

# Mathematics-I Solutions

DI01000021 – Summer 2025

Semester 1 Study Material

*Detailed Solutions and Explanations*

## Question 1 [14 marks]

Fill in the blanks/MCQs using appropriate choice from the given options

### Q1.1 [1 mark]

$\log_3 1 =$  \_\_\_\_\_

**Solution**

**Answer:** d. 0

**Solution:** For any base  $a > 0, a \neq 1$ :  $\log_a 1 = 0$  Therefore:  $\log_3 1 = 0$

### Q1.2 [1 mark]

The modulus of the complex number  $z = 3 + 4i$  is \_\_\_\_\_

**Solution**

**Answer:** a. 5

**Solution:**  $|z| = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

### Q1.3 [1 mark]

The value of  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$  is \_\_\_\_\_

**Solution**

**Answer:** b. 1

**Solution:** Standard limit identity.

### Q1.4 [1 mark]

If  $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ , then  $|A| =$  \_\_\_\_\_

**Solution**

**Answer:** c. -3

**Solution:**  $|A| = (2)(0) - (3)(1) = 0 - 3 = -3$

### Q1.5 [1 mark]

The derivative of  $\sin x$  is \_\_\_\_\_

**Solution**

**Answer:** a.  $\cos x$

**Solution:** Standard differentiation formula.

Q1.6 [1 mark]

$\int_0^1 x dx =$  \_\_\_\_\_

**Solution**

**Answer:** b.  $1/2$

**Solution:**  $\int x dx = x^2/2$ . Value =  $[\frac{1^2}{2}] - [\frac{0^2}{2}] = 1/2 - 0 = 0.5$

Q1.7 [1 mark]

If two lines slopes  $m_1$  and  $m_2$  are perpendicular, then \_\_\_\_\_

**Solution**

**Answer:** c.  $m_1 m_2 = -1$

**Solution:** Condition for perpendicularity of two lines.

Q1.8 [1 mark]

The value of  $i^4$  is \_\_\_\_\_

**Solution**

**Answer:** a. 1

**Solution:**  $i^2 = -1$ .  $i^4 = (i^2)^2 = (-1)^2 = 1$ .

Q1.9 [1 mark]

The conjugate of  $2 - 3i$  is \_\_\_\_\_

**Solution**

**Answer:** b.  $2 + 3i$

**Solution:** To find conjugate, change sign of imaginary part.

Q1.10 [1 mark]

The radius of the circle  $x^2 + y^2 = 36$  is \_\_\_\_\_

**Solution**

**Answer:** d. 6

**Solution:** Standard form  $x^2 + y^2 = r^2$ .  $r^2 = 36 \Rightarrow r = 6$

Q1.11 [1 mark]

If vectors  $\vec{a}$  and  $\vec{b}$  are parallel, then \_\_\_\_\_

**Solution**

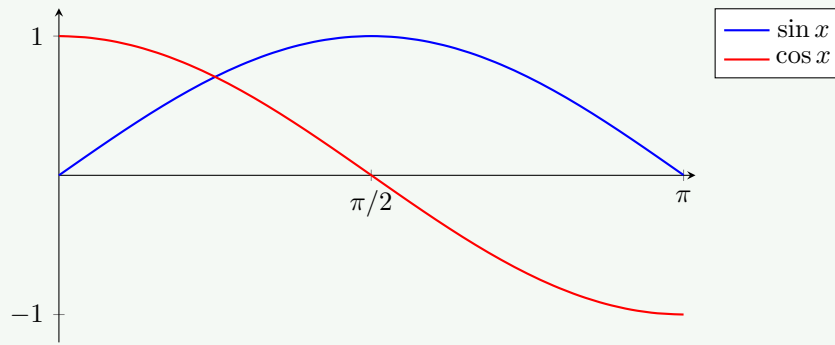
**Answer:** a.  $\vec{a} \times \vec{b} = 0$

**Solution:** Cross product of parallel vectors is zero vector.

Q1.12 [1 mark]

The dot product of  $\vec{i}$  and  $\vec{j}$  is \_\_\_\_\_

**Solution****Answer:** c. 0**Solution:** Orthogonal unit vectors have dot product zero.**Q1.13 [1 mark]****The degree of differential equation  $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 = 0$  is \_\_\_\_\_****Solution****Answer:** a. 1**Solution:** Degree is the power of the highest order derivative. Highest order derivative is  $\frac{d^2y}{dx^2}$ , its power is 1.**Q1.14 [1 mark]****The value of  $\cos(0)$  is \_\_\_\_\_****Solution****Answer:** b. 1**Solution:** Standard trigonometric value.**Question 2 [14 marks]****Q2.a [3 marks]****Evaluate the determinant:**  $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ **Solution****Solution:**  $= 1(45 - 48) - 2(36 - 42) + 3(32 - 35) = 1(-3) - 2(-6) + 3(-3) = -3 + 12 - 9 = 12 - 12 = 0$ **Q2.b [4 marks]****If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ , find  $AB$ .****Solution****Solution:**  $AB = \begin{bmatrix} (1)(5) + (2)(7) & (1)(6) + (2)(8) \\ (3)(5) + (4)(7) & (3)(6) + (4)(8) \end{bmatrix} AB = \begin{bmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{bmatrix} AB = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$ **Q2.c [7 marks]****Graph of  $\sin x$  vs  $\cos x$  in  $[0, \pi]$ .****Solution****Solution:** Plot both functions on same axis.



Intersection at  $x = \pi/4$ .

### Question 3 [14 marks]

#### Q3.a [3 marks]

Find complex conjugate and modulus of  $z = \frac{3+4i}{1-2i}$ .

##### Solution

**Solution:** Rationalize denominator:  $z = \frac{(3+4i)(1+2i)}{(1-2i)(1+2i)} = \frac{3+6i+4i+8i^2}{1-4i^2} = \frac{3+10i-8}{1+4} = \frac{-5+10i}{5} = -1 + 2i$   
 Conjugate  $\bar{z} = -1 - 2i$  Modulus  $|z| = \sqrt{(-1)^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

#### Q3.b [4 marks]

Find the angle between vectors  $\vec{a} = i + j$  and  $\vec{b} = i - j$ .

##### Solution

**Solution:**  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$  Dot product:  $(1)(1) + (1)(-1) = 1 - 1 = 0$  Magnitudes:  $|\vec{a}| = \sqrt{2}, |\vec{b}| = \sqrt{2}$   
 $0 = \sqrt{2}\sqrt{2} \cos \theta \Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$  or  $\pi/2$  radians.

#### Q3.c [7 marks]

Verify Cayley-Hamilton Theorem for  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ .

##### Solution

**Solution:** Characteristic equation:  $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)(4-\lambda) - 6 = 0 \Rightarrow 4 - \lambda - 4\lambda + \lambda^2 - 6 = 0$   
 $\lambda^2 - 5\lambda - 2 = 0$   
 By theorem,  $A$  satisfies this equation:  $A^2 - 5A - 2I = 0$   
 Calculate  $A^2$ :  $A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$   
 Substitute into equation:  $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7-5-2 & 10-10-0 \\ 15-15-0 & 22-20-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$   
 Hence Verified.

### Question 4 [14 marks]

#### Q4.a [3 marks]

Differentiate  $y = \log(\sin x)$ .

#### Solution

**Solution:** Chain rule:  $\frac{dy}{dx} = \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) = \frac{\cos x}{\sin x} = \cot x$

#### Q4.b [4 marks]

**Evaluate limit:**  $\lim_{x \rightarrow \infty} \frac{2x^2+3x}{5x^2-4}$ .

#### Solution

**Solution:** Divide numerator and denominator by highest power  $x^2$ :  $= \lim_{x \rightarrow \infty} \frac{2+3/x}{5-4/x^2}$  As  $x \rightarrow \infty, 3/x \rightarrow 0, 4/x^2 \rightarrow 0. = \frac{2+0}{5-0} = \frac{2}{5}$

#### Q4.c [7 marks]

**Find the area enclosed by  $y = x^2$  and  $y = 2x$  in the first quadrant.**

#### Solution

**Solution:** Intersection points:  $x^2 = 2x \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow x = 0, x = 2$ .  
Area  $A = \int_0^2 (y_{upper} - y_{lower}) dx$   $y_{upper} = 2x$  (line is above parabola in  $[0, 2]$ )  $y_{lower} = x^2$   
 $A = \int_0^2 (2x - x^2) dx = [x^2 - \frac{x^3}{3}]_0^2 = (2^2 - \frac{8}{3}) - (0) = 4 - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3}$  square units.

#### Question 5 [14 marks]

##### Q5.a [3 marks]

**Evaluate**  $\int_0^{\pi/2} \sin^2 x dx$ .

#### Solution

**Solution:** Use  $\sin^2 x = \frac{1-\cos 2x}{2}$ .  $= \frac{1}{2} \int_0^{\pi/2} (1-\cos 2x) dx = \frac{1}{2} [x - \frac{\sin 2x}{2}]_0^{\pi/2} = \frac{1}{2} [(\frac{\pi}{2} - \frac{\sin \pi}{2}) - (0-0)] = \frac{1}{2} [\frac{\pi}{2} - 0] = \frac{\pi}{4}$

##### Q5.b [4 marks]

**Solve differential equation**  $\frac{dy}{dx} = e^{3x-2y}$ .

#### Solution

**Solution:**  $\frac{dy}{dx} = \frac{e^{3x}}{e^{2y}}$  Separate variables:  $e^{2y} dy = e^{3x} dx$   
Integrate both sides:  $\int e^{2y} dy = \int e^{3x} dx$   $\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + C$

##### Q5.c [7 marks]

**Find the radius of curvature for curve  $y = x^2$  at origin.**

#### Solution

**Solution:** Formula:  $\rho = \frac{(1+(y')^2)^{3/2}}{|y''|}$   
Given  $y = x^2$   $y' = 2x \Rightarrow y'(0) = 0$   $y'' = 2 \Rightarrow y''(0) = 2$   
Substitute values:  $\rho = \frac{(1+0^2)^{3/2}}{|2|} = \frac{1^{3/2}}{2} = \frac{1}{2}$  Radius of Curvature = 0.5

## Formula Cheat Sheet

### Key Formula

**Complex Numbers:**  $z = a + bi, |z| = \sqrt{a^2 + b^2}, \bar{z} = a - bi, i^2 = -1$

### Key Formula

**Vectors:**  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta, \vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \theta \hat{n}$  Unit vectors:  $\hat{i} \cdot \hat{i} = 1, \hat{i} \cdot \hat{j} = 0$

### Key Formula

**Calculus:**  $\int x^n dx = \frac{x^{n+1}}{n+1}, \frac{d}{dx}(\log x) = 1/x$