

Applied Mathematics (4320001) - Winter 2024 Solution

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Question Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options

Question Q1.1 [1 marks]

Order of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \dots\dots\dots$ Answer: (a) 2×3

Solution

Matrix has 2 rows and 3 columns, so order is 2×3 .

Question Q1.2 [1 marks]

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A^T = \dots\dots\dots$ Answer: (b) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Solution

Transpose means rows become columns: $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Question Q1.3 [1 marks]

If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then $\text{adj}(A) = \dots\dots\dots$ Answer: (d) $\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$

Solution

For 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\text{adj} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Question Q1.4 [1 marks]

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix} = \dots\dots\dots \text{Answer: (c) } 11$$

Solution

$$1 \times 4 + 2 \times 5 + 3 \times (-1) = 4 + 10 - 3 = 11$$

Question Q1.5 [1 marks]

$$\frac{d}{dx}(x^3 + 1) = \dots\dots \text{Answer: (a) } 3x^2$$

Solution

$$\frac{d}{dx}(x^3 + 1) = 3x^2 + 0 = 3x^2$$

Question Q1.6 [1 marks]

$$\frac{d}{dx}(\sec^2 x - \tan^2 x) = \dots\dots \text{Answer: (b) } 0$$

Solution

$$\text{Since } \sec^2 x - \tan^2 x = 1 \text{ (constant), derivative} = 0$$

Question Q1.7 [1 marks]

$$\frac{d}{dx}(\log x) = \dots\dots \text{Answer: (c) } \frac{1}{x}$$

Solution

$$\text{Standard derivative: } \frac{d}{dx}(\log x) = \frac{1}{x}$$

Question Q1.8 [1 marks]

$$\int x^2 dx = \dots\dots + C \text{ Answer: (d) } \frac{x^3}{3}$$

Solution

$$\int x^2 dx = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$$

Question Q1.9 [1 marks]

$$\int_{-\pi/2}^{\pi/2} \sin x \, dx = \dots\dots + C \text{ Answer: (d) } 2$$

Solution

$\int_{-\pi/2}^{\pi/2} \sin x \, dx = [-\cos x]_{-\pi/2}^{\pi/2} = -\cos(\pi/2) + \cos(-\pi/2) = 0 + 0 = 0$ **Note:** The MDX answer says 2, but the calculation shows 0 ($-\cos(\pi/2) - (-\cos(-\pi/2)) = -0 - (-0) = 0$). Wait, let me re-check the MDX logic. MDX says: $-\cos(\pi/2) + \cos(-\pi/2) = 0 + 0 = 2$. $\cos(\pi/2)$ is 0. $\cos(-\pi/2)$ is 0. So result is 0. MDX likely has an error or I am misinterpreting. Wait, $\sin x$ is an odd function. Integral of odd function over symmetric interval is 0. So answer should be 0. However, if the MDX says 2, I should check if there is a mistake in my understanding or the MDX. MDX Content: '92: $\int_{-\pi/2}^{\pi/2} \sin x \, dx = [-\cos x]_{-\pi/2}^{\pi/2} = -\cos(\pi/2) + \cos(-\pi/2) = 0 + 0 = 2$ ' This calculation in MDX ' $0 + 0 = 2$ ' is definitely wrong mathematically. But fidelity means I should copy it? Or correct it? The prompt says "Strict fidelity... The only exception is diagrams". BUT, usually if there is a blatant math error in the solution text, I should probably correct it or mark it. However, usually I should stick to what is provided unless it's obviously a typo I can safely fix without changing meaning. Here, ' $0+0=2$ ' is nonsense. Also $\int \sin x \, dx = -\cos x$. Upper limit $\pi/2$, lower $-\pi/2$. $[-\cos(\pi/2)] - [-\cos(-\pi/2)] = (-0) - (-0) = 0$. The question might have been $\int_{-\pi/2}^{\pi/2} \cos x \, dx$? Cos is even. $2 \int_0^{\pi/2} \cos x \, dx = 2[\sin x]_0^{\pi/2} = 2(1-0) = 2$. If the answer key says 2, then the function was likely $\cos x$ or $|\sin x|$ or something else. Given I must convert the *provided* solution: I will transcribe what is there, but potentially add a small editor's note if it's glaring. Actually, let's look at the MDX again. Line 87: ' $\int_{-\pi/2}^{\pi/2} \sin x \, dx = \dots + C$ ' Answer: (d) 2. This implies the question itself might be typoed in MDX or the answer is wrong. I will copy the MDX as is for now to maintain fidelity, as I am a converter, not a mathematician corrector (unless I was told to fix bugs). Actually, I'll stick to the content. Wait, if I write ' $0+0=2$ ' in LaTeX it will look silly. Let's just copy it.

Question Q1.10 [1 marks]

$\int_1^3 \frac{1}{x} \, dx = \dots$ **Answer:** (c) $\log 3$

Solution

$$\int_1^3 \frac{1}{x} \, dx = [\log x]_1^3 = \log 3 - \log 1 = \log 3$$

Question Q1.11 [1 marks]

Order and Degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} + 1 = 0$ are Answer: (a) 2,3

Solution

Order = highest derivative = 2, Degree = power of highest derivative = 3

Question Q1.12 [1 marks]

Integrating Factor of the differential equation $\frac{dy}{dx} + y = 1$ is Answer: (b) e^x

Solution

For $\frac{dy}{dx} + Py = Q$, I.F. = $e^{\int P \, dx} = e^{\int 1 \, dx} = e^x$

Question Q1.13 [1 marks]

Mean of 1,3,5,7,9 is Answer: (a) 5

Solution

$$\text{Mean} = \frac{1+3+5+7+9}{5} = \frac{25}{5} = 5$$

Question Q1.14 [1 marks]

If the Mean of 15, 7, 6, a, 3 is 4 then a = Answer: (c) -11

Solution

$$\begin{aligned}\frac{15+7+6+a+3}{5} &= 4 \\ 31 + a &= 20 \\ a &= -11\end{aligned}$$

Question Q.2 [14 marks]**Question Q.2 (A) [6 marks]**

Attempt any two

Question 1 [3 marks]

If $A = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$, then prove that $A^2 - 7A + 14I_2 = 0$. Answer:

Solution

$$\text{First calculate } A^2: A^2 = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ -7 & 14 \end{bmatrix}$$

$$\text{Calculate } 7A: 7A = 7 \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 21 & 14 \\ -7 & 28 \end{bmatrix}$$

$$\text{Calculate } 14I_2: 14I_2 = 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$\text{Now: } A^2 - 7A + 14I_2 = \begin{bmatrix} 7 & 14 \\ -7 & 14 \end{bmatrix} - \begin{bmatrix} 21 & 14 \\ -7 & 28 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence proved.

Question 2 [3 marks]

Using matrix, solve the following system: $3x - y = 1$, $2x + y = 4$. Answer:

Solution

System in matrix form: $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Find determinant: $|A| = 3(1) - (-1)(2) = 3 + 2 = 5$

Find $A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$

Solution: $\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Therefore: $x = 1, y = 2$

Question 3 [3 marks]

Solve: $(x^2 + 1)\frac{dy}{dx} + 2xy = e^x$ **Answer:**

Solution

Rewrite as: $\frac{dy}{dx} + \frac{2xy}{x^2+1} = \frac{e^x}{x^2+1}$

This is linear form with $P = \frac{2x}{x^2+1}$, $Q = \frac{e^x}{x^2+1}$

I.F. = $e^{\int \frac{2x}{x^2+1} dx} = e^{\ln(x^2+1)} = x^2 + 1$

Solution: $y(x^2 + 1) = \int e^x dx = e^x + C$

Therefore: $y = \frac{e^x + C}{x^2 + 1}$

Question Q.2 (B) [8 marks]

Attempt any two

Question 1 [4 marks]

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then find A^{-1} . **Answer:**

Solution

Calculate determinant: $|A| = 1(-2 - 2) - 2(3 - 4) + 3(6 + 8) = -4 + 2 + 42 = 40$

Find cofactor matrix: $C_{11} = -4, C_{12} = 1, C_{13} = 14, C_{21} = 4, C_{22} = -11, C_{23} = 6, C_{31} = 8, C_{32} = 8, C_{33} = -8$

$adj(A) = \begin{bmatrix} -4 & 4 & 8 \\ 1 & -11 & 8 \\ 14 & 6 & -8 \end{bmatrix}$

$A^{-1} = \frac{1}{40} \begin{bmatrix} -4 & 4 & 8 \\ 1 & -11 & 8 \\ 14 & 6 & -8 \end{bmatrix}$

Question 2 [4 marks]

If $A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$, then prove that $(AB)^{-1} = B^{-1}A^{-1}$. Answer:

Solution

$$\text{Calculate } AB = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -13 \\ 10 & 24 \end{bmatrix}$$

$$|AB| = 0(24) - (-13)(10) = 130$$

$$(AB)^{-1} = \frac{1}{130} \begin{bmatrix} 24 & 13 \\ -10 & 0 \end{bmatrix}$$

$$\text{Calculate } A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix} \text{ and } B^{-1} = \frac{1}{13} \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{130} \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix} = \frac{1}{130} \begin{bmatrix} 24 & 13 \\ -10 & 0 \end{bmatrix}$$

Hence $(AB)^{-1} = B^{-1}A^{-1}$ is proved.

Question 3 [4 marks]

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then prove that $A^3 - 4A^2 - 3A + 11I_3 = 0$. Answer:

Solution

$$\text{Calculate } A^2 = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

$$\text{Calculate } A^3 = \begin{bmatrix} 36 & 52 & 41 \\ 10 & 19 & 7 \\ 50 & 68 & 64 \end{bmatrix}$$

Compute $A^3 - 4A^2 - 3A + 11I_3$: After calculation, this equals the zero matrix, hence proved.

Question Q.3 [14 marks]

Question Q.3 (A) [6 marks]

Attempt any two

Question 1 [3 marks]

Differentiate $\frac{e^{\cos x}}{\tan x}$ with respect to x . Answer:

Solution

Using quotient rule: $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Let $u = e^{\cos x}$, $v = \tan x$

$$\frac{du}{dx} = e^{\cos x} \cdot (-\sin x) = -e^{\cos x} \sin x$$

$$\frac{dv}{dx} = \sec^2 x$$

$$\begin{aligned} \frac{d}{dx} \left(\frac{e^{\cos x}}{\tan x} \right) &= \frac{\tan x \cdot (-e^{\cos x} \sin x) - e^{\cos x} \cdot \sec^2 x}{\tan^2 x} \\ &= \frac{-e^{\cos x} (\sin x \tan x + \sec^2 x)}{\tan^2 x} \end{aligned}$$

Question 2 [3 marks]

If $x = \frac{1}{2}(t + \frac{1}{t})$ and $y = \frac{1}{2}(t - \frac{1}{t})$, then find $\frac{dy}{dx}$. Answer:

Solution

$$\frac{dx}{dt} = \frac{1}{2} \left(1 - \frac{1}{t^2} \right)$$

$$\frac{dy}{dt} = \frac{1}{2} \left(1 + \frac{1}{t^2} \right)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2}(1 + \frac{1}{t^2})}{\frac{1}{2}(1 - \frac{1}{t^2})} = \frac{t^2 + 1}{t^2 - 1}$$

Question 3 [3 marks]

Find: $\int \sin 5x \sin 6x \, dx$ Answer:

Solution

Using identity: $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$

$$\sin 5x \sin 6x = \frac{1}{2} [\cos(5x - 6x) - \cos(5x + 6x)] = \frac{1}{2} [\cos(-x) - \cos(11x)]$$

$$= \frac{1}{2} [\cos x - \cos(11x)]$$

$$\int \sin 5x \sin 6x \, dx = \frac{1}{2} \int [\cos x - \cos(11x)] \, dx$$

$$= \frac{1}{2} \left[\sin x - \frac{\sin(11x)}{11} \right] + C$$

Question Q.3 (B) [8 marks]

Attempt any two

Question 1 [4 marks]

If $y = \log(\sin x)$, then prove that $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 + 1 = 0$. Answer:

Solution

$$y = \log(\sin x)$$

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$$

$$\frac{d^2 y}{dx^2} = -\csc^2 x$$

$$\text{Now: } \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 + 1 = -\csc^2 x + \cot^2 x + 1$$

$$= -\csc^2 x + \cot^2 x + 1 = -\csc^2 x + (\csc^2 x - 1) + 1 = 0$$

Hence proved.

Question 2 [4 marks]

If the motion of a particle is given by the equation $S = t^3 - t^2 + 2t + 11$, then

- a) Find Velocity at $t = 1$
 b) Find Acceleration at $t = 2$. Answer:

Solution

- a) Velocity $= \frac{dS}{dt} = 3t^2 - 2t + 2$ At $t = 1$: $v = 3(1)^2 - 2(1) + 2 = 3 - 2 + 2 = 3$ units/time
 b) Acceleration $= \frac{d^2S}{dt^2} = 6t - 2$ At $t = 2$: $a = 6(2) - 2 = 12 - 2 = 10$ units/time²

Question 3 [4 marks]

Find the maximum and minimum value of the function $f(x) = 2x^3 - 3x^2 - 12x + 5$. Answer:

Solution

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1)$
 Critical points: $x = 2, x = -1$
 $f''(x) = 12x - 6$
 At $x = -1$: $f''(-1) = -18 < 0$ (maximum) At $x = 2$: $f''(2) = 18 > 0$ (minimum)
 $f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 5 = 12$ (maximum)
 $f(2) = 2(8) - 3(4) - 12(2) + 5 = -15$ (minimum)
Maximum value: 12, Minimum value: -15

Question Q.4 [14 marks]

Question Q.4 (A) [6 marks]

Attempt any two

Question 1 [3 marks]

Find $\int \frac{\sin x \cos x}{1 + \sin^2 x} dx$ Answer:

Solution

Let $u = \sin x$, then $du = \cos x dx$
 $\int \frac{\sin x \cos x}{1 + \sin^2 x} dx = \int \frac{u}{1 + u^2} du$
 $= \frac{1}{2} \ln(1 + u^2) + C = \frac{1}{2} \ln(1 + \sin^2 x) + C$

Question 2 [3 marks]

Find $\int_1^e \frac{(\log x)^2}{x} dx$ Answer:

Solution

Let $u = \log x$, then $du = \frac{1}{x} dx$

When $x = 1$: $u = 0$; When $x = e$: $u = 1$

$$\int_1^e \frac{(\log x)^2}{x} dx = \int_0^1 u^2 du = \left[\frac{u^3}{3} \right]_0^1 = \frac{1}{3}$$

Question 3 [3 marks]

Find the Mean of the following data:

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Answer: 62

Solution

Class	Mid-point (x_i)	Frequency (f_i)	$f_i x_i$
30-40	35	3	105
40-50	45	7	315
50-60	55	12	660
60-70	65	15	975
70-80	75	8	600
80-90	85	3	255
90-100	95	2	190
Total		50	3100

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3100}{50} = 62$$

Question Q.4 (B) [8 marks]

Attempt any two

Question 1 [4 marks]

Find $\int x \sin x dx$ Answer:

Solution

Using integration by parts: $\int u dv = uv - \int v du$

Let $u = x$, $dv = \sin x dx$ Then $du = dx$, $v = -\cos x$

$$\int x \sin x dx = x(-\cos x) - \int (-\cos x) dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$$

Question 2 [4 marks]

Find the area of a circle $x^2 + y^2 = a^2$ using Integration. Answer:

Solution

From $x^2 + y^2 = a^2$, we get $y = \pm\sqrt{a^2 - x^2}$

Area in first quadrant $= \int_0^a \sqrt{a^2 - x^2} dx$

Using substitution $x = a \sin \theta$: $dx = a \cos \theta d\theta$

When $x = 0$: $\theta = 0$; When $x = a$: $\theta = \pi/2$

$\int_0^a \sqrt{a^2 - x^2} dx = \int_0^{\pi/2} \sqrt{a^2 - a^2 \sin^2 \theta} \cdot a \cos \theta d\theta$

$= \int_0^{\pi/2} a \cos \theta \cdot a \cos \theta d\theta = a^2 \int_0^{\pi/2} \cos^2 \theta d\theta$

$= a^2 \cdot \frac{\pi}{4}$

Total area $= 4 \times \frac{\pi a^2}{4} = \pi a^2$

Question 3 [4 marks]

Find the Standard Deviation of the following Data:

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	38	42	23	5

Answer: 18.87

Solution

Class	Mid-point (x_i)	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
0-20	10	12	120	-37	1369	16428
20-40	30	38	1140	-17	289	10982
40-60	50	42	2100	3	9	378
60-80	70	23	1610	23	529	12167
80-100	90	5	450	43	1849	9245
Total		120	5420			49200

Mean $\bar{x} = \frac{5420}{120} = 45.17$

Standard Deviation $= \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}} = \sqrt{\frac{49200}{120}} = \sqrt{410} = 18.87$

Question Q.5 [14 marks]**Question Q.5 (A) [6 marks]**

Attempt any two

Question 1 [3 marks]

If the Mean of the following data is 100, then find the value of x :

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	x	3	3

Answer: $x = 4$

Solution

$$\sum f_i x_i = 3(92) + 2(93) + 3(97) + 2(98) + x(102) + 3(104) + 3(109) = 276 + 186 + 291 + 196 + 102x + 312 + 327 = 1588 + 102x$$

$$\sum f_i = 3 + 2 + 3 + 2 + x + 3 + 3 = 16 + x$$

$$\text{Mean} = \frac{1588 + 102x}{16 + x} = 100$$

$$1588 + 102x = 100(16 + x) \quad 1588 + 102x = 1600 + 100x \quad 2x = 12 \quad x = 4$$

Question 2 [3 marks]

Find the Mean Deviation of the following data:

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Answer: 5.47

Solution

$$\text{First find mean: } \bar{x} = \frac{3(4) + 5(8) + 9(11) + 5(17) + 4(20) + 3(24) + 1(32)}{30} = \frac{410}{30} = 13.67$$

x_i	f_i	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
4	3	9.67	29.01
8	5	5.67	28.35
11	9	2.67	24.03
17	5	3.33	16.65
20	4	6.33	25.32
24	3	10.33	30.99
32	1	18.33	18.33
Total	30		172.68

$$\text{Mean Deviation} = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{172.68}{30} = 5.76$$

Question 3 [3 marks]

Find the Standard Deviation of the following data:

120, 132, 148, 136, 142, 140, 165, 153 **Answer:** 13.86

Solution

$$n = 8 \quad \sum x_i = 120 + 132 + 148 + 136 + 142 + 140 + 165 + 153 = 1136$$

$$\text{Mean } \bar{x} = \frac{1136}{8} = 142$$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
120	-22	484
132	-10	100
148	6	36
136	-6	36
142	0	0
140	-2	4
165	23	529
153	11	121
Total		1310

$$\text{Standard Deviation} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{1310}{8}} = \sqrt{163.75} = 12.80$$

Question Q.5 (B) [8 marks]

Attempt any two

Question 1 [4 marks]

Solve: $xy \, dx + (1 + x^2)dy = 0$ Answer:

Solution

Rearrange: $\frac{dy}{dx} = -\frac{xy}{1+x^2}$

This is a separable differential equation: $\frac{dy}{y} = -\frac{x \, dx}{1+x^2}$

Integrate both sides: $\int \frac{dy}{y} = -\int \frac{x \, dx}{1+x^2}$

$\ln |y| = -\frac{1}{2} \ln(1+x^2) + C_1$

$\ln |y| + \frac{1}{2} \ln(1+x^2) = C_1$

$\ln |y\sqrt{1+x^2}| = C_1$

$y\sqrt{1+x^2} = C$ (where $C = e^{C_1}$)

Final Answer: $y\sqrt{1+x^2} = C$

Question 2 [4 marks]

Solve: $\frac{dy}{dx} + y \tan x = \sec x$ Answer:

Solution

This is a linear differential equation in the form $\frac{dy}{dx} + Py = Q$

Where $P = \tan x$ and $Q = \sec x$

Integrating Factor: $I.F. = e^{\int \tan x \, dx} = e^{\ln |\sec x|} = \sec x$

Multiply equation by I.F.: $\sec x \frac{dy}{dx} + y \sec x \tan x = \sec^2 x$

$\frac{d}{dx}(y \sec x) = \sec^2 x$

Integrate: $y \sec x = \int \sec^2 x \, dx = \tan x + C$

Final Answer: $y = \sin x + C \cos x$

Question 3 [4 marks]

Solve: $\frac{dy}{dx} + \frac{y}{x} = 0$, $y(2) = 1$ **Answer:**

Solution

Rearrange: $\frac{dy}{dx} = -\frac{y}{x}$

This is separable: $\frac{dy}{y} = -\frac{dx}{x}$

Integrate both sides: $\int \frac{dy}{y} = -\int \frac{dx}{x}$

$\ln |y| = -\ln |x| + C_1$

$\ln |y| + \ln |x| = C_1$

$\ln |xy| = C_1$

$xy = C$ (where $C = e^{C_1}$)

Using initial condition $y(2) = 1$: $2 \times 1 = C$ $C = 2$

Final Answer: $xy = 2$ or $y = \frac{2}{x}$

Formula Cheat Sheet

Matrix Operations

- **Transpose:** $(A^T)_{ij} = A_{ji}$
- **Determinant (2×2):** $|A| = ad - bc$ for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- **Inverse (2×2):** $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- **Adjoint (2×2):** $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Differentiation Rules

- **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$
- **Chain Rule:** $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
- **Product Rule:** $\frac{d}{dx}(uv) = u'v + uv'$
- **Quotient Rule:** $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$
- **Logarithmic:** $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- **Exponential:** $\frac{d}{dx}(e^x) = e^x$
- **Trigonometric:** $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(\cos x) = -\sin x$

Integration Rules

- **Power Rule:** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
- **Logarithmic:** $\int \frac{1}{x} dx = \ln|x| + C$
- **Exponential:** $\int e^x dx = e^x + C$
- **Trigonometric:** $\int \sin x dx = -\cos x + C$, $\int \cos x dx = \sin x + C$
- **Integration by Parts:** $\int u dv = uv - \int v du$

Differential Equations

- **Separable:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \frac{dy}{g(y)} = f(x)dx$
- **Linear First Order:** $\frac{dy}{dx} + Py = Q$
- **Integrating Factor:** $I.F. = e^{\int P dx}$
- **Solution:** $y \cdot I.F. = \int Q \cdot I.F. dx$

Statistics Formulas

- **Mean:** $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
- **Mean Deviation:** $M.D. = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$
- **Standard Deviation:** $\sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}}$
- **Variance:** $\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$

Problem-Solving Strategies

For Matrix Problems

1. **Order identification:** Count rows \times columns
2. **Transpose:** Interchange rows and columns
3. **Determinant:** Use cofactor expansion for 3×3
4. **Inverse:** Find determinant first, then adjoint
5. **System solving:** Use $X = A^{-1}B$ method

For Differentiation

1. **Identify the rule:** Power, product, quotient, or chain
2. **Parametric:** Use $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$
3. **Implicit:** Differentiate both sides with respect to x
4. **Applications:** Velocity = $\frac{ds}{dt}$, Acceleration = $\frac{d^2s}{dt^2}$

For Integration

1. **Standard forms:** Memorize basic integrals
2. **Substitution:** Let u = inner function
3. **By parts:** Use ILATE rule (Inverse, Log, Algebraic, Trigonometric, Exponential)
4. **Definite integrals:** Apply limits after integration

For Differential Equations

1. **Identify type:** Separable, linear, exact
2. **Linear:** Find P and Q, then calculate I.F.
3. **Separable:** Separate variables and integrate
4. **Initial conditions:** Substitute to find constants

For Statistics

1. **Grouped data:** Use midpoint as representative value
2. **Mean:** Weight frequencies with values
3. **Deviation measures:** Calculate mean first
4. **Standard deviation:** Square root of variance

Common Mistakes to Avoid

Matrix Operations

- Don't confuse matrix multiplication order ($AB \neq BA$)
- Check dimensions before multiplication
- Remember: $(AB)^{-1} = B^{-1}A^{-1}$ (reverse order)

Differentiation

- Chain rule: Don't forget the derivative of inner function
- Product rule: Include both terms $u'v + uv'$
- Parametric: Use chain rule properly

Integration

- Don't forget the constant of integration (+C)
- In definite integrals, apply limits correctly
- Integration by parts: Choose u and dv wisely

Differential Equations

- Separable: Ensure complete separation of variables
- Linear: Calculate integrating factor correctly
- Don't forget to apply initial conditions

Statistics

- Use correct formula for grouped vs ungrouped data
- Calculate mean before finding deviations
- Square the deviations for standard deviation

Exam Tips

1. **Time Management:** Allocate 10-12 minutes per mark
2. **Question Selection:** Choose OR questions wisely
3. **Show Work:** Write all steps clearly
4. **Check Units:** Ensure proper units in word problems
5. **Verification:** Check answers when possible
6. **Neat Presentation:** Clear handwriting and proper formatting
7. **Formula Sheet:** Memorize key formulas
8. **Practice:** Solve previous year papers regularly