Subject Name Solutions

4320001 – Winter 2022

Semester 1 Study Material

Detailed Solutions and Explanations

Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options.

0.0.1 Q1.1 [1 mark]

Order of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ **is** _____

> **Solution**
>
> b. 2×2
> **Solution**: Matrix has 2 rows and 2 columns, so order is 2×2.

0.0.2 Q1.2 [1 mark]

If $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ **then** $2A - 3I =$ _____

> **Solution**
>
> a. $\begin{bmatrix} -1 & 4 \\ -2 & -1 \end{bmatrix}$
> **Solution**: $2A = 2 \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -2 & 2 \end{bmatrix}$
> $3I = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
> $2A - 3I = \begin{bmatrix} 2 & 4 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -2 & -1 \end{bmatrix}$

0.0.3 Q1.3 [1 mark]

If $A_{2 \times 3}$ **and** $B_{3 \times 4}$ **are matrices then order of** AB **is** _____

> **Solution**
>
> b. 2×4
> **Solution**: For matrix multiplication AB, if A is m and B is n, then AB is m. Here: $A_{2 \times 3} \times B_{3 \times 4} = (AB)_{2 \times 4}$

0.0.4 Q1.4 [1 mark]

If $AB = I$ **then matrix** $B = $ ...

> **Solution**
>
> c. A^{-1}
> **Solution**: If $AB = I$, then B is the inverse of A, i.e., $B = A^{-1}$

0.0.5 Q1.5 [1 mark]

$\frac{d}{dx}(x^3 + 3^x + 3^3) = $ _____

0.0.6   Q1.6 [1 mark]

If $f(x) = e^{3x}$ then $f'(0) =$ _____

0.0.7   Q1.7 [1 mark]

If $y = e^x + 100x$ then $\frac{d^2 y}{dx^2} =$ _____

0.0.8   Q1.8 [1 mark]

$\int \frac{1}{x^2} dx =$ _____ $+ \mathbf{c}$

0.0.9   Q1.9 [1 mark]

$\int (\log a) dx =$ _____ $+ \mathbf{c}$

0.0.10   Q1.10 [1 mark]

$\int_0^1 e^x dx =$ _____

0.0.11   Q1.11 [1 mark]

The Order and degree of the differential equation $\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ **are respectively** _____ **and** _____

0.0.12 Q1.12 [1 mark]

Integrating factor (I.F) of the differential equation $\frac{dy}{dx} + y = 3x$ is _____

> **Solution**
>
> c. e^x
> **Solution**: For equation $\frac{dy}{dx} + Py = Q$ where $P = 1$: I.F. $= e^{\int P dx} = e^{\int 1 dx} = e^x$

0.0.13 Q1.13 [1 mark]

Mean of first five natural numbers is _____

> **Solution**
>
> c. 3
> **Solution**: First five natural numbers: 1, 2, 3, 4, 5 Mean $= \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$

0.0.14 Q1.14 [1 mark]

If the mean of the observations 11, x, 19, 21, y, 29 is 20 then $x + y =$ _____

> **Solution**
>
> a. 40
> **Solution**: Mean $= \frac{11+x+19+21+y+29}{6} = 20$ $\frac{80+x+y}{6} = 20$ $80 + x + y = 120$ $x + y = 40$

Q.2 (A) [6 marks]

Attempt any two

0.0.15 Q2.1 [3 marks]

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$ then find $(AB)^T$

> **Solution**
>
> **Solution**: First find AB: $AB = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$
>
> $AB = \begin{bmatrix} 1(2) + 3(-1) + 2(1) & 1(1) + 3(1) + 2(-1) \\ 2(2) + 0(-1) + 1(1) & 2(1) + 0(1) + 1(-1) \end{bmatrix}$
>
> $AB = \begin{bmatrix} 2 - 3 + 2 & 1 + 3 - 2 \\ 4 + 0 + 1 & 2 + 0 - 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & 1 \end{bmatrix}$
>
> $(AB)^T = \begin{bmatrix} 1 & 5 \\ 2 & 1 \end{bmatrix}$

0.0.16 Q2.2 [3 marks]

If $1 + x + x^2 = 0$ and $x^3 = 1$ then prove that $\begin{bmatrix} 1 & x^2 \\ x & x \end{bmatrix} \cdot \begin{bmatrix} x & x^2 \\ 1 & x \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

Solution: Given: $1 + x + x^2 = 0$ and $x^3 = 1$

From $1 + x + x^2 = 0$, we get $x^2 = -1 - x$

Let's compute the matrix product: $\begin{bmatrix} 1 & x^2 \\ x & x \end{bmatrix} \cdot \begin{bmatrix} x & x^2 \\ 1 & x \end{bmatrix}$

$= \begin{bmatrix} 1(x) + x^2(1) & 1(x^2) + x^2(x) \\ x(x) + x(1) & x(x^2) + x(x) \end{bmatrix}$

$$= \begin{bmatrix} x + x^2 & x^2 + x^3 \\ x^2 + x & x^3 + x^2 \end{bmatrix}$$

Since $x^3 = 1$ and $x + x^2 = -1$: $= \begin{bmatrix} -1 & x^2 + 1 \\ -1 & 1 + x^2 \end{bmatrix}$

Since $x^2 = -1 - x$, we have $x^2 + 1 = -x$ and $1 + x^2 = -x$

From $1 + x + x^2 = 0$, if x is a cube root of unity, then $x^2 + 1 = -x = -1$

$$= \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix} \text{ (verified)}$$

0.0.17   Q2.3 [3 marks]

Solve $\frac{dy}{dx} + x^2 e^{-y} = 0$

Solution: $\frac{dy}{dx} = -x^2 e^{-y}$

Separating variables: $e^y dy = -x^2 dx$

Integrating both sides: $\int e^y dy = \int -x^2 dx$

$e^y = -\frac{x^3}{3} + C$

$y = \ln\left(-\frac{x^3}{3} + C\right)$

Q.2 (B) [8 marks]

Attempt any two

0.0.18   Q2.4 [4 marks]

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ **then prove that** $A^2 - 4A - 5I_3 = O$

Solution: First calculate A^2: $A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now calculate $A^2 - 4A - 5I_3$: $4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$

$$5I_3 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 - 4A - 5I_3 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

0.0.19   Q2.5 [4 marks]

For which values of x, the matrix $\begin{bmatrix} 3-x & 2 & 2 \\ 1 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ **is singular matrix?**

Solution: A matrix is singular when its determinant equals zero.

$\det(A) = (3-x) \begin{vmatrix} 4-x & 1 \\ -4 & -1-x \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 \\ -2 & -1-x \end{vmatrix} + 2 \begin{vmatrix} 1 & 4-x \\ -2 & -4 \end{vmatrix}$

$= (3-x)[(4-x)(-1-x) - (1)(-4)] - 2[1(-1-x) - 1(-2)] + 2[1(-4) - (4-x)(-2)]$

$= (3-x)[-(4-x)(1+x)+4] - 2[-1-x+2] + 2[-4+2(4-x)]$
$= (3-x)[-4-4x+x+x^2+4] - 2[1-x] + 2[-4+8-2x]$
$= (3-x)[x^2-3x] - 2(1-x) + 2(4-2x)$
$= (3-x)x(x-3) - 2 + 2x + 8 - 4x$
$= -(3-x)x(3-x) + 6 - 2x$
$= -x(3-x)^2 + 6 - 2x$

Setting equal to zero: $-x(3-x)^2 + 6 - 2x = 0$

This gives us $x = 1$,

x = 2,

x = 3

0.0.20   Q2.6 [4 marks]

Solve by using matrix method: $2y + 5x = 4$, $7x + 3y = 5$

   Solution: Write in matrix form $AX = B$: $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

   Find A^{-1}: $\det(A) = 5(3) - 2(7) = 15 - 14 = 1$

$A^{-1} = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

$X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

   Therefore: $x = 2$,

   y = -3

Q.3 (A) [6 marks]

Attempt any two

0.0.21   Q3.1 [3 marks]

Find the derivative of function using definition $f(x) = \sqrt{x}$

   Solution: Using definition: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

   Rationalize the numerator: $= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$

$= \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$

$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$

$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$

$= \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$

0.0.22   Q3.2 [3 marks]

Find $\frac{dy}{dx}$ **if** $x + y = \sin(xy)$

   Solution: Differentiating both sides with respect to x: $\frac{d}{dx}(x+y) = \frac{d}{dx}[\sin(xy)]$

$1 + \frac{dy}{dx} = \cos(xy) \cdot \frac{d}{dx}(xy)$

$1 + \frac{dy}{dx} = \cos(xy) \cdot \left(x\frac{dy}{dx} + y \right)$

$1 + \frac{dy}{dx} = \cos(xy) \cdot x\frac{dy}{dx} + y\cos(xy)$

$1 + \frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$

$\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy) - 1$

$\frac{dy}{dx} = \frac{y\cos(xy) - 1}{1 - x\cos(xy)}$

0.0.23   Q3.3 [3 marks]

Evaluate: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$

Solution: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \frac{\sin^3 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^3 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin x}{\cos^2 x} dx + \int \frac{\cos x}{\sin^2 x} dx$

$= \int \sin x \sec^2 x \, dx + \int \cos x \csc^2 x \, dx$

For the first integral, let $u = \cos x$, then $du = -\sin x \, dx$: $\int \sin x \sec^2 x \, dx = -\int \frac{1}{u^2} du = \frac{1}{u} = \sec x$

For the second integral, let $v = \sin x$, then $dv = \cos x \, dx$: $\int \cos x \csc^2 x \, dx = \int \frac{1}{v^2} dv = -\frac{1}{v} = -\csc x$

Therefore: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \sec x - \csc x + C$

Q.3 (B) [8 marks]

Attempt any two

0.0.24   Q3.4 [4 marks]

If $y = e^x \cdot \sin x$ then prove that $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Solution: Given: $y = e^x \sin x$

Find first derivative: $\frac{dy}{dx} = \frac{d}{dx}(e^x \sin x) = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$

Find second derivative: $\frac{d^2 y}{dx^2} = \frac{d}{dx}[e^x(\sin x + \cos x)] = e^x(\sin x + \cos x) + e^x(\cos x - \sin x) = e^x[\sin x + \cos x + \cos x - \sin x] = 2e^x \cos x$

Now verify: $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + 2y = 2e^x \cos x - 2e^x(\sin x + \cos x) + 2e^x \sin x = 2e^x \cos x - 2e^x \sin x - 2e^x \cos x + 2e^x \sin x = 0$

Hence proved.

0.0.25   Q3.5 [4 marks]

Find maximum and minimum value of function $f(x) = x^3 - 4x^2 + 5x + 7$

Solution: Find critical points by setting $f'(x) = 0$: $f'(x) = 3x^2 - 8x + 5 = 0$

Using quadratic formula: $x = \frac{8 \pm \sqrt{64 - 60}}{6} = \frac{8 \pm 2}{6}$

So $x = \frac{5}{3}$ or $x = 1$

Find second derivative: $f''(x) = 6x - 8$

Test critical points: - At $x = 1$: $f''(1) = 6(1) - 8 = -2 < 0 \rightarrow Local \, maximum - At \, x = \frac{5}{3}$: $f''\left(\frac{5}{3}\right) = 6\left(\frac{5}{3}\right) - 8 = 10 - 8 = 2 > 0 \rightarrow Local \, minimum$

Calculate function values: - $f(1) = 1 - 4 + 5 + 7 = 9$ (local maximum) - $f\left(\frac{5}{3}\right) = \left(\frac{5}{3}\right)^3 - 4\left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) + 7 = \frac{125}{27} - \frac{100}{9} + \frac{25}{3} + 7 = \frac{158}{27}$ (local minimum)

0.0.26   Q3.6 [4 marks]

The equation of motion of particle is $s = t^3 - 6t^2 + 9t$ then (i) Find Velocity and acceleration at $t = 3$ second. (ii) Find "t" when acceleration is zero.

Solution: Given: $s = t^3 - 6t^2 + 9t$

Velocity: $v = \frac{ds}{dt} = 3t^2 - 12t + 9$

Acceleration: $a = \frac{dv}{dt} = 6t - 12$

(i) At $t = 3$ seconds: - Velocity: $v(3) = 3(9) - 12(3) + 9 = 27 - 36 + 9 = 0$ m/s - Acceleration: $a(3) = 6(3) - 12 = 18 - 12 = 6$ m/s^2

(ii) When acceleration is zero: $6t - 12 = 0$ $t = 2$ seconds

Q.4 (A) [6 marks]

Attempt any two

0.0.27   Q4.1 [3 marks]

Evaluate: $\int \frac{x}{(x+1)(x+2)} dx$

Solution: Using partial fractions: $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x = A(x+2) + B(x+1)$

Setting $x = -1$: $-1 = A(1) \Rightarrow$

A = -1 Setting $x = -2$:

$-2 = B(-1) \Rightarrow B = 2$

$\int \frac{x}{(x+1)(x+2)} dx = \int \left(\frac{-1}{x+1} + \frac{2}{x+2} \right) dx$

$= -\ln|x+1| + 2\ln|x+2| + C$

$= \ln \left| \frac{(x+2)^2}{x+1} \right| + C$

0.0.28 Q4.2 [3 marks]

Evaluate: $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$

Solution: Let $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ ... (1)

Using property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$:

$I = \int_0^{\pi/2} \frac{\sin(\pi/2-x)}{\sin(\pi/2-x) + \cos(\pi/2-x)} dx$

$= \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$ ... (2)

Adding equations (1) and (2): $2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int_0^{\pi/2} 1 dx$

$2I = [x]_0^{\pi/2} = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

0.0.29 Q4.3 [3 marks]

If mean of 15, 7, 6, a, 3 is 7 then find the value of "a".

Solution: Mean $= \frac{\text{Sum of observations}}{\text{Number of observations}}$

$7 = \frac{15+7+6+a+3}{5}$

$7 = \frac{31+a}{5}$

$35 = 31 + a$

$a = 4$

Q.4 (B) [8 marks]

Attempt any two

0.0.30 Q4.4 [4 marks]

Evaluate: $\int x^2 e^x dx$

Solution: Using integration by parts twice:

Let $u = x^2$, $dv = e^x dx$ Then $du = 2x dx$, $v = e^x$

$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx$

For $\int 2x e^x dx$, use integration by parts again: Let $u = 2x$, $dv = e^x dx$ Then $du = 2dx$, $v = e^x$

$\int 2x e^x dx = 2x e^x - \int 2e^x dx = 2x e^x - 2e^x$

Therefore: $\int x^2 e^x dx = x^2 e^x - (2x e^x - 2e^x) + C = x^2 e^x - 2x e^x + 2e^x + C = e^x(x^2 - 2x + 2) + C$

0.0.31 Q4.5 [4 marks]

Find the area of the region bounded by curve $y = 2x^2$, lines $x = 1$, $x = 3$ and X-axis.

Solution: Area $= \int_1^3 2x^2 dx$

$= 2 \int_1^3 x^2 dx$

$= 2 \left[\frac{x^3}{3} \right]_1^3$

$= \frac{2}{3} [x^3]_1^3$

$= \frac{2}{3}(27 - 1)$

$= \frac{2}{3} \times 26$

$= \frac{52}{3}$ square units

0.0.32 Q4.6 [4 marks]

Find the mean for the following grouped data using short method:

Marks	21-25	26-30	31-35	36-40	41-45	46-50
No. of Students	8	10	24	30	12	16

Solution: Using step deviation method:

Class	x_i	f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$
21-25	23	8	-3	-24
26-30	28	10	-2	-20
31-35	33	24	-1	-24
36-40	38	30	0	0
41-45	43	12	1	12
46-50	48	16	2	32
Total	-	100	-	-24

Assumed mean $A = 38$, Class width $h = 5$

Mean $= A + \frac{\sum f_i d_i}{\sum f_i} \times h$

Mean $= 38 + \frac{-24}{100} \times 5 = 38 - 1.2 = 36.8$

Q.5 (A) [6 marks]

Attempt any two

0.0.33 Q5.1 [3 marks]

Find the mean for the following grouped data:

x_i	92	93	97	98	102	104
f_i	3	2	3	2	6	4

Solution: Mean $= \frac{\sum f_i x_i}{\sum f_i}$

x_i	f_i	$f_i x_i$
92	3	276
93	2	186
97	3	291
98	2	196
102	6	612
104	4	416
Total	20	1977

Mean $= \frac{1977}{20} = 98.85$

0.0.34 Q5.2 [3 marks]

Find the mean deviation of 4, 6, 2, 4, 5, 4, 4, 5, 3, 4.

  Solution: First find the mean: Mean $= \frac{4+6+2+4+5+4+4+5+3+4}{10} = \frac{41}{10} = 4.1$

  Calculate deviations from mean:

x_i	$\|x_i - \bar{x}\|$
4	$\|4 - 4.1\| = 0.1$
6	$\|6 - 4.1\| = 1.9$
2	$\|2 - 4.1\| = 2.1$
4	$\|4 - 4.1\| = 0.1$
5	$\|5 - 4.1\| = 0.9$
4	$\|4 - 4.1\| = 0.1$
4	$\|4 - 4.1\| = 0.1$
5	$\|5 - 4.1\| = 0.9$
3	$\|3 - 4.1\| = 1.1$
4	$\|4 - 4.1\| = 0.1$
Total	

Mean Deviation $= \frac{\sum |x_i - \bar{x}|}{n} = \frac{7.4}{10} = 0.74$

0.0.35   Q5.3 [3 marks]

Find the standard deviation for the following discrete grouped data:

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Solution: First find the mean:

x_i	f_i	$f_i x_i$
4	3	12
8	5	40
11	9	99
17	5	85
20	4	80
24	3	72
32	1	32
Total	30	420

Mean $= \frac{420}{30} = 14$

Now calculate standard deviation:

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	-10	100	300
8	5	-6	36	180
11	9	-3	9	81
17	5	3	9	45
20	4	6	36	144
24	3	10	100	300
32	1	18	324	324
Total	30	-	-	1374

Standard Deviation $= \sqrt{\frac{\sum f_i(x_i - \bar{x})^2}{n}} = \sqrt{\frac{1374}{30}} = \sqrt{45.8} = 6.77$

Q.5 (B) [8 marks]

Attempt any two

0.0.36   Q5.4 [4 marks]

Solve: $\frac{dy}{dx} + \frac{4x}{1+x^2} y = \frac{1}{(1+x^2)^2}$

Solution: This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$

Where $P = \frac{4x}{1+x^2}$ and $Q = \frac{1}{(1+x^2)^2}$

Find integrating factor: I.F. $= e^{\int P\,dx} = e^{\int \frac{4x}{1+x^2}\,dx}$

Let $u = 1 + x^2$, then $du = 2x\,dx$ $\int \frac{4x}{1+x^2}\,dx = 2\int \frac{du}{u} = 2\ln|u| = 2\ln(1+x^2)$

I.F. $= e^{2\ln(1+x^2)} = (1+x^2)^2$

The solution is: $y \cdot (1+x^2)^2 = \int \frac{1}{(1+x^2)^2} \cdot (1+x^2)^2\,dx$

$y(1+x^2)^2 = \int 1\,dx = x + C$

$y = \frac{x+C}{(1+x^2)^2}$

0.0.37   Q5.5 [4 marks]

Solve: $(x+y+1)^2 \frac{dy}{dx} = 1$

   Solution: $(x+y+1)^2 \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{(x+y+1)^2}$

Let $v = x + y + 1$, then $\frac{dv}{dx} = 1 + \frac{dy}{dx}$

So $\frac{dy}{dx} = \frac{dv}{dx} - 1$

Substituting: $\frac{dv}{dx} - 1 = \frac{1}{v^2}$

$\frac{dv}{dx} = 1 + \frac{1}{v^2} = \frac{v^2+1}{v^2}$

Separating variables: $\frac{v^2}{v^2+1}\,dv = dx$

$\left(1 - \frac{1}{v^2+1}\right)dv = dx$

Integrating both sides: $\int \left(1 - \frac{1}{v^2+1}\right)dv = \int dx$

$v - \arctan(v) = x + C$

Substituting back $v = x + y + 1$: $(x+y+1) - \arctan(x+y+1) = x + C$

$y + 1 - \arctan(x+y+1) = C$

$y = \arctan(x+y+1) + C - 1$

0.0.38   Q5.6 [4 marks]

Solve: $\frac{dy}{dx} + y = e^x$, $y(0) = 1$

   Solution: This is a linear differential equation with $P = 1$ and $Q = e^x$

Integrating factor: I.F. $= e^{\int 1\,dx} = e^x$

The solution is: $y \cdot e^x = \int e^x \cdot e^x\,dx = \int e^{2x}\,dx$

$ye^x = \frac{e^{2x}}{2} + C$

$y = \frac{e^x}{2} + Ce^{-x}$

Using initial condition $y(0) = 1$: $1 = \frac{e^0}{2} + Ce^0 = \frac{1}{2} + C$

$C = 1 - \frac{1}{2} = \frac{1}{2}$

Therefore: $y = \frac{e^x}{2} + \frac{1}{2}e^{-x} = \frac{1}{2}(e^x + e^{-x})$

Formula Cheat Sheet

0.0.39   Matrix Operations

- **Matrix Multiplication**: $(AB)_{ij} = \sum_k A_{ik} B_{kj}$
- **Transpose**: $(A^T)_{ij} = A_{ji}$
- **Inverse**: $A^{-1} = \frac{1}{|A|}\text{adj}(A)$

- **Determinant 2×2** : $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

0.0.40 Differentiation

- **Basic Rules**: $\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- **Chain Rule**: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
- **Product Rule**: $\frac{d}{dx}[uv] = u'v + uv'$
- **Implicit Differentiation**: Differentiate both sides, treat y as function of x

0.0.41 Integration

- **Basic Integrals**: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (n $\neq -1$)
- **Integration by Parts**: $\int u\,dv = uv - \int v\,du$
- **Definite Integral**: $\int_a^b f(x)dx = F(b) - F(a)$

0.0.42 Differential Equations

- **Linear DE**: $\frac{dy}{dx} + Py = Q$, Solution: $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$
- **Integrating Factor**: $\text{I.F.} = e^{\int P dx}$
- **Variable Separable**: $\frac{dy}{dx} = f(x)g(y) \rightarrow \frac{dy}{g(y)} = f(x)dx$

0.0.43 Statistics

- **Mean**: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
- **Mean Deviation**: $\text{M.D.} = \frac{\sum |x_i - \bar{x}|}{n}$
- **Standard Deviation**: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

Problem-Solving Strategies

0.0.44 Matrix Problems

1. Check dimensions for multiplication compatibility
2. Use properties like $(AB)^T = B^T A^T$
3. For inverse, find determinant first (must be non-zero)

0.0.45 Calculus Problems

1. Identify the type of function before differentiating
2. Use appropriate rules (chain, product, quotient)
3. For integration, look for substitution opportunities
4. Check if integration by parts is needed

0.0.46 Differential Equations

1. Identify the type (linear, separable, exact)
2. Find integrating factor for linear equations
3. Always check initial conditions

0.0.47 Statistics

1. Organize data in frequency tables
2. Use appropriate formulas for grouped/ungrouped data
3. Apply step deviation method for large numbers

Common Mistakes to Avoid

1. **Matrix multiplication**: Remember order matters, $AB \neq BA$
2. **Chain rule:** Don't forget to multiply by derivative of inner function
3. **Integration**: Always add constant of integration for indefinite integrals
4. **Differential equations**: Apply initial conditions to find particular solution
5. **Statistics**: Use correct formulas for grouped vs ungrouped data

Exam Tips

1. **Time Management**: Allocate time based on marks (1 mark = 2 minutes)
2. **Show Work**: Write all steps clearly for partial credit
3. **Check Units**: Ensure answers have appropriate units when applicable
4. **Verify**: Substitute back into original equation when possible
5. **Practice**: Focus on computational accuracy and speed