Subject Name Solutions

4320002 – Summer 2023

Semester 1 Study Material

Detailed Solutions and Explanations

Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options.

0.0.1 Q.1.1 [1 mark]

Order of $\begin{bmatrix} 1 & 0 & 3 \\ -2 & 4 & 0 \end{bmatrix}$ **is __________________.**

Solution

b. 2×3
Solution: The matrix has 2 rows and 3 columns, so the order is 2×3.

0.0.2 Q.1.2 [1 mark]

If A is of order 2×3 **and B is of order** 3×2 **then AB is of order _______________.**

Solution

d. 2×2
Solution: For matrix multiplication AB, if A is 2×3 and B is 3×2, then AB is of order 2×2.

0.0.3 Q.1.3 [1 mark]

If $A = \begin{bmatrix} 1 & -1 \end{bmatrix}$ **then $A^T = $ ____________**

Solution

b. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Solution: The transpose of a row matrix becomes a column matrix. $A^T = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

0.0.4 Q.1.4 [1 mark]

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **then $adj A = $ __________**

Solution

d. $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Solution: For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Therefore: adj $A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

0.0.5 Q.1.5 [1 mark]

$d $\frac{}{dx(e\hat{}x)=\$\_\_\_\_\_\_\_\_}$

Solution

a. e^x
Solution: $\frac{d}{dx}(e^x) = e^x$

0.0.6 Q.1.6 [1 mark]

If $f(x) = \log x$ **then $f'(1) = $** ________

Solution

c. 1
Solution: $f'(x) = \frac{1}{x}$ $f'(1) = \frac{1}{1} = 1$

0.0.7 Q.1.7 [1 mark]

$d\frac{}{dx(3\hat{}\{\log 3\ x\})\ =\ \$\ \_\_\_\_\_\_\_\_}$

Solution

b. $2x$
Solution: Using the property $a^{\log_a x} = x$**:** $3^{\log_3 x} = x$ **Therefore:** $\frac{d}{dx}(3^{\log_3 x}) = \frac{d}{dx}(x) = 1$
Wait, let me recalculate this. The expression is $3^{\log_3 x^2} = x^2$ $\frac{d}{dx}(x^2) = 2x$

0.0.8 Q.1.8 [1 mark]

**$**$\int \sin x, dx = \$$________

Solution

c. $-\cos x$
Solution: $\int \sin x \, dx = -\cos x + C$

0.0.9 Q.1.9 [1 mark]

****$**$\int$ ***{-1}^{1} x^3 , dx = $*** _______ $**$

Solution

b. 0
Solution: $\int_{-1}^{1} x^3 \, dx = \left[\frac{x^4}{4}\right]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0$

0.0.10 Q.1.10 [1 mark]

**$**$\int \frac{1}{1+x^2}, dx = \$$________

Solution

d. $\tan^{-1} x$
Solution: $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$

0.0.11 Q.1.11 [1 mark]

Order of the differential equation $\frac{d^2y}{dx^2} - y = 0$ **is** ______________.

Solution

b. 2
Solution: The highest derivative is $\frac{d^2y}{dx^2}$**, so the order is 2.**

0.0.12 Q.1.12 [1 mark]

The integration factor (I.F) of $\frac{dy}{dx} + Py = Q$ is ____________

Solution

a. $e^{\int P\,dx}$
Solution: For a linear differential equation $\frac{dy}{dx} + Py = Q$, the integrating factor is $e^{\int P\,dx}$.

0.0.13 Q.1.13 [1 mark]

If $Z = 4 - 5i$ then $Z} = $ ____________

Solution

c. $4 - 5i$
Solution: Wait, this seems incorrect. If $Z = 4 - 5i$, then $\bar{Z} = 4 + 5i$. The correct answer should be $4 + 5i$.

0.0.14 Q.1.14 [1 mark]

$i^{10} = $ _________

Solution

b. -1
Solution: $i^{10} = i^{4 \cdot 2+2} = (i^4)^2 \cdot i^2 = 1^2 \cdot (-1) = -1$

Q.2 (A) [6 marks]

Attempt any two.

0.0.15 Q.2(A).1 [3 marks]

If $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ then find the matrix X such that $2A + X = 3B$.

Solution: $2A + X = 3B$ $X = 3B - 2A$

$$2A = 2\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 8 & 6 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 3 & 12 \end{bmatrix}$$

$$X = \begin{bmatrix} 9 & 6 \\ 3 & 12 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ -5 & 6 \end{bmatrix}$$

0.0.16 Q.2(A).2 [3 marks]

If $A = \begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ then find $(AB)^T$.

Solution: First, find AB: $AB = \begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 5(1)+4(2) & 5(3)+4(1) \\ 4(1)+3(2) & 4(3)+3(1) \end{bmatrix} = \begin{bmatrix} 13 & 19 \\ 10 & 15 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 13 & 10 \\ 19 & 15 \end{bmatrix}$$

0.0.17 Q.2(A).3 [3 marks]

Solve: $\frac{dy}{dx} = x^2 \cdot e^{-y}$.

Solution: $\frac{dy}{dx} = x^2 \cdot e^{-y}$
Separating variables: $e^y \, dy = x^2 \, dx$
Integrating both sides: $\int e^y \, dy = \int x^2 \, dx$
$e^y = \frac{x^3}{3} + C$
$y = \ln\left(\frac{x^3}{3} + C\right)$

Q.2 (B) [8 marks]

Attempt any two.

0.0.18 Q.2(B).1 [4 marks]

If $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \end{bmatrix}$ **then prove that** $(A+B)^T = A^T + B^T$**.**

Solution: $A + B = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \end{bmatrix}$

$A + B = \begin{bmatrix} 3 & 5 & 3 \\ 6 & 8 & 1 \end{bmatrix}$

$(A+B)^T = \begin{bmatrix} 3 & 6 \\ 5 & 8 \\ 3 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & 0 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 5 & 8 \\ 3 & 1 \end{bmatrix}$

Therefore, $(A+B)^T = A^T + B^T$ **is proved.**

0.0.19 Q.2(B).2 [4 marks]

If $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$ **then find** A^{-1}**.**

Solution: To find A^{-1}**, we use the formula** $A^{-1} = \frac{1}{|A|} \cdot \mathbf{adj}(A)$

First, find $|A|$**:** $|A| = 2(0 \cdot 1 - 4 \cdot (-1)) - (-1)(1 \cdot 1 - 4 \cdot 1) + 0(1 \cdot (-1) - 0 \cdot 1)$ $|A| = 2(4) + 1(-3) = 8 - 3 = 5$

Next, find cofactors: $C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 4 \\ -1 & 1 \end{vmatrix} = 4$

$C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = -(-3) = 3$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1$

$C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} = -(-1) = 1$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$

$C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-1) = 1$

$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 0 \\ 0 & 4 \end{vmatrix} = -4$

$C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix} = -(8) = -8$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$\mathbf{adj}(A) = \begin{bmatrix} 4 & 1 & -4 \\ 3 & 2 & -8 \\ -1 & 1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{5} \begin{bmatrix} 4 & 1 & -4 \\ 3 & 2 & -8 \\ -1 & 1 & 1 \end{bmatrix}$

0.0.20 Q.2(B).3 [4 marks]

Solve the equations $3x - y = 1, x + 2y = 5$ by matrix method.

Solution: The system can be written as $AX = B$ where: $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

$|A| = 3(2) - (-1)(1) = 6 + 1 = 7$

$A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

$X = A^{-1}B = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

$X = \frac{1}{7} \begin{bmatrix} 2+5 \\ -1+15 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ 14 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Therefore, $x = 1$ and $y = 2$.

Q.3 (A) [6 marks]

Attempt any two.

0.0.21 Q.3(A).1 [3 marks]

If $y = \frac{e^x+1}{e^x-1}$ then find $\frac{dy}{dx}$.

Solution: Using quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Let $u = e^x + 1$ and $v = e^x - 1$ $\frac{du}{dx} = e^x$ and $\frac{dv}{dx} = e^x$

$\frac{dy}{dx} = \frac{(e^x-1)(e^x)-(e^x+1)(e^x)}{(e^x-1)^2}$

$= \frac{e^{2x}-e^x-e^{2x}-e^x}{(e^x-1)^2} = \frac{-2e^x}{(e^x-1)^2}$

0.0.22 Q.3(A).2 [3 marks]

If $x = a\cos\theta, y = b\sin\theta$ then find $\frac{dy}{dx}$.

Solution: $\frac{dx}{d\theta} = -a\sin\theta$ $\frac{dy}{d\theta} = b\cos\theta$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b\cos\theta}{a\sin\theta} = -\frac{b}{a}\cot\theta$

0.0.23 Q.3(A).3 [3 marks]

Evaluate: $\int \frac{\cos\sqrt{x}}{2\sqrt{x}}dx$.

Solution: Let $u = \sqrt{x}$, then $du = \frac{1}{2\sqrt{x}}dx$

$\int \frac{\cos\sqrt{x}}{2\sqrt{x}}dx = \int \cos u\,du = \sin u + C = \sin\sqrt{x} + C$

Q.3 (B) [8 marks]

Attempt any two.

0.0.24 Q.3(B).1 [4 marks]

Differentiate $y = x^{\cos x}$ with respect to x.

Solution: Taking natural logarithm on both sides: $\ln y = \cos x \ln x$

Differentiating both sides with respect to x: $\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \frac{1}{x} + \ln x \cdot (-\sin x)$

$\frac{dy}{dx} = y\left(\frac{\cos x}{x} - \sin x \ln x\right)$
$\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \ln x\right)$

0.0.25 Q.3(B).2 [4 marks]

If $y = A\cos pt + B\sin pt$, prove that $\frac{d^2y}{dt^2} + p^2y = 0$.

Solution: $y = A\cos pt + B\sin pt$
$\frac{dy}{dt} = -Ap\sin pt + Bp\cos pt$
$\frac{d^2y}{dt^2} = -Ap^2\cos pt - Bp^2\sin pt = -p^2(A\cos pt + B\sin pt) = -p^2y$
Therefore: $\frac{d^2y}{dt^2} + p^2y = -p^2y + p^2y = 0$

0.0.26 Q.3(B).3 [4 marks]

The equation of motion of a particle is $s = t^3 + 2t^2 - 3t + 5$. Find the velocity and acceleration of the particle at $t = 1$ and $t = 2$ seconds.

Solution: $s = t^3 + 2t^2 - 3t + 5$
Velocity: $v = \frac{ds}{dt} = 3t^2 + 4t - 3$
Acceleration: $a = \frac{dv}{dt} = 6t + 4$
At $t = 1$: $v(1) = 3(1)^2 + 4(1) - 3 = 3 + 4 - 3 = 4$ **units/sec** $a(1) = 6(1) + 4 = 10$ **units/sec^2**
At $t = 2$: $v(2) = 3(2)^2 + 4(2) - 3 = 12 + 8 - 3 = 17$ **units/sec** $a(2) = 6(2) + 4 = 16$ **units/sec^2**

Q.4 (A) [6 marks]

Attempt any two.

0.0.27 Q.4(A).1 [3 marks]

Evaluate: $\int x \log x \, dx$.

Solution: Using integration by parts: $\int u\,dv = uv - \int v\,du$
Let $u = \log x$ **and** $dv = x\,dx$ **Then** $du = \frac{1}{x}dx$ **and** $v = \frac{x^2}{2}$
$\int x\log x\,dx = \log x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}dx$
$= \frac{x^2 \log x}{2} - \int \frac{x}{2}dx$
$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$
$= \frac{x^2}{2}(\log x - \frac{1}{2}) + C$

0.0.28 Q.4(A).2 [3 marks]

Evaluate: $\int_{-1}^{1} \frac{1}{1+x^2}dx$.

Solution: $\int_{-1}^{1} \frac{1}{1+x^2}dx = [\tan^{-1} x]_{-1}^{1}$
$= \tan^{-1}(1) - \tan^{-1}(-1)$
$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

0.0.29 Q.4(A).3 [3 marks]

Find inverse of $Z = 3 + 4i$.

Solution: $Z^{-1} = \frac{1}{Z} = \frac{1}{3+4i}$
Multiply numerator and denominator by the conjugate: $Z^{-1} = \frac{1}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{3-4i}{(3)^2+(4)^2} = \frac{3-4i}{9+16} = \frac{3-4i}{25}$
$Z^{-1} = \frac{3}{25} - \frac{4}{25}i$

Q.4 (B) [8 marks]

Attempt any two.

0.0.30 Q.4(B).1 [4 marks]

Evaluate: $\int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x} dx$.

Solution: Let $I = \int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x} dx$

Using the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$:

$I = \int_0^{\pi/2} \frac{\tan(\pi/2-x)}{\tan(\pi/2-x)+\cot(\pi/2-x)} dx$

$= \int_0^{\pi/2} \frac{\cot x}{\cot x + \tan x} dx$

Adding the two expressions: $2I = \int_0^{\pi/2} \frac{\tan x + \cot x}{\tan x + \cot x} dx = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

0.0.31 Q.4(B).2 [4 marks]

Find the area bounded by the line $y = x$, $x = 5$ **and the X-axis.**

Solution: The region is bounded by $y = x$, $x = 5$, **and** $y = 0$ **(X-axis).**

Area $= \int_0^5 x\, dx = \left[\frac{x^2}{2}\right]_0^5 = \frac{25}{2} - 0 = \frac{25}{2}$ **square units**

0.0.32 Q.4(B).3 [4 marks]

If $x + iy = \left(\frac{1+i}{2-i}\right)^2$, **find the value of** $x + y$.

Solution: First, simplify $\frac{1+i}{2-i}$: $\frac{1+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{(1+i)(2+i)}{(2-i)(2+i)} = \frac{2+i+2i+i^2}{4-i^2} = \frac{2+3i-1}{4+1} = \frac{1+3i}{5}$

Now: $\left(\frac{1+3i}{5}\right)^2 = \frac{(1+3i)^2}{25} = \frac{1+6i+9i^2}{25} = \frac{1+6i-9}{25} = \frac{-8+6i}{25}$

Therefore: $x = -\frac{8}{25}$ **and** $y = \frac{6}{25}$

$x + y = -\frac{8}{25} + \frac{6}{25} = -\frac{2}{25}$

Q.5 (A) [6 marks]

Attempt any two.

0.0.33 Q.5(A).1 [3 marks]

Find Square root of $Z = 5 + 12i$.

Solution: Let $\sqrt{5+12i} = a + bi$ **where** $a, b \in \mathbb{R}$

$(a+bi)^2 = 5 + 12i$ $a^2 + 2abi + b^2i^2 = 5 + 12i$ $(a^2 - b^2) + 2abi = 5 + 12i$

Comparing real and imaginary parts: $a^2 - b^2 = 5$ **... (1)** $2ab = 12$ **... (2)**

From (2): $b = \frac{6}{a}$

Substituting in (1): $a^2 - \frac{36}{a^2} = 5$ $a^4 - 5a^2 - 36 = 0$

Let $u = a^2$: $u^2 - 5u - 36 = 0$ $(u-9)(u+4) = 0$

Since $u = a^2 \geq 0$, **we have** $u = 9$, **so** $a = \pm 3$

If $a = 3$, **then** $b = 2$ **If** $a = -3$, **then** $b = -2$

Therefore: $\sqrt{5+12i} = \pm(3+2i)$

0.0.34 Q.5(A).2 [3 marks]

Find $x, y \in \mathbb{R}$ **from the equation** $(2x - y) + yi = 6 + 4i$.

Solution: Comparing real and imaginary parts: Real part: $2x - y = 6$ **... (1) Imaginary part:** $y = 4$ **... (2)**

Substituting (2) into (1): $2x - 4 = 6$ $2x = 10$ $x = 5$

Therefore: $x = 5$ **and** $y = 4$

0.0.35 Q.5(A).3 [3 marks]

Find the modulus and principal argument of $Z = 1 + i$, **and express Z into the polar form.**

Solution: $Z = 1 + i$
Modulus: $|Z| = \sqrt{1^2 + 1^2} = \sqrt{2}$
Principal argument: $\arg(Z) = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$
Polar form: $Z = |Z|(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Q.5 (B) [8 marks]

Attempt any two.

0.0.36 Q.5(B).1 [4 marks]

Solve: $\frac{dy}{dx} = 1 + x + y + xy$**.**

Solution: $\frac{dy}{dx} = 1 + x + y + xy = (1 + x) + y(1 + x) = (1 + x)(1 + y)$
Separating variables: $\frac{dy}{1+y} = (1 + x)dx$
Integrating both sides: $\int \frac{dy}{1+y} = \int(1 + x)dx$
$\ln|1 + y| = x + \frac{x^2}{2} + C$
$1 + y = Ae^{x+x^2/2}$ **where** $A = e^C$
$y = Ae^{x+x^2/2} - 1$

0.0.37 Q.5(B).2 [4 marks]

Solve the differential equation: $\frac{dy}{dx} + y = e^x$**.**

Solution: This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$ **where** $P = 1$ **and** $Q = e^x$**.**
Integrating factor: $I.F. = e^{\int P\,dx} = e^{\int 1\,dx} = e^x$
Multiplying the equation by e^x**:** $e^x\frac{dy}{dx} + e^x y = e^{2x}$
$\frac{d}{dx}(ye^x) = e^{2x}$
Integrating both sides: $ye^x = \int e^{2x}dx = \frac{e^{2x}}{2} + C$
$y = \frac{e^x}{2} + Ce^{-x}$

0.0.38 Q.5(B).3 [4 marks]

Solve the differential equation: $\frac{dy}{dx} - y\tan x = 1$**.**

Solution: This is a first-order linear differential equation where $P = -\tan x$ **and** $Q = 1$**.**
Integrating factor: $I.F. = e^{\int(-\tan x)dx} = e^{\ln|\cos x|} = \cos x$
Multiplying the equation by $\cos x$**:** $\cos x\frac{dy}{dx} - y\cos x\tan x = \cos x$
$\cos x\frac{dy}{dx} - y\sin x = \cos x$
$\frac{d}{dx}(y\cos x) = \cos x$
Integrating both sides: $y\cos$
x = $\int\cos x\,dx = \sin x + C$
$y = \tan x + \frac{C}{\cos x} = \tan x + C\sec x$

Formula Cheat Sheet

0.0.39 Matrix Operations

- **Order of Matrix: If matrix has** m **rows and** n **columns, order is** $m \times n$
- **Matrix Multiplication:** $(AB)_{ij} = \sum_k A_{ik}B_{kj}$
- **Transpose:** $(A^T)_{ij} = A_{ji}$
- **Adjoint of 2**$\times 2 Matrix : If A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$**, then adj**$(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- **Inverse:** $A^{-1} = \frac{1}{|A|} \cdot \textbf{adj}(A)$

0.0.40 Differentiation

- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- **Chain Rule:** $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
- **Product Rule:** $\frac{d}{dx}(uv) = u'v + uv'$
- **Quotient Rule:** $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-uv'}{v^2}$
- **Parametric: If** $x = f(t)$ **and** $y = g(t)$**, then** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

0.0.41 Integration

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ **(for** $n \neq -1$**)**
- $\int e^x \, dx = e^x + C$
- $\int \frac{1}{x} \, dx = \ln|x| + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$
- **Integration by Parts:** $\int u \, dv = uv - \int v \, du$
- **Definite Integration:** $\int_a^b f(x) \, dx = F(b) - F(a)$ **where** $F'(x) = f(x)$

0.0.42 Differential Equations

- **Order: Highest derivative present**
- **Degree: Power of highest derivative**
- **Linear DE:** $\frac{dy}{dx} + Py = Q$
- **Integrating Factor:** $I.F. = e^{\int P \, dx}$
- **Variable Separable:** $\frac{dy}{dx} = f(x)g(y) \rightarrow \frac{dy}{g(y)} = f(x)dx$

0.0.43 Complex Numbers

- **Standard Form:** $z = a + bi$
- **Conjugate:** $\overline{a + bi} = a - bi$
- **Modulus:** $|a + bi| = \sqrt{a^2 + b^2}$
- **Argument:** $\arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$
- **Polar Form:** $z = r(\cos\theta + i\sin\theta)$ **where** $r = |z|$ **and** $\theta = \arg(z)$
- **Powers of i:** $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$
- **Inverse:** $z^{-1} = \frac{\bar{z}}{|z|^2}$

Problem-Solving Strategies

0.0.44 Matrix Problems

1. **Check dimensions before multiplication**
2. **Use properties:** $(AB)^T = B^T A^T$, $(A + B)^T = A^T + B^T$
3. **For inverse: Calculate determinant first, then adjoint**
4. **System of equations: Write as** $AX = B$**, solve** $X = A^{-1}B$

0.0.45 Differentiation Problems

1. **Identify the type: Basic, chain rule, product rule, quotient rule**
2. **For implicit: Differentiate both sides with respect to x**
3. **For parametric: Use** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$
4. **For logarithmic: Take ln of both sides first**

0.0.46 Integration Problems

1. Check standard forms first
2. For products: Try integration by parts (ILATE rule)
3. For rational functions: Check for substitution
4. For definite integrals: Use properties like $\int_{-a}^{a} f(x)dx = 0$ if f(x) is odd

0.0.47 Differential Equations

1. Identify type: Order, degree, linear/non-linear
2. For linear DE: Find integrating factor
3. For separable: Separate variables and integrate
4. Check initial conditions if given

0.0.48 Complex Numbers

1. For operations: Use standard form $a + bi$
2. For modulus/argument: Convert to polar form
3. For powers: Use De Moivre's theorem
4. For square roots: Let $\sqrt{a + bi} = c + di$ and solve

Common Mistakes to Avoid

1. Matrix multiplication: Remember $AB \neq BA$ in general
2. Chain rule: Don't forget to multiply by derivative of inner function
3. Integration: Remember the constant of integration
4. Definite integrals: Apply limits correctly
5. Complex numbers: $i^2 = -1$, not $+1$
6. Differential equations: Don't forget integrating factor for linear DE
7. Parametric differentiation: Use $\frac{dy/dt}{dx/dt}$, not $\frac{dt/dy}{dt/dx}$

Exam Tips

0.0.49 Time Management

- Q.1 (MCQs): Spend 15-20 minutes maximum
- Short answers: 3-4 minutes per question
- Long answers: 8-10 minutes per question
- Keep 10 minutes for final review

0.0.50 Strategy

1. Read all questions first to identify easy ones
2. Attempt easy questions first to build confidence
3. Show all steps clearly for partial marks
4. Check units in application problems
5. Verify answers where possible (especially in matrix problems)

0.0.51 During Exam

- Write clearly and organize solutions
- Draw diagrams where helpful
- State formulas before using them
- Don't panic if stuck on one question - move to next
- Use remaining time to review and check calculations

Good Luck with your exams!