Mathematics-I (DI01000021) - Winter 2024 Solution

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Question Q.1 [14 marks]

Fill in the blanks/MCQs using appropriate choice from the given options.

Question Q1.1 [1 marks]

$\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} =$ ____________

Solution

Answer: b. 13

Solution: For 2×2 determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} = (5 \times 3) - (1 \times 2) = 15 - 2 = 13$

Question Q1.2 [1 marks]

If $\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = 0$ **then** $x =$ ____________

Solution

Answer: b. 2

Solution: $\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = x \times 1 - 1 \times 2 = x - 2 = 0$

Therefore, $x = 2$

Question Q1.3 [1 marks]

If $f(x) = x^2$ **then** $f(-1) =$ ____________

Solution

Answer: a. 1

Solution: $f(x) = x^2$ $f(-1) = (-1)^2 = 1$

Question Q1.4 [1 marks]

$\log_{10} 1 =$ ____________

Solution

Answer: b. 0
Solution: By logarithm property: $\log_a 1 = 0$ for any base $a > 0$ Therefore, $\log_{10} 1 = 0$

Question Q1.5 [1 marks]

$\sin\frac{\pi}{2} + \cos\frac{\pi}{2} =$ ____________

Solution

Answer: c. 1
Solution: $\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$ Therefore, $\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1 + 0 = 1$

Question Q1.6 [1 marks]

$\tan^{-1}(1) =$ ____________

Solution

Answer: a. $\frac{\pi}{4}$
Solution: $\tan\frac{\pi}{4} = 1$ Therefore, $\tan^{-1}(1) = \frac{\pi}{4}$

Question Q1.7 [1 marks]

$\frac{2\pi}{3}$ radian = ____________ degree

Solution

Answer: d. 120
Solution: To convert radians to degrees: degrees = radians $\times \frac{180}{\pi}$ $\frac{2\pi}{3} \times \frac{180}{\pi} = \frac{2\times180}{3} = \frac{360}{3} = 120$

Question Q1.8 [1 marks]

$\hat{i} \times \hat{j} =$ ____________

Solution

Answer: c. $\hat{k}$
Solution: By right-hand rule for cross product: $\hat{i} \times \hat{j} = \hat{k}$

Question Q1.9 [1 marks]

$|\hat{i} + \hat{j} + \hat{k}| =$ ____________

Solution

Answer: d. $\sqrt{3}$
Solution: $|\hat{i}+\hat{j}+\hat{k}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

Question Q1.10 [1 marks]

Slope of line $2x + y - 3 = 0$ **is** ____________

Solution

Answer: a. -2
Solution: Convert to slope-intercept form: $y = -2x + 3$ Slope = coefficient of $x = -2$

Question Q1.11 [1 marks]

Radius of circle $x^2 + y^2 = 81$ **is** ____________

Solution

Answer: b. 9
Solution: Standard form: $x^2 + y^2 = r^2$ Here, $r^2 = 81$, so $r = 9$

Question Q1.12 [1 marks]

$\lim_{n\to\infty} \frac{1}{n} =$ ____________

Solution

Answer: c. 0
Solution: As n approaches infinity, $\frac{1}{n}$ approaches 0

Question Q1.13 [1 marks]

$\lim_{x\to 1}(x^2 + x + 1) =$ ____________

Solution

Answer: a. 3
Solution: Direct substitution: $(1)^2 + (1) + 1 = 1 + 1 + 1 = 3$

Question Q1.14 [1 marks]

$\lim_{\theta\to 0} \frac{\tan\theta}{\theta} =$ ____________

Solution

Answer: b. 1
Solution: This is a standard limit: $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$

Question Q.2 (A) [6 marks]

Attempt any two

Question Q2.1 [3 marks]

Find the value of $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 4 & -2 & 5 \end{vmatrix}$

Solution

Answer:

Solution: Using expansion along second row (has zero): $= -2\begin{vmatrix} 3 & 1 \\ -2 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 4 & 5 \end{vmatrix} + 0$
$= -2(15+2) - 1(5-4) = -2(17) - 1(1) = -34 - 1 = -35$

Table 1. Determinant Calculation Steps

Step	Calculation	Result
Minor 1	$(3 \times 5) - (1 \times -2)$	17
Minor 2	$(1 \times 5) - (1 \times 4)$	1
Final	$-2(17) - 1(1)$	-35

Question Q2.2 [3 marks]

If $f(x) = x^3 + 5$ **then find** $f(0)$, $f(1)$ **and** $f(-1)$

Solution

Answer:
Solution: Given: $f(x) = x^3 + 5$
$f(0) = (0)^3 + 5 = 0 + 5 = 5$ $f(1) = (1)^3 + 5 = 1 + 5 = 6$ $f(-1) = (-1)^3 + 5 = -1 + 5 = 4$

Table 2. Function Values

Input	Calculation	Output
$f(0)$	$0^3 + 5$	5
$f(1)$	$1^3 + 5$	6
$f(-1)$	$(-1)^3 + 5$	4

Question Q2.3 [3 marks]

Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

Solution

Answer:
Solution: Using formula: $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$
Let $a = \frac{1}{2}$, $b = \frac{1}{3}$
$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)$
$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$
Hence proved.

Question Q.2 (B) [8 marks]

Attempt any two

Question Q2.1 [4 marks]

If $f(x) = \frac{x-1}{x+1}$ **then prove that** $f(x) \cdot f(-x) = 1$

Solution

Answer:
Solution: Given: $f(x) = \frac{x-1}{x+1}$
First find $f(-x)$: $f(-x) = \frac{(-x)-1}{(-x)+1} = \frac{-x-1}{-x+1} = \frac{-(x+1)}{-(x-1)} = \frac{x+1}{x-1}$
Now calculate $f(x) \cdot f(-x)$: $f(x) \cdot f(-x) = \frac{x-1}{x+1} \cdot \frac{x+1}{x-1} = \frac{(x-1)(x+1)}{(x+1)(x-1)} = 1$
Hence proved.

Question Q2.2 [4 marks]

If $\log\left(\frac{x+y}{2}\right) = \frac{1}{2}(\log x + \log y)$ **then prove that** $x = y$

Solution

Answer:
Solution: Given: $\log\left(\frac{x+y}{2}\right) = \frac{1}{2}(\log x + \log y)$
Using logarithm properties: $\frac{1}{2}(\log x + \log y) = \frac{1}{2}\log(xy) = \log\sqrt{xy}$
So: $\log\left(\frac{x+y}{2}\right) = \log\sqrt{xy}$
Taking antilog: $\frac{x+y}{2} = \sqrt{xy}$
Squaring both sides: $\left(\frac{x+y}{2}\right)^2 = xy$
$\frac{(x+y)^2}{4} = xy$
$(x+y)^2 = 4xy$
$x^2 + 2xy + y^2 = 4xy$
$x^2 - 2xy + y^2 = 0$
$(x-y)^2 = 0$
Therefore, $x = y$. Hence proved.

Question Q2.3 [4 marks]

Solve $\log(x+3) + \log(x-3) = \log 27$

Solution

Answer:
Solution: Given: $\log(x+3)+\log(x-3)=\log 27$
Using logarithm property: $\log a+\log b=\log(ab)$ $\log[(x+3)(x-3)]=\log 27$
Taking antilog: $(x+3)(x-3)=27$
$x^2-9=27$
$x^2=36$
$x=\pm 6$
Check validity:

- For $x=6$: $x+3=9>0$ and $x-3=3>0$
- For $x=-6$: $x+3=-3<0$ (invalid for logarithm)

Therefore, $x=6$

Question Q.3 (A) [6 marks]

Attempt any two

Question Q3.1 [3 marks]

Prove that $\frac{\sin\left(\frac{\pi}{2}+\theta\right)}{\cos(\pi-\theta)}+\frac{\tan(\pi-\theta)}{\cot\left(\frac{3\pi}{2}-\theta\right)}+\frac{\mathbf{cosec}\left(\frac{\pi}{2}-\theta\right)}{\sec(\pi+\theta)}=-3$

Solution

Answer:
Solution: Using trigonometric identities:
$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$ $\cos(\pi-\theta)=-\cos\theta$ $\tan(\pi-\theta)=-\tan\theta$ $\cot\left(\frac{3\pi}{2}-\theta\right)=\tan\theta$ $\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$ $\sec(\pi+\theta)=-\sec\theta$
Substituting: $\frac{\cos\theta}{-\cos\theta}+\frac{-\tan\theta}{\tan\theta}+\frac{\sec\theta}{-\sec\theta}$
$=-1+(-1)+(-1)=-3$
Hence proved.

Question Q3.2 [3 marks]

Prove that $\tan 55=\frac{\cos 10+\sin 10}{\cos 10-\sin 10}$

Solution

Answer:
Solution: We know that $\tan 55=\tan(45+10)$
Using formula: $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$
$\tan 55=\frac{\tan 45+\tan 10}{1-\tan 45\tan 10}=\frac{1+\tan 10}{1-\tan 10}$
Now, $\tan 10=\frac{\sin 10}{\cos 10}$
$\tan 55=\frac{1+\frac{\sin 10}{\cos 10}}{1-\frac{\sin 10}{\cos 10}}=\frac{\cos 10+\sin 10}{\cos 10-\sin 10}$
Hence proved.

Question Q3.3 [3 marks]

If $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ **and** $\vec{c}=3\hat{i}+\hat{j}+\hat{k}$ **then find** $2\vec{a}+\vec{b}-\vec{c}$

Solution

Answer:
Solution: Given: $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ $\vec{c} = 3\hat{i} + \hat{j} + \hat{k}$
$2\vec{a} = 2(2\hat{i} + 3\hat{j} + \hat{k}) = 4\hat{i} + 6\hat{j} + 2\hat{k}$
$2\vec{a} + \vec{b} - \vec{c} = (4\hat{i} + 6\hat{j} + 2\hat{k}) + (\hat{i} + \hat{j} + \hat{k}) - (3\hat{i} + \hat{j} + \hat{k})$
$= (4 + 1 - 3)\hat{i} + (6 + 1 - 1)\hat{j} + (2 + 1 - 1)\hat{k}$
$= 2\hat{i} + 6\hat{j} + 2\hat{k}$

Question Q.3 (B) [8 marks]

Attempt any two

Question Q3.1 [4 marks]

Prove that $\frac{\sin(x-y)}{\cos x \cos y} + \frac{\sin(y-z)}{\cos y \cos z} + \frac{\sin(z-x)}{\cos z \cos x} = 0$

Solution

Answer:
Solution: Using identity: $\sin(A - B) = \sin A \cos B - \cos A \sin B$
$\frac{\sin(x-y)}{\cos x \cos y} = \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y} = \tan x - \tan y$
Similarly: $\frac{\sin(y-z)}{\cos y \cos z} = \tan y - \tan z$ $\frac{\sin(z-x)}{\cos z \cos x} = \tan z - \tan x$
Adding all three: $(\tan x - \tan y) + (\tan y - \tan z) + (\tan z - \tan x) = 0$
Hence proved.

Question Q3.2 [4 marks]

Draw graph of $y = \cos x$ **for** $0 \le x \le \pi$

Solution

Answer:
Solution:

Table 3. Table of values

x	**0**	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1

Question Q3.3 [4 marks]

Find equation of line passing through (1, 2) and (-3, 1)

Solution

Answer:
Solution: Given points: $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (-3, 1)$
Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{-3-1} = \frac{-1}{-4} = \frac{1}{4}$
Using point-slope form: $y - y_1 = m(x - x_1)$ $y - 2 = \frac{1}{4}(x - 1)$ $4(y - 2) = x - 1$ $4y - 8 = x - 1$ $x - 4y + 7 = 0$

Equation: $x - 4y + 7 = 0$

Question Q.4 (A) [6 marks]

Attempt any two

Question Q4.1 [3 marks]

Find unit vector perpendicular to $\vec{a} = \hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$

Solution

Answer:

Solution: Cross product: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 2 & 1 & 2 \end{vmatrix}$
$= \hat{i}[(-3)(2) - (1)(1)] - \hat{j}[(1)(2) - (1)(2)] + \hat{k}[(1)(1) - (-3)(2)] = \hat{i}(-6-1) - \hat{j}(2-2) + \hat{k}(1+6) = -7\hat{i} + 0\hat{j} + 7\hat{k}$
Magnitude: $|\vec{a} \times \vec{b}| = \sqrt{(-7)^2 + 0^2 + 7^2} = \sqrt{49 + 49} = 7\sqrt{2}$
Unit vector: $\hat{n} = \frac{-7\hat{i}+7\hat{k}}{7\sqrt{2}} = \frac{-\hat{i}+\hat{k}}{\sqrt{2}}$

Question Q4.2 [3 marks]

Forces (1, 2, 1) and (2, -1, 3) act on a particle and the particle moves from point (2, 3, 1) to (4, 6, 2). Find the work done.

Solution

Answer:
Solution: Resultant force: $\vec{F} = (1, 2, 1) + (2, -1, 3) = (3, 1, 4)$
Displacement: $\vec{s} = (4, 6, 2) - (2, 3, 1) = (2, 3, 1)$
Work done: $W = \vec{F} \cdot \vec{s} = (3)(2) + (1)(3) + (4)(1) = 6 + 3 + 4 = 13$ units

Question Q4.3 [3 marks]

Show that lines $2x - 3y + 5 = 0$ and $8x - 12y - 3 = 0$ are parallel lines.

Solution

Answer:
Solution: For line $2x - 3y + 5 = 0$: slope $m_1 = \frac{2}{3}$ For line $8x - 12y - 3 = 0$: slope $m_2 = \frac{8}{12} = \frac{2}{3}$
Since $m_1 = m_2 = \frac{2}{3}$, the lines are parallel.

Table 4. Parallel Lines Comparison

Line	Standard Form	Slope
Line 1	$2x - 3y + 5 = 0$	$\frac{2}{3}$
Line 2	$8x - 12y - 3 = 0$	$\frac{2}{3}$

Question Q.4 (B) [8 marks]

Attempt any two

Question Q4.1 [4 marks]

Show that angle between $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ is $\sin^{-1}\left(\frac{\sqrt{26}}{27}\right)$

Solution

Answer:
Solution: $\vec{a} \cdot \vec{b} = (1)(2) + (1)(-2) + (-1)(1) = 2 - 2 - 1 = -1$
$|\vec{a}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$ $|\vec{b}| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$
$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{-1}{\sqrt{3}\times 3} = \frac{-1}{3\sqrt{3}}$
$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{27} = \frac{26}{27}$
Therefore, $\sin\theta = \frac{\sqrt{26}}{3\sqrt{3}} = \frac{\sqrt{26}}{\sqrt{27}}$
Hence, $\theta = \sin^{-1}\left(\frac{\sqrt{26}}{\sqrt{27}}\right)$

Question Q4.2 [4 marks]

If $\vec{a} = (1, 1, 1)$, $\vec{b} = (2, 0, 1)$ and $\vec{c} = (-2, 1, 0)$ then find $\vec{a} \cdot (\vec{b} \times \vec{c})$

Solution

Answer:
Solution: First find $\vec{b} \times \vec{c}$: $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ -2 & 1 & 0 \end{vmatrix}$
$= \hat{i}(0 \times 0 - 1 \times 1) - \hat{j}(2 \times 0 - 1 \times (-2)) + \hat{k}(2 \times 1 - 0 \times (-2)) = \hat{i}(-1) - \hat{j}(2) + \hat{k}(2) = -\hat{i} - 2\hat{j} + 2\hat{k}$
Now find $\vec{a} \cdot (\vec{b} \times \vec{c})$: $\vec{a} \cdot (\vec{b} \times \vec{c}) = (1, 1, 1) \cdot (-1, -2, 2) = (1)(-1) + (1)(-2) + (1)(2) = -1 - 2 + 2 = -1$

Question Q4.3 [4 marks]

Evaluate $\lim_{\theta\to 0} \frac{\sin 4\theta}{\theta}$

Solution

Answer:
Solution: $\lim_{\theta\to 0} \frac{\sin 4\theta}{\theta} = \lim_{\theta\to 0} \frac{\sin 4\theta}{4\theta} \times 4$
Using standard limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$:
Let $u = 4\theta$, then as $\theta \to 0$, $u \to 0$
$\lim_{\theta\to 0} \frac{\sin 4\theta}{4\theta} = \lim_{u\to 0} \frac{\sin u}{u} = 1$
Therefore, $\lim_{\theta\to 0} \frac{\sin 4\theta}{\theta} = 4 \times 1 = 4$

Question Q.5 (A) [6 marks]

Attempt any two

Question Q5.1 [3 marks]

Evaluate $\lim_{x \to 9} \frac{x^2-81}{x-9}$

Solution

Answer:
Solution: Direct substitution gives $\frac{0}{0}$ form.
Factor the numerator: $x^2 - 81 = (x-9)(x+9)$
$\lim_{x \to 9} \frac{x^2-81}{x-9} = \lim_{x \to 9} \frac{(x-9)(x+9)}{x-9}$
$= \lim_{x \to 9}(x+9) = 9+9 = 18$

Question Q5.2 [3 marks]

Evaluate $\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{2x}$

Solution

Answer:
Solution: Let $y = \left(1 + \frac{3}{x}\right)^{2x}$
Taking natural logarithm: $\ln y = 2x \ln\left(1 + \frac{3}{x}\right)$
As $x \to \infty$, $\frac{3}{x} \to 0$
Using $\ln(1+u) \approx u$ for small u: $\ln y = 2x \times \frac{3}{x} = 6$
Therefore, $y = e^6$

Question Q5.3 [3 marks]

Evaluate $\lim_{x \to 1} \frac{x-1}{x^2+x-2}$

Solution

Answer:
Solution: Factor the denominator: $x^2 + x - 2 = (x+2)(x-1)$
$\lim_{x \to 1} \frac{x-1}{x^2+x-2} = \lim_{x \to 1} \frac{x-1}{(x+2)(x-1)}$
$= \lim_{x \to 1} \frac{1}{x+2} = \frac{1}{1+2} = \frac{1}{3}$

Question Q.5 (B) [8 marks]

Attempt any two

Question Q5.1 [4 marks]

Find the equation of line passing through the point (2, -3) and having slope 4.

Solution

Answer:
Solution: Using point-slope form: $y - y_1 = m(x - x_1)$
Given: $(x_1, y_1) = (2, -3)$ and slope $m = 4$
$y - (-3) = 4(x - 2)$ $y + 3 = 4x - 8$ $y = 4x - 11$
Equation: $y = 4x - 11$ or $4x - y - 11 = 0$

Question Q5.2 [4 marks]

For what value of m, lines $7x + y - 1 = 0$ and $3x - my + 2 = 0$ are perpendicular to each other.

Solution

Answer:
Solution: For perpendicular lines, product of slopes = -1
For line $7x + y - 1 = 0$: slope $m_1 = -7$ For line $3x - my + 2 = 0$: slope $m_2 = \frac{3}{m}$
Condition: $m_1 \times m_2 = -1$ $(-7) \times \frac{3}{m} = -1$ $\frac{-21}{m} = -1$ $21 = m$
Therefore, $m = 21$

Table 5. Perpendicular Lines

Line	Standard Form	Slope
Line 1	$7x + y - 1 = 0$	-7
Line 2	$3x - my + 2 = 0$	$\frac{3}{m}$

Verification: When $m = 21$, slopes are -7 and $\frac{3}{21} = \frac{1}{7}$ Product: $(-7) \times \frac{1}{7} = -1$

Question Q5.3 [4 marks]

Find the centre and radius of the circle $4x^2 + 4y^2 + 8x - 12y - 3 = 0$

Solution

Answer:
Solution: First, divide by 4 to get standard form: $x^2 + y^2 + 2x - 3y - \frac{3}{4} = 0$
Complete the square for x and y terms: $x^2 + 2x = (x + 1)^2 - 1$ $y^2 - 3y = \left(y - \frac{3}{2}\right)^2 - \frac{9}{4}$
Substituting: $(x + 1)^2 - 1 + \left(y - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{3}{4} = 0$
$(x + 1)^2 + \left(y - \frac{3}{2}\right)^2 = 1 + \frac{9}{4} + \frac{3}{4} = 1 + 3 = 4$
Centre: $(-1, \frac{3}{2})$ **Radius:** $r = \sqrt{4} = 2$

Table 6. Circle Properties

Component	Value
Centre (h,k)	$(-1, \frac{3}{2})$
Radius	2
Standard Form	$(x + 1)^2 + (y - \frac{3}{2})^2 = 4$

Formula Cheat Sheet

Determinants

- **2×2 Determinant:** $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
- **3×3 Determinant:** Expand along any row/column

Functions & Logarithms

- **Basic:** $\log_a 1 = 0$, $\log_a a = 1$
- **Properties:** $\log(ab) = \log a + \log b$, $\log\left(\frac{a}{b}\right) = \log a - \log b$

Trigonometry

- **Basic Values:** $\sin 0 = 0$, $\sin 30 = \frac{1}{2}$, $\sin 45 = \frac{\sqrt{2}}{2}$, $\sin 60 = \frac{\sqrt{3}}{2}$, $\sin 90 = 1$
- **Conversion:** Radians to degrees: $\times\frac{180}{\pi}$
- **Identities:** $\sin^2\theta + \cos^2\theta = 1$
- **Inverse:** $\tan^{-1}(1) = \frac{\pi}{4}$

Vectors

- **Magnitude:** $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$
- **Dot Product:** $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$
- **Cross Product:** $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$
- **Work Done:** $W = \vec{F} \cdot \vec{s}$

Coordinate Geometry

- **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$
- **Parallel Lines:** Same slope
- **Perpendicular Lines:** Product of slopes = -1
- **Circle:** $(x - h)^2 + (y - k)^2 = r^2$

Limits

- **Standard Limits:** $\lim_{x \to 0} \frac{\sin x}{x} = 1$, $\lim_{x \to 0} \frac{\tan x}{x} = 1$
- **Factorization:** Use for $\frac{0}{0}$ forms
- **L'Hôpital's Rule:** For indeterminate forms

Quick Reference Table

Table 7. Key Formulas

Topic	Key Formula	Example
Determinant 2×2	$ad - bc$	$\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 8 - 3 = 5$
Slope	$\frac{y_2 - y_1}{x_2 - x_1}$	Points (1,2), (3,8): $m = \frac{8-2}{3-1} = 3$
Circle	$(x - h)^2 + (y - k)^2 = r^2$	Center (1,2), radius 3
Limit	$\lim_{x \to a} f(x)$	Direct substitution or factoring