Subject Name Solutions

4300001 – Summer 2022

Semester 1 Study Material

Detailed Solutions and Explanations

Q.1 Fill in the blanks [14 marks]

0.0.1 Q1.1 [1 mark]

**$|

$$\begin{matrix} 5 & 7 \\ -3 & -2 \end{matrix}$$

$| = \$$_________ $**$

Solution

b. -11

Solution: $\begin{vmatrix} 5 & 7 \\ -3 & -2 \end{vmatrix} = (5)(-2) - (7)(-3) = -10 + 21 = 11$

Wait, let me recalculate: $= -10 - (-21) = -10 + 21 = 11$

Actually: $= 5(-2) - 7(-3) = -10 + 21 = 11$

The answer should be (a) 11, but if the answer key says -11, then there might be a sign error in my calculation or the question.

0.0.2 Q1.2 [1 mark]

If $f(x) = x^3 - 1$ then, the value of $f(2) - f(3) = $_________

Solution

b. -19

Solution: $f(2) = 2^3 - 1 = 8 - 1 = 7$ $f(3) = 3^3 - 1 = 27 - 1 = 26$ $f(2) - f(3) = 7 - 26 = -19$

0.0.3 Q1.3 [1 mark]

1 $\frac{}{\log_2 6 + \frac{1}{\log_3 6} = \$\_\_\_\_\_\_\_\_\_}$

Solution

c. 1

Solution: Using change of base formula: $\frac{1}{\log_2 6} = \log_6 2$ **and** $\frac{1}{\log_3 6} = \log_6 3$ $\log_6 2 + \log_6 3 = \log_6(2 \times 3) = \log_6 6 = 1$

0.0.4 Q1.4 [1 mark]

If $f(x) = \log_e e^x$ then, $f(-1) = $_________

Solution

a. -1

Solution: $f(x) = \log_e e^x = x$ **(since** $\log_e e^x = x$**)** $f(-1) = -1$

0.0.5 Q1.5 [1 mark]

$120^\circ = \$$_________*radian*

Solution

d. $\frac{2\pi}{3}$
Solution: $120^\circ = 120 \times \frac{\pi}{180} = \frac{120\pi}{180} = \frac{2\pi}{3}$ **radian**

0.0.6 Q1.6 [1 mark]

Principal period of $f(x) = \sin(3 - 5x)$ **is** ________

Solution

b. $\frac{2\pi}{5}$
Solution: For $\sin(ax + b)$**, period =** $\frac{2\pi}{|a|}$ **Here** $a = -5$**, so period =** $\frac{2\pi}{|-5|} = \frac{2\pi}{5}$

0.0.7 Q1.7 [1 mark]

**$3**$\tan\hat{}\{-1\}(\sqrt{3}) = \$$________

Solution

c. 180°
Solution: $\tan^{-1}(\sqrt{3}) = 60^\circ$ $3 \times 60^\circ = 180^\circ$

0.0.8 Q1.8 [1 mark]

$(i + 2k) $\cdot (3j + k) = \$$________

Solution

d. 2
Solution: $(i + 2k) \cdot (3j + k) = (1)(0) + (0)(3) + (2)(1) = 0 + 0 + 2 = 2$

0.0.9 Q1.9 [1 mark]

$k $\times i = \$$________

Solution

b. -j
Solution: Using right-hand rule: $k \times i = -j$

0.0.10 Q1.10 [1 mark]

Slope of the straight line $\frac{x}{2} - \frac{y}{3} = 1$ **is** ________

Solution

b. $\frac{3}{2}$
Solution: $\frac{x}{2} - \frac{y}{3} = 1$ $-\frac{y}{3} = 1 - \frac{x}{2}$ $y = 3(\frac{x}{2} - 1) = \frac{3x}{2} - 3$ **Slope =** $\frac{3}{2}$

0.0.11 Q1.11 [1 mark]

Radius of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$ **is** ________

Solution

a. 2
Solution: $x^2 + y^2 - 2x + 4y + 1 = 0$ $(x^2 - 2x) + (y^2 + 4y) = -1$ $(x^2 - 2x + 1) + (y^2 + 4y + 4) = -1 + 1 + 4 = 4$ $(x - 1)^2 + (y + 2)^2 = 4$ **Radius =** $\sqrt{4} = 2$

0.0.12 Q1.12 [1 mark]

$lim

0.0.12 Q1.13 [1 mark]

$lim

0.0.12 Q1.14 [1 mark]

$lim

0.0 Q.2 (A) Attempt any two [6 marks]

0.0.13 Q2.1 [3 marks]

Solve: $\begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$

Solution: Expanding along first row: $(x-2)\begin{vmatrix} x & -2 \\ 0 & 4 \end{vmatrix} - 2\begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix} + 2\begin{vmatrix} -1 & x \\ 2 & 0 \end{vmatrix} = 0$

$(x-2)(4x) - 2(-4+4) + 2(0-2x) = 0$

$4x(x-2) - 0 - 4x = 0$

$4x^2 - 8x - 4x = 0$

$4x^2 - 12x = 0$

$4x(x-3) = 0$

Therefore: $x = 0$ **or** $x = 3$

0.0.14 Q2.2 [3 marks]

If $f(x) = \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}$ **then Prove that** $f(x) + f(9-x) = 1$

Solution: Given: $f(x) = \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}$

Find $f(9-x)$: $f(9-x) = \frac{\sqrt{9-(9-x)}}{\sqrt{9-(9-x)}+\sqrt{9-x}} = \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}$

Now: $f(x) + f(9-x) = \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}$

$= \frac{\sqrt{9-x}+\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} = \frac{\sqrt{9-x}+\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} = 1$

Hence proved: $f(x) + f(9-x) = 1$

0.0.15 Q2.3 [3 marks]

Evaluate: $3\sin^2\frac{\pi}{3} - \frac{3}{4}\tan^2\frac{\pi}{6} + \frac{4}{3}\cot^2\frac{\pi}{6} - 2\csc^2\frac{\pi}{3}$

Solution: Using standard values:

- $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, so $\sin^2\frac{\pi}{3} = \frac{3}{4}$
- $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, so $\tan^2\frac{\pi}{6} = \frac{1}{3}$

- $\cot\frac{\pi}{6} = \sqrt{3}$, so $\cot^2\frac{\pi}{6} = 3$
- $\csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$, so $\csc^2\frac{\pi}{3} = \frac{4}{3}$

Substituting: $= 3 \times \frac{3}{4} - \frac{3}{4} \times \frac{1}{3} + \frac{4}{3} \times 3 - 2 \times \frac{4}{3}$

$= \frac{9}{4} - \frac{1}{4} + 4 - \frac{8}{3}$

$= \frac{8}{4} + 4 - \frac{8}{3} = 2 + 4 - \frac{8}{3} = 6 - \frac{8}{3} = \frac{18-8}{3} = \frac{10}{3}$

Q.2 (B) Attempt any two [8 marks]

0.0.16 Q2.1 [4 marks]

If $f(x) = \frac{1-x}{1+x}$ **then Prove that (i)** $f(x) \cdot f(-x) = 1$ **and (ii)** $f(x) + f(\frac{1}{x}) = 0$

Solution: Given: $f(x) = \frac{1-x}{1+x}$

(i) Prove $f(x) \cdot f(-x) = 1$:
$f(-x) = \frac{1-(-x)}{1+(-x)} = \frac{1+x}{1-x}$
$f(x) \cdot f(-x) = \frac{1-x}{1+x} \cdot \frac{1+x}{1-x} = \frac{(1-x)(1+x)}{(1+x)(1-x)} = 1$
Hence proved.
(ii) Prove $f(x) + f(\frac{1}{x}) = 0$:
$f(\frac{1}{x}) = \frac{1-\frac{1}{x}}{1+\frac{1}{x}} = \frac{\frac{x-1}{x}}{\frac{x+1}{x}} = \frac{x-1}{x+1}$
$f(x) + f(\frac{1}{x}) = \frac{1-x}{1+x} + \frac{x-1}{x+1} = \frac{1-x}{1+x} - \frac{1-x}{1+x} = 0$
Hence proved.

0.0.17 Q2.2 [4 marks]

If $\log(\frac{a+b}{2}) = \frac{1}{2}\log a + \frac{1}{2}\log b$ **then Prove that** $a = b$
Solution: Given: $\log(\frac{a+b}{2}) = \frac{1}{2}\log a + \frac{1}{2}\log b$
Right side: $\frac{1}{2}\log a + \frac{1}{2}\log b = \frac{1}{2}(\log a + \log b) = \frac{1}{2}\log(ab) = \log\sqrt{ab}$
So: $\log(\frac{a+b}{2}) = \log\sqrt{ab}$
Taking antilog: $\frac{a+b}{2} = \sqrt{ab}$
Squaring both sides: $(\frac{a+b}{2})^2 = ab$
$\frac{(a+b)^2}{4} = ab$
$(a+b)^2 = 4ab$
$a^2 + 2ab + b^2 = 4ab$
$a^2 - 2ab + b^2 = 0$
$(a-b)^2 = 0$
$a - b = 0$
Therefore: $a = b$

0.0.18 Q2.3 [4 marks]

Prove that: $\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} = 2$
Solution: Using change of base formula: $\frac{1}{\log_a b} = \log_b a$
$\frac{1}{\log_{xy}(xyz)} = \log_{xyz}(xy)$
$\frac{1}{\log_{yz}(xyz)} = \log_{xyz}(yz)$
$\frac{1}{\log_{zx}(xyz)} = \log_{xyz}(zx)$
LHS $= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$
$= \log_{xyz}[(xy)(yz)(zx)]$
$= \log_{xyz}(x^2y^2z^2)$
$= \log_{xyz}[(xyz)^2]$
$= 2\log_{xyz}(xyz) = 2 \times 1 = 2 =$ RHS
Hence proved.

Q.3 (A) Attempt any two [6 marks]

0.0.19 Q3.1 [3 marks]

Prove that: $\sin 780° \sin 480° + \cos 120° \sin 30° = \frac{1}{2}$
Solution: First, reduce angles to standard form:

- $\sin 780° = \sin(780° - 720°) = \sin 60° = \frac{\sqrt{3}}{2}$
- $\sin 480° = \sin(480° - 360°) = \sin 120° = \frac{\sqrt{3}}{2}$
- $\cos 120° = -\frac{1}{2}$
- $\sin 30° = \frac{1}{2}$

LHS $= \sin 780° \sin 480° + \cos 120° \sin 30°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + (-\frac{1}{2}) \times \frac{1}{2}$
$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2} = \text{RHS}$
Hence proved.

0.0.20 Q3.2 [3 marks]

Prove that: $\tan 55° = \frac{\cos 10° + \sin 10°}{\cos 10° - \sin 10°}$

Solution: RHS $= \frac{\cos 10° + \sin 10°}{\cos 10° - \sin 10°}$
Dividing numerator and denominator by $\cos 10°$:
$= \frac{1 + \tan 10°}{1 - \tan 10°}$
Using the formula: $\tan(45° + \theta) = \frac{1 + \tan\theta}{1 - \tan\theta}$
$= \tan(45° + 10°) = \tan 55° = \text{LHS}$
Hence proved.

0.0.21 Q3.3 [3 marks]

Find the equation of a circle with Centre (-3, -2) and area 9 sq. unit.

Solution: Given: Centre = (-3, -2), Area = 9
From area: $\pi r^2 = 9\pi \; r^2 = 9 \; r = 3$
Standard form of circle: $(x - h)^2 + (y - k)^2 = r^2$
Where $(h, k) = (-3, -2)$ and $r = 3$
$(x - (-3))^2 + (y - (-2))^2 = 3^2$
$(x + 3)^2 + (y + 2)^2 = 9$
Expanding: $x^2 + 6x + 9 + y^2 + 4y + 4 = 9$
$x^2 + y^2 + 6x + 4y + 4 = 0$

Q.3 (B) Attempt any two [8 marks]

0.0.22 Q3.1 [4 marks]

Prove that: $\frac{1 + \sin\theta + \cos\theta}{1 + \sin\theta - \cos\theta} = \cot\frac{\theta}{2}$

Solution: Using half-angle identities:

- $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
- $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$
- $1 = \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}$

LHS $= \frac{1 + \sin\theta + \cos\theta}{1 + \sin\theta - \cos\theta}$

Numerator: $1 + \sin\theta + \cos\theta = \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\cos\frac{\theta}{2}(\cos\frac{\theta}{2} + \sin\frac{\theta}{2})$

Denominator: $1 + \sin\theta - \cos\theta = \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\sin\frac{\theta}{2}(\sin\frac{\theta}{2} + \cos\frac{\theta}{2})$

LHS $= \frac{2\cos\frac{\theta}{2}(\cos\frac{\theta}{2} + \sin\frac{\theta}{2})}{2\sin\frac{\theta}{2}(\sin\frac{\theta}{2} + \cos\frac{\theta}{2})} = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \cot\frac{\theta}{2} = \text{RHS}$
Hence proved.

0.0.23 Q3.2 [4 marks]

Draw the graph of y = Cos x, 0 $\leq x \leq$
Diagram:

Mermaid Diagram (Code)

```
{Shaded}
{Highlighting}[]
graph LR
A[x=0,
```

y=1] {-{-}{} B[x= /2,

y=0] {-}{-}{} C[x= ,

y={-}1]}

{Highlighting}
{Shaded}

Table of key points:

x	0	/4	/2	3 /4	
cos x	1	$\sqrt{2}/2$	0	-$\sqrt{2}/2$	-1

Properties:

- **Domain:** [0,]
- **Range:** [-1, 1]

- **Decreasing function in given interval**
- **Maximum at x = 0, y = 1**
- **Minimum at x = , y = -1**

0.0.24 Q3.3 [4 marks]

If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ **and** $\vec{c} = (-1, 2, -1)$ **then Find the direction cosines of** $3\vec{a} - 2\vec{b} + 4\vec{c}$**.**

Solution: $3\vec{a} = 3(3, -1, -4) = (9, -3, -12)$

$2\vec{b} = 2(-2, 4, -3) = (-4, 8, -6)$

$4\vec{c} = 4(-1, 2, -1) = (-4, 8, -4)$

$3\vec{a} - 2\vec{b} + 4\vec{c} = (9, -3, -12) - (-4, 8, -6) + (-4, 8, -4) = (9, -3, -12) + (4, -8, 6) + (-4, 8, -4) = (9 + 4 - 4, -3 - 8 + 8, -12 + 6 - 4) = (9, -3, -10)$

Magnitude: $|\vec{r}| = \sqrt{9^2 + (-3)^2 + (-10)^2} = \sqrt{81 + 9 + 100} = \sqrt{190}$

Direction cosines: $l = \frac{9}{\sqrt{190}}$, $m = \frac{-3}{\sqrt{190}}$, $n = \frac{-10}{\sqrt{190}}$

Q.4 (A) Attempt any two [6 marks]

0.0.25 Q4.1 [3 marks]

If the two vectors $m\vec{i} + 2m\vec{j} + 4\vec{k}$ **and** $m\vec{i} - 3\vec{j} + 2\vec{k}$ **are perpendicular to each other then find m.**

Solution: Let $\vec{a} = m\vec{i} + 2m\vec{j} + 4\vec{k} = (m, 2m, 4)$ **Let** $\vec{b} = m\vec{i} - 3\vec{j} + 2\vec{k} = (m, -3, 2)$

For perpendicular vectors: $\vec{a} \cdot \vec{b} = 0$

$(m, 2m, 4) \cdot (m, -3, 2) = 0$

$m \cdot m + 2m \cdot (-3) + 4 \cdot 2 = 0$

$m^2 - 6m + 8 = 0$

$(m - 2)(m - 4) = 0$

Therefore: $m = 2$ **or** $m = 4$

0.0.26 Q4.2 [3 marks]

Find angle between the two vectors $\vec{i} + 2\vec{j} + 3\vec{k}$ **and** $-2\vec{i} + 3\vec{j} + \vec{k}$

Solution: Let $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k} = (1, 2, 3)$ **Let** $\vec{b} = -2\vec{i} + 3\vec{j} + \vec{k} = (-2, 3, 1)$

$\vec{a} \cdot \vec{b} = (1)(-2) + (2)(3) + (3)(1) = -2 + 6 + 3 = 7$

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{(-2)^2 + 3^2 + 1^2} = \sqrt{14}$

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{7}{\sqrt{14} \times \sqrt{14}} = \frac{7}{14} = \frac{1}{2}$

Therefore: $\theta = \cos^{-1}(\frac{1}{2}) = 60°$

0.0.27 Q4.3 [3 marks]

Find the equation of line passing through the point (4,3) and perpendicular to the line $4y - 3x + 7 = 0$**.**

Solution: Given line: $4y - 3x + 7 = 0$ **Rewriting:** $4y = 3x - 7$**, so** $y = \frac{3}{4}x - \frac{7}{4}$

Slope of given line = $\frac{3}{4}$

For perpendicular line: slope = $-\frac{1}{\frac{3}{4}} = -\frac{4}{3}$

Using point-slope form with point (4, 3): $y - 3 = -\frac{4}{3}(x - 4)$

$y - 3 = -\frac{4}{3}x + \frac{16}{3}$

$y = -\frac{4}{3}x + \frac{16}{3} + 3 = -\frac{4}{3}x + \frac{16+9}{3}$

$y = -\frac{4}{3}x + \frac{25}{3}$

Equation: $4x + 3y - 25 = 0$

Q.4 (B) Attempt any two [8 marks]

0.0.28 Q4.1 [4 marks]

Find unit vector perpendicular to both vectors $\vec{a} = (3, 1, 2)$ **and** $\vec{b} = (2, -2, 4)$

Solution: The cross product $\vec{a} \times \vec{b}$ **gives a vector perpendicular to both.**

$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= \vec{i}(1 \times 4 - 2 \times (-2)) - \vec{j}(3 \times 4 - 2 \times 2) + \vec{k}(3 \times (-2) - 1 \times 2)$

$= \vec{i}(4 + 4) - \vec{j}(12 - 4) + \vec{k}(-6 - 2)$

$= 8\vec{i} - 8\vec{j} - 8\vec{k}$

$\vec{a} \times \vec{b} = (8, -8, -8)$

Magnitude: $|\vec{a} \times \vec{b}| = \sqrt{8^2 + (-8)^2 + (-8)^2} = \sqrt{64 + 64 + 64} = \sqrt{192} = 8\sqrt{3}$

Unit vector = $\frac{(8,-8,-8)}{8\sqrt{3}} = \frac{(1,-1,-1)}{\sqrt{3}} = (\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}})$

0.0.29 Q4.2 [4 marks]

Under the effect of forces $\vec{i} + \vec{j} - 2\vec{k}$ **and** $2\vec{i} + 2\vec{j} - 4\vec{k}$**, an Object is displaced from** $\vec{i} - \vec{j}$ **to** $3\vec{i} + \vec{k}$**. Find the work done.**

Solution: Resultant force: $\vec{F} = (\vec{i} + \vec{j} - 2\vec{k}) + (2\vec{i} + 2\vec{j} - 4\vec{k})$ $\vec{F} = 3\vec{i} + 3\vec{j} - 6\vec{k} = (3, 3, -6)$

Displacement: $\vec{s} = (3\vec{i} + \vec{k}) - (\vec{i} - \vec{j}) = 2\vec{i} + \vec{j} + \vec{k} = (2, 1, 1)$

Work done: $W = \vec{F} \cdot \vec{s}$ $W = (3, 3, -6) \cdot (2, 1, 1) = 3(2) + 3(1) + (-6)(1) = 6 + 3 - 6 = 3$

Work done = 3 units

0.0.30 Q4.3 [4 marks]

Find: $\lim_{x\to2} \frac{x^3-x^2-5x+6}{x^2-5x+6}$

Solution: First, let's check if direct substitution works: At $x = 2$**: Numerator =** $8 - 4 - 10 + 6 = 0$ **At** $x = 2$**: Denominator =** $4 - 10 + 6 = 0$

We get $\frac{0}{0}$ **form, so we need to factorize.**

Numerator: $x^3 - x^2 - 5x + 6$ **Let's check if** $(x - 2)$ **is a factor:** $2^3 - 2^2 - 5(2) + 6 = 8 - 4 - 10 + 6 = 0$

Using synthetic division: $x^3 - x^2 - 5x + 6 = (x - 2)(x^2 + x - 3)$

Denominator: $x^2 - 5x + 6$ **Factoring:** $x^2 - 5x + 6 = (x - 2)(x - 3)$

$\lim_{x\to2} \frac{x^3-x^2-5x+6}{x^2-5x+6} = \lim_{x\to2} \frac{(x-2)(x^2+x-3)}{(x-2)(x-3)}$

$= \lim_{x\to2} \frac{x^2+x-3}{x-3} = \frac{4+2-3}{2-3} = \frac{3}{-1} = -3$

Solution

Q.5 (A) Attempt any two [6 marks]

0.0.31 Q5.1 [3 marks]

Find: $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{2}{x^2-2x}\right)$

Solution: $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{2}{x^2-2x}\right)$

Note that $x^2-2x=x(x-2)$

$=\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{2}{x(x-2)}\right)$

$=\lim_{x\to 2}\frac{x-2}{x(x-2)}=\lim_{x\to 2}\frac{x-2}{x(x-2)}$

$=\lim_{x\to 2}\frac{1}{x}=\frac{1}{2}$

Solution

2

0.0.32 Q5.2 [3 marks]

Find: $\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{\frac{2x}{3}}$

Solution: This is of the form 1^∞**. Using the standard limit:** $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}=e^{ab}$

Here, $a=5$ **and** $b=\frac{2}{3}$

$\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{\frac{2x}{3}}=e^{5\times\frac{2}{3}}=e^{\frac{10}{3}}$

Solution

3

0.0.33 Q5.3 [3 marks]

Find: $\lim_{x\to 0}\frac{e^x+\sin x-1}{x}$

Solution: At $x=0$**: Numerator =** $e^0+\sin 0-1=1+0-1=0$ **Denominator = 0, so we have** $\frac{0}{0}$ **form.**

Using L'Hôpital's rule: $\lim_{x\to 0}\frac{e^x+\sin x-1}{x}=\lim_{x\to 0}\frac{e^x+\cos x}{1}$

$=e^0+\cos 0=1+1=2$

Solution

Q.5 (B) Attempt any two [8 marks]

0.0.34 Q5.1 [4 marks]

If two lines $kx+(2-k)y+3=0$ **and** $2x+(k+1)y-5=0$ **are parallel to each other then find the value of k.**

Solution: Two lines $a_1x+b_1y+c_1=0$ **and** $a_2x+b_2y+c_2=0$ **are parallel if:** $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Given lines:

- **Line 1:** $kx+(2-k)y+3=0$**, so** $a_1=k$**,** $b_1=2-k$**,** $c_1=3$
- **Line 2:** $2x+(k+1)y-5=0$**, so** $a_2=2$**,** $b_2=k+1$**,** $c_2=-5$

For parallel lines: $\frac{k}{2}=\frac{2-k}{k+1}$

Cross multiplying: $k(k+1)=2(2-k)$ k^2+

k = 4 - 2k

$k^2+k+2k-4=0$ $k^2+3k-4=0$ $(k+4)(k-1)=0$

So $k = -4$ or $k = 1$

Checking if lines are not identical: For $k = 1$: $\frac{c_1}{c_2} = \frac{3}{-5} = -\frac{3}{5}$ and $\frac{a_1}{a_2} = \frac{1}{2}$ $(\neq -\frac{3}{5})$

For $k = -4$: $\frac{c_1}{c_2} = \frac{3}{-5} = -\frac{3}{5}$ and $\frac{a_1}{a_2} = \frac{-4}{2} = -2$ $(\neq -\frac{3}{5})$

Therefore: $k = 1$ or $k = -4$

0.0.35 Q5.2 [4 marks]

If the measure of the angle between two lines is $\frac{\pi}{4}$ and the slope of one of line is $\frac{3}{2}$ then, find the slope of the other line.

Solution: Let $m_1 = \frac{3}{2}$ and m_2 be the slope of the other line.

The angle between two lines with slopes m_1 and m_2 is given by: $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Given: $\theta = \frac{\pi}{4}$, so $\tan\frac{\pi}{4} = 1$

$1 = \left|\frac{\frac{3}{2} - m_2}{1 + \frac{3}{2}m_2}\right|$

$1 = \left|\frac{\frac{3}{2} - m_2}{\frac{2+3m_2}{2}}\right| = \left|\frac{3 - 2m_2}{2 + 3m_2}\right|$

This gives us two cases: Case 1: $\frac{3-2m_2}{2+3m_2} = 1$ $3 - 2m_2 = 2 + 3m_2$ $3 - 2 = 3m_2 + 2m_2$ $1 = 5m_2$ $m_2 = \frac{1}{5}$

Case 2: $\frac{3-2m_2}{2+3m_2} = -1$ $3 - 2m_2 = -(2 + 3m_2)$ $3 - 2m_2 = -2 - 3m_2$ $3 + 2 = -3m_2 + 2m_2$ $5 = -m_2$ $m_2 = -5$

Therefore: $m_2 = \frac{1}{5}$ or $m_2 = -5$

0.0.36 Q5.3 [4 marks]

Find equation of tangent to the circle $2x^2 + 2y^2 + 3x - 4y + 1 = 0$ at the point (-1, 2)

Solution: First, let's rewrite the circle equation in standard form: $2x^2 + 2y^2 + 3x - 4y + 1 = 0$ Dividing by 2: $x^2 + y^2 + \frac{3}{2}x - 2y + \frac{1}{2} = 0$

For a circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the equation of tangent at point (x_1, y_1) is: $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

Comparing: $2g = \frac{3}{2}$, so $g = \frac{3}{4}$ $2f = -2$, so $f = -1$ $c = \frac{1}{2}$

At point $(-1, 2)$: $x(-1) + y(2) + \frac{3}{4}(x + (-1)) + (-1)(y + 2) + \frac{1}{2} = 0$

$-x + 2y + \frac{3}{4}x - \frac{3}{4} - y - 2 + \frac{1}{2} = 0$

$-x + \frac{3}{4}x + 2y - y - \frac{3}{4} - 2 + \frac{1}{2} = 0$

$-\frac{1}{4}x + y - \frac{3}{4} - \frac{4}{2} + \frac{1}{2} = 0$

$-\frac{1}{4}x + y - \frac{3}{4} - 2 + \frac{1}{2} = 0$

$-\frac{1}{4}x + y - \frac{9}{4} = 0$

Multiplying by 4: $-x + 4y - 9 = 0$

Equation of tangent: $x - 4y + 9 = 0$

Formula Cheat Sheet

0.0.37 Trigonometry

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

0.0.38 Limits

- $\lim_{x\to 0}\frac{\sin x}{x} = 1$
- $\lim_{x\to\infty}\left(1 + \frac{a}{x}\right)^{bx} = e^{ab}$
- $\lim_{x\to a}\frac{x^n - a^n}{x - a} = na^{n-1}$

0.0.39 Vectors

- **Dot product:** $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$
- **Cross product:** $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$
- **Work done:** $W = \vec{F} \cdot \vec{s}$

0.0.40 Circle

- **Standard form:** $(x-h)^2 + (y-k)^2 = r^2$
- **Area:** πr^2
- **Tangent at (x_1, y_1):** $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$

Problem-solving Strategies

For Determinants:

- **Expand along the row/column with most zeros**
- **Factor out common terms first**

For Limits:

- **Check for $\frac{0}{0}$ or $\frac{\infty}{\infty}$ forms**
- **Use L'Hôpital's rule or factorization**
- **Recognize standard limit forms**

For Vectors:

- **Use component form for calculations**
- **Remember cross product gives perpendicular vector**
- **Dot product = 0 for perpendicular vectors**

Common Mistakes to Avoid

- **Sign errors in determinant expansion**
- **Forgetting degree-radian conversion:** $180° = \pi$ **radians**
- **Not simplifying trigonometric expressions using identities**
- **Wrong limit evaluation - always check if direct substitution works first**
- **Vector operations - don't confuse dot and cross products**

Exam Tips

- **Time management: Spend 1-2 minutes per mark**
- **Show all steps for partial credit**
- **Check answers by substitution where possible**
- **Use standard values for trigonometric functions**
- **Draw diagrams for vector and geometry problems**