

Mathematics-I (DI01000021) - Winter 2024 Solution

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Question Q.1 [14 marks]

Fill in the blanks/MCQs using appropriate choice from the given options.

Question Q1.1 [1 marks]

$$\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = \underline{\hspace{2cm}}$$

Solution

Answer: b. 13

Solution: For 2×2 determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = (5 \times 3) - (1 \times 2) = 15 - 2 = 13$$

Question Q1.2 [1 marks]

If $\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = 0$ then $x = \underline{\hspace{2cm}}$

Solution

Answer: b. 2

Solution: $\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = x \times 1 - 1 \times 2 = x - 2 = 0$

Therefore, $x = 2$

Question Q1.3 [1 marks]

If $f(x) = x^2$ then $f(-1) = \underline{\hspace{2cm}}$

Solution

Answer: a. 1

Solution: $f(x) = x^2$ $f(-1) = (-1)^2 = 1$

Question Q1.4 [1 marks]

$$\log_{10} 1 = \underline{\hspace{2cm}}$$

Solution

Answer: b. 0

Solution: By logarithm property: $\log_a 1 = 0$ for any base $a > 0$ Therefore, $\log_{10} 1 = 0$

Question Q1.5 [1 marks]

$$\sin \frac{\pi}{2} + \cos \frac{\pi}{2} = \underline{\hspace{2cm}}$$

Solution

Answer: c. 1

Solution: $\sin \frac{\pi}{2} = 1$ and $\cos \frac{\pi}{2} = 0$ Therefore, $\sin \frac{\pi}{2} + \cos \frac{\pi}{2} = 1 + 0 = 1$

Question Q1.6 [1 marks]

$$\tan^{-1}(1) = \underline{\hspace{2cm}}$$

Solution

Answer: a. $\frac{\pi}{4}$

Solution: $\tan \frac{\pi}{4} = 1$ Therefore, $\tan^{-1}(1) = \frac{\pi}{4}$

Question Q1.7 [1 marks]

$$\frac{2\pi}{3} \text{ radian} = \underline{\hspace{2cm}} \text{ degree}$$

Solution

Answer: d. 120

Solution: To convert radians to degrees: $\text{degrees} = \text{radians} \times \frac{180}{\pi}$ $\frac{2\pi}{3} \times \frac{180}{\pi} = \frac{2 \times 180}{3} = \frac{360}{3} = 120$

Question Q1.8 [1 marks]

$$\hat{i} \times \hat{j} = \underline{\hspace{2cm}}$$

Solution

Answer: c. \hat{k}

Solution: By right-hand rule for cross product: $\hat{i} \times \hat{j} = \hat{k}$

Question Q1.9 [1 marks]

$$|\hat{i} + \hat{j} + \hat{k}| = \underline{\hspace{2cm}}$$

Solution**Answer:** d. $\sqrt{3}$ **Solution:** $|\hat{i} + \hat{j} + \hat{k}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$ **Question Q1.10 [1 marks]**Slope of line $2x + y - 3 = 0$ is _____**Solution****Answer:** a. -2**Solution:** Convert to slope-intercept form: $y = -2x + 3$ Slope = coefficient of $x = -2$ **Question Q1.11 [1 marks]**Radius of circle $x^2 + y^2 = 81$ is _____**Solution****Answer:** b. 9**Solution:** Standard form: $x^2 + y^2 = r^2$ Here, $r^2 = 81$, so $r = 9$ **Question Q1.12 [1 marks]** $\lim_{n \rightarrow \infty} \frac{1}{n} =$ _____**Solution****Answer:** c. 0**Solution:** As n approaches infinity, $\frac{1}{n}$ approaches 0**Question Q1.13 [1 marks]** $\lim_{x \rightarrow 1} (x^2 + x + 1) =$ _____**Solution****Answer:** a. 3**Solution:** Direct substitution: $(1)^2 + (1) + 1 = 1 + 1 + 1 = 3$ **Question Q1.14 [1 marks]** $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} =$ _____

Solution**Answer:** b. 1**Solution:** This is a standard limit: $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1$ **Question Q.2 (A) [6 marks]**

Attempt any two

Question Q2.1 [3 marks]Find the value of $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 4 & -2 & 5 \end{vmatrix}$ **Solution****Answer:****Solution:** Using expansion along second row (has zero): $= -2 \begin{vmatrix} 3 & 1 \\ -2 & 5 \end{vmatrix} + (-1) \begin{vmatrix} 1 & 1 \\ 4 & 5 \end{vmatrix} + 0$
 $= -2(15 + 2) - 1(5 - 4) = -2(17) - 1(1) = -34 - 1 = -35$ **Table 1.** Determinant Calculation Steps

Step	Calculation	Result
Minor 1	$(3 \times 5) - (1 \times -2)$	17
Minor 2	$(1 \times 5) - (1 \times 4)$	1
Final	$-2(17) - 1(1)$	-35

Question Q2.2 [3 marks]If $f(x) = x^3 + 5$ then find $f(0)$, $f(1)$ and $f(-1)$ **Solution****Answer:****Solution:** Given: $f(x) = x^3 + 5$ $f(0) = (0)^3 + 5 = 0 + 5 = 5$ $f(1) = (1)^3 + 5 = 1 + 5 = 6$ $f(-1) = (-1)^3 + 5 = -1 + 5 = 4$ **Table 2.** Function Values

Input	Calculation	Output
$f(0)$	$0^3 + 5$	5
$f(1)$	$1^3 + 5$	6
$f(-1)$	$(-1)^3 + 5$	4

Question Q2.3 [3 marks]Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

Solution**Answer:****Solution:** Using formula: $\tan^{-1} a + \tan^{-1} b = \tan^{-1} \left(\frac{a+b}{1-ab} \right)$ Let $a = \frac{1}{2}$, $b = \frac{1}{3}$

$$\tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{3} \right) = \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{5}{6}}{1 - \frac{1}{6}} \right) = \tan^{-1} \left(\frac{\frac{5}{6}}{\frac{5}{6}} \right) = \tan^{-1}(1) = \frac{\pi}{4}$$

Hence proved.

Question Q.2 (B) [8 marks]

Attempt any two

Question Q2.1 [4 marks]If $f(x) = \frac{x-1}{x+1}$ then prove that $f(x) \cdot f(-x) = 1$ **Solution****Answer:****Solution:** Given: $f(x) = \frac{x-1}{x+1}$

First find $f(-x)$: $f(-x) = \frac{(-x)-1}{(-x)+1} = \frac{-x-1}{-x+1} = \frac{-(x+1)}{-(x-1)} = \frac{x+1}{x-1}$

Now calculate $f(x) \cdot f(-x)$: $f(x) \cdot f(-x) = \frac{x-1}{x+1} \cdot \frac{x+1}{x-1} = \frac{(x-1)(x+1)}{(x+1)(x-1)} = 1$

Hence proved.

Question Q2.2 [4 marks]If $\log \left(\frac{x+y}{2} \right) = \frac{1}{2}(\log x + \log y)$ then prove that $x = y$ **Solution****Answer:****Solution:** Given: $\log \left(\frac{x+y}{2} \right) = \frac{1}{2}(\log x + \log y)$

Using logarithm properties: $\frac{1}{2}(\log x + \log y) = \frac{1}{2} \log(xy) = \log \sqrt{xy}$

So: $\log \left(\frac{x+y}{2} \right) = \log \sqrt{xy}$

Taking antilog: $\frac{x+y}{2} = \sqrt{xy}$

Squaring both sides: $\left(\frac{x+y}{2} \right)^2 = xy$

$$\frac{(x+y)^2}{4} = xy$$

$$(x+y)^2 = 4xy$$

$$x^2 + 2xy + y^2 = 4xy$$

$$x^2 - 2xy + y^2 = 0$$

$$(x-y)^2 = 0$$

Therefore, $x = y$. Hence proved.**Question Q2.3 [4 marks]**Solve $\log(x+3) + \log(x-3) = \log 27$

Solution**Answer:****Solution:** Given: $\log(x+3) + \log(x-3) = \log 27$ Using logarithm property: $\log a + \log b = \log(ab)$ $\log[(x+3)(x-3)] = \log 27$ Taking antilog: $(x+3)(x-3) = 27$

$$x^2 - 9 = 27$$

$$x^2 = 36$$

$$x = \pm 6$$

Check validity:

- For $x = 6$: $x+3 = 9 > 0$ and $x-3 = 3 > 0$
- For $x = -6$: $x+3 = -3 < 0$ (invalid for logarithm)

Therefore, $x = 6$ **Question Q.3 (A) [6 marks]**

Attempt any two

Question Q3.1 [3 marks]

Prove that $\frac{\sin(\frac{\pi}{2} + \theta)}{\cos(\pi - \theta)} + \frac{\tan(\pi - \theta)}{\cot(\frac{3\pi}{2} - \theta)} + \frac{\operatorname{cosec}(\frac{\pi}{2} - \theta)}{\sec(\pi + \theta)} = -3$

Solution**Answer:****Solution:** Using trigonometric identities:

$$\sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta \quad \cos(\pi - \theta) = -\cos \theta \quad \tan(\pi - \theta) = -\tan \theta \quad \cot\left(\frac{3\pi}{2} - \theta\right) = \tan \theta \quad \operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec \theta \quad \sec(\pi + \theta) = -\sec \theta$$

$$\text{Substituting: } \frac{\cos \theta}{-\cos \theta} + \frac{-\tan \theta}{\tan \theta} + \frac{\sec \theta}{-\sec \theta}$$

$$= -1 + (-1) + (-1) = -3$$

Hence proved.

Question Q3.2 [3 marks]

Prove that $\tan 55 = \frac{\cos 10 + \sin 10}{\cos 10 - \sin 10}$

Solution**Answer:****Solution:** We know that $\tan 55 = \tan(45 + 10)$ Using formula: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\tan 55 = \frac{\tan 45 + \tan 10}{1 - \tan 45 \tan 10} = \frac{1 + \tan 10}{1 - \tan 10}$$

$$\text{Now, } \tan 10 = \frac{\sin 10}{\cos 10}$$

$$\tan 55 = \frac{1 + \frac{\sin 10}{\cos 10}}{1 - \frac{\sin 10}{\cos 10}} = \frac{\cos 10 + \sin 10}{\cos 10 - \sin 10}$$

Hence proved.

Question Q3.3 [3 marks]

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + \hat{k}$ then find $2\vec{a} + \vec{b} - \vec{c}$

Solution**Answer:****Solution:** Given: $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ $\vec{c} = 3\hat{i} + \hat{j} + \hat{k}$

$$2\vec{a} = 2(2\hat{i} + 3\hat{j} + \hat{k}) = 4\hat{i} + 6\hat{j} + 2\hat{k}$$

$$2\vec{a} + \vec{b} - \vec{c} = (4\hat{i} + 6\hat{j} + 2\hat{k}) + (\hat{i} + \hat{j} + \hat{k}) - (3\hat{i} + \hat{j} + \hat{k})$$

$$= (4 + 1 - 3)\hat{i} + (6 + 1 - 1)\hat{j} + (2 + 1 - 1)\hat{k}$$

$$= 2\hat{i} + 6\hat{j} + 2\hat{k}$$

Question Q.3 (B) [8 marks]

Attempt any two

Question Q3.1 [4 marks]Prove that $\frac{\sin(x-y)}{\cos x \cos y} + \frac{\sin(y-z)}{\cos y \cos z} + \frac{\sin(z-x)}{\cos z \cos x} = 0$ **Solution****Answer:****Solution:** Using identity: $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$\frac{\sin(x-y)}{\cos x \cos y} = \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y} = \tan x - \tan y$$

$$\text{Similarly: } \frac{\sin(y-z)}{\cos y \cos z} = \tan y - \tan z \quad \frac{\sin(z-x)}{\cos z \cos x} = \tan z - \tan x$$

$$\text{Adding all three: } (\tan x - \tan y) + (\tan y - \tan z) + (\tan z - \tan x) = 0$$

Hence proved.

Question Q3.2 [4 marks]Draw graph of $y = \cos x$ for $0 \leq x \leq \pi$ **Solution****Answer:****Solution:****Table 3.** Table of values

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1

Question Q3.3 [4 marks]

Find equation of line passing through (1, 2) and (-3, 1)

Solution**Answer:****Solution:** Given points: $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (-3, 1)$

$$\text{Slope: } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{-3 - 1} = \frac{-1}{-4} = \frac{1}{4}$$

$$\text{Using point-slope form: } y - y_1 = m(x - x_1) \quad y - 2 = \frac{1}{4}(x - 1) \quad 4(y - 2) = x - 1 \quad 4y - 8 = x - 1 \quad x - 4y + 7 = 0$$

Equation: $x - 4y + 7 = 0$

Question Q.4 (A) [6 marks]

Attempt any two

Question Q4.1 [3 marks]

Find unit vector perpendicular to $\vec{a} = \hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$

Solution

Answer:

Solution: Cross product: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$$= \hat{i}[(-3)(2) - (1)(1)] - \hat{j}[(1)(2) - (1)(2)] + \hat{k}[(1)(1) - (-3)(2)] = \hat{i}(-6 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 6) = -7\hat{i} + 0\hat{j} + 7\hat{k}$$

Magnitude: $|\vec{a} \times \vec{b}| = \sqrt{(-7)^2 + 0^2 + 7^2} = \sqrt{49 + 49} = 7\sqrt{2}$

Unit vector: $\hat{n} = \frac{-7\hat{i} + 7\hat{k}}{7\sqrt{2}} = \frac{-\hat{i} + \hat{k}}{\sqrt{2}}$

Question Q4.2 [3 marks]

Forces (1, 2, 1) and (2, -1, 3) act on a particle and the particle moves from point (2, 3, 1) to (4, 6, 2). Find the work done.

Solution

Answer:

Solution: Resultant force: $\vec{F} = (1, 2, 1) + (2, -1, 3) = (3, 1, 4)$

Displacement: $\vec{s} = (4, 6, 2) - (2, 3, 1) = (2, 3, 1)$

Work done: $W = \vec{F} \cdot \vec{s} = (3)(2) + (1)(3) + (4)(1) = 6 + 3 + 4 = 13$ units

Question Q4.3 [3 marks]

Show that lines $2x - 3y + 5 = 0$ and $8x - 12y - 3 = 0$ are parallel lines.

Solution

Answer:

Solution: For line $2x - 3y + 5 = 0$: slope $m_1 = \frac{2}{3}$ For line $8x - 12y - 3 = 0$: slope $m_2 = \frac{8}{12} = \frac{2}{3}$
 Since $m_1 = m_2 = \frac{2}{3}$, the lines are parallel.

Table 4. Parallel Lines Comparison

Line	Standard Form	Slope
Line 1	$2x - 3y + 5 = 0$	$\frac{2}{3}$
Line 2	$8x - 12y - 3 = 0$	$\frac{2}{3}$

Question Q.4 (B) [8 marks]

Attempt any two

Question Q4.1 [4 marks]

Show that angle between $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ is $\sin^{-1} \left(\frac{\sqrt{26}}{27} \right)$

Solution

Answer:

Solution: $\vec{a} \cdot \vec{b} = (1)(2) + (1)(-2) + (-1)(1) = 2 - 2 - 1 = -1$

$|\vec{a}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$ $|\vec{b}| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$

$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{-1}{\sqrt{3} \times 3} = \frac{-1}{3\sqrt{3}}$

$\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{1}{27} = \frac{26}{27}$

Therefore, $\sin \theta = \frac{\sqrt{26}}{3\sqrt{3}} = \frac{\sqrt{26}}{\sqrt{27}}$

Hence, $\theta = \sin^{-1} \left(\frac{\sqrt{26}}{\sqrt{27}} \right)$

Question Q4.2 [4 marks]

If $\vec{a} = (1, 1, 1)$, $\vec{b} = (2, 0, 1)$ and $\vec{c} = (-2, 1, 0)$ then find $\vec{a} \cdot (\vec{b} \times \vec{c})$

Solution

Answer:

Solution: First find $\vec{b} \times \vec{c}$: $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ -2 & 1 & 0 \end{vmatrix}$

$= \hat{i}(0 \times 0 - 1 \times 1) - \hat{j}(2 \times 0 - 1 \times (-2)) + \hat{k}(2 \times 1 - 0 \times (-2)) = \hat{i}(-1) - \hat{j}(2) + \hat{k}(2) = -\hat{i} - 2\hat{j} + 2\hat{k}$

Now find $\vec{a} \cdot (\vec{b} \times \vec{c})$: $\vec{a} \cdot (\vec{b} \times \vec{c}) = (1, 1, 1) \cdot (-1, -2, 2) = (1)(-1) + (1)(-2) + (1)(2) = -1 - 2 + 2 = -1$

Question Q4.3 [4 marks]

Evaluate $\lim_{\theta \rightarrow 0} \frac{\sin 4\theta}{\theta}$

Solution

Answer:

Solution: $\lim_{\theta \rightarrow 0} \frac{\sin 4\theta}{\theta} = \lim_{\theta \rightarrow 0} \frac{\sin 4\theta}{4\theta} \times 4$

Using standard limit $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$:

Let $u = 4\theta$, then as $\theta \rightarrow 0$, $u \rightarrow 0$

$\lim_{\theta \rightarrow 0} \frac{\sin 4\theta}{4\theta} = \lim_{u \rightarrow 0} \frac{\sin u}{u} = 1$

Therefore, $\lim_{\theta \rightarrow 0} \frac{\sin 4\theta}{\theta} = 4 \times 1 = 4$

Question Q.5 (A) [6 marks]

Attempt any two

Question Q5.1 [3 marks]

Evaluate $\lim_{x \rightarrow 9} \frac{x^2 - 81}{x - 9}$

Solution

Answer:

Solution: Direct substitution gives $\frac{0}{0}$ form.

Factor the numerator: $x^2 - 81 = (x - 9)(x + 9)$

$$\begin{aligned} \lim_{x \rightarrow 9} \frac{x^2 - 81}{x - 9} &= \lim_{x \rightarrow 9} \frac{(x - 9)(x + 9)}{x - 9} \\ &= \lim_{x \rightarrow 9} (x + 9) = 9 + 9 = 18 \end{aligned}$$

Question Q5.2 [3 marks]

Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^{2x}$

Solution

Answer:

Solution: Let $y = \left(1 + \frac{3}{x}\right)^{2x}$

Taking natural logarithm: $\ln y = 2x \ln \left(1 + \frac{3}{x}\right)$

As $x \rightarrow \infty$, $\frac{3}{x} \rightarrow 0$

Using $\ln(1 + u) \approx u$ for small u : $\ln y = 2x \times \frac{3}{x} = 6$

Therefore, $y = e^6$

Question Q5.3 [3 marks]

Evaluate $\lim_{x \rightarrow 1} \frac{x - 1}{x^2 + x - 2}$

Solution

Answer:

Solution: Factor the denominator: $x^2 + x - 2 = (x + 2)(x - 1)$

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x - 1}{x^2 + x - 2} &= \lim_{x \rightarrow 1} \frac{x - 1}{(x + 2)(x - 1)} \\ &= \lim_{x \rightarrow 1} \frac{1}{x + 2} = \frac{1}{1 + 2} = \frac{1}{3} \end{aligned}$$

Question Q.5 (B) [8 marks]

Attempt any two

Question Q5.1 [4 marks]

Find the equation of line passing through the point (2, -3) and having slope 4.

Solution**Answer:****Solution:** Using point-slope form: $y - y_1 = m(x - x_1)$ Given: $(x_1, y_1) = (2, -3)$ and slope $m = 4$

$$y - (-3) = 4(x - 2) \quad y + 3 = 4x - 8 \quad y = 4x - 11$$

Equation: $y = 4x - 11$ or $4x - y - 11 = 0$

Question Q5.2 [4 marks]For what value of m , lines $7x + y - 1 = 0$ and $3x - my + 2 = 0$ are perpendicular to each other.**Solution****Answer:****Solution:** For perpendicular lines, product of slopes = -1For line $7x + y - 1 = 0$: slope $m_1 = -7$ For line $3x - my + 2 = 0$: slope $m_2 = \frac{3}{m}$

Condition: $m_1 \times m_2 = -1 \quad (-7) \times \frac{3}{m} = -1 \quad \frac{-21}{m} = -1 \quad 21 = m$

Therefore, $m = 21$ **Table 5.** Perpendicular Lines

Line	Standard Form	Slope
Line 1	$7x + y - 1 = 0$	-7
Line 2	$3x - my + 2 = 0$	$\frac{3}{m}$

Verification: When $m = 21$, slopes are -7 and $\frac{3}{21} = \frac{1}{7}$ Product: $(-7) \times \frac{1}{7} = -1$ **Question Q5.3 [4 marks]**Find the centre and radius of the circle $4x^2 + 4y^2 + 8x - 12y - 3 = 0$ **Solution****Answer:****Solution:** First, divide by 4 to get standard form: $x^2 + y^2 + 2x - 3y - \frac{3}{4} = 0$ Complete the square for x and y terms: $x^2 + 2x = (x + 1)^2 - 1$ $y^2 - 3y = (y - \frac{3}{2})^2 - \frac{9}{4}$

Substituting: $(x + 1)^2 - 1 + (y - \frac{3}{2})^2 - \frac{9}{4} - \frac{3}{4} = 0$

$$(x + 1)^2 + (y - \frac{3}{2})^2 = 1 + \frac{9}{4} + \frac{3}{4} = 1 + 3 = 4$$

Centre: $(-1, \frac{3}{2})$ **Radius:** $r = \sqrt{4} = 2$ **Table 6.** Circle Properties

Component	Value
Centre (h,k)	$(-1, \frac{3}{2})$
Radius	2
Standard Form	$(x + 1)^2 + (y - \frac{3}{2})^2 = 4$

Formula Cheat Sheet

Determinants

- **2×2 Determinant:** $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
- **3×3 Determinant:** Expand along any row/column

Functions & Logarithms

- **Basic:** $\log_a 1 = 0$, $\log_a a = 1$
- **Properties:** $\log(ab) = \log a + \log b$, $\log\left(\frac{a}{b}\right) = \log a - \log b$

Trigonometry

- **Basic Values:** $\sin 0 = 0$, $\sin 30 = \frac{1}{2}$, $\sin 45 = \frac{\sqrt{2}}{2}$, $\sin 60 = \frac{\sqrt{3}}{2}$, $\sin 90 = 1$
- **Conversion:** Radians to degrees: $\times \frac{180}{\pi}$
- **Identities:** $\sin^2 \theta + \cos^2 \theta = 1$
- **Inverse:** $\tan^{-1}(1) = \frac{\pi}{4}$

Vectors

- **Magnitude:** $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$
- **Dot Product:** $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$
- **Cross Product:** $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$
- **Work Done:** $W = \vec{F} \cdot \vec{s}$

Coordinate Geometry

- **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$
- **Parallel Lines:** Same slope
- **Perpendicular Lines:** Product of slopes = -1
- **Circle:** $(x - h)^2 + (y - k)^2 = r^2$

Limits

- **Standard Limits:** $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$
- **Factorization:** Use for $\frac{0}{0}$ forms
- **L'Hôpital's Rule:** For indeterminate forms

Quick Reference Table

Table 7. Key Formulas

Topic	Key Formula	Example
Determinant 2×2	$ad - bc$	$\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 8 - 3 = 5$
Slope	$\frac{y_2 - y_1}{x_2 - x_1}$	Points (1,2), (3,8): $m = \frac{8-2}{3-1} = 3$
Circle	$(x - h)^2 + (y - k)^2 = r^2$	Center (1,2), radius 3
Limit	$\lim_{x \rightarrow a} f(x)$	Direct substitution or factoring