$$

$$\text{Using L'Hôpital's rule: } = \lim_{x \rightarrow 0} \frac{\frac{3/7}{1 + \frac{3x}{7}}}{1} = \frac{3}{7}$$

$$\text{Therefore: } \lim_{x \rightarrow 0} y = e^{3/7}$$

0.0.32 Q5(A).2 [3 marks]**Evaluate:** $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9}$ **Solution:** Factoring numerator: $x^2 - 5x + 6 = (x - 2)(x - 3)$ Factoring denominator: $x^2 - 9 = (x - 3)(x + 3)$

$$\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9} = \lim_{x \rightarrow 3} \frac{(x-2)(x-3)}{(x-3)(x+3)} = \lim_{x \rightarrow 3} \frac{x-2}{x+3} = \frac{3-2}{3+3} = \frac{1}{6}$$

0.0.33 Q5(A).3 [3 marks]**Evaluate:** $\lim_{x \rightarrow 0} \frac{\sqrt{4+x}-2}{x}$ **Solution:** Rationalizing the numerator:

$$\lim_{x \rightarrow 0} \frac{\sqrt{4+x}-2}{x} \times \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$$

$$= \lim_{x \rightarrow 0} \frac{(4+x)-4}{x(\sqrt{4+x}+2)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{4+x}+2)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{4+x}+2} = \frac{1}{2+2} = \frac{1}{4}$$

Q.5 (B) [8 marks]**Attempt any two****0.0.34 Q5(B).1 [4 marks]****Find out equation of the line passing through points (1, 2) and (2, 1).****Solution:** Using two-point form: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$$\frac{y-2}{1-2} = \frac{x-1}{2-1}$$

$$\frac{y-2}{-1} = \frac{x-1}{1}$$

$$y - 2 = -(x - 1) = -x + 1$$

$$x + y = 3$$

0.0.35 Q5(B).2 [4 marks]**Find equation of the line that passes through (-3, 2) and parallel to the line $x - 2y + 1 = 0$** **Solution:** The given line $x - 2y + 1 = 0$ has slope $m = \frac{1}{2}$ Since parallel lines have the same slope, required line has slope $m = \frac{1}{2}$ Using point-slope form: $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{1}{2}(x - (-3))$$

$$y - 2 = \frac{1}{2}(x + 3)$$

$$2y - 4 = x + 3$$

$$x - 2y + 7 = 0$$

0.0.36 Q5(B).3 [4 marks]**Find out center and radius of the circle: $x^2 + y^2 + 6x - 4y - 3 = 0$** **Solution:** Completing the square:

$$x^2 + 6x + y^2 - 4y = 3$$

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 3 + 9 + 4$$

$$(x + 3)^2 + (y - 2)^2 = 16$$

Center: $(-3, 2)$ **Radius:** $r = \sqrt{16} = 4$ **Formula Cheat Sheet****0.0.37 Logarithms**

- $\log_a 1 = 0$

- $\log_a a = 1$
- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

0.0.38 Trigonometry

- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin(180^\circ - x) = \sin x$, $\cos(90^\circ + x) = -\sin x$

0.0.39 Vectors

- $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
- $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos$
- For perpendicular vectors: $\vec{a} \cdot \vec{b} = 0$

0.0.40 Coordinate Geometry

- Two-point form: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
- Circle: $(x-h)^2 + (y-k)^2 = r^2$
- Parallel lines have equal slopes

0.0.41 Limits

- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x \rightarrow \infty} \frac{ax+b}{cx+d} = \frac{a}{c}$

Problem-Solving Strategies

1. **Logarithms:** Use properties to simplify expressions
2. **Trigonometry:** Apply compound angle formulas and identities
3. **Vectors:** Remember dot and cross product properties

Common Mistakes to Avoid

0.0.42 Logarithms

- **Mistake:** Confusing $\log_a b$ with $\log_b a$
- **Solution:** Remember change of base: $\frac{1}{\log_a b} = \log_b a$

0.0.43 Trigonometry

- **Mistake:** Wrong angle conversions between degrees and radians
- **Solution:** Always use $180^\circ = \pi$ radians for conversion

0.0.44 Vectors

- **Mistake:** Confusing dot product with cross product
- **Solution:** Dot product gives scalar, cross product gives vector

0.0.45 Limits

- **Mistake:** Direct substitution in indeterminate forms
- **Solution:** Use algebraic manipulation, L'Hôpital's rule, or standard limits

0.0.46 Determinants

- **Mistake:** Sign errors in expansion
- **Solution:** Follow the checkerboard pattern carefully

Exam Tips

0.0.47 Time Management

- **Q1 (14 marks):** 20-25 minutes - Quick calculations
- **Q2-Q5:** 35-40 minutes each - Show all steps clearly

0.0.48 Strategy

1. **Read all questions first** - Choose easier OR options
2. **Start with Q1** - Build confidence with MCQs
3. **Show work clearly** - Partial credit is available
4. **Use standard formulas** - Don't derive unless asked

0.0.49 Key Points to Remember

- Always write the final answer clearly
- Use proper mathematical notation
- Draw diagrams where helpful
- Check units in physics-related problems (work, force)

0.0.50 Calculator Usage

- Scientific calculator allowed
- Use for complex arithmetic only
- Show the setup before calculating
- Round final answers appropriately

0.0.51 Common Formula Applications

Standard Limits (Memory aids)

$\lim_{x \rightarrow 0} \sin(x)/x = 1$ "Sine over x is one"
 $\lim_{x \rightarrow 0} (e^x - 1)/x = 1$ "e minus one over x is one"
 $\lim_{x \rightarrow 0} (a^x - 1)/x = \ln(a)$ "General exponential form"

Trigonometric Identities (Quick Reference)

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A["sin2 + cos2 = 1"] --- B["1 + tan2 = sec2"]
    A --- C["1 + cot2 = cosec2"]
    D["sin(A) cos(B) + cos(A) sin(B)"]
    E["cos(A) cos(B) - sin(A) sin(B)"]
    F["cos(A) sin(B) + sin(A) cos(B)"]
    G["sin(A) cos(B) - cos(A) sin(B)"]
{Highlighting}
{Shaded}
```

Vector Operations (Step-by-step)

1. **Magnitude:** $|\vec{a}| = \sqrt{\text{sum of squares}}$
2. **Dot Product:** $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
3. **Angle:** $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

Circle Equations (Forms)

Form	Equation	When to Use
Standard	$(x - h)^2 + (y - k)^2 = r^2$	Given center and radius
General	$x^2 + y^2 + Dx + Ey + F = 0$	Need to find center/radius
Complete Square	$(x + D/2)^2 + (y + E/2)^2 = (D^2 + E^2 - 4F)/4$	Converting general to standard

0.0.52 Problem-Specific Strategies

For Determinant Problems

1. Look for zeros to simplify expansion
2. Use row/column operations if allowed
3. Remember: if two rows/columns are proportional, determinant = 0

For Limit Problems

```

Start with limit
|
Direct substitution?
/      {}
Yes      No (0/0, /, etc.)
|          |
Answer    Try factoring/
          rationalization
          |
          Still indeterminate?
          |
          L'Hôpital's Rule
          |
          Find answer

```

For Vector Problems

- **Step 1:** Write vectors in component form
- **Step 2:** Apply required operation (dot/cross product)
- **Step 3:** Simplify and find magnitude if needed
- **Step 4:** Check perpendicularity condition ($\vec{a} \cdot \vec{b} = 0$)

For Coordinate Geometry

- **Line problems:** Identify what's given (points, slope, parallel/perpendicular)
- **Circle problems:** Identify center and radius from given information
- **Always** check your equation by substituting known points

0.0.53 Memory Techniques

Logarithm Properties (MNEMONIC: "PLUS")

- **Product:** $\log(ab) = \log a + \log b$
- **Limit:** $\log_a 1 = 0$
- **Unity:** $\log_a a = 1$
- **Subtraction:** $\log(a/b) = \log a - \log b$

Trigonometric Values ($30^\circ, 45^\circ, 60^\circ$)

Angle	sin	cos	tan
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$

Memory aid: “1, 2, 3” under square roots for sin values (30° to 60°)

0.0.54 Final Review Checklist

Before submitting your paper:

- ☐ All questions attempted as required
- ☐ Final answers clearly marked
- ☐ Units included where applicable

- ☐ No arithmetic errors in simple calculations
- ☐ Proper mathematical notation used
- ☐ Diagrams labeled clearly (if drawn)

0.0.55 Quick Problem Solving Guide

If you're stuck on a problem:

1. **Read the problem again** - Often missed details become clear
2. **Try a different approach** - Multiple methods usually exist
3. **Work backwards** - Start from what you want to prove/find
4. **Use elimination** - In MCQs, eliminate obviously wrong options
5. **Move on and return** - Don't spend too much time on one problem

Last 15 minutes strategy:

- Focus on completing MCQs in Q1
- Check arithmetic in longer problems
- Ensure all final answers are clearly marked
- Review any skipped parts of questions

Remember: This exam tests fundamental concepts. Focus on understanding rather than memorizing, and always show your reasoning clearly for maximum partial credit.