Subject Name Solutions

4320002 – Winter 2022

Semester 1 Study Material

Detailed Solutions and Explanations

Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options.

0.0.1 Q1.1 [1 mark]

If A = $\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ **then adj.A = __________.**

Solution

(d) $\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

Solution: For a $2\times 2 matrix A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adj.A = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$adj.A = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$

0.0.2 Q1.2 [1 mark]

If A is 2$\times 3 and B is 3 \times 4 matrices then AB is$__________$matrix$

Solution

(b) 2$\times$4

Solution: Matrix multiplication rule: $(m \times n) \times (n \times p) = (m \times p)$ $(2 \times 3) \times (3 \times 4) = (2 \times 4)$

0.0.3 Q1.3 [1 mark]

If $\begin{bmatrix} 0 & x \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & 4 \end{bmatrix}$ **then x = ___________**

Solution

(b) 4

Solution: Comparing corresponding elements: $x = 4$

0.0.4 Q1.4 [1 mark]

If A is non singular matrix then __________

Solution

(d) $|A| \neq 0$

Solution: A matrix is non-singular if its determinant is non-zero.

0.0.5 Q1.5 [1 mark]

**$d**$\frac{}{dx(e\hat{}\{-\log x\})=\$____________________}$

Solution

(d) x
Solution: $e^{-\log x} = e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$ $\frac{d}{dx}(\frac{1}{x}) = -\frac{1}{x^2}$

0.0.6 Q1.6 [1 mark]

If $f(x) = \log\sqrt{x^2+1}$**, then $f'(0) = $** ____________________

Solution

(a) 0
Solution: $f(x) = \frac{1}{2}\log(x^2+1)$ $f'(x) = \frac{1}{2}\cdot\frac{2x}{x^2+1} = \frac{x}{x^2+1}$ $f'(0) = \frac{0}{0+1} = 0$

0.0.7 Q1.7 [1 mark]

If $x = \sec\theta + \tan\theta$ **and** $y = \sec\theta - \tan\theta$ **then $dy** $_{dx=\$}$ ____________________

Solution

(d) 1
Solution: $xy = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \sec^2\theta - \tan^2\theta = 1$ **Differentiating:** $x\frac{dy}{dx} + y = 0$ $\frac{dy}{dx} = -\frac{y}{x}$

0.0.8 Q1.8 [1 mark]

**$**$\int e\hat{}x(\sin x + \cos x)dx = \$$_______________

Solution

(b) $e^x \sin x + c$
Solution: Using integration by parts or standard result: $\int e^x(\sin x + \cos x)dx = e^x \sin x + c$

0.0.9 Q1.9 [1 mark]

$**$\int$ *{-1}^{1} x^2 + 1 dx = $*** ________

Solution

(d) $\frac{8}{3}$
Solution: $\int_{-1}^{1}(x^2+1)dx = [\frac{x^3}{3}+x]_{-1}^{1} = (\frac{1}{3}+1) - (\frac{-1}{3}-1) = \frac{8}{3}$

0.0.10 Q1.10 [1 mark]

**$**$\int \cot x dx = \$$____________________ $+c$

Solution

(a) $\log|\sin x|$
Solution: $\int \cot x dx = \int \frac{\cos x}{\sin x}dx = \log|\sin x| + c$

0.0.11 Q1.11 [1 mark]

The order & degree of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 3y = 0$ **are respectively**____________ **and** ____________

Solution

(a) 2, 1
Solution: Order = highest order derivative = 2 Degree = power of highest order derivative = 1

0.0.12 Q1.12 [1 mark]

The integrating factor for the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$ is _______

Solution

(b) x
Solution: For $\frac{dy}{dx} + P(x)y = Q(x)$**, where** $P(x) = \frac{1}{x}$ **I.F.** $= e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

0.0.13 Q1.13 [1 mark]

$i + i^2 + i^3 + i^4 = $ __________

Solution

(d) 0
Solution: $i + i^2 + i^3 + i^4 = i + (-1) + (-i) + 1 = 0$

0.0.14 Q1.14 [1 mark]

arg(-1) = __________________

Solution

(a)
Solution: $-1 = \cos\pi + i\sin\pi$**, so** $\arg(-1) = \pi$

Q.2(a) [6 marks]

Attempt any two.

0.0.15 Q2(a).1 [3 marks]

If $A = \begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 \\ -2 & 3 \end{bmatrix}$ **then find matrix X from equation 3(X+B) + 5A = 0**

Solution: $3(X+B) + 5A = 0$ $3X + 3B + 5A = 0$ $3X = -3B - 5A$ $X = -B - \frac{5A}{3}$

$$5A = 5\begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ -15 & 10 \end{bmatrix}$$

$$X = -\begin{bmatrix} 5 & 6 \\ -2 & 3 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 5 & 10 \\ -15 & 10 \end{bmatrix}$$

$$X = \begin{bmatrix} -5 & -6 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} \frac{5}{3} & \frac{10}{3} \\ -5 & \frac{10}{3} \end{bmatrix}$$

$$X = \begin{bmatrix} -\frac{20}{3} & -\frac{28}{3} \\ 7 & -\frac{19}{3} \end{bmatrix}$$

0.0.16 Q2(a).2 [3 marks]

If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ **then Prove that** $A^2 - 4A - 5I = 0$

Solution: $A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

$$4A = 4\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$$

$$5I = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$= \begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence proved.

0.0.17 Q2(a).3 [3 marks]

Solve differential equation $\frac{dy}{dx} = (x+y)^2$

Solution: Let $v = x + y$**, then** $\frac{dv}{dx} = 1 + \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{dv}{dx} - 1$

Substituting: $\frac{dv}{dx} - 1 = v^2$ $\frac{dv}{dx} = v^2 + 1$ $\frac{dv}{v^2+1} = dx$

Integrating: $\int \frac{dv}{v^2+1} = \int dx$ $\tan^{-1} v = x + c$ $\tan^{-1}(x+y) = x + c$ $x + y = \tan(x+c)$ $y = \tan(x+c) - x$

Q.2(b) [8 marks]

Attempt any two.

0.0.18 Q2(b).1 [4 marks]

If $A = \begin{bmatrix} 3 & -1 \\ 4 & 1 \\ 5 & 0 \end{bmatrix}$ **then find** A^{-1}

Solution: This is a 3$\times 2 matrix, which is non-square. Inverse doesn't exist for non-square matrices.$

Alternative interpretation - if it's $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$**:**

Using adjoint method: $|A| = 3(1-0) + 1(4+5) + 2(0-5) = 3 + 9 - 10 = 2$

Calculate cofactors and adjoint, then $A^{-1} = \frac{1}{|A|} \times adj(A)$

0.0.19 Q2(b).2 [4 marks]

Solve Equation 3X-2Y=8 and 5X+4Y=6 using matrices method.

Solution: $\begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$

$|A| = 3(4) - (-2)(5) = 12 + 10 = 22$

$A^{-1} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 32+12 \\ -40+18 \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 44 \\ -22 \end{bmatrix}$

$X = 2, Y = -1$

0.0.20 Q2(b).3 [4 marks]

If $M = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$**,** $N = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$ **then Prove that** $(MN)^T = N^T M^T$

Solution: $MN = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 11 \\ 2 & 1 \end{bmatrix}$

$(MN)^T = \begin{bmatrix} 12 & 2 \\ 11 & 1 \end{bmatrix}$

$M^T = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$, $N^T = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

$N^T M^T = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ 11 & 1 \end{bmatrix}$

Hence $(MN)^T = N^T M^T$ **is proved.**

Q.3(a) [6 marks]

Attempt any two.

0.0.21 Q3(a).1 [3 marks]

Differentiate $\sqrt{x}$ using the definition.

Solution: $f(x) = \sqrt{x} = x^{1/2}$

Using definition: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

Rationalizing: $f'(x) = \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

0.0.22 Q3(a).2 [3 marks]

If $y = \log(x+\sqrt{1+x^2})$ **then Find** $\frac{dy}{dx}$

Solution: $y = \log(x+\sqrt{1+x^2})$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}} \cdot \frac{d}{dx}(x+\sqrt{1+x^2})$

$\frac{d}{dx}(x+\sqrt{1+x^2}) = 1 + \frac{1}{2\sqrt{1+x^2}} \cdot 2x = 1 + \frac{x}{\sqrt{1+x^2}}$

$= \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$

$= \frac{1}{\sqrt{1+x^2}}$

0.0.23 Q3(a).3 [3 marks]

$\int \frac{4+3\cos x}{\sin^2 x} dx$

Solution: $\int \frac{4+3\cos x}{\sin^2 x} dx = \int \frac{4}{\sin^2 x} dx + \int \frac{3\cos x}{\sin^2 x} dx$

$= 4\int \csc^2 x dx + 3\int \frac{\cos x}{\sin^2 x} dx$

$= -4\cot x + 3\int \sin^{-2} x \cos x dx$

For the second integral, let $u = \sin x$, $du = \cos x dx$ $3\int u^{-2} du = 3(-u^{-1}) = -\frac{3}{\sin x}$

$\int \frac{4+3\cos x}{\sin^2 x} dx = -4\cot x - 3\csc x + c$

Q.3(b) [8 marks]

Attempt any two.

0.0.24 Q3(b).1 [4 marks]

If $y = \log(\sin x)$ **then prove that** $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 + 1 = 0$

Solution: $y = \log(\sin x)$

$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$

$\frac{d^2y}{dx^2} = \frac{d}{dx}(\cot x) = -\csc^2 x$

Now, $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 + 1 = -\csc^2 x + \cot^2 x + 1$

Using identity: $\csc^2 x - \cot^2 x = 1$ $-\csc^2 x + \cot^2 x + 1 = -(\csc^2 x - \cot^2 x) = -1 + 1 = 0$

Hence proved.

0.0.25 Q3(b).2 [4 marks]

If $x + y = \sin(xy)$ **then Find** $\frac{dy}{dx}$

Solution: $x + y = \sin(xy)$

Differentiating both sides with respect to x: $1+\frac{dy}{dx}=\cos(xy)\cdot\frac{d}{dx}(xy)$

$1+\frac{dy}{dx}=\cos(xy)\cdot(y+x\frac{dy}{dx})$

$1+\frac{dy}{dx}=y\cos(xy)+x\cos(xy)\frac{dy}{dx}$

$1+\frac{dy}{dx}-x\cos(xy)\frac{dy}{dx}=y\cos(xy)$

$\frac{dy}{dx}(1-x\cos(xy))=y\cos(xy)-1$

$\frac{dy}{dx}=\frac{y\cos(xy)-1}{1-x\cos(xy)}$

0.0.26 Q3(b).3 [4 marks]

A particle has motion of $s=t^3-5t^2+3t$ **Find the acceleration when particle comes to rest?**

Solution: Given: $s=t^3-5t^2+3t$

Velocity: $v=\frac{ds}{dt}=3t^2-10t+3$

Acceleration: $a=\frac{dv}{dt}=6t-10$

At rest, $v=0$**:** $3t^2-10t+3=0$

Using quadratic formula: $t=\frac{10\pm\sqrt{100-36}}{6}=\frac{10\pm 8}{6}$

$t=3$ **or** $t=\frac{1}{3}$

At $t=3$**:** $a=6(3)-10=8$ **At** $t=\frac{1}{3}$**:** $a=6(\frac{1}{3})-10=-8$

The accelerations are 8 **and** -8 **respectively.**

Q.4(a) [6 marks]

Attempt any two.

0.0.27 Q4(a).1 [3 marks]

$\int x\sin x dx$

Solution: Using integration by parts: $\int udv=uv-\int vdu$

Let $u=x$**,** $dv=\sin x dx$ $du=dx$**,** $v=-\cos x$

$\int x\sin x dx=x(-\cos x)-\int(-\cos x)dx=-x\cos x+\int\cos x dx=-x\cos x+\sin x+c$

0.0.28 Q4(a).2 [3 marks]

$\int\frac{2x+1}{(x+1)(x-3)}dx$

Solution: Using partial fractions: $\frac{2x+1}{(x+1)(x-3)}=\frac{A}{x+1}+\frac{B}{x-3}$

$2x+1=A(x-3)+B(x+1)$

At $x=-1$**:** $-2+1=A(-4)\Rightarrow$

A = 1$_{\overline{4}}$ **At**

$x=3$**:** $6+1=B(4)\Rightarrow B=\frac{7}{4}$

$\int\frac{2x+1}{(x+1)(x-3)}dx=\frac{1}{4}\int\frac{1}{x+1}dx+\frac{7}{4}\int\frac{1}{x-3}dx$

$=\frac{1}{4}\log|x+1|+\frac{7}{4}\log|x-3|+c$

0.0.29 Q4(a).3 [3 marks]

Find square root of complex number $z=7+24i$

Solution: Let $\sqrt{7+24i}=a+bi$

$(a+bi)^2=7+24i$ $a^2-b^2+2abi=7+24i$

Comparing: $a^2-b^2=7$ **and** $2ab=24$ **From second equation:** $b=\frac{12}{a}$

Substituting: $a^2-\frac{144}{a^2}=7$ $a^4-7a^2-144=0$

Let $u=a^2$**:** $u^2-7u-144=0$ $(u-16)(u+9)=0$ $u=16$ **(taking positive value)** $a^2=16\Rightarrow a=4$ $b=\frac{12}{4}=3$

Therefore: $\sqrt{7+24i}=4+3i$ **or** $-(4+3i)$

Q.4(b) [8 marks]

Attempt any two.

0.0.30 Q4(b).1 [4 marks]

$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Solution: Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Using property: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}} dx$

$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

Adding both expressions: $2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \int_0^{\pi/2} 1dx = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

0.0.31 Q4(b).2 [4 marks]

Find the area of the region bounded by the curve $y = 3x^2$, x axis and the line $x = 2$ and $x = 3$

Solution: Area = $\int_2^3 ydx = \int_2^3 3x^2 dx$

$= 3\int_2^3 x^2 dx = 3[\frac{x^3}{3}]_2^3$

$= [x^3]_2^3 = 3^3 - 2^3 = 27 - 8 = 19$

Area = 19 square units

0.0.32 Q4(b).3 [4 marks]

Simplify $\frac{(\cos 2\theta+i\sin 2\theta)^{-3}\cdot(\cos 3\theta-i\sin 3\theta)^2}{(\cos 2\theta-i\sin 2\theta)^{-7}\cdot(\cos 5\theta-i\sin 5\theta)^3}$

Solution: Using Euler's formula: $\cos\theta + i\sin\theta = e^{i\theta}$

$(\cos 2\theta + i\sin 2\theta)^{-3} = e^{-6i\theta}$ $(\cos 3\theta - i\sin 3\theta)^2 = e^{-6i\theta}$ $(\cos 2\theta - i\sin 2\theta)^{-7} = e^{14i\theta}$ $(\cos 5\theta - i\sin 5\theta)^3 = e^{-15i\theta}$

Expression = $\frac{e^{-6i\theta}\cdot e^{-6i\theta}}{e^{14i\theta}\cdot e^{-15i\theta}} = \frac{e^{-12i\theta}}{e^{-i\theta}} = e^{-11i\theta}$

$= \cos(-11\theta) + i\sin(-11\theta) = \cos(11\theta) - i\sin(11\theta)$

Q.5(a) [6 marks]

Attempt any two.

0.0.33 Q5(a).1 [3 marks]

Convert $\frac{4+2i}{(3+2i)(5-3i)}$ **in a+ib form.**

Solution: First, simplify the denominator: $(3+2i)(5-3i) = 15 - 9i + 10i - 6i^2 = 15 + i + 6 = 21 + i$

Now: $\frac{4+2i}{21+i}$

Multiply by conjugate: $\frac{4+2i}{21+i} \cdot \frac{21-i}{21-i}$

$= \frac{(4+2i)(21-i)}{(21+i)(21-i)} = \frac{84-4i+42i-2i^2}{441-i^2}$

$= \frac{84+38i+2}{441+1} = \frac{86+38i}{442} = \frac{43+19i}{221}$

0.0.34 Q5(a).2 [3 marks]

Convert $z = 1 - \sqrt{3}i$ in polar form.

Solution: $z = 1 - \sqrt{3}i$

$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$

$\arg(z) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$ **(since z is in 4th quadrant)**

Therefore: $z = 2(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})) = 2e^{-i\pi/3}$

0.0.35 Q5(a).3 [3 marks]

Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n(\frac{\theta}{2})\cos(\frac{n\theta}{2})$

Solution: $1+\cos\theta+i\sin\theta=1+e^{i\theta}=1+\cos\theta+i\sin\theta$

Using identity: $1+\cos\theta=2\cos^2(\frac{\theta}{2})$

$1+\cos\theta+i\sin\theta=2\cos^2(\frac{\theta}{2})+2i\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})$

$=2\cos(\frac{\theta}{2})[\cos(\frac{\theta}{2})+i\sin(\frac{\theta}{2})]=2\cos(\frac{\theta}{2})e^{i\theta/2}$

Similarly: $1+\cos\theta-i\sin\theta=2\cos(\frac{\theta}{2})e^{-i\theta/2}$

$(1+\cos\theta+i\sin\theta)^n=2^n\cos^n(\frac{\theta}{2})e^{in\theta/2}$

$(1+\cos\theta-i\sin\theta)^n=2^n\cos^n(\frac{\theta}{2})e^{-in\theta/2}$

Sum $=2^n\cos^n(\frac{\theta}{2})[e^{in\theta/2}+e^{-in\theta/2}]=2^n\cos^n(\frac{\theta}{2})\cdot 2\cos(\frac{n\theta}{2})$

$=2^{n+1}\cos^n(\frac{\theta}{2})\cos(\frac{n\theta}{2})$

Hence proved.

Q.5(b) [8 marks]

Attempt any two.

0.0.36 Q5(b).1 [4 marks]

Solve differential equation $x\log x\frac{dy}{dx}+y=\log x^2$

Solution: $x\log x\frac{dy}{dx}+y=2\log x$

Dividing by $x\log x$**:** $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$

This is a linear differential equation: $\frac{dy}{dx}+P(x)y=Q(x)$

Where $P(x)=\frac{1}{x\log x}$ **and** $Q(x)=\frac{2}{x}$

Integrating Factor: $e^{\int P(x)dx}=e^{\int\frac{1}{x\log x}dx}$

Let $u=\log x$**, then** $du=\frac{1}{x}dx$ $\int\frac{1}{x\log x}dx=\int\frac{1}{u}du=\log u=\log(\log x)$

I.F. $=e^{\log(\log x)}=\log x$

Solution: $y\cdot\log x=\int\frac{2}{x}\cdot\log x dx$

$=2\int\frac{\log x}{x}dx=2\cdot\frac{(\log x)^2}{2}=(\log x)^2$

Therefore: $y=\frac{(\log x)^2}{\log x}=\log x$

0.0.37 Q5(b).2 [4 marks]

Solve differential equation $\frac{dy}{dx}-\frac{y}{x}=e^x$

Solution: This is a linear differential equation: $\frac{dy}{dx}+P(x)y=Q(x)$

Where $P(x)=-\frac{1}{x}$ **and** $Q(x)=e^x$

Integrating Factor: $e^{\int P(x)dx}=e^{\int-\frac{1}{x}dx}=e^{-\log x}=\frac{1}{x}$

Solution: $y\cdot\frac{1}{x}=\int e^x\cdot\frac{1}{x}dx$

The integral $\int\frac{e^x}{x}dx$ **cannot be expressed in elementary functions.**

Alternative approach - assuming it's $\frac{dy}{dx}+\frac{y}{x}=e^x$**:**

I.F. $=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

$y\cdot x=\int e^x\cdot xdx$

Using integration by parts: $\int xe^xdx=xe^x-\int e^xdx=xe^x-e^x=e^x(x-1)$

Therefore: $xy=e^x(x-1)+c$ $y=\frac{e^x(x-1)+c}{x}$

0.0.38 Q5(b).3 [4 marks]

Solve differential equation $\sec^2x\tan ydx+\sec^2y\tan xdy=0$**,** $y(\frac{\pi}{4})=\frac{\pi}{4}$

Solution: $\sec^2x\tan ydx+\sec^2y\tan xdy=0$

Rearranging: $\frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$
$\frac{\cos x}{\sin x \cos^2 x}dx + \frac{\cos y}{\sin y \cos^2 y}dy = 0$
$\frac{1}{\sin x \cos x}dx + \frac{1}{\sin y \cos y}dy = 0$
$\frac{2}{\sin 2x}dx + \frac{2}{\sin 2y}dy = 0$
$\csc(2x)dx + \csc(2y)dy = 0$
Integrating: $\int \csc(2x)dx + \int \csc(2y)dy = c$
$-\frac{1}{2}\log|\csc(2x) + \cot(2x)| - \frac{1}{2}\log|\csc(2y) + \cot(2y)| = c$
$\log|\csc(2x) + \cot(2x)| + \log|\csc(2y) + \cot(2y)| = -2c = k$
$|\csc(2x) + \cot(2x)| \cdot |\csc(2y) + \cot(2y)| = e^k$
Using initial condition $y(\frac{\pi}{4}) = \frac{\pi}{4}$**: At** $x = \frac{\pi}{4}$**,** $y = \frac{\pi}{4}$
$|\csc(\frac{\pi}{2}) + \cot(\frac{\pi}{2})| \cdot |\csc(\frac{\pi}{2}) + \cot(\frac{\pi}{2})| = |1 + 0| \cdot |1 + 0| = 1$
Therefore: $(\csc(2x) + \cot(2x))(\csc(2y) + \cot(2y)) = 1$

Complete Formula Cheat Sheet

0.0.39 Matrix Operations

Operation	Formula
Adjoint (2$\times$2)	**If** $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$**, then** $adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
Inverse	$A^{-1} = \frac{1}{\lvert A\rvert} \times adj(A)$
Matrix Multiplication	$(AB)_{ij} = \sum_k A_{ik}B_{kj}$
Transpose Property	$(AB)^T = B^T A^T$

0.0.40 Differentiation

Function	Derivative
x^n	nx^{n-1}
$\log x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
Product Rule	$(uv)' = u'v + uv'$
Quotient Rule	$(\frac{u}{v})' = \frac{u'v - uv'}{v^2}$

0.0.41 Integration

Function	Integral
x^n	$\frac{x^{n+1}}{n+1} + c$
$\frac{1}{x}$	$\log\lvert x\rvert + c$
e^x	$e^x + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
$\sec^2 x$	$\tan x + c$
$\csc^2 x$	$-\cot x + c$
Integration by Parts	$\int udv = uv - \int vdu$

0.0.42 Differential Equations

Type	Method	Solution
Variable Separable	$\frac{dy}{dx} = f(x)g(y)$	$\int \frac{dy}{g(y)} = \int f(x)dx$
Linear DE	$\frac{dy}{dx} + Py = Q$	$y \cdot I.F. = \int Q \cdot I.F.dx$
Integrating Factor	**I.F.** $= e^{\int Pdx}$	-

0.0.43 Complex Numbers

Operation	Formula
Modulus	$\lvert a + bi \rvert = \sqrt{a^2 + b^2}$
Argument	$\arg(z) = \tan^{-1}(\frac{b}{a})$
Polar Form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$
Powers	$i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$
De Moivre's	$(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$

Problem-Solving Strategies

0.0.44 For Matrix Problems:

1. **Check dimensions first for multiplication**
2. **Use determinant to check if inverse exists**
3. **Apply properties like** $(AB)^T = B^T A^T$
4. **Substitute and verify your answers**

0.0.45 For Differentiation:

1. **Identify the type (composite, product, quotient)**
2. **Apply appropriate rule systematically**
3. **Simplify step by step**
4. **Check using basic derivatives**

0.0.46 For Integration:

1. **Look for standard forms first**
2. **Try substitution if composite function**
3. **Use integration by parts for products**
4. **Apply partial fractions for rational functions**

0.0.47 For Differential Equations:

1. **Identify the type (separable, linear, etc.)**
2. **Find integrating factor for linear DEs**
3. **Separate variables when possible**
4. **Apply initial conditions to find constants**

Common Mistakes to Avoid

0.0.48 Matrix Operations:

- **Wrong dimension calculation in multiplication**
- **Forgetting to transpose in** $(AB)^T = B^T A^T$
- **Not checking if matrix is invertible before finding inverse**

0.0.49 Differentiation:

- Missing chain rule in composite functions
- Sign errors in trigonometric derivatives
- Forgetting product rule in multiplied functions

0.0.50 Integration:

- Wrong limits in definite integrals
- Missing constant of integration
- Incorrect substitution bounds

0.0.51 Complex Numbers:

- Wrong quadrant in argument calculation
- Modulus calculation errors
- Forgetting to rationalize denominators

Exam Tips

0.0.52 Time Management:

- Attempt Q.1 first (14 marks, quick fill-ups)
- Choose easier sub-questions in each section
- Leave difficult calculations for the end

0.0.53 Answer Presentation:

- Show all steps clearly
- Box final answers
- Use proper mathematical notation
- Draw diagrams where helpful

0.0.54 Verification:

- Check dimensions in matrix problems
- Verify differentiation by differentiating your answer
- Substitute back in differential equations
- Check modulus and argument for complex numbers

0.0.55 Key Formulas to Remember:

- Matrix inverse formula
- Integration by parts
- Linear DE solution method
- Complex number polar form
- Standard derivatives and integrals

Remember: Practice is key to mastering these concepts. Work through similar problems and focus on understanding the underlying principles rather than just memorizing formulas.