Subject Name Solutions

4331101 – Winter 2022

Semester 1 Study Material

Detailed Solutions and Explanations

Question 1(a) [3 marks]

Define: 1) Branch 2) Junction 3) Mesh

Solution

- **Branch**: A branch is a single circuit element or a combination of elements connected between two nodes of a network.
- **Junction**: A junction (or node) is a point in a circuit where two or more circuit elements are connected together.
- **Mesh**: A mesh is a closed path in a network where no other closed path exists inside it.

Mnemonic

"BJM: Branches Join at junctions to Make meshes"

Question 1(b) [4 marks]

Write voltage division and current division rule with necessary circuit diagram

Solution

Voltage Division Rule: In a series circuit, voltage across any component is proportional to its resistance.

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A(({+}) {-}{-}{-} B[R1] {-}{-}{-} C[R2] {-}{-}{-} D((-))}
    E[V1] {-.{-} B}
    F[V2] {-.{-} C}
    G[VS] {-.{-} A}
{Highlighting}
{Shaded}
```

- **Formula**: $V_1 = VS \times (R_1/(R_1 + R_2))$
- **Application**: Used to find individual voltage drops across series components

Current Division Rule: In a parallel circuit, current through any branch is inversely proportional to its resistance.

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A(({+}) {-}{-}{-} B {-}{-}{-} C((-))}
    B {-{-}{-} D[R1] {-}{-}{-} C}
    B {-{-}{-} E[R2] {-}{-}{-} C}
    F[I1] {-.{-} D}
    G[I2] {-.{-} E}
    H[IS] {-.{-} A}
{Highlighting}
{Shaded}
```

- **Formula**: $I_1 = IS \times (R_2/(R_1 + R_2))$
- **Key concept**: Current takes path of least resistance

Mnemonic

"VoSe CuPa: Voltage divides in Series, Current divides in Parallel"

Question 1(c) [7 marks]

Draw Graph and Tree for a network shown in fig(1). Show link currents on a graph. Also write Tie-set schedule for a tree of network shown in fig. (1)

Solution

Graph of the Network:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((A)) {-{-}{-} B((B))}
    A {-{-}{-} C((C))}
    A {-{-}{-} D((D))}
    B {-{-}{-} C}
    B {-{-}{-} D}
    C {-{-}{-} D}
    A {-{-} 1 {-}{-}{-} B}
    A {-{-} 3 {-}{-}{-} C}
    B {-{-} 2 {-}{-}{-} D}
    C {-{-} 5 {-}{-}{-} D}
    B {-{-} 6 {-}{-}{-} C}
    A {-{-} 7 {-}{-}{-} D}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style B fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style D fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Tree of the Network (shown with bold edges):

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((A)) {-{-}{-} B((B))}
    A {-{-}{-} C((C))}
    C {-{-}{-} D((D))}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style B fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style D fill:\#f9f,stroke:\#333,stroke{-width:2px}
    linkStyle 0 stroke{-width:4px,stroke:green}
    linkStyle 1 stroke{-width:4px,stroke:green}
    linkStyle 2 stroke{-width:4px,stroke:green}
{Highlighting}
{Shaded}
```

Link Currents (shown on remaining branches that are not part of the tree):

- Link 1: Branch 2 (BD)

- Link 2: Branch 6 (BC)
- Link 3: Branch 7 (AD)
- Link 4: Branch 5 (CD)

Tie-set Schedule:

Link/Tree Branch	Branch 1 (AB)	Branch 3 (AC)	Branch 4 (CD)	Branch 2 (BD)	Branch 6 (BC)	Branch 7 (AD)	Branch 5 (CD)
Link 1 (BD)	1	0	0	1	0	0	0
Link 2 (BC)	1	1	0	0	1	0	0
Link 3 (AD)	0	0	1	0	0	1	0
Link 4 (CD)	0	0	1	0	0	0	1

Mnemonic

"TGLT: Trees Generate Link-current Tie-sets"

Question 1(c) OR [7 marks]

Draw Graph and Tree for a network shown in fig(1). Show branch voltages on tree. Also write cut-set schedule for a tree of network shown on fig.(1)

Solution

Graph of the Network:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((A)) {-{-}{-} B((B))}
    A {-{-}{-} C((C))}
    A {-{-}{-} D((D))}
    B {-{-}{-} C}
    B {-{-}{-} D}
    C {-{-}{-} D}
    A {-{-} 1 {-}{-}{-} B}
    A {-{-} 3 {-}{-}{-} C}
    B {-{-} 2 {-}{-}{-} D}
    C {-{-} 5 {-}{-}{-} D}
    B {-{-} 6 {-}{-}{-} C}
    A {-{-} 7 {-}{-}{-} D}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style B fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style D fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Tree of the Network (shown with bold edges and branch voltages):

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((A)) {-{-}"V_{1}"{-}{-}{} B((B))}
    A {-{-}"V_{3}"{-}{-}{} C((C))}
    C {-{-}"V_{4}"{-}{-}{} D((D))}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style B fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
```

```
    style D fill:\#f9f,stroke:\#333,stroke{-width:2px}
    linkStyle 0 stroke{-width:4px,stroke:green}
    linkStyle 1 stroke{-width:4px,stroke:green}
    linkStyle 2 stroke{-width:4px,stroke:green}
{Highlighting}
{Shaded}
```

Cut-set Schedule:

Cut-set/Branch	Branch 1 (AB)	Branch 3 (AC)	Branch 4 (CD)	Branch 2 (BD)	Branch 6 (BC)	Branch 7 (AD)	Branch 5 (CD)
Cut-set 1 (AB)	1	0	0	-1	-1	0	0
Cut-set 2 (AC)	0	1	0	0	1	-1	0
Cut-set 3 (CD)	0	0	1	1	0	1	1

Mnemonic

"CGVS: Cut-sets Generate Voltage Sources"

Question 2(a) [3 marks]

Define: 1) Active and passive network 2)Unilateral and Bilateral network.

Solution

- **Active Network**: A network containing one or more sources of EMF (voltage/current sources) that supply energy to the circuit.
- **Passive Network**: A network containing only passive elements like resistors, capacitors, and inductors with no energy sources.
- **Unilateral Network**: A network in which the properties and performance change when input and output terminals are interchanged.
- **Bilateral Network**: A network in which the properties and performance remain unchanged when input and output terminals are interchanged.

Diagram:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    subgraph "Network Types"
    A[Active: Contains sources]
    B[Passive: No sources]
    C[Unilateral: Diodes/Transistors]
    D[Bilateral: R, L, C elements]
    end
{Highlighting}
{Shaded}
```

Mnemonic

"APUB: Active Provides energy, Unilateral Blocks reversal"

Question 2(b) [4 marks]

Write equation for Z parameter and derive Z11, Z12, Z21, Z22 from that equation.

Solution

Z-parameters define the relationship between port voltages and currents in a two-port network:
Equations:

- $V_1 = Z_{11}I_1 + Z_{12}I_2$
- $V_2 = Z_{21}I_1 + Z_{22}I_2$

Derivation:

- $\mathbf{Z}_{11} = V_1/I_1(with I_2 = 0) : Input impedance with output port open - circuited$
- $\mathbf{Z}_{12} = V_1/I_2(with I_1 = 0) : Reverse transfer impedance with input port open - circuited$
- $\mathbf{Z}_{21} = V_2/I_1(with I_2 = 0) : Forward transfer impedance with output port open - circuited$
- $\mathbf{Z}_{22} = V_2/I_2(with I_1 = 0) : Output impedance with input port open - circuited$

Mnemonic

"Z Impedance: Open circuit gives correct Parameters"

Question 2(c) [7 marks]

Derive equation of characteristic impedance(ZOT) for a standard T network.

Solution

For a standard T-network:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((Port{-1)) {-}{-}{-} B[Z1] {-}{-}{-} C((Junction))}
    C {-{-}{-} D[Z2] {-}{-}{-} E((Port{-}2))}
    C {-{-}{-} F[Z3] {-}{-}{-} G((Ground))}
{Highlighting}
{Shaded}
```

Derivation Steps:

1. For a symmetric T-network, $Z_1 = Z_2$
1. Under matched condition, input impedance equals characteristic impedance
2. $Z_{0t} = Z_1 + (Z_{13})/(Z_1 + Z_3)$
2. For balanced T-network where $Z_1 = Z_2 = Z/2 and Z_3 = Z :$
2. $Z_{0t} = Z/2 + (Z/2)/(Z/2 + Z)$
2. $Z_{0t} = Z/2 + (Z^2/2)/(Z + Z/2)$
2. $Z_{0t} = Z/2 + (Z^2/2)/(3Z/2)$
2. $Z_{0t} = Z/2 + Z^2/3Z$
2. $Z_{0t} = Z/2 + Z/3$
2. $Z_{0t} = (3Z + 2Z)/6$
2. $Z_{0t} = \sqrt{(}Z_1(Z_1 + 2Z_3))$

Final Equation: $Z_{0t} = \sqrt{(}Z_1(Z_1 + 2Z_3))$

Mnemonic

"TO Impedance: Two arms Over middle branch"

Question 2(a) OR [3 marks]

Define: 1)Driving point impedance 2) Transfer impedance

Solution

- **Driving Point Impedance**: The ratio of voltage to current at the same port/pair of terminals when all other independent sources are set to zero.
- **Transfer Impedance**: The ratio of voltage at one port to the current at another port when all other

independent sources are set to zero.
Diagram:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    subgraph "Impedance Types"
    A[Driving Point: V_{1/I_{1} or V_{2}/I_{2}]}
    B[Transfer: V_{2/I_{1} or V_{1}/I_{2}]}
    end
{Highlighting}
{Shaded}
```

Mnemonic

"DTSS: Driving at Terminal Same, Transfer at Separate"

Question 2(b) OR [4 marks]

Explain Kirchhoff's voltage law with example.

Solution

Kirchhoff's Voltage Law (KVL): The algebraic sum of all voltages around any closed loop in a circuit is zero.
Mathematically: = 0 (around a closed loop)
Circuit Example:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A(({+)) {-}{-}"10V"{-}{-}{} B}
    B {-{-}"R_{1} = 2Ω"{-}{-}{} C}
    C {-{-}"R_{2} = 3Ω"{-}{-}{} D}
    D {-{-}"R_{3} = 5Ω"{-}{-}{} A}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style B fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style D fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

If I = 1A, then:
- $V_1 = 1A \times 2 = 2V$
- $V_2 = 1A \times 3 = 3V$
- $V_3 = 1A \times 5 = 5V$

Applying KVL: 10V - 2V - 3V - 5V = 0

Mnemonic

"VACZ: Voltages Around Closed loop are Zero"

Question 2(c) OR [7 marks]

Derive equation to convert network into T network.

Solution

Network to T Network Conversion:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph TD
    subgraph "  Network"
    A1((A)) {-{-}{-} B1((B))}
    A1 {-{-}{-} Y1[Ya] {-}{-}{-} C1}
    B1 {-{-}{-} Y2[Yb] {-}{-}{-} C1}
    A1 {-{-}{-} Y3[Yc] {-}{-}{-} B1}
    C1((C))
    end

    subgraph "T Network"
    A2((A)) {-{-}{-} Z1[Za] {-}{-}{-} D2((D))}
    B2((B)) {-{-}{-} Z2[Zb] {-}{-}{-} D2}
    D2 {-{-}{-} Z3[Zc] {-}{-}{-} C2((C))}
    end
{Highlighting}
{Shaded}
```

Conversion Equations:

1. Za = (Ya $\times Yc)/Y$
1. Zb = (Yb $\times Yc)/Y$
1. Zc = (Ya $\times Yb)/Y$

Where Y = Ya + Yb + Yc

Derivation:

1. Start with Y-parameters of -network
2. Express Y-parameters in terms of branch admittances
3. Convert to Z-parameters using matrix inversion
4. Express T-network impedances in terms of Z-parameters
5. Simplify to get the conversion formulas above

Mnemonic

"PIE to TEA: Product over sum for opposite branch"

Question 3(a) [3 marks]

Explain Kirchhoff's current law with example.

Solution

Kirchhoff's Current Law (KCL): The algebraic sum of all currents entering and leaving a node must equal zero.

Mathematically: = 0 (at any node)

Circuit Example:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph TD
    A[I_{1 = 5A] {-}{-}{} B((Node))}
    C[I_{2 = 2A] {-}{-}{} B}
    B {-{-}{} D[I_{3} = 3A]}
    B {-{-}{} E[I_{4} = 4A]}
    style B fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Applying KCL at node B:

- Currents entering: $\mathrm{I}_1 + I_2 = 5A + 2A = 7A$
- Currents leaving: $\mathrm{I}_3 + I_4 = 3A + 4A = 7A$
- Therefore: $\mathrm{I}_1 + I_2 - I_3 - I_4 = 5 + 2 - 3 - 4 = 0$

Mnemonic

"CuNoZ: Currents at Node are Zero"

Question 3(b) [4 marks]

Explain mesh analysis with required equations.

Solution

Mesh Analysis: A circuit analysis technique that uses mesh currents as variables to solve a circuit with multiple loops.

Steps:

1. Identify all meshes (closed loops) in the circuit
2. Assign a mesh current to each mesh
3. Apply KVL to each mesh
4. Solve the resulting system of equations

Example Circuit:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((A)) {-{-} R_{1} {-}{-}{-} B((B))}
    B {-{-} R_{3} {-}{-}{-} C((C))}
    A {-{-} R_{2} {-}{-}{-} C}
    A {-{-} V_{1} {-}{-}{-} D}
    D {-{-} {}+ {-}{-}{-} A}
    C {-{-} V_{2} {-}{-}{-} E}
    E {-{-} {}+ {-}{-}{-} C}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style B fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Equations:

- Mesh 1: $\mathrm{V}_1 = I_1R_1 + I_1R_2 - I_2R_2$
- Mesh 2: $\mathrm{V}_2 = I_2R_2 + I_2R_3 - I_1R_2$

Mnemonic

"MILK: Mesh Is Loop with KVL"

Question 3(c) [7 marks]

State and explain Thevenin's theorem.

Solution

Thevenin's Theorem: Any linear network with voltage and current sources can be replaced by an equivalent circuit consisting of a voltage source (VTH) in series with a resistance (RTH).

Mermaid Diagram (Code)

```
{Shaded}
```

```
{Highlighting}[]
graph TD
    subgraph "Original Network"
    A((A)) {-{-}{-} B[Complex Network] {-}{-}{-} C((B))}
    end
    subgraph "Thevenin Equivalent"
    D((A)) {-{-}{-} E[VTH] {-}{-}{-} F(({}+))}
    F {-{-}{-} G[RTH] {-}{-}{-} H((B))}
    end
{Highlighting}
{Shaded}
```

Steps to Find Thevenin Equivalent:

1. Remove the load from the terminals of interest
2. Calculate the open-circuit voltage (VOC) across these terminals (= VTH)
3. Calculate the resistance looking back into the circuit with all sources replaced by their internal resistances (= RTH)
4. The Thevenin equivalent consists of VTH in series with RTH

Example Application:

- Original complex circuit with load RL
- Remove RL and find VOC = VTH
- Deactivate sources and find RTH
- Reconnect RL to simplified Thevenin equivalent

Mnemonic

"TORV: Thevenin's Open-circuit Resistance and Voltage"

Question 3(a) OR [3 marks]

State and explain reciprocity theorem.

Solution

Reciprocity Theorem: In a linear, bilateral network, if a voltage source in one branch produces a current in another branch, then the same voltage source, if placed in the second branch, will produce the same current in the first branch.

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    subgraph "Original Circuit"
    direction LR
    A((A)) {-{-}{-} B[V] {-}{-}{-} C((B))}
    C {-{-}{-} D[Network] {-}{-}{-} E((C))}
    E {-{-}{-} F[Ammeter] {-}{-}{-} A}
    end

    subgraph "Reciprocal Circuit"
    direction LR
    G((A)) {-{-}{-} H[Ammeter] {-}{-}{-} I((B))}
    I {-{-}{-} J[Network] {-}{-}{-} K((C))}
    K {-{-}{-} L[V] {-}{-}{-} G}
    end
{Highlighting}
{Shaded}
```

Mathematically: If a voltage $V_1 in branch 1 produces current I_2 in branch 2, then voltage V_1 in branch 2 will produce current I_2 inb$

Limitations: Applies only to networks with:

- Linear elements

- Bilateral elements (no diodes, transistors)
- Single independent source

Mnemonic

"RESWAP: REciprocity SWAPs Position with identical results"

Question 3(b) OR [4 marks]

Explain nodal analysis with required equations.

Solution

Nodal Analysis: A circuit analysis technique that uses node voltages as variables to solve a circuit.

Steps:

1. Choose a reference node (ground)
2. Assign voltage variables to remaining nodes
3. Apply KCL at each non-reference node
4. Solve the resulting system of equations

Example Circuit:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((Node 1)) {-{-} G_{1} {-}{-}{-} B((Ground))}
    C((Node 2)) {-{-} G_{2} {-}{-}{-} B}
    A {-{-} G_{3} {-}{-}{-} C}
    A {-{-} I_{1} {-}{-}{} B}
    C {-{-} I_{2} {-}{-}{} B}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style B fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Equations:

- Node 1: $\mathrm{I}_1 = V_1 G_1 + (V_1 - V_2)G_3$
- Node 2: $\mathrm{I}_2 = V_2 G_2 + (V_2 - V_1)G_3$

Mnemonic

"NKCV: Nodal uses KCL with Voltage variables"

Question 3(c) OR [7 marks]

State and prove maximum power transfer theorem.

Solution

Maximum Power Transfer Theorem: A load connected to a source will extract maximum power when its resistance equals the internal resistance of the source.

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A(({+)) {-}{-}{-} B[VS] {-}{-}{-} C((X))}
    C {-{-}{-} D[RS] {-}{-}{-} E((Y))}
    E {-{-}{-} F[RL] {-}{-}{-} G((Z))}
```

```
    G {-{-}{-} A}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style E fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Proof:

1. Current in the circuit: I = VS/(RS + RL)
2. Power delivered to load:
   P = I$^2 RL = (VS^2 RL)/(RS + RL)^2$

3. For maximum power, dP/dRL = 0
4. Solving: (VS$^2 (RS + RL)^2 - VS^2 RL 2(RS + RL))/(RS + RL)^4 = 0$
4. Simplifying: (RS + RL)$^2 = 2RL(RS + RL)$
4. Further simplifying: RS + RL = 2RL
5. Therefore: RS = RL

Maximum Power: Pmax = VS$^2/(4RS)$

Mnemonic

"MaRLRS: Maximum power when load Resistance equals Source Resistance"

Question 4(a) [3 marks]

Why series resonance circuit act as voltage amplifier and parallel resonance circuit act as current amplifier?

Solution

Series Resonance as Voltage Amplifier:

- At resonance, series circuit impedance is minimum (just R)
- Voltage across L or C can be much larger than source voltage
- Voltage magnification factor = Q = XL/R = 1/R$\sqrt{(L/C)}$
- Voltage across L or C = Q $\times Source voltage$

Parallel Resonance as Current Amplifier:

- At resonance, parallel circuit impedance is maximum
- Current in L or C can be much larger than source current
- Current magnification factor = Q = R/XL = R$\sqrt{(C/L)}$
- Current through L or C = Q $\times Source current$

Table:

Circuit Type	Impedance at Resonance	Amplification
Series	Minimum (R only)	Voltage (VL or VC = Q)
Parallel	Maximum (R$^2/r$)	Current (IL or IC = Q)

Mnemonic

"SeVoPa: Series Voltage, Parallel current amplification"

Question 4(b) [4 marks]

Derive equation of Q of coil.

Solution

Q-factor of a Coil:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
```

```
    A((A)) {-{-}{-} B[R] {-}{-}{-} C((B))}
    C {-{-}{-} D[L] {-}{-}{-} A}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Derivation:

1. Q-factor is defined as: Q = Energy stored / Energy dissipated per cycle
2. Energy stored in inductor = (1/2)LI^2
2. Power dissipated in resistor = I^2R
2. Energy dissipated per cycle = Power $\times Timeperiod = I^2R \times (1/f)$
2. Therefore: Q = ((1/2)LI^2)$/(I^2R \times (1/f))$
2. Simplifying: Q = 2 $\times (1/2)LI^2 \times f/(I^2R)$
2. Q = 2 f $\times L/$

R = L / R

Final Equation: Q = L / R = 2 fL / R = XL / R

Mnemonic

"QualityEDR: Quality equals Energy stored Divided by energy lost per Radian"

Question 4(c) [7 marks]

Derive equation of series resonance frequency for series R-L-C circuit.

Solution

Series R-L-C Circuit:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((Input)) {-{-}{-} B[R] {-}{-}{-} C[L] {-}{-}{-} D[C] {-}{-}{-} E((Output))}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style E fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Derivation:

1. Impedance of series RLC circuit: Z = R + j(XL - XC)
2. Where: XL = L and XC = 1/ C
3. At resonance, XL = XC (inductive and capacitive reactances are equal)
4. Therefore: L = 1/ C
5. Solving for : $^2 = 1/LC$
5. Resonant frequency: $_0 = 1/\sqrt(LC)$
5. In terms of frequency f: f$_0 = 1/(2\sqrt(LC))$

Characteristics at Resonance:

- Impedance is minimum (purely resistive: Z = R)
- Current is maximum (I = V/R)
- Power factor is unity (circuit appears resistive)
- Voltages across L and C are equal and opposite

Mnemonic

"RES: Reactances Equal at Series resonance"

Question 4(a) OR [3 marks]

What is coupled circuits? Define self-inductance and mutual inductance.

Solution

Coupled Circuits: Two or more circuits that are magnetically linked such that energy can be transferred between them through their mutual magnetic field.

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph TD
    subgraph "Primary"
    A((A)) {-{-}{-} B[L1] {-}{-}{-} C((B))}
    end

    subgraph "Secondary"
    D((C)) {-{-}{-} E[L2] {-}{-}{-} F((D))}
    end

    G[M] {-.{-}{} B}
    G {-.{-}{} E}
{Highlighting}
{Shaded}
```

Self-inductance (L): The property of a circuit whereby a change in current produces a self-induced EMF in the same circuit. L = Φ/I (ratio of magnetic flux to the current producing it)
Mutual inductance (M): The property of a circuit whereby a change in current in one circuit induces an EMF in another circuit. M = $\Phi_{21}/I_1(ratio of flux in circuit 2 due to current in circuit 1)$

Mnemonic

"SiMu: Self in Mine, Mutual in Yours"

Question 4(b) OR [4 marks]

Derive equation for co-efficient of coupling (K).

Solution

Coefficient of Coupling (k):

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    subgraph "Coupled Coils"
    A((A)) {-{-}{-} B[L1] {-}{-}{-} C((B))}
    D((C)) {-{-}{-} E[L2] {-}{-}{-} F((D))}
    G[M] {-.{-}{} B}
    G {-.{-}{} E}
    end
{Highlighting}
{Shaded}
```

Derivation:

1. The mutual inductance (M) between two coils depends on:
    - Self-inductances of the coils (L$_1 and L_2$)
    - Physical arrangement (proximity and orientation)
2. Maximum possible mutual inductance: M$_{max} = \sqrt(L_1 L_2)$
2. Coefficient of coupling is defined as: k = M/M$_{max}$
2. Therefore: k = M/$\sqrt(L_1 L_2)$

Characteristics:

- k ranges from 0 (no coupling) to 1 (perfect coupling)

- k depends on geometry, orientation, and medium
- Typical transformers: k = 0.95 to 0.99
- Air-core coils: k = 0.01 to 0.5

Mnemonic

"KMutual: K Measures Mutual linkage proportion"

Question 4(c) OR [7 marks]

A series RLC circuit has R=30Ω, L=0.5H, and C=5μF. Calculate (i) series resonance frequency (2) Q Factor (3)BW

Solution

Given:

- Resistance, R = 30Ω
- Inductance, L = 0.5H
- Capacitance,
  C = 5μF = $5\times10^{-6}F$

Calculations:

(i) Series Resonance Frequency:

- $f_0 = 1/(2\sqrt(LC))$
- $f_0 = 1/(2\sqrt(0.5 \times 5 \times 10^{-6}))$
- $f_0 = 1/(2\sqrt(2.5 \times 10^{-6}))$
- $f_0 = 1/(2 \times 1.58 \times 10^{-3})$
- $f_0 = 1/(9.9 \times 10^{-3})$
- $f_0 = 100.76Hz$
- $f_0 \approx 100Hz$

(ii) Q Factor:

- $Q = (1/R)\sqrt(L/C)$
- $Q = (1/30)\sqrt(0.5/(5 \times 10^{-6}))$
- $Q = (1/30)\sqrt(100,000)$
- $Q = (1/30) \times 316.23$
- $Q = 10.54$

(iii) Bandwidth (BW):

- $BW = f_0/Q$
- BW = 100.76/10.54
- BW = 9.56 Hz

Table:

Parameter	Formula	Value
Resonant Frequency (f_0)	$1/(2\sqrt(LC))$	100 Hz
Quality Factor (Q)	$(1/R)\sqrt(L/C)$	10.54
Bandwidth (BW)	f_0/Q	9.56 Hz

Mnemonic

"RQB: Resonance Quality determines Bandwidth"

Question 5(a) [3 marks]

Classify various types of attenuators.

Solution

Attenuators: Network of resistors designed to reduce (attenuate) signal level without distortion.

Types of Attenuators:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph TD
    A[Attenuators] {-{-}{} B[Fixed Attenuators]}
    A {-{-}{} C[Variable Attenuators]}
    B {-{-}{} D[T{-}type]}
    B {-{-}{} E[ {-}type]}
    B {-{-}{} F[Bridged{-}T]}
    B {-{-}{} G[Lattice]}
    C {-{-}{} H[Step Attenuators]}
    C {-{-}{} I[Continuously Variable]}
{Highlighting}
{Shaded}
```

Based on configuration:

- **T-type**: Three resistor T-shaped configuration
- **-type**: Three resistor -shaped configuration
- **Bridged-T**: T-type with a resistor bridging across
- **Lattice**: Balanced configuration with four resistors

Based on symmetry:

- **Symmetrical**: Equal input and output impedance
- **Asymmetrical**: Different input and output impedance

Mnemonic

"ATP Fixed: Attenuator Types include Pad, Tee, Lattice"

Question 5(b) [4 marks]

Derive relation between attenuator and neper.

Solution

Relationship between Attenuation and Neper:

- **Attenuation ( )**: Ratio of input voltage (or current) to output voltage (or current), expressed in different units.
- **Neper (Np)**: Natural logarithmic unit of ratios, used mainly in transmission line theory.

Derivation:

1. For a voltage ratio V_1/V_2 :
   - Attenuation in Nepers = $\ln(\mathrm{V}_1/V_2)$
   - Attenuation in Decibels = $20\log_{10}(V_1/V_2)$
2. For a power ratio P_1/P_2 :
   - Attenuation in Nepers = $(1/2)\ln(\mathrm{P}_1/P_2)$
   - Attenuation in Decibels = $10\log_{10}(P_1/P_2)$
3. Relationship between dB and Neper:
   - 1 Neper = 8.686 dB
   - 1 dB = 0.115 Neper

Table:

Unit	Voltage Ratio	Power Ratio
Neper (Np)	$\ln(\mathrm{V}_1/V_2)$	$(1/2)\ln(\mathrm{P}_1/P_2)$
Decibel (dB)	$20\log_{10}(V_1/V_2)$	$10\log_{10}(P_1/P_2)$

Mnemonic

"NED: Neper Equals Decibel divided by 8.686"

Question 5(c) [7 marks]

Derive equations of R1 and R2 for symmetrical T attenuator.

Solution

Symmetrical T Attenuator:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
    A((Input)) {-{-}{-} B[R1] {-}{-}{-} C((Junction))}
    C {-{-}{-} D[R1] {-}{-}{-} E((Output))}
    C {-{-}{-} F[R2] {-}{-}{-} G((Ground))}
    style A fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style E fill:\#f9f,stroke:\#333,stroke{-width:2px}
    style C fill:\#f9f,stroke:\#333,stroke{-width:2px}
{Highlighting}
{Shaded}
```

Derivation:

1. For a symmetrical T-attenuator with characteristic impedance Z_0 :
   - Input and output impedance must both equal Z_0
   - Attenuation ratio
     $N = V_1/V_2 = I_2/I_1$
2. From circuit analysis:
   - $Z_0 = R_1 + (R_2(R_1))/(R_2 + R_1)$
   - $N = (R_1 + R_2 + R_1)/R_2 = (2R_1 + R_2)/R_2$
3. Solving for $R_1 and R_2$:
   - $R_1 = Z_0(N-1)/(N+1)$
   - $R_2 = 2Z_0N/(N^2-1)$
4. For attenuation in dB ( ):
   - N = 10^(/20)
   - $R_1 = Z_0 tanh(/2)$
   - $R_2 = Z_0/sinh()$

Final Equations:

- $R_1 = Z_0(N-1)/(N+1)$
- $R_2 = 2Z_0N/(N^2-1)$

Mnemonic

"TSR: T-attenuator Symmetry Requires equal R1 values"

Question 5(a) OR [3 marks]

Draw circuit diagram of symmetrical Bridge T and symmetrical Lattice attenuator.

Solution

Symmetrical Bridge-T Attenuator:

```
               R1
A o{-{-}{-}{-}{-}{-}{-}{-}///{-}{-}{-}{-}{-}{-}{-}{-}o B}
           |      |
           |      |
           {      /}
           /  R3  {}
    R2     {      /}
o{-{-}{-}///{-}{-}+     +{-}{-}{-}{-}o}
|                      |
o{-{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}o}
C                      D
```

Symmetrical Lattice Attenuator:

```
        R1
```

```
A o{-{-}{-}{-}///{-}{-}{-}{-}o B}
   {            /}
    {          /}
     {        /}
  R2  {      / R2}
       {    /}
        {  /}
         {/}
         /{}
        /  {}
       /    {}
      /      {}
 R1  /        {}
  C o{-{-}///{-}{-}o D}
```

Characteristics:

1. **Bridge-T**: Combines features of T and attenuators, suitable for high-frequency applications
2. **Lattice**: Balanced configuration with excellent phase and frequency response, commonly used in balanced lines

Mnemonic

"BL-BA: Bridge Ladder, Balanced Attenuators"

Question 5(b) OR [4 marks]

Write classification of filter based on frequency with their frequency responses showing pass band and stop band.

Solution

Classification of Filters Based on Frequency:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph TD
    A[Passive Filters] {-{-}{} B[Low Pass Filter]}
    A {-{-}{} C[High Pass Filter]}
    A {-{-}{} D[Band Pass Filter]}
    A {-{-}{} E[Band Stop Filter]}
    A {-{-}{} F[All Pass Filter]}
{Highlighting}
{Shaded}
```

Frequency Responses:

1. **Low Pass Filter**: Passes frequencies below cutoff, attenuates above

```
Gain |
   1 |****
     |    ****
     |        ****
   0 |------------****----
     |
     +----------------------
       0     fc        f \rightarrow
```

2. **High Pass Filter**: Passes frequencies above cutoff, attenuates below

```
Gain |
   1 |            ****
     |        ****
     |    ****
```

```
      0 |****-----------------
        |
        +---------------------
           0     fc        f \rightarrow
```

3. **Band Pass Filter**: Passes frequencies within a specific band

```
Gain |
   1 |        ****
     |    ****    ****
     |   *          *
   0 |***-------------***--
     |
     +---------------------
        0   f1   f2     f \rightarrow
```

4. **Band Stop Filter**: Rejects frequencies within a specific band

```
Gain |
  1 |***              ***
     |   *            *
     |    ***     ***
  0 |        *****
     |
     +---------------------
        0   f1   f2     f \rightarrow
```

Mnemonic

"LHBBA: Low High Band-pass Band-stop All-pass"

Question 5(c) OR [7 marks]

Draw the circuit for T-section and -section constant-K low pass filter and Derive equation of cut-off frequency.

Solution

T-section Constant-K Low Pass Filter:

```
        L/2            L/2
o{-{-}{-}{-}{-}{-}UUUUUU{-}{-}{-}{-}{-}{-}UUUUUU{-}{-}{-}{-}{-}{-}{-}o}
|                               |
|                               |
|                               |
|              C                |
|              |                |
o{-{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}+{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}o}
Input                        Output
```

-section Constant-K Low Pass Filter:

```
       L
o{-{-}{-}{-}{-}{-}UUUUUU{-}{-}{-}{-}{-}{-}{-}{-}{-}o}
|                     |
|                     |
|                     |
|                     |
{-{-}{-}                  {-}{-}{-}}
{-{-}{-} C/2              {-}{-}{-} C/2}
|                     |
|                     |
o{-{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}{-}o}
Input              Output
```

Derivation of Cutoff Frequency:

1. For a constant-K filter:
   - $Z_1 \times Z_2 = R_0^2 (characteristicimpedancesquared)$
   - $Z_1 = jL(seriesimpedance)$
   - $Z_2 = 1/jC(shuntimpedance)$
2. Therefore:
   - $R_0^2 = Z_1 \times Z_2 = jL \times 1/jC = L/C$
   - $R_0 = \sqrt{(}L/C)$
3. Pass band condition:
   - $-1 < Z_1/4Z_2 < 0$
   - $-1 <$ j $L/(4 \times 1/jC) < 0$
   - $-1 < - {}^2LC/4 < 0$
4. At cutoff frequency:
   - ${}^2LC/4 = 1$
   - $c^2 = 4/LC$
   - $c = 2/\sqrt{(}LC)$
   - fc = c/2 = $1/ \sqrt{(}LC)$

Final Equation:

- Cutoff frequency fc = $1/ \sqrt{(}LC)$

Mnemonic

"KCLP: Konstant-k Cutoff in Low Pass depends on L and C product"