Mathematics-I Solutions

DI01000021 – Summer 2025

Semester 1 Study Material

Detailed Solutions and Explanations

Question 1 [14 marks]

Fill in the blanks/MCQs using appropriate choice from the given options

Q1.1 [1 mark]

$\log_3 1 =$ ___________

Solution

Answer: d. 0
Solution: For any base $a > 0, a \neq 1$: $\log_a 1 = 0$ Therefore: $\log_3 1 = 0$

Q1.2 [1 mark]

The modulus of the complex number $z = 3 + 4i$ **is** ___________

Solution

Answer: a. 5
Solution: $|z| = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Q1.3 [1 mark]

The value of $\lim_{x \to 0} \frac{\tan x}{x}$ **is** ___________

Solution

Answer: b. 1
Solution: Standard limit identity.

Q1.4 [1 mark]

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$**, then** $|A| =$ ___________

Solution

Answer: c. -3
Solution: $|A| = (2)(0) - (3)(1) = 0 - 3 = -3$

Q1.5 [1 mark]

The derivative of $\sin x$ **is** ___________

Solution

Answer: a. $\cos x$
Solution: Standard differentiation formula.

Q1.6 [1 mark]

$\int_0^1 xdx =$ ___________

Solution

Answer: b. 1/2
Solution: $\int xdx = x^2/2$. Value $= [\frac{1^2}{2}] - [\frac{0^2}{2}] = 1/2 - 0 = 0.5$

Q1.7 [1 mark]

If two lines slopes m_1 and m_2 are perpendicular, then ___________

Solution

Answer: c. $m_1 m_2 = -1$
Solution: Condition for perpendicularity of two lines.

Q1.8 [1 mark]

The value of i^4 is ___________

Solution

Answer: a. 1
Solution: $i^2 = -1$. $i^4 = (i^2)^2 = (-1)^2 = 1$.

Q1.9 [1 mark]

The conjugate of $2 - 3i$ is ___________

Solution

Answer: b. $2 + 3i$
Solution: To find conjugate, change sign of imaginary part.

Q1.10 [1 mark]

The radius of the circle $x^2 + y^2 = 36$ is ___________

Solution

Answer: d. 6
Solution: Standard form $x^2 + y^2 = r^2$. $r^2 = 36 \Rightarrow r = 6$

Q1.11 [1 mark]

If vectors $\vec{a}$ and $\vec{b}$ are parallel, then ___________

Solution

Answer: a. $\vec{a} \times \vec{b} = 0$
Solution: Cross product of parallel vectors is zero vector.

Q1.12 [1 mark]

The dot product of $\vec{i}$ and $\vec{j}$ is ___________

Solution

Answer: c. 0
Solution: Orthogonal unit vectors have dot product zero.

Q1.13 [1 mark]

The degree of differential equation $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 = 0$ **is** ____________

Solution

Answer: a. 1
Solution: Degree is the power of the highest order derivative. Highest order derivative is $\frac{d^2y}{dx^2}$, its power is 1.

Q1.14 [1 mark]

The value of $\cos(0)$ **is** ____________

Solution

Answer: b. 1
Solution: Standard trigonometric value.

Question 2 [14 marks]

Q2.a [3 marks]

Evaluate the determinant: $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

Solution

Solution: $= 1(45 - 48) - 2(36 - 42) + 3(32 - 35) = 1(-3) - 2(-6) + 3(-3) = -3 + 12 - 9 = 12 - 12 = 0$

Q2.b [4 marks]

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$**, find** AB**.**

Solution

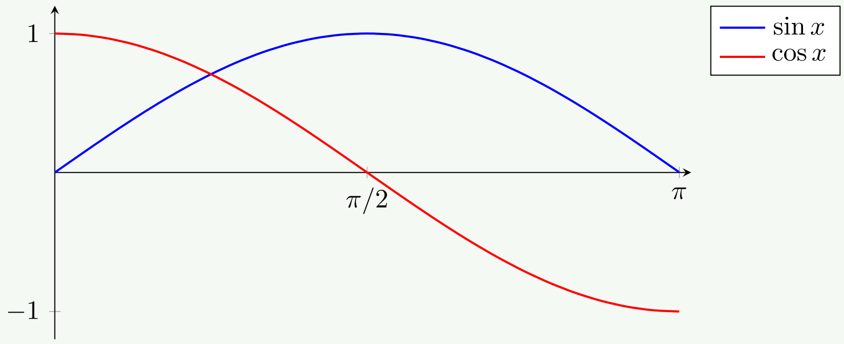
Solution: $AB = \begin{bmatrix} (1)(5) + (2)(7) & (1)(6) + (2)(8) \\ (3)(5) + (4)(7) & (3)(6) + (4)(8) \end{bmatrix}$ $AB = \begin{bmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{bmatrix}$ $AB = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

Q2.c [7 marks]

Graph of $\sin x$ **vs** $\cos x$ **in** $[0, \pi]$**.**

Solution

Solution: Plot both functions on same axis.



Intersection at $x = \pi/4$.

Question 3 [14 marks]

Q3.a [3 marks]

Find complex conjugate and modulus of $z = \frac{3+4i}{1-2i}$.

Solution

Solution: Rationalize denominator: $z = \frac{(3+4i)(1+2i)}{(1-2i)(1+2i)} = \frac{3+6i+4i+8i^2}{1-4i^2}$ $z = \frac{3+10i-8}{1+4} = \frac{-5+10i}{5} = -1+2i$
Conjugate $\bar{z} = -1-2i$ Modulus $|z| = \sqrt{(-1)^2+2^2} = \sqrt{1+4} = \sqrt{5}$

Q3.b [4 marks]

Find the angle between vectors $\vec{a} = i + j$ and $\vec{b} = i - j$.

Solution

Solution: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$ Dot product: $(1)(1) + (1)(-1) = 1 - 1 = 0$ Magnitudes: $|\vec{a}| = \sqrt{2}, |\vec{b}| = \sqrt{2}$
$0 = \sqrt{2}\sqrt{2}\cos\theta \Rightarrow \cos\theta = 0$ $\theta = 90°$ or $\pi/2$ radians.

Q3.c [7 marks]

Verify Cayley-Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Solution

Solution: Characteristic equation: $|A - \lambda I| = 0$ $\begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0$ $(1-\lambda)(4-\lambda) - 6 = 0$ $4 - \lambda - 4\lambda + \lambda^2 - 6 = 0$ $\lambda^2 - 5\lambda - 2 = 0$
By theorem, A satisfies this equation: $A^2 - 5A - 2I = 0$
Calculate A^2: $A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$
Substitute into equation: $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7-5-2 & 10-10-0 \\ 15-15-0 & 22-20-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$
Hence Verified.

Question 4 [14 marks]

Q4.a [3 marks]

Differentiate $y = \log(\sin x)$.

Solution

Solution: Chain rule: $\frac{dy}{dx} = \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) = \frac{\cos x}{\sin x} = \cot x$

Q4.b [4 marks]

Evaluate limit: $\lim_{x\to\infty} \frac{2x^2+3x}{5x^2-4}$.

Solution

Solution: Divide numerator and denominator by highest power x^2: $= \lim_{x\to\infty} \frac{2+3/x}{5-4/x^2}$ As $x \to \infty, 3/x \to 0, 4/x^2 \to 0$. $= \frac{2+0}{5-0} = \frac{2}{5}$

Q4.c [7 marks]

Find the area enclosed by $y = x^2$ and $y = 2x$ in the first quadrant.

Solution

Solution: Intersection points: $x^2 = 2x \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0$ $x = 0, x = 2$.
Area $A = \int_0^2 (y_{upper} - y_{lower})dx$ $y_{upper} = 2x$ (line is above parabola in $[0, 2]$) $y_{lower} = x^2$
$A = \int_0^2 (2x - x^2)dx = [x^2 - \frac{x^3}{3}]_0^2 = (2^2 - \frac{8}{3}) - (0) = 4 - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3}$ square units.

Question 5 [14 marks]

Q5.a [3 marks]

Evaluate $\int_0^{\pi/2} \sin^2 x dx$.

Solution

Solution: Use $\sin^2 x = \frac{1-\cos 2x}{2}$. $= \frac{1}{2}\int_0^{\pi/2}(1-\cos 2x)dx = \frac{1}{2}[x - \frac{\sin 2x}{2}]_0^{\pi/2} = \frac{1}{2}[(\frac{\pi}{2} - \frac{\sin \pi}{2}) - (0-0)] = \frac{1}{2}[\frac{\pi}{2} - 0] = \frac{\pi}{4}$

Q5.b [4 marks]

Solve differential equation $\frac{dy}{dx} = e^{3x-2y}$.

Solution

Solution: $\frac{dy}{dx} = \frac{e^{3x}}{e^{2y}}$ Separate variables: $e^{2y}dy = e^{3x}dx$
Integrate both sides: $\int e^{2y}dy = \int e^{3x}dx$ $\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + C$

Q5.c [7 marks]

Find the radius of curvature for curve $y = x^2$ at origin.

Solution

Solution: Formula: $\rho = \frac{(1+(y')^2)^{3/2}}{|y''|}$
Given $y = x^2$ $y' = 2x \Rightarrow y'(0) = 0$ $y'' = 2 \Rightarrow y''(0) = 2$
Substitute values: $\rho = \frac{(1+0^2)^{3/2}}{|2|} = \frac{1^{3/2}}{2} = \frac{1}{2}$ Radius of Curvature $= 0.5$

Formula Cheat Sheet

Key Formula

Complex Numbers: $z = a + bi, |z| = \sqrt{a^2 + b^2}, \bar{z} = a - bi\ i^2 = -1$

Key Formula

Vectors: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta\ \vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \hat{n}$ Unit vectors: $\hat{i} \cdot \hat{i} = 1, \hat{i} \cdot \hat{j} = 0$

Key Formula

Calculus: $\int x^n dx = \frac{x^{n+1}}{n+1}\ \frac{d}{dx}(\log x) = 1/x$