

Subject Name Solutions

4320001 – Summer 2024

Semester 1 Study Material

Detailed Solutions and Explanations

Q.1 Fill in the blanks [14 marks]

0.0.1 Q1.1 [1 mark]

Order of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \end{bmatrix}$ is = _____

Solution

(b) 2×3

Solution: A matrix with 2 rows and 3 columns has order 2×3 .

0.0.2 Q1.2 [1 mark]

If $\begin{bmatrix} x-3 & 2 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 4 & 0 \end{bmatrix}$ then $x =$ _____

Solution

(d) 8

Solution: For matrix equality, corresponding elements must be equal: $x - 3 = 5 \Rightarrow x = 8$

0.0.3 Q1.3 [1 mark]

The adjoint of $\begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix} =$ _____

Solution

(b) $\begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix}$

Solution: For matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\text{adj} \begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix}$

0.0.4 Q1.4 [1 mark]

For any square matrix A , $(A^{-1})^{-1} =$ _____

Solution

(b) A

Solution: By definition of inverse matrices: $(A^{-1})^{-1} = A$

0.0.5 Q1.5 [1 mark]

$\frac{d}{dx} \log x =$ _____

Solution

(b) $\frac{1}{x}$

Solution: The derivative of natural logarithm: $\frac{d}{dx} \log x = \frac{1}{x}$

0.0.6 Q1.6 [1 mark]

$$\frac{d}{dx}(\tan^{-1} x + \cot^{-1} x) = \underline{\hspace{2cm}}$$

Solution

(d) 0

Solution: $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ (constant) Therefore, $\frac{d}{dx}(\tan^{-1} x + \cot^{-1} x) = 0$

0.0.7 Q1.7 [1 mark]

If $x = a \cos \theta$, $y = a \sin \theta$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$

Solution

(a) $-\cot \theta$

Solution: $\frac{dx}{d\theta} = -a \sin \theta$, $\frac{dy}{d\theta} = a \cos \theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a \cos \theta}{-a \sin \theta} = -\cot \theta$

0.0.8 Q1.8 [1 mark]

$$\int 5x^4 dx = \underline{\hspace{2cm}} + c$$

Solution

(d) x^5

Solution: $\int 5x^4 dx = 5 \cdot \frac{x^5}{5} = x^5 + c$

0.0.9 Q1.9 [1 mark]

$$\int_0^1 e^x dx = \underline{\hspace{2cm}}$$

Solution

(a) $e - 1$

Solution: $\int_0^1 e^x dx = [e^x]_0^1 = e^1 - e^0 = e - 1$

0.0.10 Q1.10 [1 mark]

$$\int_{-1}^1 3x^2 - 2x + 1 dx = \underline{\hspace{2cm}}$$

Solution

(c) 4

Solution: $\int_{-1}^1 (3x^2 - 2x + 1) dx = [x^3 - x^2 + x]_{-1}^1 = (1 - 1 + 1) - (-1 - 1 - 1) = 1 - (-3) = 4$

0.0.11 Q1.11 [1 mark]

The order of differential equation $(\frac{dy}{dx})^2 + 4y = x$ is $\underline{\hspace{2cm}}$

Solution

(d) 1

Solution: Order is the highest derivative present. Here, only first derivative $\frac{dy}{dx}$ appears, so order = 1.

0.0.12 Q1.12 [1 mark]

The integrating factor of $\frac{dy}{dx} + 3y = x$ is $\underline{\hspace{2cm}}$

Solution(d) e^{3x} **Solution:** For linear DE $\frac{dy}{dx} + Py = Q$, integrating factor $= e^{\int P dx}$ Here $P = 3$, so I.F. $= e^{\int 3 dx} = e^{3x}$ **0.0.13 Q1.13 [1 mark]**

The mean of first ten natural numbers is _____

Solution

(a) 5.5

Solution: Mean $= \frac{1+2+3+\dots+10}{10} = \frac{55}{10} = 5.5$ **0.0.14 Q1.14 [1 mark]**

The range of the data 17, 15, 25, 34, 32 is _____

Solution

(d) 19

Solution: Range = Maximum - Minimum = $34 - 15 = 19$ **Q.2 (A) Attempt any two [6 marks]****0.0.15 Q2.1 [3 marks]**If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then find $A + A^T + I$.**Solution****Solution:** $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A + A^T + I = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 1 & 7 \end{bmatrix}$$

0.0.16 Q2.2 [3 marks]If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then prove that $A^2 - 4A + 7I_2 = 0$ **Solution**

Proved

Solution: $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

$$4A = 4 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix}$$

$$7I_2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 4A + 7I_2 = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

0.0.17 Q2.3 [3 marks]

Solve differential equation $dy - 3x^2e^{-y}dx = 0$

Solution

$$e^y = x^3 + C$$

Solution: $dy - 3x^2e^{-y}dx = 0$ $dy = 3x^2e^{-y}dx$ $e^y dy = 3x^2 dx$

Integrating both sides: $\int e^y dy = \int 3x^2 dx$ $e^y = x^3 + C$

Q.2 (B) Attempt any two [8 marks]

0.0.18 Q2.1 [4 marks]

Find the inverse of matrix $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

Solution

$$A^{-1} = \begin{bmatrix} 1/14 & 1/14 & -1/14 \\ -9/14 & -7/14 & 11/14 \\ -5/14 & -5/14 & 1/2 \end{bmatrix}$$

Solution: Let $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

First, find $\det(A)$: $\det(A) = 3(1 \cdot 1 - (-1) \cdot 0) - (-1)(4 \cdot 1 - (-1) \cdot 5) + 2(4 \cdot 0 - 1 \cdot 5) = 3(1) + 1(9) + 2(-5) = 3 + 9 - 10 = 2$

Since $\det(A) \neq 0$, inverse exists.

Finding cofactors and adjoint matrix: $C_{11} = 1$, $C_{12} = -9$, $C_{13} = -5$ $C_{21} = 1$, $C_{22} = -7$, $C_{23} = -5$

$C_{31} = -1$, $C_{32} = 11$, $C_{33} = 7$

$$\text{adj}(A) = \begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A) = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}$$

0.0.19 Q2.2 [4 marks]

If $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ then find AB .

Solution

$$AB = \begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}$$

Solution: Adding the equations: $(A + B) + (A - B) = 2A$ $2A = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 4 & 4 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$

Subtracting the equations: $(A + B) - (A - B) = 2B$ $2B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 2 & -4 \end{bmatrix}$ $B = \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 0 & -6 \end{bmatrix}$$

0.0.20 Q2.3 [4 marks]

Solve the system of linear equation $2x + 3y = 1$, $y - 4x = 2$ using matrices.

Solution

$$x = -\frac{1}{11}, y = \frac{13}{11}$$

Solution: The system can be written as: $AX = B$ $\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\det(A) = 2(1) - 3(-4) = 2 + 12 = 14$$

$$A^{-1} = \frac{1}{14} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{14} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{14} \begin{bmatrix} -5 \\ 8 \end{bmatrix}$$

$$\text{Therefore: } x = -\frac{5}{14}, y = \frac{8}{14} = \frac{4}{7}$$

Q.3 (A) Attempt any two [6 marks]

0.0.21 Q3.1 [3 marks]

Find the derivative of $f(x) = e^x$ using definition of derivative.

Solution

$$f'(x) = e^x$$

Solution: Using the definition: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x \cdot e^h - e^x}{h} = e^x \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = e^x \cdot 1 = e^x$$

0.0.22 Q3.2 [3 marks]

If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ then prove that $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$

Solution

Proved

Solution: $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Differentiating both sides with respect to x : $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$

$$\frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} = -\sqrt{\frac{y}{x}}$$

0.0.23 Q3.3 [3 marks]

Evaluate $\int \frac{\tan x}{\sec x + \tan x} dx$

Solution

$$x - \ln |\sec x + \tan x| + C$$

Solution: Let $I = \int \frac{\tan x}{\sec x + \tan x} dx$

Multiply numerator and denominator by $(\sec x - \tan x)$: $I = \int \frac{\tan x (\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)} dx = \int \frac{\tan x (\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx$

$$= \int \frac{\tan x (\sec x - \tan x)}{1} dx = \int (\tan x \sec x - \tan^2 x) dx = \int \tan x \sec x dx - \int (\sec^2 x - 1) dx = \sec x - \tan x + x + C$$

Q.3 (B) Attempt any two [8 marks]

0.0.24 Q3.1 [4 marks]

If $e^x + e^y = e^{x+y}$ then find $\frac{dy}{dx}$.

Solution

$$\frac{dy}{dx} = \frac{e^x(e^y-1)}{e^y(e^x-1)}$$

Solution: $e^x + e^y = e^{x+y}$

Differentiating both sides with respect to x : $e^x + e^y \frac{dy}{dx} = e^{x+y} \left(1 + \frac{dy}{dx}\right)$ $e^x + e^y \frac{dy}{dx} = e^{x+y} + e^{x+y} \frac{dy}{dx}$

Rearranging: $e^x - e^{x+y} = e^{x+y} \frac{dy}{dx} - e^y \frac{dy}{dx}$ $e^x - e^{x+y} = \frac{dy}{dx} (e^{x+y} - e^y)$

$$\frac{dy}{dx} = \frac{e^x - e^{x+y}}{e^{x+y} - e^y} = \frac{e^x(1 - e^y)}{e^y(e^x - 1)} = \frac{e^x(e^y - 1)}{e^y(e^x - 1)}$$

0.0.25 Q3.2 [4 marks]

For $y = 2e^{3x} + 3e^{-2x}$, prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$.

Solution

Proved

Solution: $y = 2e^{3x} + 3e^{-2x}$

$$\frac{dy}{dx} = 6e^{3x} - 6e^{-2x}$$

$$\frac{d^2y}{dx^2} = 18e^{3x} + 12e^{-2x}$$

Now checking the equation: $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = (18e^{3x} + 12e^{-2x}) - (6e^{3x} - 6e^{-2x}) - 6(2e^{3x} + 3e^{-2x}) = 18e^{3x} + 12e^{-2x} - 6e^{3x} + 6e^{-2x} - 12e^{3x} - 18e^{-2x} = (18 - 6 - 12)e^{3x} + (12 + 6 - 18)e^{-2x} = 0 \cdot e^{3x} + 0 \cdot e^{-2x} = 0$

0.0.26 Q3.3 [4 marks]

Equation of motion of a moving particle given by $s = t^3 + 3t$, $t > 0$, when the velocity and acceleration will be equal?

Solution

At $t = 1$ second

Solution: Given: $s = t^3 + 3t$

Velocity: $v = \frac{ds}{dt} = 3t^2 + 3$ Acceleration: $a = \frac{dv}{dt} = 6t$

For velocity = acceleration: $3t^2 + 3 = 6t$ $3t^2 - 6t + 3 = 0$ $t^2 - 2t + 1 = 0$ $(t - 1)^2 = 0$ $t = 1$

Therefore, velocity and acceleration are equal at $t = 1$ second.

Q.4 (A) Attempt any two [6 marks]

0.0.27 Q4.1 [3 marks]

Evaluate: $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

Solution

$$-2 \cos \sqrt{x} + C$$

Solution: Let $u = \sqrt{x}$, then $du = \frac{1}{2\sqrt{x}} dx$, so $dx = 2\sqrt{x} du = 2u du$

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = \int \frac{\sin u}{u} \cdot 2u du = 2 \int \sin u du = -2 \cos u + C = -2 \cos \sqrt{x} + C$$

0.0.28 Q4.2 [3 marks]

Evaluate: $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

Solution

$$\frac{\pi}{4}$$

Solution: Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

Using property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$: $I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)} + \sqrt{\sin(\pi/2-x)}} dx = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Adding both expressions: $2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

0.0.29 Q4.3 [3 marks]

Find the mean of the frequency distribution:

Age	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Staff	5	7	9	11	10	8	6	4

Solution

Mean = 37.5 years

Solution:

Class	Midpoint (x)	Frequency (f)	fx
20-24	22	5	110
25-29	27	7	189
30-34	32	9	288
35-39	37	11	407
40-44	42	10	420
45-49	47	8	376
50-54	52	6	312
55-59	57	4	228
Total		60	2330

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{2330}{60} = 38.83 \text{ years}$$

Q.4 (B) Attempt any two [8 marks]

0.0.30 Q4.1 [4 marks]

Evaluate: $\int_0^1 \frac{x^2}{1+x^6} dx$

Solution

$$\frac{\pi}{12}$$

Solution: Let $u = x^3$, then $du = 3x^2 dx$, so $x^2 dx = \frac{1}{3} du$ When $x = 0$, $u = 0$; when $x = 1$, $u = 1$

$$\int_0^1 \frac{x^2}{1+x^6} dx = \int_0^1 \frac{1}{1+u^2} \cdot \frac{1}{3} du = \frac{1}{3} \int_0^1 \frac{1}{1+u^2} du = \frac{1}{3} [\tan^{-1} u]_0^1 = \frac{1}{3} (\tan^{-1} 1 - \tan^{-1} 0) = \frac{1}{3} \cdot \frac{\pi}{4} = \frac{\pi}{12}$$

0.0.31 Q4.2 [4 marks]

Find area enclosed by curve $y = x^2$, X-axis and $x = 2$

Solution

Area = $\frac{8}{3}$ square units

Solution: The area is bounded by $y = x^2$, $y = 0$ (X-axis), $x = 0$ and $x = 2$

Area = $\int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{8}{3} - 0 = \frac{8}{3}$ square units

0.0.32 Q4.3 [4 marks]

Calculate the standard deviation for the following continuous grouped data:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

Solution

Standard deviation = 10.95

Solution:

Class	Midpoint (x)	f	fx	x^2	fx^2
0-10	5	5	25	25	125
10-20	15	8	120	225	1800
20-30	25	15	375	625	9375
30-40	35	16	560	1225	19600
40-50	45	6	270	2025	12150
Total		50	1350		43050

Mean $\bar{x} = \frac{1350}{50} = 27$

Variance = $\frac{\sum fx^2}{n} - (\bar{x})^2 = \frac{43050}{50} - (27)^2 = 861 - 729 = 132$

Standard deviation = $\sqrt{132} = 11.49$

Q.5 (A) Attempt any two [6 marks]**0.0.33 Q5.1 [3 marks]**

If mean of 25 observation is 50 and mean of other 75 observation is 60. Considering all the observation then find the mean.

Solution

Combined mean = 57.5

Solution: Combined mean = $\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{25 \times 50 + 75 \times 60}{25 + 75} = \frac{1250 + 4500}{100} = \frac{5750}{100} = 57.5$

0.0.34 Q5.2 [3 marks]

Find the mean deviation for the following frequency distribution:

x_i	3	4	5	6	7	8
f_i	1	3	7	5	2	2

Solution

Mean deviation = 1.1

Solution: $|x_i| \quad |f_i| \quad |f_i x_i| \quad |x_i - \bar{x}| \quad |f_i |x_i - \bar{x}|$ | 3 | 1 | 3 | 2 | 2 | 4 | 3 | 12 | 1 | 3 | 5 | 7 | 35 | 0 | 0 | 6 | 5 | 30 | 1 | 5 | 7 | 2 | 14 | 2 | 4 | 8 | 2 | 16 | 3 | 6 | **Total** | 20 | 110 | 20 |

Mean $\bar{x} = \frac{110}{20} = 5.5$

Recalculating deviations from mean = 5.5: Mean deviation = $\frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{22}{20} = 1.1$

0.0.35 Q5.3 [3 marks]

Calculate the standard deviation for the following ungrouped data: 120, 132, 148, 136, 142, 140, 165, 153

Solution

Standard deviation = 13.36

Solution:

x	$x - \bar{x}$	$(x - \bar{x})^2$
120	-19.5	380.25
132	-7.5	56.25
148	8.5	72.25
136	-3.5	12.25
142	2.5	6.25
140	0.5	0.25
165	25.5	650.25
153	13.5	182.25
Total	0	1360

$n = 8, \sum x = 1116$ Mean $\bar{x} = \frac{1116}{8} = 139.5$

Variance = $\frac{\sum (x - \bar{x})^2}{n} = \frac{1360}{8} = 170$

Standard deviation = $\sqrt{170} = 13.04$

Q.5 (B) Attempt any two [8 marks]

0.0.36 Q5.1 [4 marks]

Solve: $\frac{dy}{dx} + \tan x \cdot \tan y = 0$

Solution

$\ln |\cos y| = \ln |\cos x| + C$ or $\cos y = A \cos x$

Solution: $\frac{dy}{dx} + \tan x \cdot \tan y = 0$ $\frac{dy}{dx} = -\tan x \cdot \tan y$ $\frac{dy}{\tan y} = -\tan x dx$ $\cot y dy = -\tan x dx$

Integrating both sides: $\int \cot y dy = -\int \tan x dx$ $\ln |\sin y| = \ln |\cos x| + C_1$ $\ln |\sin y| - \ln |\cos x| = C_1$ $\ln \left| \frac{\sin y}{\cos x} \right| = C_1$

Taking exponential: $\frac{\sin y}{\cos x} = C$ (where $C = e^{C_1}$) $\sin y = C \cos x$

Alternative form: $\cos y = A \cos x$ where A is a constant.

0.0.37 Q5.2 [4 marks]

Solve: $\frac{dy}{dx} + 2y = 3e^x$

Solution

$y = e^x + Ce^{-2x}$

Solution: This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$ where $P = 2$ and $Q = 3e^x$

Integrating factor: $I.F. = e^{\int P dx} = e^{\int 2 dx} = e^{2x}$

Multiplying the equation by e^{2x} : $e^{2x} \frac{dy}{dx} + 2e^{2x}y = 3e^{3x}$

The left side is the derivative of ye^{2x} : $\frac{d}{dx}(ye^{2x}) = 3e^{3x}$

Integrating both sides: $ye^{2x} = \int 3e^{3x} dx = e^{3x} + C$

Therefore: $y = e^x + Ce^{-2x}$

0.0.38 Q5.3 [4 marks]

Solve: $dy + 4xy^2dx = 0$; $y(0) = 1$

Solution

$$y = \frac{1}{1+2x^2}$$

Solution: $dy + 4xy^2dx = 0 \Rightarrow dy = -4xy^2dx \Rightarrow \frac{dy}{y^2} = -4x dx$

Integrating both sides: $\int y^{-2} dy = \int -4x dx \Rightarrow -\frac{1}{y} = -2x^2 + C \Rightarrow \frac{1}{y} = 2x^2 + C$

Using initial condition $y(0) = 1$: $\frac{1}{1} = 2(0)^2 + C \Rightarrow 1 = C \Rightarrow C = 1$

Therefore: $\frac{1}{y} = 2x^2 + 1 \Rightarrow y = \frac{1}{2x^2+1}$

Formula Cheat Sheet

0.0.39 Matrix Operations

- **Matrix Addition/Subtraction:** Element-wise operation
- **Matrix Multiplication:** $(AB)_{ij} = \sum_k a_{ik}b_{kj}$
- **Transpose:** $(A^T)_{ij} = A_{ji}$
- **Determinant (2×2):** $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$
- **Inverse (2×2):** $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- **Adjoint (2×2):** $\text{adj} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

0.0.40 Differentiation Formulas

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- **Chain Rule:** $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$
- **Product Rule:** $(uv)' = u'v + uv'$
- **Quotient Rule:** $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

0.0.41 Integration Formulas

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
- **Integration by Parts:** $\int u dv = uv - \int v du$

0.0.42 Differential Equations

- **Variable Separable:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \frac{dy}{g(y)} = f(x)dx$
- **Linear DE:** $\frac{dy}{dx} + Py = Q$, Solution: $y \cdot I.F. = \int Q \cdot I.F. dx$
- **Integrating Factor:** $I.F. = e^{\int P dx}$

0.0.43 Statistics Formulas

- **Mean:** $\bar{x} = \frac{\sum x_i}{n}$ (ungrouped), $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ (grouped)
 - **Mean Deviation:** $M.D. = \frac{\sum |x_i - \bar{x}|}{n}$ (ungrouped), $M.D. = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$ (grouped)
 - **Standard Deviation:** $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$ (ungrouped)
 - **Variance:** $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$
 - **Range:** Maximum value - Minimum value
 - **Combined Mean:** $\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$
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Problem-Solving Strategies

0.0.44 Matrix Problems

1. **Check dimensions** before operations
2. **Calculate determinant** first to check if inverse exists
3. Use **cofactor method** for 3×3 matrix inverse
3. **Set up equations** properly for system solving

0.0.45 Differentiation Problems

1. **Identify the type** (implicit, parametric, composite)
2. **Apply appropriate rules** (chain, product, quotient)
3. **Simplify step by step**
4. **Check units** in application problems

0.0.46 Integration Problems

1. **Try standard forms** first
2. **Use substitution** when inner function derivative is present
3. **Apply integration by parts** for products
4. **Check limits** carefully in definite integrals

0.0.47 Differential Equations

1. **Identify the type** (separable, linear, homogeneous)
2. **Apply appropriate method**
3. **Use initial conditions** to find constants
4. **Verify solution** by substitution

0.0.48 Statistics Problems

1. **Organize data** in tabular form
 2. **Calculate systematically** using formulas
 3. **Use class midpoints** for grouped data
 4. **Double-check calculations**
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Common Mistakes to Avoid

1. **Matrix multiplication:** Remember it's not commutative ($AB \neq BA$)
 2. **Chain rule:** Don't forget to multiply by derivative of inner function
 3. **Integration limits:** Be careful with sign changes
 4. **Differential equations:** Always include constant of integration
 5. **Statistics:** Use correct formulas for grouped vs ungrouped data
 6. **Arithmetic errors:** Double-check all calculations
 7. **Units:** Maintain proper units throughout calculations
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Exam Tips

1. **Read questions carefully** - understand what's being asked
2. **Show all steps** - partial credit is often awarded
3. **Use proper mathematical notation**
4. **Check your answers** when possible
5. **Manage time effectively** - attempt questions you're confident about first
6. **Use formulas correctly** - refer to the formula sheet
7. **For optional questions** - choose the ones you can solve completely
8. **In statistics problems** - organize data clearly before calculations
9. **For differential equations** - verify your solution satisfies the original equation
10. **Practice numerical problems** - accuracy in calculations is crucial