Mathematics-I (DI01000021) - Summer 2025 Solution

Date: 2025-05-30

Q.1 [14 marks]

Fill in the blanks/MCQs using appropriate choice from the given options

Q1.1 [1 mark]

$\log_3 1 =$ __________

Answer: d. 0

Solution:
For any base $a > 0, a \neq 1$: $\log_a 1 = 0$
Therefore: $\log_3 1 = 0$

Q1.2 [1 mark]

If $f(x) = e^{x-1}$ then $f(1) =$ __________

Answer: c. 1

Solution:
$f(x) = e^{x-1}$
$f(1) = e^{1-1} = e^0 = 1$

Q1.3 [1 mark]

$\log_5 125 =$ __________

Answer: b. 3

Solution:
$\log_5 125 = \log_5 5^3 = 3$
Since $5^3 = 125$

Q1.4 [1 mark]

If $f(x) = x^3 - 7$ then $f(-2) =$ __________

Answer: c. -15

Solution:
$f(x) = x^3 - 7$
$f(-2) = (-2)^3 - 7 = -8 - 7 = -15$

Q1.5 [1 mark]

Principal period of $\cos x$ is ________
Answer: c. 2π

Solution:
The cosine function repeats every 2π radians, so its principal period is 2π.

Q1.6 [1 mark]

$150° =$ ________
Answer: a. $\frac{5\pi}{6}$

Solution:
Converting degrees to radians: $150° = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$

Q1.7 [1 mark]

$\sin^{-1} x + \cos^{-1} x =$ ________
Answer: a. $\frac{\pi}{2}$

Solution:
This is a standard identity: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for $x \in [-1, 1]$

Q1.8 [1 mark]

$(1, 0, 0) \times (1, 0, 0) =$ ________
Answer: d. (0,0,0)

Solution:
Cross product of any vector with itself is zero vector:
$(1, 0, 0) \times (1, 0, 0) = (0, 0, 0)$

Q1.9 [1 mark]

If $\vec{a} = 4\hat{i} - 3\hat{j}$ then $|\vec{a}| =$ ________
Answer: b. 5

Solution:
$|\vec{a}| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

Q1.10 [1 mark]

If a line makes an angle 45° with positive x-axis then slope of the line is ________
Answer: c. 1

Solution:
Slope $m = \tan(45°) = 1$

Q1.11 [1 mark]

Radius of the circle $x^2 + y^2 = 4$ is __________
Answer: d. 2

Solution:
Standard form: $x^2 + y^2 = r^2$
Comparing: $r^2 = 4$, so $r = 2$

Q1.12 [1 mark]

$\lim_{x\to 0} \frac{e^x-1}{x} =$ __________
Answer: a. 1

Solution:
This is a standard limit: $\lim_{x\to 0} \frac{e^x-1}{x} = 1$

Q1.13 [1 mark]

$\lim_{x\to 0} \frac{\sin 3x}{x} =$ __________
Answer: d. 3

Solution:
$\lim_{x\to 0} \frac{\sin 3x}{x} = \lim_{x\to 0} \frac{\sin 3x}{3x} \times 3 = 1 \times 3 = 3$

Q1.14 [1 mark]

$\lim_{n\to\infty} \frac{5n+4}{4n+5} =$ __________
Answer: c. 5/4

Solution:
$\lim_{n\to\infty} \frac{5n+4}{4n+5} = \lim_{n\to\infty} \frac{5+\frac{4}{n}}{4+\frac{5}{n}} = \frac{5}{4}$

Q.2 (A) [6 marks]

Attempt any two

Q2(A).1 [3 marks]

Find value: $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$
Answer: 0

Solution:
$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(5 \times 9 - 6 \times 8) - 2(4 \times 9 - 6 \times 7) + 3(4 \times 8 - 5 \times 7)$
$= 1(45 - 48) - 2(36 - 42) + 3(32 - 35)$
$= 1(-3) - 2(-6) + 3(-3)$
$= -3 + 12 - 9 = 0$

Q2(A).2 [3 marks]

Prove that: $\log\left(\frac{x^p}{x^q}\right) + \log\left(\frac{x^q}{x^r}\right) + \log\left(\frac{x^r}{x^p}\right) = 0$

Solution:
$\text{LHS} = \log\left(\frac{x^p}{x^q}\right) + \log\left(\frac{x^q}{x^r}\right) + \log\left(\frac{x^r}{x^p}\right)$
Using logarithm properties:
$= \log(x^p) - \log(x^q) + \log(x^q) - \log(x^r) + \log(x^r) - \log(x^p)$
$= p\log x - q\log x + q\log x - r\log x + r\log x - p\log x$
$= 0 = \text{RHS}$

Q2(A).3 [3 marks]

Find value: $\tan(75°)$
Answer: $2 + \sqrt{3}$

Solution:
$\tan(75°) = \tan(45° + 30°)$
Using $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$:
$\tan(75°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$
$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2 + \sqrt{3}$

Q.2 (B) [8 marks]

Attempt any two

Q2(B).1 [4 marks]

Prove that: $\frac{1}{\log_{12} 120} + \frac{1}{\log_2 120} + \frac{1}{\log_5 120} = 1$

Solution:
Using change of base formula: $\frac{1}{\log_a b} = \log_b a$
$\text{LHS} = \log_{120} 12 + \log_{120} 2 + \log_{120} 5$
Using logarithm properties:
$= \log_{120}(12 \times 2 \times 5) = \log_{120} 120 = 1 = \text{RHS}$

Q2(B).2 [4 marks]

Solve: $\begin{vmatrix} x & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3$

Solution:
Expanding along third row:
$\begin{vmatrix} x & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3 \begin{vmatrix} x & 1 \\ 1 & 2 \end{vmatrix}$
$= 3(2x - 1) = 6x - 3$
Given: $6x - 3 = 3$
$6x = 6$
$x = 1$

Q2(B).3 [4 marks]

If $f(x) = \frac{1-x}{1+x}$ **prove that: (i)** $f(x) + f\left(\frac{1}{x}\right) = 0$ **(ii)** $f(x) \times f(-x) = 1$

Solution:
Given: $f(x) = \frac{1-x}{1+x}$
(i) $f\left(\frac{1}{x}\right) = \frac{1-\frac{1}{x}}{1+\frac{1}{x}} = \frac{\frac{x-1}{x}}{\frac{x+1}{x}} = \frac{x-1}{x+1} = -\frac{1-x}{1+x} = -f(x)$
Therefore: $f(x) + f\left(\frac{1}{x}\right) = f(x) + (-f(x)) = 0$
(ii) $f(-x) = \frac{1-(-x)}{1+(-x)} = \frac{1+x}{1-x}$
$f(x) \times f(-x) = \frac{1-x}{1+x} \times \frac{1+x}{1-x} = 1$

Q.3 (A) [6 marks]

Attempt any two

Q3(A).1 [3 marks]

Prove that: $\frac{\sin(180° - x) + \textbf{cosec}(180° - x) + \tan(180° + x)}{\cos(90° + x) + \sec(90° + x) + \cot(90° + x)} = -3$

Solution:
Using trigonometric identities:
- $\sin(180° - x) = \sin x$
- $\text{cosec}(180° - x) = \text{cosec } x$
- $\tan(180° + x) = \tan x$
- $\cos(90° + x) = -\sin x$
- $\sec(90° + x) = -\text{cosec } x$
- $\cot(90° + x) = -\tan x$

Numerator $= \sin x + \text{cosec } x + \tan x$
Denominator $= -\sin x - \text{cosec } x - \tan x = -(\sin x + \text{cosec } x + \tan x)$
Therefore: $\frac{\sin x + \text{cosec } x + \tan x}{-(\sin x + \text{cosec } x + \tan x)} = -1 \neq -3$
Note: There appears to be an error in the problem statement or expected answer.

Q3(A).2 [3 marks]

Prove that: $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) = 45°$

Solution:
Using $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$:
$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\right)$
$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) = 45°$

Q3(A).3 [3 marks]

Find out equation of the line whose X-intercept is 3 and Y-intercept is 2.

Solution:
Using intercept form: $\frac{x}{a} + \frac{y}{b} = 1$
Where $a = 3$ (x-intercept) and $b = 2$ (y-intercept)
$\frac{x}{3} + \frac{y}{2} = 1$
Multiplying by 6: $2x + 3y = 6$

Q.3 (B) [8 marks]

Attempt any two

Q3(B).1 [4 marks]

Prove that: $\tan(70°) = \frac{\cos(25°)+\sin(25°)}{\cos(25°)-\sin(25°)}$

Solution:
RHS $= \frac{\cos(25°)+\sin(25°)}{\cos(25°)-\sin(25°)}$
Dividing numerator and denominator by $\cos(25°)$:
$= \frac{1+\tan(25°)}{1-\tan(25°)}$
Using $\tan(45° + \theta) = \frac{1+\tan\theta}{1-\tan\theta}$:
$= \tan(45° + 25°) = \tan(70°) =$ LHS

Q3(B).2 [4 marks]

Prove that: $\frac{\sin\theta+\sin 2\theta+\sin 3\theta}{\cos\theta+\cos 2\theta+\cos 3\theta} = \tan 2\theta$

Solution:
Using sum-to-product formulas:
Numerator: $\sin\theta + \sin 3\theta + \sin 2\theta = 2\sin 2\theta\cos\theta + \sin 2\theta = \sin 2\theta(2\cos\theta + 1)$
Denominator: $\cos\theta + \cos 3\theta + \cos 2\theta = 2\cos 2\theta\cos\theta + \cos 2\theta = \cos 2\theta(2\cos\theta + 1)$
Therefore: $\frac{\sin 2\theta(2\cos\theta+1)}{\cos 2\theta(2\cos\theta+1)} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$

Q3(B).3 [4 marks]

If $\vec{a} = (1, 2, 3)$**,** $\vec{b} = (4, 0, 0)$ **and** $\vec{c} = (2, 0, 1)$ **find** $2\vec{a} + 3\vec{b} - 5\vec{c}$

Solution:
$2\vec{a} = 2(1, 2, 3) = (2, 4, 6)$
$3\vec{b} = 3(4, 0, 0) = (12, 0, 0)$
$5\vec{c} = 5(2, 0, 1) = (10, 0, 5)$
$2\vec{a} + 3\vec{b} - 5\vec{c} = (2, 4, 6) + (12, 0, 0) - (10, 0, 5)$
$= (2 + 12 - 10, 4 + 0 - 0, 6 + 0 - 5)$
$= (4, 4, 1)$

Q.4 (A) [6 marks]

Attempt any two

Q4(A).1 [3 marks]

If the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ **and** $\vec{b} = 2\hat{i} + m\hat{j} - 4\hat{k}$ **are perpendicular, find m.**

Solution:
For perpendicular vectors: $\vec{a} \cdot \vec{b} = 0$
$\vec{a} \cdot \vec{b} = (1)(2) + (-2)(m) + (3)(-4) = 2 - 2m - 12 = -10 - 2m$
Setting equal to zero: $-10 - 2m = 0$
$2m = -10$
$m = -5$

Q4(A).2 [3 marks]

Find the direction cosines and direction angles of the vector $\vec{a} = 5\hat{i} - 12\hat{k}$

Solution:
$\vec{a} = 5\hat{i} + 0\hat{j} - 12\hat{k}$
Magnitude: $|\vec{a}| = \sqrt{5^2 + 0^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$
Direction cosines:
- $l = \frac{5}{13}$
- $m = \frac{0}{13} = 0$
- $n = \frac{-12}{13}$

Direction angles:
- $\alpha = \cos^{-1}\left(\frac{5}{13}\right)$
- $\beta = \cos^{-1}(0) = 90°$
- $\gamma = \cos^{-1}\left(\frac{-12}{13}\right)$

Q4(A).3 [3 marks]

Find out equation of the circle having center at $(2, -3)$ and radius 3.

Solution:
Standard form: $(x - h)^2 + (y - k)^2 = r^2$
Where $(h, k) = (2, -3)$ and $r = 3$
$(x - 2)^2 + (y + 3)^2 = 9$
Expanding: $x^2 - 4x + 4 + y^2 + 6y + 9 = 9$
$x^2 + y^2 - 4x + 6y + 4 = 0$

Q.4 (B) [8 marks]

Attempt any two

Q4(B).1 [4 marks]

Show that the angle between vectors $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + 3\hat{k}$ is $\sin^{-1}\sqrt{\frac{46}{55}}$

Solution:
$\vec{a} \cdot \vec{b} = (1)(1) + (2)(1) + (0)(3) = 1 + 2 = 3$
$|\vec{a}| = \sqrt{1^2 + 2^2} = \sqrt{5}$
$|\vec{b}| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}$
$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{3}{\sqrt{5}\sqrt{11}} = \frac{3}{\sqrt{55}}$
$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{9}{55} = \frac{46}{55}$
Therefore: $\theta = \sin^{-1}\sqrt{\frac{46}{55}}$

Q4(B).2 [4 marks]

Under effect of the forces $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$ a particle moves from the point $(1,2,-3)$ to the point $(5,3,7)$. Find out work done.

Solution:
Net force: $\vec{F}=(2\hat{i}+\hat{j}+\hat{k})+(\hat{i}+3\hat{j}-\hat{k})=3\hat{i}+4\hat{j}$
Displacement: $\vec{s}=(5,3,7)-(1,2,-3)=(4,1,10)$
Work done: $W=\vec{F}\cdot\vec{s}=(3)(4)+(4)(1)+(0)(10)=12+4=16$ units

Q4(B).3 [4 marks]

Evaluate: $\lim_{x\to 0}\frac{2^x-5^x}{x}$

Solution:
Using L'Hôpital's rule or the derivative definition:
$\lim_{x\to 0}\frac{2^x-5^x}{x}=\lim_{x\to 0}\frac{2^x\ln 2-5^x\ln 5}{1}$
$=2^0\ln 2-5^0\ln 5=\ln 2-\ln 5=\ln\left(\frac{2}{5}\right)$

Q.5 (A) [6 marks]

Attempt any two

Q5(A).1 [3 marks]

Evaluate: $\lim_{x\to 0}\left(1+\frac{3x}{7}\right)^{\frac{1}{x}}$

Solution:
Let $y=\left(1+\frac{3x}{7}\right)^{\frac{1}{x}}$
Taking natural log: $\ln y=\frac{1}{x}\ln\left(1+\frac{3x}{7}\right)$
$\lim_{x\to 0}\ln y=\lim_{x\to 0}\frac{\ln\left(1+\frac{3x}{7}\right)}{x}$
Using L'Hôpital's rule: $=\lim_{x\to 0}\frac{\frac{3/7}{1+\frac{3x}{7}}}{1}=\frac{3}{7}$
Therefore: $\lim_{x\to 0}y=e^{3/7}$

Q5(A).2 [3 marks]

Evaluate: $\lim_{x\to 3}\frac{x^2-5x+6}{x^2-9}$

Solution:
Factoring numerator: $x^2-5x+6=(x-2)(x-3)$
Factoring denominator: $x^2-9=(x-3)(x+3)$
$\lim_{x\to 3}\frac{x^2-5x+6}{x^2-9}=\lim_{x\to 3}\frac{(x-2)(x-3)}{(x-3)(x+3)}=\lim_{x\to 3}\frac{x-2}{x+3}=\frac{3-2}{3+3}=\frac{1}{6}$

Q5(A).3 [3 marks]

Evaluate: $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

Solution:
Rationalizing the numerator:
$\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} \times \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$
$= \lim_{x\to 0} \frac{(4+x)-4}{x(\sqrt{4+x}+2)} = \lim_{x\to 0} \frac{x}{x(\sqrt{4+x}+2)} = \lim_{x\to 0} \frac{1}{\sqrt{4+x}+2} = \frac{1}{2+2} = \frac{1}{4}$

Q.5 (B) [8 marks]

Attempt any two

Q5(B).1 [4 marks]

Find out equation of the line passing through points $(1, 2)$ and $(2, 1)$.

Solution:
Using two-point form: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
$\frac{y-2}{1-2} = \frac{x-1}{2-1}$
$\frac{y-2}{-1} = \frac{x-1}{1}$
$y - 2 = -(x - 1) = -x + 1$
$x + y = 3$

Q5(B).2 [4 marks]

Find equation of the line that passes through $(-3, 2)$ and parallel to the line $x - 2y + 1 = 0$

Solution:
The given line $x - 2y + 1 = 0$ has slope $m = \frac{1}{2}$
Since parallel lines have the same slope, required line has slope $m = \frac{1}{2}$
Using point-slope form: $y - y_1 = m(x - x_1)$
$y - 2 = \frac{1}{2}(x - (-3))$
$y - 2 = \frac{1}{2}(x + 3)$
$2y - 4 = x + 3$
$x - 2y + 7 = 0$

Q5(B).3 [4 marks]

Find out center and radius of the circle: $x^2 + y^2 + 6x - 4y - 3 = 0$

Solution:
Completing the square:
$x^2 + 6x + y^2 - 4y = 3$
$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 3 + 9 + 4$
$(x + 3)^2 + (y - 2)^2 = 16$
Center: $(-3, 2)$
Radius: $r = \sqrt{16} = 4$

Formula Cheat Sheet

Logarithms

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

Trigonometry

- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin(180^\circ - x) = \sin x$, $\cos(90^\circ + x) = -\sin x$

Vectors

- $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
- $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$
- For perpendicular vectors: $\vec{a} \cdot \vec{b} = 0$

Coordinate Geometry

- Two-point form: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
- Circle: $(x-h)^2 + (y-k)^2 = r^2$
- Parallel lines have equal slopes

Limits

- $\lim_{x\to 0} \frac{\sin x}{x} = 1$
- $\lim_{x\to 0} \frac{e^x-1}{x} = 1$
- $\lim_{x\to\infty} \frac{ax+b}{cx+d} = \frac{a}{c}$