

Mathematics-I (DI01000021) - Summer 2025 Solution

Milav Dabgar

May 30, 2025

Question Q.1 [14 marks]

Fill in the blanks/MCQs using appropriate choice from the given options

Question Q1.1 [1 marks]

$\log_3 1 = \underline{\hspace{2cm}}$

Solution

Answer: d. 0

Solution: For any base $a > 0, a \neq 1$: $\log_a 1 = 0$ Therefore: $\log_3 1 = 0$

Question Q1.2 [1 marks]

If $f(x) = e^{x-1}$ then $f(1) = \underline{\hspace{2cm}}$

Solution

Answer: c. 1

Solution: $f(x) = e^{x-1}$ $f(1) = e^{1-1} = e^0 = 1$

Question Q1.3 [1 marks]

$\log_5 125 = \underline{\hspace{2cm}}$

Solution

Answer: b. 3

Solution: $\log_5 125 = \log_5 5^3 = 3$ Since $5^3 = 125$

Question Q1.4 [1 marks]

If $f(x) = x^3 - 7$ then $f(-2) = \underline{\hspace{2cm}}$

Solution**Answer:** c. -15**Solution:** $f(x) = x^3 - 7$ $f(-2) = (-2)^3 - 7 = -8 - 7 = -15$ **Question Q1.5 [1 marks]**Principal period of $\cos x$ is _____**Solution****Answer:** c. 2π **Solution:** The cosine function repeats every 2π radians, so its principal period is 2π .**Question Q1.6 [1 marks]** $150 =$ _____**Solution****Answer:** a. $\frac{5\pi}{6}$ **Solution:** Converting degrees to radians: $150 = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$ **Question Q1.7 [1 marks]** $\sin^{-1} x + \cos^{-1} x =$ _____**Solution****Answer:** a. $\frac{\pi}{2}$ **Solution:** This is a standard identity: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for $x \in [-1, 1]$ **Question Q1.8 [1 marks]** $(1, 0, 0) \times (1, 0, 0) =$ _____**Solution****Answer:** d. (0,0,0)**Solution:** Cross product of any vector with itself is zero vector: $(1, 0, 0) \times (1, 0, 0) = (0, 0, 0)$ **Question Q1.9 [1 marks]**If $\vec{a} = 4\hat{i} - 3\hat{j}$ then $|\vec{a}| =$ _____**Solution****Answer:** b. 5**Solution:** $|\vec{a}| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

Question Q1.10 [1 marks]

If a line makes an angle 45 with positive x-axis then slope of the line is _____

Solution

Answer: c. 1

Solution: Slope $m = \tan(45) = 1$

Question Q1.11 [1 marks]

Radius of the circle $x^2 + y^2 = 4$ is _____

Solution

Answer: d. 2

Solution: Standard form: $x^2 + y^2 = r^2$ Comparing: $r^2 = 4$, so $r = 2$

Question Q1.12 [1 marks]

$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} =$ _____

Solution

Answer: a. 1

Solution: This is a standard limit: $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$

Question Q1.13 [1 marks]

$\lim_{x \rightarrow 0} \frac{\sin 3x}{x} =$ _____

Solution

Answer: d. 3

Solution: $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \times 3 = 1 \times 3 = 3$

Question Q1.14 [1 marks]

$\lim_{n \rightarrow \infty} \frac{5n+4}{4n+5} =$ _____

Solution

Answer: c. 5/4

Solution: $\lim_{n \rightarrow \infty} \frac{5n+4}{4n+5} = \lim_{n \rightarrow \infty} \frac{5+\frac{4}{n}}{4+\frac{5}{n}} = \frac{5}{4}$

Question Q.2 (A) [6 marks]

Attempt any two

Question Q2(A).1 [3 marks]

Find value:
$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Solution

Answer: 0

Solution:
$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(5 \times 9 - 6 \times 8) - 2(4 \times 9 - 6 \times 7) + 3(4 \times 8 - 5 \times 7) = 1(45 - 48) - 2(36 - 42) + 3(32 - 35) \\ = 1(-3) - 2(-6) + 3(-3) = -3 + 12 - 9 = 0$$

Question Q2(A).2 [3 marks]

Prove that: $\log\left(\frac{x^p}{x^q}\right) + \log\left(\frac{x^q}{x^r}\right) + \log\left(\frac{x^r}{x^p}\right) = 0$

Solution

Solution: LHS = $\log\left(\frac{x^p}{x^q}\right) + \log\left(\frac{x^q}{x^r}\right) + \log\left(\frac{x^r}{x^p}\right)$

Using logarithm properties: $= \log(x^p) - \log(x^q) + \log(x^q) - \log(x^r) + \log(x^r) - \log(x^p) = p \log x - q \log x + q \log x - r \log x + r \log x - p \log x = 0 = \text{RHS}$

Question Q2(A).3 [3 marks]

Find value: $\tan(75)$

Solution

Answer: $2 + \sqrt{3}$

Solution: $\tan(75) = \tan(45 + 30)$

Using $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$:

$$\begin{aligned} \tan(75) &= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \\ &= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2 + \sqrt{3} \end{aligned}$$

Question Q.2 (B) [8 marks]

Attempt any two

Question Q2(B).1 [4 marks]

Prove that: $\frac{1}{\log_{12} 120} + \frac{1}{\log_2 120} + \frac{1}{\log_5 120} = 1$

Solution

Solution: Using change of base formula: $\frac{1}{\log_a b} = \log_b a$

$$\text{LHS} = \log_{120} 12 + \log_{120} 2 + \log_{120} 5$$

$$\text{Using logarithm properties: } = \log_{120}(12 \times 2 \times 5) = \log_{120} 120 = 1 = \text{RHS}$$

Question Q2(B).2 [4 marks]

Solve: $\begin{vmatrix} x & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3$

Solution

Solution: Expanding along third row: $\begin{vmatrix} x & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3 \begin{vmatrix} x & 1 \\ 1 & 2 \end{vmatrix}$

$$= 3(2x - 1) = 6x - 3$$

$$\text{Given: } 6x - 3 = 3 \quad 6x = 6 \quad x = 1$$

Question Q2(B).3 [4 marks]

If $f(x) = \frac{1-x}{1+x}$ **prove that:** (i) $f(x) + f\left(\frac{1}{x}\right) = 0$ (ii) $f(x) \times f(-x) = 1$

Solution

Solution: Given: $f(x) = \frac{1-x}{1+x}$

$$\text{(i) } f\left(\frac{1}{x}\right) = \frac{1-\frac{1}{x}}{1+\frac{1}{x}} = \frac{\frac{x-1}{x}}{\frac{x+1}{x}} = \frac{x-1}{x+1} = -\frac{1-x}{1+x} = -f(x)$$

$$\text{Therefore: } f(x) + f\left(\frac{1}{x}\right) = f(x) + (-f(x)) = 0$$

$$\text{(ii) } f(-x) = \frac{1-(-x)}{1+(-x)} = \frac{1+x}{1-x}$$

$$f(x) \times f(-x) = \frac{1-x}{1+x} \times \frac{1+x}{1-x} = 1$$

Question Q.3 (A) [6 marks]

Attempt any two

Question Q3(A).1 [3 marks]

Prove that: $\frac{\sin(180-x) + (180-x) + \tan(180+x)}{\cos(90+x) + \sec(90+x) + \cot(90+x)} = -3$

Solution**Solution:** Using trigonometric identities:

- $\sin(180 - x) = \sin x$
- $(180 - x) = x$
- $\tan(180 + x) = \tan x$
- $\cos(90 + x) = -\sin x$
- $\sec(90 + x) = -x$
- $\cot(90 + x) = -\tan x$

Numerator = $\sin x + x + \tan x$ Denominator = $-\sin x - x - \tan x = -(\sin x + x + \tan x)$ Therefore: $\frac{\sin x + x + \tan x}{-(\sin x + x + \tan x)} = -1 \neq -3$ **Note:** There appears to be an error in the problem statement or expected answer.**Question Q3(A).2 [3 marks]****Prove that:** $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) = 45$ **Solution****Solution:** Using $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$:

$$\begin{aligned} \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) &= \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \times \frac{1}{2}}\right) \\ &= \tan^{-1}\left(\frac{\frac{5}{6}}{1 - \frac{1}{6}}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) = 45 \end{aligned}$$

Question Q3(A).3 [3 marks]**Find out equation of the line whose X-intercept is 3 and Y-intercept is 2.****Solution****Solution:** Using intercept form: $\frac{x}{a} + \frac{y}{b} = 1$ Where $a = 3$ (x-intercept) and $b = 2$ (y-intercept)

$$\frac{x}{3} + \frac{y}{2} = 1$$

Multiplying by 6: $2x + 3y = 6$ **Question Q.3 (B) [8 marks]****Attempt any two****Question Q3(B).1 [4 marks]****Prove that:** $\tan(70) = \frac{\cos(25) + \sin(25)}{\cos(25) - \sin(25)}$ **Solution****Solution:** RHS = $\frac{\cos(25) + \sin(25)}{\cos(25) - \sin(25)}$ Dividing numerator and denominator by $\cos(25)$:

$$= \frac{1 + \tan(25)}{1 - \tan(25)}$$

$$\text{Using } \tan(45 + \theta) = \frac{1 + \tan \theta}{1 - \tan \theta}:$$

$$= \tan(45 + 25) = \tan(70) = \text{LHS}$$

Question Q3(B).2 [4 marks]

Prove that: $\frac{\sin \theta + \sin 2\theta + \sin 3\theta}{\cos \theta + \cos 2\theta + \cos 3\theta} = \tan 2\theta$

Solution

Solution: Using sum-to-product formulas:

$$\text{Numerator: } \sin \theta + \sin 3\theta + \sin 2\theta = 2 \sin 2\theta \cos \theta + \sin 2\theta = \sin 2\theta (2 \cos \theta + 1)$$

$$\text{Denominator: } \cos \theta + \cos 3\theta + \cos 2\theta = 2 \cos 2\theta \cos \theta + \cos 2\theta = \cos 2\theta (2 \cos \theta + 1)$$

$$\text{Therefore: } \frac{\sin 2\theta (2 \cos \theta + 1)}{\cos 2\theta (2 \cos \theta + 1)} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

Question Q3(B).3 [4 marks]

If $\vec{a} = (1, 2, 3)$, $\vec{b} = (4, 0, 0)$ and $\vec{c} = (2, 0, 1)$ find $2\vec{a} + 3\vec{b} - 5\vec{c}$

Solution

$$\text{Solution: } 2\vec{a} = 2(1, 2, 3) = (2, 4, 6) \quad 3\vec{b} = 3(4, 0, 0) = (12, 0, 0) \quad 5\vec{c} = 5(2, 0, 1) = (10, 0, 5)$$

$$2\vec{a} + 3\vec{b} - 5\vec{c} = (2, 4, 6) + (12, 0, 0) - (10, 0, 5) = (2 + 12 - 10, 4 + 0 - 0, 6 + 0 - 5) = (4, 4, 1)$$

Question Q.4 (A) [6 marks]

Attempt any two

Question Q4(A).1 [3 marks]

If the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + m\hat{j} - 4\hat{k}$ are perpendicular, find m.

Solution

Solution: For perpendicular vectors: $\vec{a} \cdot \vec{b} = 0$

$$\vec{a} \cdot \vec{b} = (1)(2) + (-2)(m) + (3)(-4) = 2 - 2m - 12 = -10 - 2m$$

$$\text{Setting equal to zero: } -10 - 2m = 0 \quad 2m = -10 \quad m = -5$$

Question Q4(A).2 [3 marks]

Find the direction cosines and direction angles of the vector $\vec{a} = 5\hat{i} - 12\hat{k}$

Solution

$$\text{Solution: } \vec{a} = 5\hat{i} + 0\hat{j} - 12\hat{k}$$

$$\text{Magnitude: } |\vec{a}| = \sqrt{5^2 + 0^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

Direction cosines:

- $l = \frac{5}{13}$

- $m = \frac{0}{13} = 0$
- $n = \frac{-12}{13}$

Direction angles:

- $\alpha = \cos^{-1}\left(\frac{5}{13}\right)$
- $\beta = \cos^{-1}(0) = 90$
- $\gamma = \cos^{-1}\left(\frac{-12}{13}\right)$

Question Q4(A).3 [3 marks]

Find out equation of the circle having center at $(2, -3)$ and radius 3.

Solution

Solution: Standard form: $(x - h)^2 + (y - k)^2 = r^2$

Where $(h, k) = (2, -3)$ and $r = 3$

$$(x - 2)^2 + (y + 3)^2 = 9$$

$$\text{Expanding: } x^2 - 4x + 4 + y^2 + 6y + 9 = 9 \quad x^2 + y^2 - 4x + 6y + 4 = 0$$

Question Q.4 (B) [8 marks]

Attempt any two

Question Q4(B).1 [4 marks]

Show that the angle between vectors $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + 3\hat{k}$ is $\sin^{-1} \sqrt{\frac{46}{55}}$

Solution

$$\text{Solution: } \vec{a} \cdot \vec{b} = (1)(1) + (2)(1) + (0)(3) = 1 + 2 = 3$$

$$|\vec{a}| = \sqrt{1^2 + 2^2} = \sqrt{5} \quad |\vec{b}| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{3}{\sqrt{5} \sqrt{11}} = \frac{3}{\sqrt{55}}$$

$$\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{9}{55} = \frac{46}{55}$$

$$\text{Therefore: } \theta = \sin^{-1} \sqrt{\frac{46}{55}}$$

Question Q4(B).2 [4 marks]

Under effect of the forces $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$ a particle moves from the point $(1, 2, -3)$ to the point $(5, 3, 7)$. Find out work done.

Solution

$$\text{Solution: Net force: } \vec{F} = (2\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 3\hat{j} - \hat{k}) = 3\hat{i} + 4\hat{j}$$

$$\text{Displacement: } \vec{s} = (5, 3, 7) - (1, 2, -3) = (4, 1, 10)$$

$$\text{Work done: } W = \vec{F} \cdot \vec{s} = (3)(4) + (4)(1) + (0)(10) = 12 + 4 = 16 \text{ units}$$

Question Q4(B).3 [4 marks]

Evaluate: $\lim_{x \rightarrow 0} \frac{2^x - 5^x}{x}$

Solution

Solution: Using L'Hôpital's rule or the derivative definition:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2^x - 5^x}{x} &= \lim_{x \rightarrow 0} \frac{2^x \ln 2 - 5^x \ln 5}{1} \\ &= 2^0 \ln 2 - 5^0 \ln 5 = \ln 2 - \ln 5 = \ln \left(\frac{2}{5} \right) \end{aligned}$$

Question Q.5 (A) [6 marks]

Attempt any two

Question Q5(A).1 [3 marks]

Evaluate: $\lim_{x \rightarrow 0} \left(1 + \frac{3x}{7} \right)^{\frac{1}{x}}$

Solution

Solution: Let $y = \left(1 + \frac{3x}{7} \right)^{\frac{1}{x}}$

Taking natural log: $\ln y = \frac{1}{x} \ln \left(1 + \frac{3x}{7} \right)$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \frac{\ln \left(1 + \frac{3x}{7} \right)}{x}$$

$$\text{Using L'Hôpital's rule: } = \lim_{x \rightarrow 0} \frac{\frac{3/7}{1 + \frac{3x}{7}}}{1} = \frac{3}{7}$$

$$\text{Therefore: } \lim_{x \rightarrow 0} y = e^{3/7}$$

Question Q5(A).2 [3 marks]

Evaluate: $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9}$

Solution

Solution: Factoring numerator: $x^2 - 5x + 6 = (x - 2)(x - 3)$ Factoring denominator: $x^2 - 9 = (x - 3)(x + 3)$

$$\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9} = \lim_{x \rightarrow 3} \frac{(x-2)(x-3)}{(x-3)(x+3)} = \lim_{x \rightarrow 3} \frac{x-2}{x+3} = \frac{3-2}{3+3} = \frac{1}{6}$$

Question Q5(A).3 [3 marks]

Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x}$

Solution

Solution: Rationalizing the numerator:

$$\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x} \times \frac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2}$$

$$= \lim_{x \rightarrow 0} \frac{(4+x)-4}{x(\sqrt{4+x}+2)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{4+x}+2)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{4+x}+2} = \frac{1}{2+2} = \frac{1}{4}$$

Question Q.5 (B) [8 marks]

Attempt any two

Question Q5(B).1 [4 marks]

Find out equation of the line passing through points $(1, 2)$ and $(2, 1)$.

Solution

Solution: Using two-point form: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$$\frac{y-2}{1-2} = \frac{x-1}{2-1}$$

$$\frac{y-2}{-1} = \frac{x-1}{1}$$

$$y-2 = -(x-1) = -x+1$$

$$x+y=3$$

Question Q5(B).2 [4 marks]

Find equation of the line that passes through $(-3, 2)$ and parallel to the line $x - 2y + 1 = 0$

Solution

Solution: The given line $x - 2y + 1 = 0$ has slope $m = \frac{1}{2}$

Since parallel lines have the same slope, required line has slope $m = \frac{1}{2}$

Using point-slope form: $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{1}{2}(x - (-3))$$

$$y - 2 = \frac{1}{2}(x + 3)$$

$$2y - 4 = x + 3$$

$$x - 2y + 7 = 0$$

Question Q5(B).3 [4 marks]

Find out center and radius of the circle: $x^2 + y^2 + 6x - 4y - 3 = 0$

Solution

Solution: Completing the square:

$$x^2 + 6x + y^2 - 4y = 3$$

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 3 + 9 + 4$$

$$(x+3)^2 + (y-2)^2 = 16$$

Center: $(-3, 2)$ **Radius:** $r = \sqrt{16} = 4$

Formula Cheat Sheet

Logarithms

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

Trigonometry

- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin(180 - x) = \sin x$, $\cos(90 + x) = -\sin x$

Trigonometric Identities Map:

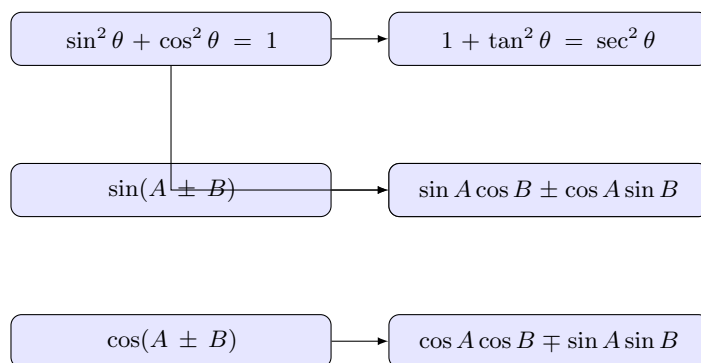


Figure 1. Trigonometric Identities

Vectors

- $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
- $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$
- For perpendicular vectors: $\vec{a} \cdot \vec{b} = 0$

Limits Strategy

Coordinate Geometry

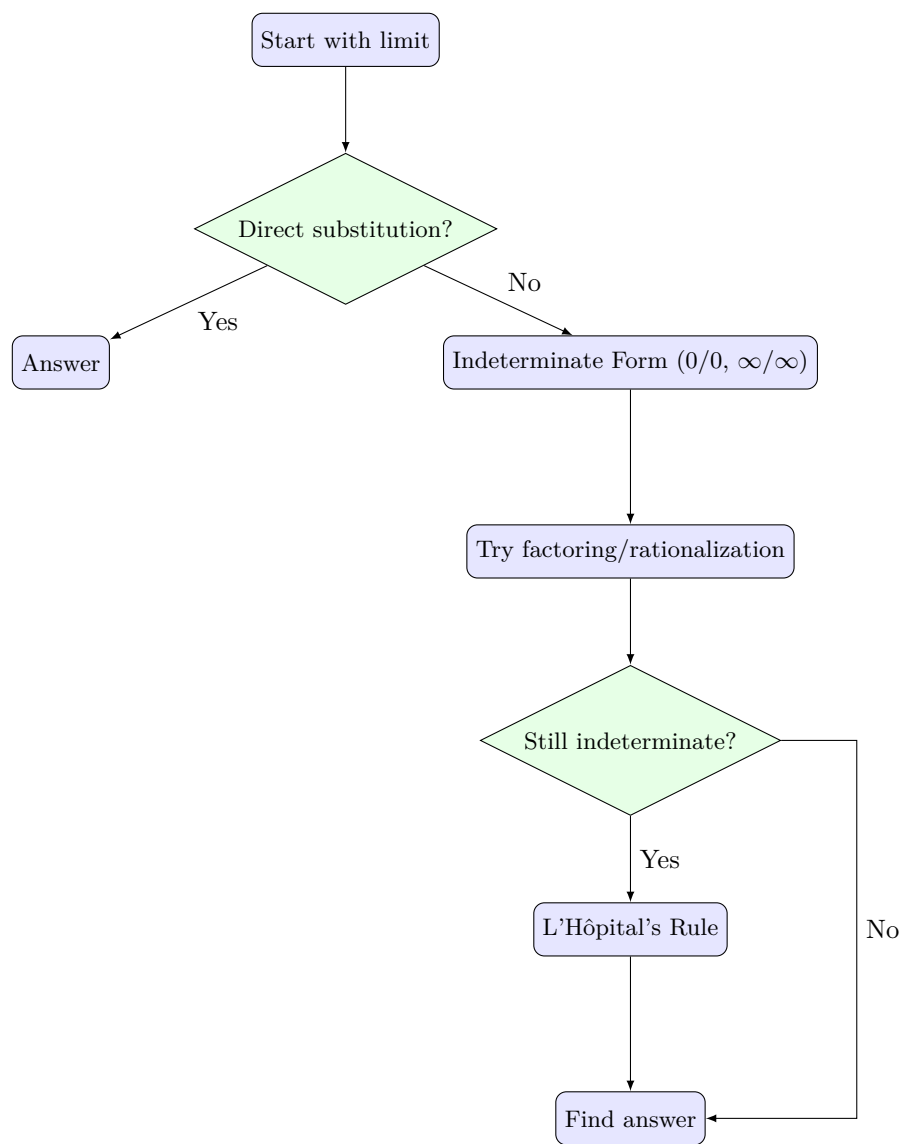
- Two-point form: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
- Circle: $(x-h)^2 + (y-k)^2 = r^2$
- Parallel lines have equal slopes

Memory Techniques (Logarithms)

Mnemonic

“PLUS: Product, Limit, Unity, Subtraction”

- **Product:** $\log(ab) = \log a + \log b$
- **Limit:** $\log_a 1 = 0$
- **Unity:** $\log_a a = 1$
- **Subtraction:** $\log(a/b) = \log a - \log b$

**Figure 2.** Limits Solving Strategy