Subject Name Solutions

unit – Study Material

Semester 1 Study Material

Detailed Solutions and Explanations

Unit-2. Electrostatics - Solutions

Part A: Definitions with Standard Units (1 or 2 marks)

0.0.1 (1) Give definitions with its standard unit:

Electric Field (E): Definition: The electric field at a point is defined as the force experienced by a unit positive charge placed at that point. It represents the effect of an electric charge in the surrounding space.

Mathematical Formula:

```
E = F / q_{0}
```

Where:

- E = Electric field intensity
- F = Electric force experienced
- $q_0 = Testcharge(verysmallpositivecharge)$

For a point charge Q at distance r:

```
E = kQ / r^{2}  or

E = Q / (4 _{0}r^{2})
```

SI Unit: Newton per Coulomb (N/C) or Volt per meter (V/m)

Direction: Electric field is a vector quantity. It points away from positive charges and towards negative charges.

Electric Potential (V): Definition: The electric potential at a point in an electric field is defined as the amount of work done in bringing a unit positive charge from infinity to that point without acceleration.

Mathematical Formula:

```
V = W / q
```

Where:

- V = Electric potential
- W = Work done
- q = Charge

For a point charge Q at distance r:

```
V = kQ / r  or

V = Q / (4 _{0}r)
```

SI Unit: Volt (V) or Joule per Coulomb (J/C)

Note: Electric potential is a scalar quantity.

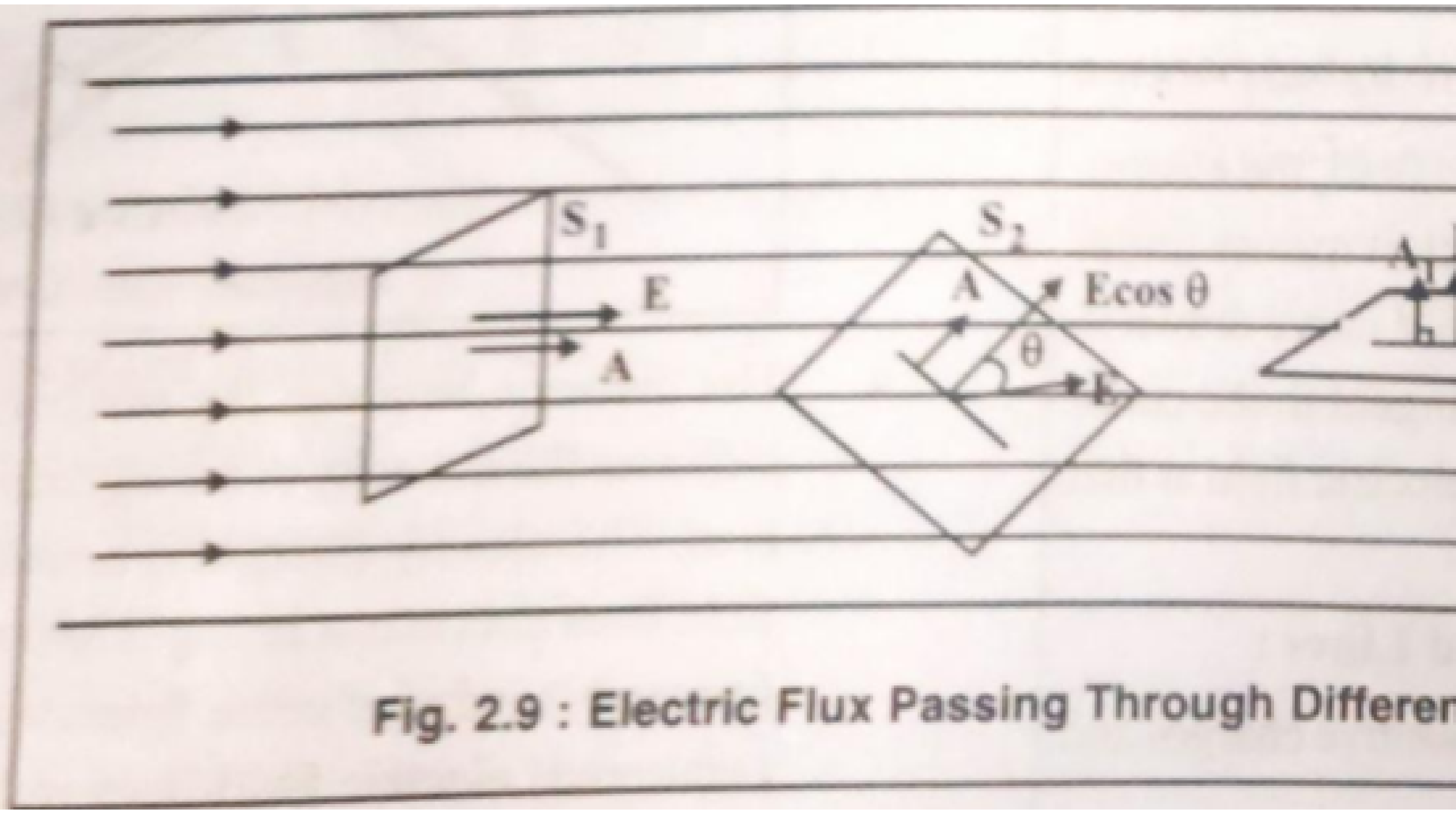

Fig. 2.9 : Electric Flux Passing Through Differen

Figure 1: Electric Flux Diagram

Electric Potential Difference (ΔV or V): **Definition:** The potential difference between two points in an electric field is defined as the work done in moving a unit positive charge from one point to another against the electric field.

Mathematical Formula:

$$V_{2} - V_{1} = W / q$$

or

$$\Delta V = W / q$$

Where:

- $V_2 - V_1 = Potentialdifferencebetweenpoints2and1$
- W = Work done
- q = Charge moved

Relation with Electric Field:

$$V = -E \times d \quad \text{(for uniform field)}$$

SI Unit: Volt (V)

Note: Potential difference is also called voltage.

Electric Flux (Φ): **Definition:** Electric flux through a surface is defined as the total number of electric field lines passing perpendicularly through that surface. It measures the quantity of electric field passing through a given area.

Figure: Electric flux through different orientations of surface area in electric field

Mathematical Formula:

$$\Phi = E \cdot$$

$$A = EA \cos$$

Where:

- Φ = Electric flux
- E = Electric field intensity
- A = Area of the surface
- = Angle between electric field and normal to the surface

Special Cases:

- When
  $= 0°(perpendicular)$:
Φ = EA (maximum)
- When
  $= 90°(parallel)$:
Φ = 0 (minimum)

SI Unit: Newton meter$^2 perCoulomb(Nm^2/C)orVoltmeter(Vm)$

Capacitor: Definition: A capacitor is an electrical device that stores electrical energy in the form of electric charge. It consists of two conducting plates separated by an insulating material (dielectric).

Construction: Two parallel conducting plates separated by a small distance with air or dielectric material between them.

Function:

- Stores electric charge and energy
- Blocks DC and allows AC
- Used in filtering, timing circuits, energy storage

Types:

1. Fixed capacitors (paper, mica, ceramic, electrolytic)
2. Variable capacitors
3. Parallel plate capacitors
4. Spherical capacitors
5. Cylindrical capacitors

Capacitance (C): Definition: Capacitance is the ability of a capacitor to store electric charge. It is defined as the ratio of charge stored on one plate to the potential difference between the plates.

Mathematical Formula:

```
C = Q / V
```

Where:

- C = Capacitance
- Q = Charge stored on one plate
- V = Potential difference between plates

For Parallel Plate Capacitor:

```
C =  _{0} A /

d =  _{0}KA / d
```

Where:

- $_0 = Permittivityoffreespace = 8.85 \times 10^{-12} F/m$
- or K = Relative permittivity (dielectric constant)
- A = Area of each plate
- d = Distance between plates

SI Unit: Farad (F)

Practical Units:

- Microfarad (F) = $10^{-6}F$
- Nanofarad (nF) = $10^{-9}F$
- Picofarad (pF) = $10^{-12}F$

Note: Capacitance depends on:

1. Area of plates (C A)
2. Distance between plates (C 1/d)
3. Dielectric medium (C K)

Part B: Detailed Answers (2 or 3 marks)

0.0.2 (1) Explain Coulomb's law with mathematical formula.

Solution

Coulomb's Law: French scientist Charles Augustin de Coulomb (1736-1806) conducted experiments to find the force between two electric charges and formulated Coulomb's law.
Figure: Electric force between two point charges $q_1 and q_2$
Statement: *"The electric force (Coulombian force) between two stationary point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. This force acts along the line joining the two charges."*
Mathematical Formula:

```
F   q_{1}q_{2}  (Force is directly proportional to product of charges)

F   1/r^{2}  (Force is inversely proportional to square of distance)

Combining both:

F = k(q_{1}q_{2})/r^{2}
```

Where:

- *F = Electric force between charges (N)*
- $q_1, q_2 = Magnitudes of two point charges (C)$
- *r = Distance between the charges (m)*
- *k = Coulomb's constant = 9* $\times 10^9 Nm^2/C^2$

Alternative Form:

```
F = (1/4 _{0}) \times (q_{1}q_{2})/r^{2}
```

Where:

- $_0 = Permittivity of free space = 8.85 \times 10^{-12} C^2/Nm^2$
- *k = 1/(4* $_0) = 8.9875 \times 10^9 \approx 9 \times 10^9 Nm^2/C^2$

In a Medium:

```
F = (1/4 _{0} ) \times (q_{1}q_{2})/r^{2} = k(q_{1}q_{2})/( r^{2})
```

Where:

- *= Relative permittivity or dielectric constant (K)*

Nature of Force:

1. ***Like charges:*** *If both charges are of the same sign (both positive or both negative), the force is* ***repulsive*** *(pushes them apart)*
2. ***Unlike charges:*** *If charges are of opposite signs (one positive, one negative), the force is* ***attractive*** *(pulls them together)*

Vector Form:

```
F_{1}_{2} = k(q_{1}q_{2})/r^{2} \times r̂_{1}_{2}
```

Where $\hat{r}_{12} is the unit vector from q_1 to q_2$.
Key Points:

1. *Coulomb's law is valid only for* ***stationary point charges***

*2. It is a **fundamental law** of nature*
3. Similar to Newton's law of gravitation in form
*4. Electric force is **much stronger** than gravitational force (~10^{39}times)*
4. The law can be applied to large charged objects if the distance between them is much larger than their size
5. Permittivity ( ) represents the resistance of the medium that impedes the electric field

Comparison with Gravitational Force:

Property	*Gravitational Force*	*Electric Force*
Formula	$F = Gm_1m_2/r^2$	$F = kq_1q_2/r^2$
Nature	*Always attractive*	*Attractive or repulsive*
Strength	*Very weak*	*Very strong*
Depends on	*Mass*	*Charge*
Constant	$G = 6.67\times10^{-11}$	$k = 9\times10^{9}$

0.0.3 (2) Explain characteristics of Electric field lines with figures.

Solution

Electric Field Lines: Michael Faraday introduced the concept of electric field lines (also called "electric lines of force"). The geometric representation of an electric field is called electric field lines.
Definition: An electric field line is a curve drawn in an electric field such that the tangent to the curve at any point gives the direction of the net electric field at that point.
Figure: Electric field lines for positive and negative charges
Figure: Electric field lines between positive and negative charges showing force direction
Characteristics of Electric Field Lines:
1. Origin and Termination:

- Electric field lines **start from positive charges** and **end at negative charges**
- For isolated positive charge: lines radiate outward to infinity
- For isolated negative charge: lines come from infinity and converge inward

```
Positive Charge (+Q):          Negative Charge (-Q):
         ↑                             ↓
         |                             |
   \rightarrow  (+Q)  \leftarrow                 \rightarrow  (-Q)  \leftarrow
         |                             |
         ↓                             ↑
```

2. Direction of Electric Field:

- The **tangent** at any point on an electric field line indicates the **direction of the electric field** at that point
- It shows the direction in which a positive test charge would move if placed at that point

Figure: Tangent at points $P_1andP_2showingelectricfielddirectionE_1andE_2$
3. Non-Intersection:

- ***Two electric field lines never intersect or cross each other***
- *If they intersect, there would be two directions of electric field at that point, which is impossible*
- *A charge at the intersection would experience force in two directions simultaneously, which contradicts the definition*

Figure: Two field lines cannot intersect as it would give two field directions at point P
4. Field Intensity and Line Density:

- *The **density** (closeness) of electric field lines indicates the **strength of the electric field***
- ***Closely spaced lines*** $\rightarrow Strongelectricfield(highintensity)$
- ***Widely spaced lines*** $\rightarrow Weakelectricfield(lowintensity)$
- *Number of lines passing through a unit area is proportional to field strength*

Figure: More field lines through $A_1 indicatesstrongerfieldthanthroughA_2$
5. Uniform Electric Field:

- Electric field lines of a **uniform electric field** are:
  - Mutually **parallel**
  - **Equidistant** from each other
  - Example: Field between two parallel charged plates

```
Uniform Electric Field:
    ++++++++++++++++
```

6. Imaginary Nature:
- Electric field lines are **imaginary**, but **electric field is real**
- They are a visual tool to represent the field
- Actual field exists continuously in space

7. Perpendicular to Conducting Surface:
- Electric field lines are always **perpendicular** to the conducting surface
- This applies both when leaving and entering the charge
- **Reason:** Electric field parallel to conducting surface is zero
- No electric force exists parallel to the conducting surface

8. Open Curves:
- Electric field lines **do not form closed loops**
- They always have a beginning (positive charge) and end (negative charge)
- Unlike magnetic field lines which form closed loops

Examples of Field Line Patterns:

a) Two Positive Charges:

```
       ↑                 ↑
       |                 |
 \rightarrow  (+Q)      \leftrightarrow    \leftrightarrow      (+Q)  \leftarrow
       |                 |
       ↓                 ↓
```

Field lines repel each other, never connect

b) Positive and Negative Charges (Dipole):

```
      \rightarrow  \rightarrow  \rightarrow  \rightarrow

 (+Q) \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow (-Q)

      \rightarrow  \rightarrow  \rightarrow  \rightarrow
```

Field lines flow from + to -

Summary Table:

Characteristic	Description
Start Point	Positive charge
End Point	Negative charge
Direction	Tangent at any point
Intersection	Never cross
Density	Indicates field strength
Conductor	Perpendicular to surface
Nature	Imaginary lines, real field
Closed Loop	No, always open curves

0.0.4 (3) Write short note on parallel plate capacitor.

Solution

Parallel Plate Capacitor: A parallel plate capacitor consists of two large conducting plates of equal area placed parallel to each other and separated by a small distance with air or a dielectric medium between them.

Construction:

```
    ++++++++++++++++  \leftarrow Plate 1 (+Q charge)
```

```
         ↓ ↓ ↓
                \leftarrow Electric field (uniform)
         ↓ ↓ ↓
              \leftarrow Plate 2 (-Q charge)
         ↑
      Distance d

    \leftarrow    A    \rightarrow  (Area of each plate)
```

Components:

1. **Two Conducting Plates:**
    - Made of metal (copper, aluminum)
    - Same area A
    - Placed parallel to each other
2. **Dielectric Medium:**
    - Insulating material between plates
    - Air, paper, mica, ceramic, plastic, etc.
    - Prevents direct contact and discharge
3. **Separation (d):**
    - Small distance between plates
    - Typically much smaller than plate dimensions
    - d «

Working Principle:

When connected to a battery:

1. One plate gets **positive charge (+Q)**
2. Other plate gets **equal negative charge (-Q)**
3. Electric field is established between plates
4. Energy is stored in the electric field

Electric Field:

Inside the capacitor (between plates):

```
E =  / _{0} = Q/( _{0}A)
```

Where:

- = Surface charge density = Q/A
- $_0 = Permittivity of free space$

Potential Difference:

```
V = Ed = Qd/( _{0}A)
```

Capacitance:

The capacitance of a parallel plate capacitor is given by:

```
C = Q/V =  _{0}A/d
```

With Dielectric:

```
C =  _{0} A/d =  _{0}KA/d =  A/d
```

Where:

- C = Capacitance (F)
- $_0 = 8.85 \times 10^{-12} F/m$
- or K = Dielectric constant
- A = Area of each plate (m^2)
- d = Distance between plates (m)
- $= \ _0 = Permittivity of medium$

Factors Affecting Capacitance:

1. **Area of Plates (A):**
    - C A
    - Larger area $\rightarrow More charge storage \rightarrow Higher capacitance$
2. **Distance Between Plates (d):**
    - C 1/d
    - Smaller distance $\rightarrow Stronger field \rightarrow Higher capacitance$
3. **Dielectric Medium:**

- C K (dielectric constant)
- Higher K → $Highercapacitance$
- Air: K = 1, Paper: K ≈ $3.7, Mica : K \approx 5.5$

Energy Stored:
The energy stored in a parallel plate capacitor:

```
U = (1/2)QV = (1/2)CV^{2} = Q^{2}/(2C)
```

Characteristics:
1. **Uniform Electric Field:**
   - Field between plates is uniform (except at edges)
   - Parallel and equally spaced field lines
2. **High Capacitance:**
   - Relatively high capacitance for given size
   - Depends on area, separation, and dielectric
3. **Linear Device:**
   - Q V (charge proportional to voltage)
   - Constant capacitance

Applications:
1. **Energy Storage:**
   - Camera flash circuits
   - Power supplies
2. **Filtering:**
   - Smoothing voltage in power supplies
   - Signal processing
3. **Timing Circuits:**
   - Oscillators
   - Timers
4. **Coupling/Decoupling:**
   - Blocking DC, allowing AC
   - Separating circuit stages
5. **Tuning:**
   - Radio and TV circuits
   - Resonant circuits

Advantages:
- Simple construction
- Predictable capacitance
- Can handle high voltages
- Low cost
- Reliable

Limitations:
- Fixed capacitance (unless variable design)
- Limited voltage rating
- Can discharge rapidly (short circuit hazard)
- Electrolytic types have polarity

Practical Considerations:
1. **Fringing Effect:**
   - Field lines curve at edges
   - Formula assumes negligible edge effects
   - Valid when d «
2. **Breakdown Voltage:**
   - Maximum voltage before dielectric breaks down
   - Depends on dielectric strength and thickness
3. **Leakage Current:**
   - Small current through dielectric
   - Causes gradual discharge

0.0.5 (4) Explain series connection of capacitors in detail.

Solution

Series Connection of Capacitors: When capacitors are connected end-to-end such that the negative plate of one capacitor is connected to the positive plate of the next capacitor, they are said to be connected in series.
Circuit Diagram:

```
     C_{1}     C_{2}     C_{3}

  +  -  +  -  +  -
  Q  Q  Q  Q  Q  Q
```

Alternative Representation:

```
       C_{1}        C_{2}        C_{3}
  +    +    +    +
          -   +        -   +


  V                             -
```

Characteristics of Series Connection:
1. Same Charge on All Capacitors:

- When connected to a battery, the same charge Q flows through each capacitor
- Charge on each capacitor: $\mathbf{Q}_1 = Q_2 = Q_3 = Q$
  Reason: When a charge +Q is stored on the positive plate of $C_1, it induces - Q on its negative plate. This - Q repels equal amount from the positive plate of C_2, leaving + Q on it, and so on.$

2. Different Potential Differences:

- Total voltage divides among capacitors
- $V = V_1 + V_2 + V_3$

Where:

- $V_1 = Q/C_1 (voltage across C_1)$
- $V_2 = Q/C_2 (voltage across C_2)$
- $V_3 = Q/C_3 (voltage across C_3)$

3. Equivalent Capacitance:
The total or equivalent capacitance $(C_s) of capacitors in series is given by:$

```
1/C_{s} = 1/C_{1} + 1/C_{2} + 1/C_{3} + ... + 1/C_{n}
```

Derivation:
Starting with the definition of capacitance:

```
C = Q/V  \rightarrow

V = Q/C
```

For series connection:

```
V = V_{1} + V_{2} + V_{3}

V = Q/C_{1} + Q/C_{2} + Q/C_{3}

V = Q(1/C_{1} + 1/C_{2} + 1/C_{3})
```

If $C_s is the equivalent capacitance:$

```
V = Q/C_{s}
```

Comparing:

```
Q/C_{s} = Q(1/C_{1} + 1/C_{2} + 1/C_{3})

1/C_{s} = 1/C_{1} + 1/C_{2} + 1/C_{3}
```

For Two Capacitors in Series:

```
1/C_{s} = 1/C_{1} + 1/C_{2} = (C_{1} + C_{2})/(C_{1}C_{2})
```

```
C_{s} = (C_{1}C_{2})/(C_{1} + C_{2})
```

```
C_{s} = Product / Sum
```

For n Equal Capacitors (C each) in Series:

```
1/C_{s} = n/C
```

```
C_{s} = C/n
```

Important Points:

1. **Equivalent Capacitance Decreases:**
   - $C_s < smallestindividualcapacitance$
   - Series connection reduces total capacitance
2. **Effective Distance Increases:**
   - Series connection is like increasing plate separation
   - Since C 1/d, capacitance decreases
3. **Voltage Distribution:**
   - Smaller capacitor gets larger voltage
   - V 1/C
   - $V_1/V_2 = C_2/C_1$
4. **Same Energy Distribution:**
   - Energy in each: $U_1 = Q^2/(2C_1)$
   - Total energy: $U = Q^2/(2C_s)$

Example Calculation:
For $C_1 = 10F, C_2 = 20F, C_3 = 30Finseries:$

```
1/C_{s} = 1/10 + 1/20 + 1/30
1/C_{s} = (6 + 3 + 2)/60 = 11/60
C_{s} = 60/11 = 5.45  F
```

Applications:

1. **Voltage Division:**
   - Obtaining different voltages from single source
   - High voltage rating circuits
2. **Increased Voltage Rating:**
   - Total voltage rating = sum of individual ratings
   - Used when higher voltage handling needed
3. **Fine Tuning:**
   - Achieving specific capacitance values
   - Precision applications

Advantages:

- Same charge on all capacitors
- Higher voltage handling capability
- Simple voltage division

Disadvantages:

- Reduced total capacitance
- If one capacitor fails (open circuit), entire circuit fails
- Unequal voltage distribution

Comparison with Parallel:

Property	Series	Parallel
Charge	Same (Q)	Different ($Q = Q_1 + Q_2 + Q_3$)
Voltage	Different ($V = V_1 + V_2 + V_3$)	Same (V)
Capacitance	$1/C = 1/C_1 + 1/C_2 + 1/C_3$	$C = C_1 + C_2 + C_3$
Effect	Decreases	Increases

0.0.6 (5) Explain parallel connection of capacitors in detail.

Solution

Parallel Connection of Capacitors: When all the positive plates of capacitors are connected to one common point and all negative plates are connected to another common point, the capacitors are said to be connected in parallel.

Circuit Diagram:

```
          C_{1}    C_{2}    C_{3}
    +            +


    V                        -
```

Alternative Representation:

```
                C_{1}
    +
               C_{2}

    V           C_{3}

              -
```

Characteristics of Parallel Connection:

1. Same Potential Difference:

- All capacitors have the same voltage across them
- $\mathbf{V}_1 = V_2 = V_3 = V$
  Reason: All positive plates connected to same point (positive terminal) and all negative plates to another point (negative terminal)

2. Different Charges:

- Total charge divides among capacitors
- Each stores charge according to its capacitance
- $\mathbf{Q} = \mathbf{Q}_1 + Q_2 + Q_3$
  Where:
- $\mathrm{Q}_1 = C_1V(chargeonC_1)$
- $\mathrm{Q}_2 = C_2V(chargeonC_2)$
- $\mathrm{Q}_3 = C_3V(chargeonC_3)$

3. Equivalent Capacitance:

The total or equivalent capacitance $(\mathrm{C}_p) ofcapacitorsinparallelisgivenby:$

```
C_{p} = C_{1} + C_{2} + C_{3} + ... + C_{n}
```

Derivation:

Starting with the definition of capacitance:

```
Q = CV
```

For parallel connection:

```
Q = Q_{1} + Q_{2} + Q_{3}

Q = C_{1}V + C_{2}V + C_{3}V

Q = V(C_{1} + C_{2} + C_{3})
```

If $\mathrm{C}_p istheequivalentcapacitance:$

```
Q = C_{p}V
```

Comparing:

```
C_{p}V = V(C_{1} + C_{2} + C_{3})

C_{p} = C_{1} + C_{2} + C_{3}
```

For n Equal Capacitors (C each) in Parallel:

```
C_{p} = nC
```

Important Points:

1. **Equivalent Capacitance Increases:**
   - $C_p > largestindividualcapacitance$
   - Parallel connection increases total capacitance
2. **Effective Area Increases:**
   - Parallel connection is like increasing plate area
   - Since C A, capacitance increases
3. **Charge Distribution:**
   - Larger capacitor stores more charge
   - Q C
   - $Q_1/Q_2 = C_1/C_2$
4. **Current Distribution:**
   - Total current: $I = I_1 + I_2 + I_3$
   - Each branch carries different current
5. **Energy Storage:**
   - Energy in each: $U_1 = C_1V^2$
   - Total energy: $U = ½C_pV^2$
   - $U = U_1 + U_2 + U_3$

Example Calculation:

For $C_1 = 10F, C_2 = 20F, C_3 = 30Finparallel:$

```
C_{p} = C_{1} + C_{2} + C_{3}
C_{p} = 10 + 20 + 30
C_{p} = 60  F
```

If V = 12V, charges are:

```
Q_{1} = C_{1}V = 10 \times 12 = 120  C
Q_{2} = C_{2}V = 20 \times 12 = 240  C
Q_{3} = C_{3}V = 30 \times 12 = 360  C
Total

Q = 120 + 240 + 360 = 720  C
```

Verification:

```
Q = C_{p}V = 60 \times 12 = 720  C
```

Applications:

1. **Increased Capacitance:**
   - Getting higher capacitance from smaller units
   - When large capacitance needed
2. **Increased Current Capacity:**
   - Current divided among capacitors
   - Reduces stress on individual capacitors
3. **Power Factor Correction:**
   - Banks of capacitors in parallel
   - Industrial power systems
4. **Energy Storage:**
   - More energy stored than individual capacitors
   - UPS systems, electric vehicles
5. **Filtering:**
   - Multiple frequency filtering
   - Different capacitors for different frequencies

Advantages:

- Increased total capacitance
- Same voltage across all
- If one fails (short circuit), others continue working

- Redundancy and reliability
- Easy to add or remove capacitors

Disadvantages:
- Takes more space
- Higher cost (more capacitors)
- If one shorts, entire circuit affected

Comparison Table:

Property	Series	Parallel
Connection	End-to-end	All +ve together, all -ve together
Charge	$Q_1 = Q_2 = Q_3 = Q$	$Q = Q_1 + Q_2 + Q_3$
Voltage	$V = V_1 + V_2 + V_3$	$V_1 = V_2 = V_3 = V$
Capacitance	$1/C = 1/C_1 + 1/C_2 + 1/C_3$	$C = C_1 + C_2 + C_3$
Effect on C	Decreases (C < smallest)	Increases (C > largest)
Analogy	Like increasing d	Like increasing A
Use	High voltage circuits	High capacitance needs

Practical Example - Three Identical Capacitors:
For $C_1 = C_2 = C_3 = C = 10F$:
Series:

```
C_{s} = C/n = 10/3 = 3.33  F
```

Parallel:

```
C_{p} = nC = 3 \times 10 = 30  F
```

This shows parallel gives 9 times more capacitance than series!
Mixed Connections:
Real circuits often use both series and parallel combinations:

```
Example: (C_{1}  C_{2}) in series with C_{3}
```

Step 1: Find parallel combination

```
C_{1}_{2} = C_{1} + C_{2}
```

Step 2: Combine in series with C_3

```
1/C_{t}_{o}_{t}_{a}_{l} = 1/C_{1}_{2} + 1/C_{3}
```

0.0.7 (6) Explain effect of dielectric material on the capacitance of parallel plate.

Solution

Dielectric Material: A dielectric is an insulating (non-conducting) material that can be polarized by an electric field. When placed between the plates of a capacitor, it significantly affects the capacitance.
Common Dielectric Materials:
- Air ($K = 1.0$)
- Paper ($K \approx 3.7$)
- Mica ($K \approx 5.5$)
- Glass ($K \approx 4.5 - 10$)
- Ceramic ($K \approx 6 - 1000$)
- Polyester ($K \approx 3.3$)
- Teflon ($K \approx 2.1$)

Effect of Dielectric on Capacitance:
1. Without Dielectric (Air/Vacuum):
Capacitance of parallel plate capacitor:

```
C_{0} =  _{0}A/d
```

Where:

- $C_0 = Capacitancewithair/vacuum$
- $_0 = 8.85 \times 10^{-12} F/m(permittivityoffreespace)$
- A = Area of plates
- d = Distance between plates

2. With Dielectric Material:

When dielectric is inserted:

```
C =  _{0} A/d =  _{0}KA/d

C = KC_{0}
```

Where:

- C = New capacitance with dielectric
- or K = Relative permittivity (dielectric constant)
- K > 1 for all materials except vacuum

Dielectric Constant (K):

```
K = C/C_{0} = Capacitance with dielectric / Capacitance without dielectric
```

Key Observation:

- **Capacitance increases by factor K**
- **K is always** $\geq 1(K = 1forvacuum)$
- **Higher K** $\rightarrow Highercapacitance$

How Dielectric Increases Capacitance:

Physical Mechanism:

1. **Polarization:**
   - Dielectric molecules align in electric field
   - Positive charges slightly toward negative plate
   - Negative charges slightly toward positive plate

```
Without Dielectric:          With Dielectric:
+++++++++++                  +++++++++++
                         \oplus   \oplus   \oplus
           \rightarrow              \oplus   \oplus   \oplus
                         \oplus   \oplus   \oplus


    E_{0}                           E
```

1. **Induced Surface Charges:**
   - Negative charges appear on dielectric surface near positive plate
   - Positive charges appear on surface near negative plate
   - These are called bound charges (cannot move freely)
2. **Reduced Electric Field:**
   - Internal electric field opposes applied field
   - Net field: $E = E_0/K$
   - External field partially cancelled
3. **Same Charge, Lower Voltage:**
   - Voltage: V = Ed
   - Since E decreases, V decreases
   - $V = V_0/K$
   - Capacitance: C = Q/V increases

Mathematical Analysis:

At Constant Charge (Q constant):

Without dielectric:

```
V_{0} = Q/C_{0}
E_{0} = V_{0}/d
```

With dielectric:

```
E = E_{0}/K
V = Ed = (E_{0}/K)d = V_{0}/K
C = Q/V = Q/(V_{0}/K) = K(Q/V_{0}) = KC_{0}
```

At Constant Voltage (Battery connected):
Without dielectric:

```
Q_{0} = C_{0}V
```

With dielectric:

```
C = KC_{0}
Q = CV = KC_{0}V = KQ_{0}
```

More charge flows from battery!
Effects of Dielectric:

Property	Without Dielectric	With Dielectric (K)	Change
Capacitance	C_0	KC_0	Increases K times
Electric Field	E_0	E_0/K	Decreases K times
Voltage (Q constant)	V_0	V_0/K	Decreases K times
Charge (V constant)	Q_0	KQ_0	Increases K times
Energy (Q constant)	U_0	U_0/K	Decreases K times
Energy (V constant)	U_0	KU_0	Increases K times

Energy Considerations:
Case 1: Battery Disconnected (Q constant):

```
U_{0} = Q^{2}/(2C_{0})
U = Q^{2}/(2C) = Q^{2}/(2KC_{0}) = U_{0}/K
```

Energy decreases! Where does it go?

- Converted to mechanical work (dielectric pulled in)
- Heat due to molecular alignment

Case 2: Battery Connected (V constant):

```
U_{0} = ½C_{0}V^{2}
U = ½CV^{2} = ½(KC_{0})V^{2} = KU_{0}
```

Energy increases! From where?

- Battery supplies additional energy
- Work done in polarizing dielectric

Advantages of Using Dielectric:

1. **Increased Capacitance:**
   - Get more capacitance without increasing size
   - C = $KC_0 where K can be 2 - 1000$
2. **Higher Breakdown Voltage:**
   - Dielectric can withstand higher fields than air
   - Prevents spark/discharge between plates
   - Typical: Air ~3 kV/mm, Mica ~200 kV/mm
3. **Reduced Physical Size:**
   - For given capacitance, can reduce area or increase distance
   - More compact capacitors
4. **Mechanical Support:**
   - Keeps plates at fixed distance
   - Prevents short circuit
   - Structural stability
5. **Protection:**
   - Prevents moisture, dust entry
   - Longer life

Practical Applications:

1. **Commercial Capacitors:**
   - Paper: K $\approx 3.7 (cheap, low voltage)$
   - Mica: K $\approx 5.5 (precision, stable)$
   - Ceramic: K up to 1000 (compact, high C)
2. **Electrolytic Capacitors:**
   - Aluminum oxide: K ≈ 8
   - Tantalum oxide: K ≈ 25
   - Very high capacitance in small size
3. **Variable Capacitors:**
   - Changing dielectric position changes C
   - Used in radio tuning

Breakdown and Dielectric Strength:
Dielectric Strength: Maximum electric field a dielectric can withstand before breaking down (conducting).

```
E_max = V_max/d
```

Material	Dielectric Strength (kV/mm)
Air	3
Paper	16
Mica	200
Teflon	60
Glass	30

Summary Formula:

```
C =  _{0} A/d =  _{0}KA/d =  A/d

Where:
-

  =  _{0}  = absolute permittivity of medium

-  _{0} = 8.85 \times 10^{-}^{1}^{2} F/m
-    = K = dielectric constant (\geq 1)
```

Conclusion:
Inserting a dielectric between capacitor plates: Increases capacitance by factor K Decreases electric field by factor K Increases breakdown voltage Makes capacitors more compact Provides mechanical support and protection
This makes dielectric materials essential for practical capacitor design and applications.

Part C: Numerical Solutions (3 marks)

0.0.8 (1) Two charges with value of 20 μC and 10 μC are separated 0.02 m distance in air. Find electric force or coulomb force between these charges. K value is 9 $\times 10^9 Nm^2/C^2$.

Given:

- Charge 1: q$_1 = 20C = 20 \times 10^{-6}C$
- Charge 2: q$_2 = 10C = 10 \times 10^{-6}C$
- Distance:
  r = 0.02
  m = 2 $\times 10^{-2}m$

- Coulomb's constant: k = 9 $\times 10^9 Nm^2/C^2$
- Medium: Air (= 1)

To Find:

- Electric force: F = ?

Formula:

```
F = k(q_{1}q_{2})/r^{2}
```

Solution:

```
F = k(q_{1}q_{2})/r^{2}

F = (9 \times 10^{9}) \times (20 \times 10^{-}^{6}) \times (10 \times 10^{-}^{6}) / (0.02)^{2}

F = (9 \times 10^{9}) \times (200 \times 10^{-}^{1}^{2}) / (4 \times 10^{-}^{4})

F = (1800 \times 10^{-}^{3}) / (4 \times 10^{-}^{4})

F = (1800 \times 10^{-}^{3}) \times (10^{4}/4)

F = 1800 \times 10^{1} / 4

F = 18000 / 4

F = 4500 N

F = 4.5 \times 10^{3} N
```

Alternative Method:

```
F = (9 \times 10^{9}) \times (20 \times 10^{-}^{6}) \times (10 \times 10^{-}^{6}) / (2 \times 10^{-}^{2})^
```

```
F = (9 \times 20 \times 10) \times 10^{9}^{-}^{6}^{-}^{6} / (4 \times 10^{-}^{4})
```

```
F = 1800 \times 10^{-}^{3} / (4 \times 10^{-}^{4})
```

```
F = 1800/4 \times 10^{-}^{3}^{+}^{4}
```

```
F = 450 \times 10^{1}
```

```
F = 4500 N
```

Solution

The electric force (Coulomb force) between the two charges is **4500 N or 4.5** $\times 10^3 N$.
Nature of Force: Since both charges are positive, the force is repulsive.

0.0.9 (2) 1600 Joule of work is done in moving a charge 25 coulomb from one point to the other. Calculate the potential difference between the points.

Given:

- Work done: W = 1600 J
- Charge moved: q = 25 C

To Find:

- Potential difference: V = ?

Formula:

```
V = W/q
```

Where:

- V = Potential difference (Volt)
- W = Work done (Joule)
- q = Charge (Coulomb)

Solution:

```
V = W/q
```

```
V = 1600/25
```

```
V = 64 V
```

Solution

The potential difference between the two points is **64 Volts**.
Physical Meaning:

- 64 Joules of work is needed to move 1 Coulomb of charge between these points
- This also means the electric potential at the first point is 64 V higher than at the second point

---

0.0.10 (3) A capacitor gets a charge 60 μC when it is connected to a battery of e.m.f. 12 V. Calculate the capacitance of the capacitor.

Given:

- Charge stored:
  Q = 60 C = 60 $\times 10^{-6} C$
- Voltage applied: V = 12 V

To Find:

- Capacitance: C = ?

Formula:

```
C = Q/V
```

Where:

- C = Capacitance (Farad)
- Q = Charge stored (Coulomb)
- V = Voltage (Volt)

Solution:

```
C = Q/V

C = (60 \times 10^{-}^{6})/12

C = 5 \times 10^{-}^{6} F

C = 5  F
```

Solution

The capacitance of the capacitor is **5 F (microfarad)** or **5** $\times 10^{-6} F$.
Verification:

```
Q = CV = 5 \times 10^{-}^{6} \times 12 = 60 \times 10^{-}^{6}

C = 60  C
```

0.0.11 (4) Three capacitors of 10µF are connected in series and parallel connections in circuit. Find out total capacitance in both cases.

Given:

- Three identical capacitors
- $C_1 = C_2 = C_3 = C = 10F$
- n = 3 (number of capacitors)

To Find:

- 1. Total capacitance in series: C_s =?
- 1. Total capacitance in parallel: C_p =?

Case (a): Series Connection

Formula:

```
For n identical capacitors in series:
C_{s} = C/n
```

Or using general formula:

```
1/C_{s} = 1/C_{1} + 1/C_{2} + 1/C_{3}
```

Solution:

Method 1 (Direct):

```
C_{s} = C/n

C_{s} = 10/3

C_{s} = 3.33  F
```

Method 2 (General formula):

```
1/C_{s} = 1/C_{1} + 1/C_{2} + 1/C_{3}

1/C_{s} = 1/10 + 1/10 + 1/10

1/C_{s} = 3/10

C_{s} = 10/3

C_{s} = 3.33  F
```

Solution

Total capacitance in series = **3.33 F** or **10/3 F**

Case (b): Parallel Connection

Formula:

```
For n identical capacitors in parallel:
C_{p} = nC
```

Or using general formula:

```
C_{p} = C_{1} + C_{2} + C_{3}
```

Solution:

Method 1 (Direct):

```
C_{p} = nC

C_{p} = 3 \times 10

C_{p} = 30  F
```

Method 2 (General formula):

```
C_{p} = C_{1} + C_{2} + C_{3}

C_{p} = 10 + 10 + 10

C_{p} = 30  F
```

Solution

Total capacitance in parallel = **30 F**

Summary:

Connection	Formula	Total Capacitance
Series	$C_s = C/n$	3.33 F
Parallel	$C_p = nC$	30 F

Observation:

- Parallel gives 9 times more capacitance than series ($30/3.33 \approx 9$)
- Series: $C_s < individualcapacitance$
- Parallel: $C_p > individualcapacitance$

0.0.12 (5) Plate area of one parallel plate capacitor is 10 mm^2, $which are separated with 1 mm distance in air. Calculateca$

Given:

- Area of plates:
  A = 10 mm^2 = $10 \times 10^{-6} m^2 = 10^{-5} m^2$
- Distance between plates:
  d = 1 mm = 1 $\times 10^{-3}$
  m = $10^{-3} m$
- Medium: Air (= 1,
  K = 1)
- Permittivity of free space: $_0 = 8.85 \times 10^{-12} F/m$

To Find:

- Capacitance: C = ?

Formula:

```
C =  _{0} A/d =  _{0}A/d  (for air,   = 1)
```

Solution:

```
C =  _{0}A/d
```

```
C = (8.85 \times 10^{-}^{1}^{2}) \times (10^{-}^{5}) / (10^{-}^{3})
```

```
C = 8.85 \times 10^{-}^{1}^{2}^{-}^{5}^{+}^{3}
```

```
C = 8.85 \times 10^{-}^{1}^{4} F
```

Converting to picofarads:

```
C = 8.85 \times 10^{-}^{1}^{4} F
C = 0.0885 \times 10^{-}^{1}^{2} F
C = 0.0885 pF
```

Solution

The capacitance of the capacitor is **8.85** $\times 10^{-14} F or$ **0.0885 pF (picofarad)**.
Note: This is a very small capacitance due to the small plate area. Practical capacitors have much larger plate areas.

0.0.13 (6) The distance between the plates is 1mm, if we want to get capacitance of 1F, how much area of plate should be?

Given:

- Distance between plates:
  d = 1 mm = $10^{-3} m$
- Required capacitance: C = 1 F
- Medium: Air (assuming air, = 1)
- Permittivity of free space: $_0 = 8.85 \times 10^{-12} F/m$

To Find:

- Area of plates: A = ?

Formula:

```
C =  _{0}A/d
```

Rearranging for A:

```
A = Cd/ _{0}
```

Solution:

```
A = Cd/ _{0}

A = (1) \times (10^{-}^{3}) / (8.85 \times 10^{-}^{1}^{2})

A = 10^{-}^{3} / (8.85 \times 10^{-}^{1}^{2})

A = (1/8.85) \times 10^{-}^{3}^{+}^{1}^{2}

A = 0.113 \times 10^{9} m^{2}

A = 1.13 \times 10^{8} m^{2}

A = 113 \times 10^{6} m^{2}

A = 113,000,000 m^{2}
```

Converting to square kilometers:

```
A = 113 \times 10^{6} m^{2}
A = 113 km^{2}
```

0.0.14 (7) As per the below circuit, calculate total capacitance value.

Note: Since the circuit diagram is referenced but not visible in the text, I'll provide solutions for common circuit configurations:

Case A: Series-Parallel Combination

Assuming circuit: $(\mathrm{C}_1 C_2) inserieswith C_3$

Example: $\mathrm{C}_1 = 10F, C_2 = 20F, C_3 = 30F$

```
           C_{1} (10 F)
 +
           C_{2} (20 F)

          C_{3} (30 F)
```

Solution:

Step 1: Find parallel combination of $\mathrm{C}_1 and C_2$

```
C_{1}_{2} = C_{1} + C_{2}
C_{1}_{2} = 10 + 20 = 30  F
```

Step 2: Combine $\mathrm{C}_{12} inserieswith C_3$

```
1/C_{t}_{o}_{t}_{a}_{l} = 1/C_{1}_{2} + 1/C_{3}
1/C_{t}_{o}_{t}_{a}_{l} = 1/30 + 1/30
1/C_{t}_{o}_{t}_{a}_{l} = 2/30
C_{t}_{o}_{t}_{a}_{l} = 30/2 = 15  F
```

Solution

Total capacitance = **15 F**

Case B: Series-Parallel Combination (Alternative)

Assuming circuit: $(\mathrm{C}_1 inserieswith C_2) parallelwith C_3$

Example: $\mathrm{C}_1 = 6F, C_2 = 6F, C_3 = 4F$

```
        C_{1}     C_{2}
+
         C_{3}
```

Solution:

Step 1: Find series combination of $C_1 and C_2$

```
1/C_{1}_{2} = 1/C_{1} + 1/C_{2}
1/C_{1}_{2} = 1/6 + 1/6 = 2/6
C_{1}_{2} = 6/2 = 3  F
```

Step 2: Combine $C_{12} in parallel with C_3$

```
C_{t}_{o}_{t}_{a}_{l} = C_{1}_{2} + C_{3}
C_{t}_{o}_{t}_{a}_{l} = 3 + 4 = 7  F
```

Solution

Total capacitance = **7 F**

Case C: Complex Network

Assuming bridge network or Wheatstone arrangement

General Approach:

1. **Identify series and parallel sections**
2. **Simplify step by step** from outermost to innermost
3. **Use equivalent capacitance formulas** at each step
4. **Continue until single equivalent capacitance** remains

Steps:

- Mark parallel combinations and add: $C = C_1 + C_2 + \ldots$
- Mark series combinations and use: $1/C = 1/C_1 + 1/C_2 + \ldots$
- Replace simplified sections with equivalent values
- Repeat until circuit reduces to single capacitor

Common Circuit Configurations:

Configuration 1: Three in Series

```
   C_{1}   C_{2}   C_{3}

Result: 1/C = 1/C_{1} + 1/C_{2} + 1/C_{3}
```

Configuration 2: Three in Parallel

```
     C_{1}

     C_{2}

      C_{3}

Result: C = C_{1} + C_{2} + C_{3}
```

Configuration 3: Mixed (Two in Series, then Parallel with Third)

```
      C_{1}  C_{2}
+
       C_{3}
```

```
Step 1: C_{1}_{2} = (C_{1}C_{2})/(C_{1}+C_{2})
Step 2: C_{t}_{o}_{t}_{a}_{l} = C_{1}_{2} + C_{3}
```

Please provide the specific circuit diagram for an exact solution!

Additional Important Formulas

0.0.15 Electric Charge:

```
Q = ne  (n = 1, 2, 3, ...)
e = 1.6 \times 10^{-}^{1}^{9} C (elementary charge)
```

0.0.16 Coulomb's Law:

```
F = kq_{1}q_{2}/r^{2}
k = 9 \times 10^{9} N·m^{2}/C^{2}
 _{0} = 8.85 \times 10^{-}^{1}^{2} F/m
k = 1/(4  _{0})
```

0.0.17 Electric Field:

```
E = F/q_{0} = kQ/r^{2}
E (uniform) = V/d
```

0.0.18 Electric Potential:

```
V = kQ/r
V = W/q
ΔV = V_{2} - V_{1}
```

0.0.19 Electric Flux:

```
Φ = E·A = EA cos
```

0.0.20 Capacitance:

```
C = Q/V
C (parallel plate) =  _{0} A/d
C (with dielectric) = KC_{0}
```

0.0.21 Series Capacitors:

```
1/C_{s} = 1/C_{1} + 1/C_{2} + 1/C_{3} + ...
For 2: C_{s} = C_{1}C_{2}/(C_{1}+C_{2})
For n equal: C_{s} = C/n
```

0.0.22 Parallel Capacitors:

```
C_{p} = C_{1} + C_{2} + C_{3} + ...
For n equal: C_{p} = nC
```

0.0.23 Energy in Capacitor:

```
U = ½QV = ½CV^{2} = Q^{2}/(2C)
```

0.0.24 Constants:

```
e = 1.6 \times 10^{-}^{1}^{9} C
k = 9 \times 10^{9} N·m^{2}/C^{2}
 _{0} = 8.85 \times 10^{-}^{1}^{2} F/m
1  F = 10^{-}^{6} F
1 nF = 10^{-}^{9} F
1 pF = 10^{-}^{1}^{2} F
```

End of Solutions