Mathematics-I (DI01000021) - Winter 2024 Solution

Date: 2025-01-02

Q.1 [14 marks]

Fill in the blanks/MCQs using appropriate choice from the given options.

Q1.1 [1 mark]

$\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} =$ __________

Answer: b. 13

Solution:
For 2Œ2 determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
$\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = (5 \times 3) - (1 \times 2) = 15 - 2 = 13$

Q1.2 [1 mark]

If $\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = 0$ then $x =$ __________

Answer: b. 2

Solution:
$\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = x \times 1 - 1 \times 2 = x - 2 = 0$
Therefore, $x = 2$

Q1.3 [1 mark]

If $f(x) = x^2$ then $f(-1) =$ __________

Answer: a. 1

Solution:
$f(x) = x^2$
$f(-1) = (-1)^2 = 1$

Q1.4 [1 mark]

$\log_{10} 1 =$ __________

Answer: b. 0

Solution:
By logarithm property: $\log_a 1 = 0$ for any base $a > 0$
Therefore, $\log_{10} 1 = 0$

Q1.5 [1 mark]

$\sin\frac{\pi}{2} + \cos\frac{\pi}{2} =$ __________
Answer: c. 1

Solution:
$\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$
Therefore, $\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1 + 0 = 1$

Q1.6 [1 mark]

$\tan^{-1}(1) =$ __________
Answer: a. $\frac{\pi}{4}$

Solution:
$\tan\frac{\pi}{4} = 1$
Therefore, $\tan^{-1}(1) = \frac{\pi}{4}$

Q1.7 [1 mark]

$\frac{2\pi}{3}$ radian = __________ degree
Answer: d. 120

Solution:
To convert radians to degrees: degrees = radians $\times \frac{180}{\pi}$
$\frac{2\pi}{3} \times \frac{180}{\pi} = \frac{2\times 180}{3} = \frac{360}{3} = 120°$

Q1.8 [1 mark]

$\hat{i} \times \hat{j} =$ __________
Answer: c. $\hat{k}$

Solution:
By right-hand rule for cross product:
$\hat{i} \times \hat{j} = \hat{k}$

Q1.9 [1 mark]

$|\hat{i} + \hat{j} + \hat{k}| =$ __________
Answer: d. $\sqrt{3}$

Solution:
$|\hat{i} + \hat{j} + \hat{k}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Q1.10 [1 mark]

Slope of line $2x + y - 3 = 0$ is __________
Answer: a. -2

Solution:
Convert to slope-intercept form: $y = -2x + 3$
Slope = coefficient of $x = -2$

Q1.11 [1 mark]

Radius of circle $x^2 + y^2 = 81$ is __________
Answer: b. 9

Solution:
Standard form: $x^2 + y^2 = r^2$
Here, $r^2 = 81$, so $r = 9$

Q1.12 [1 mark]

$\lim_{n\to\infty} \frac{1}{n} =$ __________
Answer: c. 0

Solution:
As n approaches infinity, $\frac{1}{n}$ approaches 0

Q1.13 [1 mark]

$\lim_{x\to 1}(x^2 + x + 1) =$ __________
Answer: a. 3

Solution:
Direct substitution: $(1)^2 + (1) + 1 = 1 + 1 + 1 = 3$

Q1.14 [1 mark]

$\lim_{\theta\to 0} \frac{\tan\theta}{\theta} =$ __________
Answer: b. 1

Solution:
This is a standard limit: $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$

Q.2 (A) [6 marks]

Attempt any two

Q2.1 [3 marks]

Find the value of $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 4 & -2 & 5 \end{vmatrix}$

Solution:
Using expansion along second row (has zero):
$= -2\begin{vmatrix} 3 & 1 \\ -2 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 4 & 5 \end{vmatrix} + 0$
$= -2(15+2) - 1(5-4)$
$= -2(17) - 1(1)$
$= -34 - 1 = -35$

Step	Calculation	Result
Minor 1	$(3 \times 5) - (1 \times -2)$	17
Minor 2	$(1 \times 5) - (1 \times 4)$	1
Final	$-2(17) - 1(1)$	-35

Q2.2 [3 marks]

If $f(x) = x^3 + 5$ then find $f(0)$, $f(1)$ and $f(-1)$

Solution:
Given: $f(x) = x^3 + 5$
$f(0) = (0)^3 + 5 = 0 + 5 = 5$
$f(1) = (1)^3 + 5 = 1 + 5 = 6$
$f(-1) = (-1)^3 + 5 = -1 + 5 = 4$

Input	Calculation	Output
$f(0)$	$0^3 + 5$	5
$f(1)$	$1^3 + 5$	6
$f(-1)$	$(-1)^3 + 5$	4

Q2.3 [3 marks]

Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

Solution:
Using formula: $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$
Let $a = \frac{1}{2}$, $b = \frac{1}{3}$
$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)$
$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$
Hence proved.

Q.2 (B) [8 marks]

Attempt any two

Q2.1 [4 marks]

If $f(x) = \frac{x-1}{x+1}$ **then prove that** $f(x) \cdot f(-x) = 1$

Solution:
Given: $f(x) = \frac{x-1}{x+1}$
First find $f(-x)$:
$f(-x) = \frac{(-x)-1}{(-x)+1} = \frac{-x-1}{-x+1} = \frac{-(x+1)}{-(x-1)} = \frac{x+1}{x-1}$
Now calculate $f(x) \cdot f(-x)$:
$f(x) \cdot f(-x) = \frac{x-1}{x+1} \cdot \frac{x+1}{x-1} = \frac{(x-1)(x+1)}{(x+1)(x-1)} = 1$
Hence proved.

Q2.2 [4 marks]

If $\log\left(\frac{x+y}{2}\right) = \frac{1}{2}(\log x + \log y)$ **then prove that** $x = y$

Solution:
Given: $\log\left(\frac{x+y}{2}\right) = \frac{1}{2}(\log x + \log y)$
Using logarithm properties:
$\frac{1}{2}(\log x + \log y) = \frac{1}{2}\log(xy) = \log\sqrt{xy}$
So: $\log\left(\frac{x+y}{2}\right) = \log\sqrt{xy}$
Taking antilog: $\frac{x+y}{2} = \sqrt{xy}$
Squaring both sides: $\left(\frac{x+y}{2}\right)^2 = xy$
$\frac{(x+y)^2}{4} = xy$
$(x+y)^2 = 4xy$
$x^2 + 2xy + y^2 = 4xy$
$x^2 - 2xy + y^2 = 0$
$(x-y)^2 = 0$
Therefore, $x = y$. Hence proved.

Q2.3 [4 marks]

Solve $\log(x+3) + \log(x-3) = \log 27$

Solution:
Given: $\log(x+3) + \log(x-3) = \log 27$
Using logarithm property: $\log a + \log b = \log(ab)$
$\log[(x+3)(x-3)] = \log 27$
Taking antilog: $(x+3)(x-3) = 27$
$x^2 - 9 = 27$
$x^2 = 36$
$x = \pm 6$
Check validity:
- For $x = 6$: $x + 3 = 9 > 0$ and $x - 3 = 3 > 0$ ✓
- For $x = -6$: $x + 3 = -3 < 0$ (invalid for logarithm)
Therefore, $x = 6$

Q.3 (A) [6 marks]

Attempt any two

Q3.1 [3 marks]

Prove that $\frac{\sin\left(\frac{\pi}{2}+\theta\right)}{\cos(\pi-\theta)} + \frac{\tan(\pi-\theta)}{\cot\left(\frac{3\pi}{2}-\theta\right)} + \frac{\mathbf{cosec}\left(\frac{\pi}{2}-\theta\right)}{\sec(\pi+\theta)} = -3$

Solution:
Using trigonometric identities:
$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$
$\cos(\pi-\theta) = -\cos\theta$
$\tan(\pi-\theta) = -\tan\theta$
$\cot\left(\frac{3\pi}{2}-\theta\right) = \tan\theta$
$\text{cosec}\left(\frac{\pi}{2}-\theta\right) = \sec\theta$
$\sec(\pi+\theta) = -\sec\theta$
Substituting:
$\frac{\cos\theta}{-\cos\theta} + \frac{-\tan\theta}{\tan\theta} + \frac{\sec\theta}{-\sec\theta}$
$= -1 + (-1) + (-1) = -3$
Hence proved.

Q3.2 [3 marks]

Prove that $\tan 55° = \frac{\cos 10°+\sin 10°}{\cos 10°-\sin 10°}$

Solution:
We know that $\tan 55° = \tan(45° + 10°)$
Using formula: $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$
$\tan 55° = \frac{\tan 45°+\tan 10°}{1-\tan 45°\tan 10°} = \frac{1+\tan 10°}{1-\tan 10°}$
Now, $\tan 10° = \frac{\sin 10°}{\cos 10°}$
$\tan 55° = \frac{1+\frac{\sin 10°}{\cos 10°}}{1-\frac{\sin 10°}{\cos 10°}} = \frac{\cos 10°+\sin 10°}{\cos 10°-\sin 10°}$
Hence proved.

Q3.3 [3 marks]

If $\vec{a} = 2\hat{i}+3\hat{j}+\hat{k}$**,** $\vec{b} = \hat{i}+\hat{j}+\hat{k}$ **and** $\vec{c} = 3\hat{i}+\hat{j}+\hat{k}$ **then find** $2\vec{a}+\vec{b}-\vec{c}$

Solution:
Given:
$\vec{a} = 2\hat{i}+3\hat{j}+\hat{k}$
$\vec{b} = \hat{i}+\hat{j}+\hat{k}$
$\vec{c} = 3\hat{i}+\hat{j}+\hat{k}$
$2\vec{a} = 2(2\hat{i}+3\hat{j}+\hat{k}) = 4\hat{i}+6\hat{j}+2\hat{k}$
$2\vec{a}+\vec{b}-\vec{c} = (4\hat{i}+6\hat{j}+2\hat{k}) + (\hat{i}+\hat{j}+\hat{k}) - (3\hat{i}+\hat{j}+\hat{k})$
$= (4+1-3)\hat{i} + (6+1-1)\hat{j} + (2+1-1)\hat{k}$
$= 2\hat{i}+6\hat{j}+2\hat{k}$

Q.3 (B) [8 marks]

Attempt any two

Q3.1 [4 marks]

Prove that $\frac{\sin(x-y)}{\cos x\cos y} + \frac{\sin(y-z)}{\cos y\cos z} + \frac{\sin(z-x)}{\cos z\cos x} = 0$

Solution:
Using identity: $\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\frac{\sin(x-y)}{\cos x\cos y} = \frac{\sin x\cos y - \cos x\sin y}{\cos x\cos y} = \tan x - \tan y$
Similarly:
$\frac{\sin(y-z)}{\cos y\cos z} = \tan y - \tan z$
$\frac{\sin(z-x)}{\cos z\cos x} = \tan z - \tan x$
Adding all three:
$(\tan x - \tan y) + (\tan y - \tan z) + (\tan z - \tan x) = 0$
Hence proved.

Q3.2 [4 marks]

Draw graph of $y = \cos x$ **for** $0 \leq x \leq \pi$

Solution:

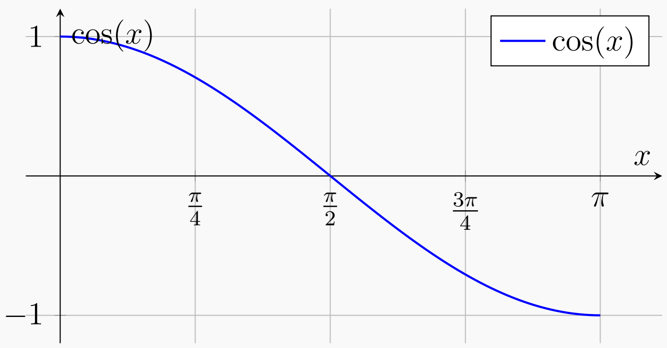

Table of values:

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1

Q3.3 [4 marks]

Find equation of line passing through (1, 2) and (-3, 1)

Solution:
Given points: $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (-3, 1)$
Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{-3-1} = \frac{-1}{-4} = \frac{1}{4}$
Using point-slope form: $y - y_1 = m(x - x_1)$
$y - 2 = \frac{1}{4}(x - 1)$
$4(y - 2) = x - 1$
$4y - 8 = x - 1$
$x - 4y + 7 = 0$
Equation: $x - 4y + 7 = 0$

Q.4 (A) [6 marks]

Attempt any two

Q4.1 [3 marks]

Find unit vector perpendicular to $\vec{a} = \hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$

Solution:
Cross product: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 2 & 1 & 2 \end{vmatrix}$
$= \hat{i}[(-3)(2) - (1)(1)] - \hat{j}[(1)(2) - (1)(2)] + \hat{k}[(1)(1) - (-3)(2)]$
$= \hat{i}(-6 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 6)$
$= -7\hat{i} + 0\hat{j} + 7\hat{k}$
Magnitude: $|\vec{a} \times \vec{b}| = \sqrt{(-7)^2 + 0^2 + 7^2} = \sqrt{49 + 49} = 7\sqrt{2}$
Unit vector: $\hat{n} = \frac{-7\hat{i}+7\hat{k}}{7\sqrt{2}} = \frac{-\hat{i}+\hat{k}}{\sqrt{2}}$

Q4.2 [3 marks]

Forces (1, 2, 1) and (2, -1, 3) act on a particle and the particle moves from point (2, 3, 1) to (4, 6, 2). Find the work done.

Solution:
Resultant force: $\vec{F} = (1, 2, 1) + (2, -1, 3) = (3, 1, 4)$
Displacement: $\vec{s} = (4, 6, 2) - (2, 3, 1) = (2, 3, 1)$
Work done: $W = \vec{F} \cdot \vec{s} = (3)(2) + (1)(3) + (4)(1) = 6 + 3 + 4 = 13$ units

Q4.3 [3 marks]

Show that lines $2x - 3y + 5 = 0$ and $8x - 12y - 3 = 0$ are parallel lines.

Solution:
For line $2x - 3y + 5 = 0$: slope $m_1 = \frac{2}{3}$
For line $8x - 12y - 3 = 0$: slope $m_2 = \frac{8}{12} = \frac{2}{3}$
Since $m_1 = m_2 = \frac{2}{3}$, the lines are parallel.

Line	Standard Form	Slope
Line 1	$2x - 3y + 5 = 0$	$\frac{2}{3}$
Line 2	$8x - 12y - 3 = 0$	$\frac{2}{3}$

Q.4 (B) [8 marks]

Attempt any two

Q4.1 [4 marks]

Show that angle between $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ is $\sin^{-1}\left(\frac{\sqrt{26}}{27}\right)$

Solution:
$\vec{a} \cdot \vec{b} = (1)(2) + (1)(-2) + (-1)(1) = 2 - 2 - 1 = -1$
$|\vec{a}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$
$|\vec{b}| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$
$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{-1}{\sqrt{3}\times 3} = \frac{-1}{3\sqrt{3}}$
$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{27} = \frac{26}{27}$
Therefore, $\sin\theta = \frac{\sqrt{26}}{3\sqrt{3}} = \frac{\sqrt{26}}{\sqrt{27}}$
Hence, $\theta = \sin^{-1}\left(\frac{\sqrt{26}}{\sqrt{27}}\right)$

Q4.2 [4 marks]

If $\vec{a} = (1, 1, 1)$, $\vec{b} = (2, 0, 1)$ and $\vec{c} = (-2, 1, 0)$ then find $\vec{a} \cdot (\vec{b} \times \vec{c})$

Solution:
First find $\vec{b} \times \vec{c}$:
$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ -2 & 1 & 0 \end{vmatrix}$$
$= \hat{i}(0 \times 0 - 1 \times 1) - \hat{j}(2 \times 0 - 1 \times (-2)) + \hat{k}(2 \times 1 - 0 \times (-2))$
$= \hat{i}(-1) - \hat{j}(2) + \hat{k}(2)$
$= -\hat{i} - 2\hat{j} + 2\hat{k}$
Now find $\vec{a} \cdot (\vec{b} \times \vec{c})$:
$\vec{a} \cdot (\vec{b} \times \vec{c}) = (1, 1, 1) \cdot (-1, -2, 2)$
$= (1)(-1) + (1)(-2) + (1)(2) = -1 - 2 + 2 = -1$

Q4.3 [4 marks]

Evaluate $\lim_{\theta\to 0} \frac{\sin 4\theta}{\theta}$

Solution:
$\lim_{\theta\to 0}\frac{\sin 4\theta}{\theta} = \lim_{\theta\to 0}\frac{\sin 4\theta}{4\theta} \times 4$
Using standard limit $\lim_{x\to 0}\frac{\sin x}{x} = 1$:
Let $u = 4\theta$, then as $\theta \to 0$, $u \to 0$
$\lim_{\theta\to 0}\frac{\sin 4\theta}{4\theta} = \lim_{u\to 0}\frac{\sin u}{u} = 1$
Therefore, $\lim_{\theta\to 0}\frac{\sin 4\theta}{\theta} = 4 \times 1 = 4$

Q.5 (A) [6 marks]

Attempt any two

Q5.1 [3 marks]

Evaluate $\lim_{x\to 9}\frac{x^2-81}{x-9}$

Solution:
Direct substitution gives $\frac{0}{0}$ form.
Factor the numerator: $x^2 - 81 = (x-9)(x+9)$
$\lim_{x\to 9}\frac{x^2-81}{x-9} = \lim_{x\to 9}\frac{(x-9)(x+9)}{x-9}$
$= \lim_{x\to 9}(x+9) = 9+9 = 18$

Q5.2 [3 marks]

Evaluate $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{2x}$

Solution:
Let $y = \left(1+\frac{3}{x}\right)^{2x}$
Taking natural logarithm:
$\ln y = 2x\ln\left(1+\frac{3}{x}\right)$
As $x\to\infty$, $\frac{3}{x}\to 0$
Using $\ln(1+u)\approx u$ for small u:
$\ln y = 2x \times \frac{3}{x} = 6$
Therefore, $y = e^6$

Q5.3 [3 marks]

Evaluate $\lim_{x\to 1}\frac{x-1}{x^2+x-2}$

Solution:
Factor the denominator: $x^2+x-2 = (x+2)(x-1)$
$\lim_{x\to 1}\frac{x-1}{x^2+x-2} = \lim_{x\to 1}\frac{x-1}{(x+2)(x-1)}$
$= \lim_{x\to 1}\frac{1}{x+2} = \frac{1}{1+2} = \frac{1}{3}$

Q.5 (B) [8 marks]

Attempt any two

Q5.1 [4 marks]

Find the equation of line passing through the point (2, -3) and having slope 4.

Solution:
Using point-slope form: $y - y_1 = m(x - x_1)$
Given: $(x_1, y_1) = (2, -3)$ and slope $m = 4$
$y - (-3) = 4(x - 2)$
$y + 3 = 4x - 8$
$y = 4x - 11$
Equation: $y = 4x - 11$ or $4x - y - 11 = 0$

Q5.2 [4 marks]

For what value of m, lines $7x + y - 1 = 0$ and $3x - my + 2 = 0$ are perpendicular to each other.

Solution:
For perpendicular lines, product of slopes = -1
For line $7x + y - 1 = 0$: slope $m_1 = -7$
For line $3x - my + 2 = 0$: slope $m_2 = \frac{3}{m}$
Condition: $m_1 \times m_2 = -1$
$(-7) \times \frac{3}{m} = -1$
$\frac{-21}{m} = -1$
$21 = m$
Therefore, $m = 21$

Q5.3 [4 marks]

Find the centre and radius of the circle $4x^2 + 4y^2 + 8x - 12y - 3 = 0$

Solution:
First, divide by 4 to get standard form:
$x^2 + y^2 + 2x - 3y - \frac{3}{4} = 0$
Complete the square for x and y terms:
$x^2 + 2x = (x + 1)^2 - 1$
$y^2 - 3y = \left(y - \frac{3}{2}\right)^2 - \frac{9}{4}$
Substituting:
$(x + 1)^2 - 1 + \left(y - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{3}{4} = 0$
$(x + 1)^2 + \left(y - \frac{3}{2}\right)^2 = 1 + \frac{9}{4} + \frac{3}{4} = 1 + 3 = 4$
Centre: $(-1, \frac{3}{2})$
Radius: $r = \sqrt{4} = 2$

Formula Cheat Sheet

Determinants

- **2Œ2 Determinant:** $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
- **3Œ3 Determinant:** Expand along any row/column

Functions & Logarithms

- **Basic:** $\log_a 1 = 0$, $\log_a a = 1$
- **Properties:** $\log(ab) = \log a + \log b$, $\log\left(\frac{a}{b}\right) = \log a - \log b$

Trigonometry

- **Basic Values:** $\sin 0° = 0$, $\sin 30° = \frac{1}{2}$, $\sin 45° = \frac{\sqrt{2}}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$, $\sin 90° = 1$
- **Conversion:** Radians to degrees: $\times\frac{180}{\pi}$
- **Identities:** $\sin^2\theta + \cos^2\theta = 1$
- **Inverse:** $\tan^{-1}(1) = \frac{\pi}{4}$

Vectors

- **Magnitude:** $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$
- **Dot Product:** $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$
- **Cross Product:** $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$
- **Work Done:** $W = \vec{F} \cdot \vec{s}$

Coordinate Geometry

- **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$
- **Parallel Lines:** Same slope
- **Perpendicular Lines:** Product of slopes = -1
- **Circle:** $(x-h)^2 + (y-k)^2 = r^2$

Limits

- **Standard Limits:** $\lim_{x\to 0} \frac{\sin x}{x} = 1$, $\lim_{x\to 0} \frac{\tan x}{x} = 1$
- **Factorization:** Use for $\frac{0}{0}$ forms
- **L'Hôpital's Rule:** For indeterminate forms