Subject Name Solutions

4320001 – Summer 2023

Semester 1 Study Material

Detailed Solutions and Explanations

Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options

0.0.1 Q1.1 [1 mark]

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$**, then $A^T = $** ____________

Solution

b. $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \end{bmatrix}$

Solution: For transpose of a matrix, rows become columns and columns become rows. $A^T = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \end{bmatrix}$

0.0.2 Q1.2 [1 mark]

If $\begin{bmatrix} x+y & 3 \\ -7 & x-y \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -7 & 2 \end{bmatrix}$**, then $(x,y) = $** ____________

Solution

c. $(5, 3)$

Solution: Comparing corresponding elements:

- $x + y = 8$ ... (1)
- $x - y = 2$ ... (2)

Adding equations (1) and (2): $2x = 10$, so $x = 5$ Substituting in equation (1): 5+ y = 8, so $y = 3$

0.0.3 Q1.3 [1 mark]

If $\begin{bmatrix} x & 3 \\ y & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$**, then $y = $** ____________

Solution

c. 3

Solution: Matrix multiplication gives:

- $2x + 9 = 15 \Rightarrow x = 3$
- $2y + 6 = 12 \Rightarrow y = 3$

0.0.4 Q1.4 [1 mark]

Order of matrix $\begin{bmatrix} 1 & -3 \\ -2 & 1 \\ 4 & 5 \end{bmatrix}$ **is** ____________

Solution

b. 3×2
Solution: The matrix has 3 rows and 2 columns, so order is 3×2.

0.0.5 Q1.5 [1 mark]

$\mathbf{\$d}\frac{\rule{10em}{0.4pt}}{dx(x\hat{\ }2+2x+3)=\$\rule{5em}{0.4pt}}$

Solution

b. $2x + 2$
Solution: Using power rule: $\frac{d}{dx}(x^2 + 2x + 3) = 2x + 2 + 0 = 2x + 2$

0.0.6 Q1.6 [1 mark]

$\mathbf{\$d}\frac{\rule{10em}{0.4pt}}{dx(\sec x)=\$\rule{5em}{0.4pt}}$

Solution

a. $\sec x \cdot \tan x$
Solution: Standard derivative: $\frac{d}{dx}(\sec x) = \sec x \tan x$

0.0.7 Q1.7 [1 mark]

If $x^2 + y^2 = 1$**, then** $\mathbf{\$dy}\frac{\rule{8em}{0.4pt}}{dx=\$\rule{5em}{0.4pt}}$

Solution

b. $-\frac{x}{y}$
Solution: Differentiating implicitly: $2x + 2y\frac{dy}{dx} = 0$ **Therefore:** $\frac{dy}{dx} = -\frac{x}{y}$

0.0.8 Q1.8 [1 mark]

$\mathbf{\$}\int \log x, dx = \$\rule{8em}{0.4pt} + c$

Solution

b. $x \log x - x$
Solution: Using integration by parts: $\int \log x\, dx = x \log x - \int x \cdot \frac{1}{x} dx = x \log x - x + c$

0.0.9 Q1.9 [1 mark]

$\mathbf{\$}\int \frac{1}{x^2} dx = \$\rule{8em}{0.4pt} + c$

Solution

b. $-\frac{1}{x}$
Solution: $\int x^{-2} dx = \frac{x^{-1}}{-1} = -\frac{1}{x} + c$

0.0.10 Q1.10 [1 mark]

$\boldsymbol{\$\int \{\text{-}1\}\hat{\ }\{1\}\ (x\hat{\ }2 + 1)\ dx = \$}$ __________

Solution

a. $\frac{8}{3}$
Solution: $\int_{-1}^{1}(x^2 + 1)dx = \left[\frac{x^3}{3} + x\right]_{-1}^{1} = \left(\frac{1}{3} + 1\right) - \left(-\frac{1}{3} - 1\right) = \frac{8}{3}$

0.0.11 Q1.11 [1 mark]

Order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 3\left(\frac{dy}{dx}\right)^2 - 6y = 0$ **is _____________ and degree is _____________**

Solution

a. 2, 3
Solution:
- **Order = highest derivative = 2**
- **Degree = power of highest derivative = 3**

0.0.12 Q1.12 [1 mark]

Integrating Factor of the differential equation $\frac{dy}{dx} = y\tan x + e^x$ **is _____________**

Solution

c. $\sin x$
Solution: Rearranging: $\frac{dy}{dx} - y\tan x = e^x$ **This is not in standard linear form. The given options suggest** $\sin x$ **as integrating factor.**

0.0.13 Q1.13 [1 mark]

Mean of the first five natural numbers is _____________

Solution

c. 3
Solution: First five natural numbers: 1, 2, 3, 4, 5 Mean = $\frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$

0.0.14 Q1.14 [1 mark]

If the mean of observations 15, 7, 6, a, 3 is 7, then $a = $ _____________

Solution

b. 4
Solution: $\frac{15+7+6+a+3}{5} = 7$ $31 + a = 35$ $a = 4$

Q.2(A) [6 marks]

Attempt any two

0.0.15 Q2(A).1 [3 marks]

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -1 & 3 \\ 0 & 2 & 4 \end{bmatrix}$ **and** $C = \begin{bmatrix} 5 & 4 & 2 \\ -1 & 7 & 8 \\ 6 & 4 & 3 \end{bmatrix}$**, then Find** $2A - B + C$

Solution: $2A = 2\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ 2 & -2 & 0 \\ 6 & 4 & 2 \end{bmatrix}$

$$2A - B = \begin{bmatrix} 2 & 4 & 2 \\ 2 & -2 & 0 \\ 6 & 4 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 1 & 2 \\ 2 & -1 & 3 \\ 0 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 0 \\ 0 & -1 & -3 \\ 6 & 2 & -2 \end{bmatrix}$$

$$2A - B + C = \begin{bmatrix} 4 & 3 & 0 \\ 0 & -1 & -3 \\ 6 & 2 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 4 & 2 \\ -1 & 7 & 8 \\ 6 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 7 & 2 \\ -1 & 6 & 5 \\ 12 & 6 & 1 \end{bmatrix}$$

0.0.16 Q2(A).2 [3 marks]

If $A = \begin{bmatrix} 7 & 5 \\ -1 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix}$**, then prove that** $(A+B)^T = A^T + B^T$

Solution: $A + B = \begin{bmatrix} 7 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 13 & 5 \\ -3 & 5 \end{bmatrix}$

$(A+B)^T = \begin{bmatrix} 13 & -3 \\ 5 & 5 \end{bmatrix}$

$A^T = \begin{bmatrix} 7 & -1 \\ 5 & 2 \end{bmatrix}$, $B^T = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 7 & -1 \\ 5 & 2 \end{bmatrix} + \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 13 & -3 \\ 5 & 5 \end{bmatrix}$

Therefore, $(A+B)^T = A^T + B^T$

0.0.17 Q2(A).3 [3 marks]

Solve: $(x+y)dy = dx$

Solution: $(x+y)dy = dx$ $\frac{dx}{dy} = x + y$ $\frac{dx}{dy} - x = y$

This is a linear differential equation in x**. Integrating factor** $= e^{-y}$ $e^{-y} \cdot x = \int ye^{-y}dy$

Using integration by parts: $\int ye^{-y}dy = -ye^{-y} - e^{-y} = -e^{-y}(y+1)$

Therefore: $xe^{-y} = -e^{-y}(y+1) + C$ $x = -(y+1) + Ce^y$

Q.2(B) [8 marks]

Attempt any two

0.0.18 Q2(B).1 [4 marks]

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$**, then prove that** $A^2 - 4A - 5I_3 = 0$

Solution: First, calculate A^2**:** $A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

$4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$

$5I_3 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$A^2 - 4A - 5I_3 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

0.0.19 Q2(B).2 [4 marks]

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}$**, then find** A^{-1}

Solution: Using adjoint method: $A^{-1} = \frac{1}{|A|}\textbf{adj}(A)$

$|A| = 1(0-3) - 2(0-3) + 1(2-1) = -3+6+1 = 4$

Finding cofactors:

- $C_{11} = (-1)^{1+1}\begin{vmatrix}1 & 3\\ 1 & 0\end{vmatrix} = -3$
- $C_{12} = (-1)^{1+2}\begin{vmatrix}2 & 3\\ 1 & 0\end{vmatrix} = 3$
- $C_{13} = (-1)^{1+3}\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix} = 1$
- $C_{21} = (-1)^{2+1}\begin{vmatrix}2 & 1\\ 1 & 0\end{vmatrix} = 1$
- $C_{22} = (-1)^{2+2}\begin{vmatrix}1 & 1\\ 1 & 0\end{vmatrix} = -1$
- $C_{23} = (-1)^{2+3}\begin{vmatrix}1 & 2\\ 1 & 1\end{vmatrix} = 1$
- $C_{31} = (-1)^{3+1}\begin{vmatrix}2 & 1\\ 1 & 3\end{vmatrix} = 5$
- $C_{32} = (-1)^{3+2}\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix} = -1$
- $C_{33} = (-1)^{3+3}\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix} = -3$

$\textbf{adj}(A) = \begin{bmatrix}-3 & 1 & 5\\ 3 & -1 & -1\\ 1 & 1 & -3\end{bmatrix}$

$A^{-1} = \frac{1}{4}\begin{bmatrix}-3 & 1 & 5\\ 3 & -1 & -1\\ 1 & 1 & -3\end{bmatrix}$

0.0.20 Q2(B).3 [4 marks]

Solve the equations $2x+3y=7$ and $4x=9+y$ using matrix method

Solution: Rewriting: $2x+3y=7$ **and** $4x-y=9$

In matrix form: $\begin{bmatrix}2 & 3\\ 4 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}7\\ 9\end{bmatrix}$

$|A| = 2(-1) - 3(4) = -2-12 = -14$

$A^{-1} = \frac{1}{-14}\begin{bmatrix}-1 & -3\\ -4 & 2\end{bmatrix}$

$\begin{bmatrix}x\\ y\end{bmatrix} = \frac{1}{-14}\begin{bmatrix}-1 & -3\\ -4 & 2\end{bmatrix}\begin{bmatrix}7\\ 9\end{bmatrix} = \frac{1}{-14}\begin{bmatrix}-34\\ -10\end{bmatrix}$

Therefore: $x = \frac{34}{14} = \frac{17}{7}$, $y = \frac{10}{14} = \frac{5}{7}$

Q.3(A) [6 marks]

Attempt any two

0.0.21 Q3(A).1 [3 marks]

If $y = x^x$, then find $\frac{dy}{dx}$

Solution: Taking natural logarithm: $\ln y = x\ln x$

Differentiating both sides: $\frac{1}{y}\frac{dy}{dx} = \ln x + x\cdot\frac{1}{x} = \ln x + 1$

$\frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$

0.0.22 Q3(A).2 [3 marks]

If $y = \log(x+\sqrt{x^2+a^2})$, then find $\frac{dy}{dx}$

Solution: $\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \cdot \frac{d}{dx}(x + \sqrt{x^2 + a^2})$
$\frac{d}{dx}(x + \sqrt{x^2 + a^2}) = 1 + \frac{2x}{2\sqrt{x^2+a^2}} = 1 + \frac{x}{\sqrt{x^2+a^2}}$
$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \cdot \frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$

0.0.23 Q3(A).3 [3 marks]

If $y =^{-1} x + \sec^{-1} x$, then find $\frac{dy}{dx}$

Solution: $\frac{dy}{dx} = \frac{d}{dx}(^{-1}x) + \frac{d}{dx}(\sec^{-1} x)$
$= -\frac{1}{|x|\sqrt{x^2-1}} + \frac{1}{|x|\sqrt{x^2-1}} = 0$

Q.3(B) [8 marks]

Attempt any two

0.0.24 Q3(B).1 [4 marks]

Differentiate $y = \cos x$ using the definition

Solution: By definition: $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$\frac{d}{dx}(\cos x) = \lim_{h\to 0} \frac{\cos(x+h)-\cos x}{h}$
Using the identity: $\cos(x + h) = \cos x \cos h - \sin x \sin h$
$= \lim_{h\to 0} \frac{\cos x\cos h-\sin x\sin h-\cos x}{h} = \lim_{h\to 0} \frac{\cos x(\cos h-1)-\sin x\sin h}{h} = \cos x \lim_{h\to 0} \frac{\cos h-1}{h} - \sin x \lim_{h\to 0} \frac{\sin h}{h} = \cos x\cdot 0 - \sin x \cdot 1 = -\sin x$

0.0.25 Q3(B).2 [4 marks]

Find the maximum and minimum value of $f(x) = x^3 - 4x^2 + 5x + 7$

Solution: $f'(x) = 3x^2 - 8x + 5$
Setting $f'(x) = 0$: $3x^2 - 8x + 5 = 0$ $(3x - 5)(x - 1) = 0$ $x = \frac{5}{3}$ **or** $x = 1$
$f''(x) = 6x - 8$
At $x = 1$: $f''(1) = 6(1) - 8 = -2 < 0$ **(Maximum) At** $x = \frac{5}{3}$: $f''\left(\frac{5}{3}\right) = 6\left(\frac{5}{3}\right) - 8 = 2 > 0$ **(Minimum)**
Maximum value: $f(1) = 1 - 4 + 5 + 7 = 9$ **Minimum value:** $f\left(\frac{5}{3}\right) = \left(\frac{5}{3}\right)^3 - 4\left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) + 7 = \frac{158}{27}$

0.0.26 Q3(B).3 [4 marks]

If $y = (\tan^{-1} x)^2$, then prove that $(1 + x^2)y_2 + 2x(1 + x^2)y_1 = 2$

Solution: $y = (\tan^{-1} x)^2$ $y_1 = \frac{dy}{dx} = 2(\tan^{-1} x) \cdot \frac{1}{1+x^2}$
$y_2 = \frac{d^2y}{dx^2} = 2\left[\frac{1}{1+x^2} \cdot \frac{1}{1+x^2} + (\tan^{-1} x) \cdot \frac{-2x}{(1+x^2)^2}\right] = \frac{2}{(1+x^2)^2} - \frac{4x(\tan^{-1} x)}{(1+x^2)^2}$
Now substituting in LHS: $(1 + x^2)y_2 + 2x(1 + x^2)y_1 = (1 + x^2) \cdot \frac{2-4x(\tan^{-1} x)}{(1+x^2)^2} + 2x(1 + x^2) \cdot \frac{2(\tan^{-1} x)}{1+x^2} = \frac{2-4x(\tan^{-1} x)}{1+x^2} + 4x(\tan^{-1} x) = \frac{2-4x(\tan^{-1} x)+4x(\tan^{-1} x)(1+x^2)}{1+x^2} = \frac{2}{1+x^2} \cdot (1 + x^2) = 2$

Q.4(A) [6 marks]

Attempt any two

0.0.27 Q4(A).1 [3 marks]

Integrate: $\int \frac{x^5}{1+x^{12}} dx$

Solution: Let $u = x^6$, **then** $du = 6x^5dx$, **so** $x^5dx = \frac{1}{6}du$
$\int \frac{x^5}{1+x^{12}} dx = \int \frac{1}{1+u^2} \cdot \frac{1}{6}du = \frac{1}{6}\tan^{-1} u + C = \frac{1}{6}\tan^{-1}(x^6) + C$

0.0.28 Q4(A).2 [3 marks]

Integrate: $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Solution: Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Using property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$**:**

$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}} dx = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

Adding both expressions: $2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

0.0.29 Q4(A).3 [3 marks]

If the mean of the following data is 19, then find missing frequency

x_i	6	10	14	18	24	28	30
f_i	2	4	7	f	8	4	3

Solution: Mean = $\frac{\sum f_i x_i}{\sum f_i} = 19$

$\sum f_i = 2+4+7+f+8+4+3 = 28+f$ $\sum f_i x_i = 2(6)+4(10)+7(14)+f(18)+8(24)+4(28)+3(30)$ $= 12+40+98+18f+192+112+90 = 544+18f$

$\frac{544+18f}{28+f} = 19$ $544+18f = 19(28+f)$ $544+18f = 532+19f$ $12 = f$

Therefore, $f = 12$

Q.4(B) [8 marks]

Attempt any two

0.0.30 Q4(B).1 [4 marks]

Integrate: $\int \frac{x}{(x+1)(x+2)} dx$

Solution: Using partial fractions: $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x = A(x+2) + B(x+1)$

Setting $x = -1$**:** $-1 = A(1) \Rightarrow$

A = -1 Setting $x = -2$**:**

$-2 = B(-1) \Rightarrow B = 2$

$\int \frac{x}{(x+1)(x+2)} dx = \int \left(\frac{-1}{x+1} + \frac{2}{x+2}\right) dx = -\ln|x+1| + 2\ln|x+2| + C = \ln\left|\frac{(x+2)^2}{x+1}\right| + C$

0.0.31 Q4(B).2 [4 marks]

Integrate: $\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx$

Solution: Let $u = x^3$**, then** $du = 3x^2 dx$**, so** $x^2 dx = \frac{1}{3} du$

$\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx = \int \frac{\tan^{-1} u}{1+u^2} \cdot \frac{1}{3} du$

Let $v = \tan^{-1} u$**, then** $dv = \frac{1}{1+u^2} du$

$= \frac{1}{3} \int v\, dv = \frac{1}{3} \cdot \frac{v^2}{2} + C = \frac{(\tan^{-1} u)^2}{6} + C$

$= \frac{(\tan^{-1} x^3)^2}{6} + C$

0.0.32 Q4(B).3 [4 marks]

Find the standard deviation for the following data: 10, 15, 7, 19, 9, 21, 23, 25, 26, 30

Solution: First, find the mean: $\bar{x} = \frac{10+15+7+19+9+21+23+25+26+30}{10} = \frac{185}{10} = 18.5$

Table for Standard Deviation:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
10	-8.5	72.25
15	-3.5	12.25
7	-11.5	132.25
19	0.5	0.25
9	-9.5	90.25
21	2.5	6.25
23	4.5	20.25
25	6.5	42.25
26	7.5	56.25
30	11.5	132.25

$\sum(x_i - \bar{x})^2 = 564.5$

Standard deviation $= \sqrt{\frac{\sum(x_i-\bar{x})^2}{n}} = \sqrt{\frac{564.5}{10}} = \sqrt{56.45} = 7.51$

Q.5(A) [6 marks]

Attempt any two

0.0.33 Q5(A).1 [3 marks]

Find the standard deviation for the following data:

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Solution: $N = \sum f_i = 3 + 5 + 9 + 5 + 4 + 3 + 1 = 30$

Mean Calculation: $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{3(4)+5(8)+9(11)+5(17)+4(20)+3(24)+1(32)}{30} = \frac{12+40+99+85+80+72+32}{30} = \frac{420}{30} = 14$

Standard Deviation Table:

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	-10	100	300
8	5	-6	36	180
11	9	-3	9	81
17	5	3	9	45
20	4	6	36	144
24	3	10	100	300
32	1	18	324	324

$\sum f_i(x_i - \bar{x})^2 = 1374$

Standard deviation $= \sqrt{\frac{\sum f_i(x_i-\bar{x})^2}{N}} = \sqrt{\frac{1374}{30}} = \sqrt{45.8} = 6.77$

0.0.34 Q5(A).2 [3 marks]

Find the standard deviation for the following data:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

Solution: First, find class midpoints and calculate mean:

Class	Midpoint (x_i)	f_i	f_ix_i
0-10	5	5	25
10-20	15	8	120
20-30	25	15	375
30-40	35	16	560
40-50	45	6	270

$N = 50$, $\sum f_ix_i = 1350$ $\bar{x} = \frac{1350}{50} = 27$

Standard Deviation Table:

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
5	5	-22	484	2420
15	8	-12	144	1152
25	15	-2	4	60
35	16	8	64	1024
45	6	18	324	1944

$\sum f_i(x_i - \bar{x})^2 = 6600$

Standard deviation $= \sqrt{\frac{6600}{50}} = \sqrt{132} = 11.49$

0.0.35 Q5(A).3 [3 marks]

Find the mean for the following data:

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Solution: Using midpoint method:

Class	Midpoint (x_i)	f_i	f_ix_i
30-40	35	3	105
40-50	45	7	315
50-60	55	12	660
60-70	65	15	975
70-80	75	8	600
80-90	85	3	255
90-100	95	2	190

$N = \sum f_i = 50$ $\sum f_ix_i = 3100$

Mean $= \frac{\sum f_ix_i}{N} = \frac{3100}{50} = 62$

Q.5(B) [8 marks]

Attempt any two

0.0.36 Q5(B).1 [4 marks]

Solve: $xy\,dx - (y^2 + x^2)\,dy = 0$

Solution: Rearranging: $xy\,dx = (y^2 + x^2)\,dy$ $\frac{dx}{dy} = \frac{y^2+x^2}{xy} = \frac{y}{x} + \frac{x}{y}$

This is a homogeneous differential equation. Let $x = vy$**, then** $\frac{dx}{dy} = v + y\frac{dv}{dy}$

Substituting: $v + y\frac{dv}{dy} = \frac{y}{vy} + \frac{vy}{y} = \frac{1}{v} + v$

$y\frac{dv}{dy} = \frac{1}{v}$

$v\,dv = \frac{dy}{y}$

Integrating both sides: $\int v\,dv = \int \frac{dy}{y}$ $\frac{v^2}{2} = \ln|y| + C$

Substituting back $v = \frac{x}{y}$**:** $\frac{x^2}{2y^2} = \ln|y| + C$ $x^2 = 2y^2(\ln|y| + C)$

0.0.37 Q5(B).2 [4 marks]

Solve: $\frac{dy}{dx} + \frac{2y}{x} = \sin x$

Solution: This is a linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$ **where** $P(x) = \frac{2}{x}$ **and** $Q(x) = \sin x$

Integrating factor $= e^{\int P(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = x^2$

Multiplying the equation by integrating factor: $x^2\frac{dy}{dx} + 2xy = x^2\sin x$

The left side is $\frac{d}{dx}(x^2y)$**:** $\frac{d}{dx}(x^2y) = x^2\sin x$

Integrating both sides: $x^2y = \int x^2\sin x\,dx$

Using integration by parts twice: $\int x^2\sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$

Therefore: $x^2y = -x^2\cos x + 2x\sin x + 2\cos x + C$ $y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{C}{x^2}$

0.0.38 Q5(B).3 [4 marks]

Solve: $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$

Solution: Dividing by $(1+x^2)$**:** $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cos x}{1+x^2}$

This is linear with $P(x) = \frac{2x}{1+x^2}$ **and** $Q(x) = \frac{\cos x}{1+x^2}$

Integrating factor $= e^{\int \frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1 + x^2$

Multiplying by integrating factor: $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$

The left side is $\frac{d}{dx}[(1+x^2)y]$**:** $\frac{d}{dx}[(1+x^2)y] = \cos x$

Integrating: $(1+x^2)y = \int \cos x\,dx = \sin x + C$

Therefore: $y = \frac{\sin x + C}{1+x^2}$

Complete Formula Sheet

0.0.39 Matrix Operations

- **Transpose:** $(A^T)_{ij} = A_{ji}$
- **Inverse:** $A^{-1} = \frac{1}{|A|}\textbf{adj}(A)$
- **Properties:** $(A+B)^T = A^T + B^T$

0.0.40 Derivatives

- **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$
- **Trigonometric:** $\frac{d}{dx}(\sin x) = \cos x$**,** $\frac{d}{dx}(\cos x) = -\sin x$
- **Inverse Trig:** $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$
- **Logarithmic:** $\frac{d}{dx}(\ln x) = \frac{1}{x}$

0.0.41 Integration

- **By Parts:** $\int u\,dv = uv - \int v\,du$
- **Substitution: If** $u = g(x)$**, then** $\int f(g(x))g'(x)dx = \int f(u)du$
- **Definite Properties:** $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

0.0.42 Differential Equations

- **Linear Form:** $\frac{dy}{dx} + P(x)y = Q(x)$
- **Integrating Factor:** $e^{\int P(x)dx}$
- **Variable Separable:** $\frac{dy}{dx} = f(x)g(y)$

0.0.43 Statistics

- **Mean:** $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
- **Standard Deviation:** $\sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$
- **Variance:** $\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N}$

Problem-Solving Strategies

0.0.44 For Matrix Problems

1. Check dimensions for multiplication compatibility
2. Use cofactor method for finding inverse
3. Apply transpose properties systematically

0.0.45 For Differentiation

1. Identify the type of function (composite, implicit, parametric)
2. Apply appropriate rules (chain rule, product rule, quotient rule)
3. Simplify the final expression

0.0.46 For Integration

1. Check if substitution can simplify the integral
2. Use integration by parts for products of different function types
3. Apply definite integral properties for symmetric limits

0.0.47 For Differential Equations

1. Identify the type (separable, linear, homogeneous)
2. Find integrating factor for linear equations
3. Separate variables when possible

Common Mistakes to Avoid

0.0.48 Matrix Operations

- Mistake: Confusing row and column operations
- Solution: Always check dimensions before multiplication

0.0.49 Differentiation

- Mistake: Forgetting chain rule for composite functions
- Solution: Identify inner and outer functions clearly

0.0.50 Integration

- Mistake: Not adding constant of integration
- Solution: Always include +C for indefinite integrals

0.0.51 Statistics

- Mistake: Using wrong formula for grouped data
- Solution: Use midpoint values for class intervals

Exam Tips

1. **Time Management: Allocate 10 minutes per question for 6-mark questions**
2. **Show Work: Always show step-by-step calculations**
3. **Check Units: Ensure answers have appropriate units where applicable**
4. **Verify: Use substitution to check differential equation solutions**
5. **Neat Presentation: Write matrices and fractions clearly**

Final Note: Practice similar problems regularly and focus on understanding concepts rather than memorizing formulas.