

# Subject Name Solutions

4331101 – Winter 2022

Semester 1 Study Material

*Detailed Solutions and Explanations*

## Question 1(a) [3 marks]

Define: 1) Branch 2) Junction 3) Mesh

### Solution

- **Branch:** A branch is a single circuit element or a combination of elements connected between two nodes of a network.
- **Junction:** A junction (or node) is a point in a circuit where two or more circuit elements are connected together.
- **Mesh:** A mesh is a closed path in a network where no other closed path exists inside it.

### Mnemonic

“BJM: Branches Join at junctions to Make meshes”

## Question 1(b) [4 marks]

Write voltage division and current division rule with necessary circuit diagram

### Solution

**Voltage Division Rule:** In a series circuit, voltage across any component is proportional to its resistance.

#### Mermaid Diagram (Code)

```
{Shaded}
{Highlighting} []
graph LR
    A(({}+)) --> B[R1]
    B --> C[R2]
    C --> D(({}-))
    E[V1] --> B
    F[V2] --> C
    G[VS] --> A
{Highlighting}
{Shaded}
```

- **Formula:**  $V_1 = VS \times (R_1 / (R_1 + R_2))$
- **Application:** Used to find individual voltage drops across series components

**Current Division Rule:** In a parallel circuit, current through any branch is inversely proportional to its resistance.

#### Mermaid Diagram (Code)

```
{Shaded}
{Highlighting} []
graph LR
    A(({}+)) --> B
    B --> C(({}-))
    D[R1] --> B
    C --> E[R2]
    E --> C
    F[I1] --> D
    G[I2] --> E
    H[IS] --> A
{Highlighting}
{Shaded}
```

- **Formula:**  $I_1 = IS \times (R_2 / (R_1 + R_2))$
- **Key concept:** Current takes path of least resistance

### Mnemonic

“VoSe CuPa: Voltage divides in Series, Current divides in Parallel”

### Question 1(c) [7 marks]

Draw Graph and Tree for a network shown in fig(1). Show link currents on a graph. Also write Tie-set schedule for a tree of network shown in fig. (1)

### Solution

#### Graph of the Network:

#### Mermaid Diagram (Code)

```
{Shaded}
{Highlighting} []
graph LR
    A((A)) --- B((B))
    A --- C((C))
    A --- D((D))
    B --- C
    B --- D
    C --- D
    A --- 1 --- B
    A --- 3 --- C
    B --- 2 --- D
    C --- 5 --- D
    B --- 6 --- C
    A --- 7 --- D
    style A fill:#f9f,stroke:#333,stroke-width:2px
    style B fill:#f9f,stroke:#333,stroke-width:2px
    style C fill:#f9f,stroke:#333,stroke-width:2px
    style D fill:#f9f,stroke:#333,stroke-width:2px
{Highlighting}
{Shaded}
```

#### Tree of the Network (shown with bold edges):

#### Mermaid Diagram (Code)

```
{Shaded}
{Highlighting} []
graph LR
    A((A)) --- B((B))
    A --- C((C))
    A --- D((D))
    style A fill:#f9f,stroke:#333,stroke-width:2px
    style B fill:#f9f,stroke:#333,stroke-width:2px
    style C fill:#f9f,stroke:#333,stroke-width:2px
    style D fill:#f9f,stroke:#333,stroke-width:2px
    linkStyle 0 stroke-width:4px,stroke:green
    linkStyle 1 stroke-width:4px,stroke:green
    linkStyle 2 stroke-width:4px,stroke:green
{Highlighting}
{Shaded}
```

#### Link Currents (shown on remaining branches that are not part of the tree):

- Link 1: Branch 2 (BD)

- Link 2: Branch 6 (BC)
- Link 3: Branch 7 (AD)
- Link 4: Branch 5 (CD)

**Tie-set Schedule:**

Link/Tree Branch	Branch 1 (AB)	Branch 3 (AC)	Branch 4 (CD)	Branch 2 (BD)	Branch 6 (BC)	Branch 7 (AD)	Branch 5 (CD)
Link 1 (BD)	1	0	0	1	0	0	0
Link 2 (BC)	1	1	0	0	1	0	0
Link 3 (AD)	0	0	1	0	0	1	0
Link 4 (CD)	0	0	1	0	0	0	1

**Mnemonic**

“TGLT: Trees Generate Link-current Tie-sets”

**Question 1(c) OR [7 marks]**

Draw Graph and Tree for a network shown in fig(1). Show branch voltages on tree. Also write cut-set schedule for a tree of network shown on fig.(1)

**Solution**

**Graph of the Network:**

**Mermaid Diagram (Code)**

```
{Shaded}
{Highlighting} []
graph LR
    A((A)) --- B((B))
    A --- C((C))
    A --- D((D))
    B --- C
    B --- D
    C --- D
    A --- 1 --- B
    A --- 3 --- C
    B --- 2 --- D
    C --- 5 --- D
    B --- 6 --- C
    A --- 7 --- D
    style A fill:#f9f,stroke:#333,stroke-width:2px
    style B fill:#f9f,stroke:#333,stroke-width:2px
    style C fill:#f9f,stroke:#333,stroke-width:2px
    style D fill:#f9f,stroke:#333,stroke-width:2px
{Highlighting}
{Shaded}
```

**Tree of the Network** (shown with bold edges and branch voltages):

**Mermaid Diagram (Code)**

```
{Shaded}
{Highlighting} []
graph LR
    A((A)) --- "V_1" --- B((B))
    A --- "V_3" --- C((C))
    C --- "V_4" --- D((D))
    style A fill:#f9f,stroke:#333,stroke-width:2px
    style B fill:#f9f,stroke:#333,stroke-width:2px
    style C fill:#f9f,stroke:#333,stroke-width:2px
```

```

style D fill:\#f9f,stroke:\#333,stroke{-width:2px}
linkStyle 0 stroke{-width:4px,stroke:green}
linkStyle 1 stroke{-width:4px,stroke:green}
linkStyle 2 stroke{-width:4px,stroke:green}
{Highlighting}
{Shaded}

```

### Cut-set Schedule:

Cut-set/Branch	Branch 1 (AB)	Branch 3 (AC)	Branch 4 (CD)	Branch 2 (BD)	Branch 6 (BC)	Branch 7 (AD)	Branch 5 (CD)
Cut-set 1 (AB)	1	0	0	-1	-1	0	0
Cut-set 2 (AC)	0	1	0	0	1	-1	0
Cut-set 3 (CD)	0	0	1	1	0	1	1

### Mnemonic

“CGVS: Cut-sets Generate Voltage Sources”

### Question 2(a) [3 marks]

Define: 1) Active and passive network 2) Unilateral and Bilateral network.

### Solution

- **Active Network:** A network containing one or more sources of EMF (voltage/current sources) that supply energy to the circuit.
- **Passive Network:** A network containing only passive elements like resistors, capacitors, and inductors with no energy sources.
- **Unilateral Network:** A network in which the properties and performance change when input and output terminals are interchanged.
- **Bilateral Network:** A network in which the properties and performance remain unchanged when input and output terminals are interchanged.

Diagram:

### Mermaid Diagram (Code)

```

{Shaded}
{Highlighting} []
graph LR
    subgraph "Network Types"
        A[Active: Contains sources]
        B[Passive: No sources]
        C[Unilateral: Diodes/Transistors]
        D[Bilateral: R, L, C elements]
        end
    {Highlighting}
    {Shaded}

```

### Mnemonic

“APUB: Active Provides energy, Unilateral Blocks reversal”

### Question 2(b) [4 marks]

Write equation for Z parameter and derive Z<sub>11</sub>, Z<sub>12</sub>, Z<sub>21</sub>, Z<sub>22</sub> from that equation.

## Solution

Z-parameters define the relationship between port voltages and currents in a two-port network:

### Equations:

- $V_1 = Z_{11}I_1 + Z_{12}I_2$
- $V_2 = Z_{21}I_1 + Z_{22}I_2$

### Derivation:

- $\mathbf{Z}_{11} = V_1/I_1$  (with  $I_2 = 0$ ) : Input impedance without output port open-circuited
- $\mathbf{Z}_{12} = V_1/I_2$  (with  $I_1 = 0$ ) : Reverse transfer impedance within input port open-circuited
- $\mathbf{Z}_{21} = V_2/I_1$  (with  $I_2 = 0$ ) : Forward transfer impedance without output port open-circuited
- $\mathbf{Z}_{22} = V_2/I_2$  (with  $I_1 = 0$ ) : Output impedance within input port open-circuited

## Mnemonic

"Z Impedance: Open circuit gives correct Parameters"

## Question 2(c) [7 marks]

Derive equation of characteristic impedance(ZOT) for a standard T network.

## Solution

For a standard T-network:

### Mermaid Diagram (Code)

```
{Shaded}  
{Highlighting} []  
graph LR  
    A((Port{-1})) --- B[Z1] --- C((Junction))  
    C --- D[Z2] --- E((Port{-2}))  
    C --- F[Z3] --- G((Ground))  
{Highlighting}  
{Shaded}
```

### Derivation Steps:

1. For a symmetric T-network,  $Z_1 = Z_2$
1. Under matched condition, input impedance equals characteristic impedance
2.  $Z_{0t} = Z_1 + (Z_{13})/(Z_1 + Z_3)$
2. For balanced T-network where  $Z_1 = Z_2 = Z/2$  and  $Z_3 = Z$  :
  2.  $Z_{0t} = Z/2 + (Z/2)/(Z/2 + Z)$
  2.  $Z_{0t} = Z/2 + (Z^2/2)/(Z + Z/2)$
  2.  $Z_{0t} = Z/2 + (Z^2/2)/(3Z/2)$
  2.  $Z_{0t} = Z/2 + Z^2/3Z$
  2.  $Z_{0t} = Z/2 + Z/3$
  2.  $Z_{0t} = (3Z + 2Z)/6$
  2.  $Z_{0t} = \sqrt{(Z_1(Z_1 + 2Z_3))}$

**Final Equation:**  $Z_{0t} = \sqrt{(Z_1(Z_1 + 2Z_3))}$

## Mnemonic

"TO Impedance: Two arms Over middle branch"

## Question 2(a) OR [3 marks]

Define: 1) Driving point impedance 2) Transfer impedance

## Solution

- **Driving Point Impedance:** The ratio of voltage to current at the same port/pair of terminals when all other independent sources are set to zero.
- **Transfer Impedance:** The ratio of voltage at one port to the current at another port when all other

independent sources are set to zero.

Diagram:

#### Mermaid Diagram (Code)

```
{Shaded}  
{Highlighting} []  
graph LR  
    subgraph "Impedance Types"  
        A[Driving Point:  $V_{1/I_1}$  or  $V_2/I_2$ ]  
        B[Transfer:  $V_2/I_1$  or  $V_1/I_2$ ]  
    end  
{Highlighting}  
{Shaded}
```

#### Mnemonic

“DTSS: Driving at Terminal Same, Transfer at Separate”

### Question 2(b) OR [4 marks]

Explain Kirchhoff's voltage law with example.

#### Solution

**Kirchhoff's Voltage Law (KVL):** The algebraic sum of all voltages around any closed loop in a circuit is zero.

**Mathematically:**  $\sum V = 0$  (around a closed loop)

**Circuit Example:**

#### Mermaid Diagram (Code)

```
{Shaded}  
{Highlighting} []  
graph LR  
    A((+)) --> B  
    B --> C  
    C --> D  
    D --> A  
    style A fill:#f9f,stroke:#333,stroke-width:2px  
    style B fill:#f9f,stroke:#333,stroke-width:2px  
    style C fill:#f9f,stroke:#333,stroke-width:2px  
    style D fill:#f9f,stroke:#333,stroke-width:2px  
{Highlighting}  
{Shaded}
```

If  $I = 1A$ , then:

- $V_1 = 1A \times 2 = 2V$
- $V_2 = 1A \times 3 = 3V$
- $V_3 = 1A \times 5 = 5V$

Applying KVL:  $10V - 2V - 3V - 5V = 0$

#### Mnemonic

“VACZ: Voltages Around Closed loop are Zero”

### Question 2(c) OR [7 marks]

Derive equation to convert network into T network.

## Solution

Network to T Network Conversion:

Mermaid Diagram (Code)

```
{Shaded}  
{Highlighting}[]  
graph TD  
    subgraph " Network"  
        A1((A)) --- B1((B))  
        A1 --- Y1[Ya] --- C1  
        B1 --- Y2[Yb] --- C1  
        A1 --- Y3[Yc] --- B1  
        C1((C))  
    end  
  
    subgraph "T Network"  
        A2((A)) --- Z1[Za] --- D2((D))  
        B2((B)) --- Z2[Zb] --- D2  
        D2 --- Z3[Zc] --- C2((C))  
    end  
{Highlighting}  
{Shaded}
```

Conversion Equations:

1.  $Za = (Ya \times Yc) / Y$
1.  $Zb = (Yb \times Yc) / Y$
1.  $Zc = (Ya \times Yb) / Y$

Where  $Y = Ya + Yb + Yc$

Derivation:

1. Start with Y-parameters of -network
2. Express Y-parameters in terms of branch admittances
3. Convert to Z-parameters using matrix inversion
4. Express T-network impedances in terms of Z-parameters
5. Simplify to get the conversion formulas above

## Mnemonic

"PIE to TEA: Product over sum for opposite branch"

## Question 3(a) [3 marks]

Explain Kirchhoff's current law with example.

## Solution

**Kirchhoff's Current Law (KCL):** The algebraic sum of all currents entering and leaving a node must equal zero.

**Mathematically:**  $\sum I = 0$  (at any node)

**Circuit Example:**

Mermaid Diagram (Code)

```
{Shaded}  
{Highlighting}[]  
graph TD  
    A[I_1 = 5A] --- B((Node))  
    C[I_2 = 2A] --- B  
    B --- D[I_3 = 3A]  
    B --- E[I_4 = 4A]  
    style B fill:#f9f,stroke:#333,stroke-width:2px  
{Highlighting}  
{Shaded}
```

Applying KCL at node B:

- Currents entering:  $I_1 + I_2 = 5A + 2A = 7A$
- Currents leaving:  $I_3 + I_4 = 3A + 4A = 7A$
- Therefore:  $I_1 + I_2 - I_3 - I_4 = 5 + 2 - 3 - 4 = 0$

### Mnemonic

“CuNoZ: Currents at Node are Zero”

## Question 3(b) [4 marks]

Explain mesh analysis with required equations.

### Solution

**Mesh Analysis:** A circuit analysis technique that uses mesh currents as variables to solve a circuit with multiple loops.

#### Steps:

1. Identify all meshes (closed loops) in the circuit
2. Assign a mesh current to each mesh
3. Apply KVL to each mesh
4. Solve the resulting system of equations

#### Example Circuit:

### Mermaid Diagram (Code)

```
{Shaded}  
{Highlighting} []  
graph LR  
    A((A)) --> R1[R1]  
    R1 --> B((B))  
    B --> R3[R3]  
    R3 --> C((C))  
    C --> R2[R2]  
    R2 --> D((D))  
    D --> V1[V1]  
    V1 --> E((E))  
    E --> A  
    style A fill:#f9f,stroke:#333,stroke-width:2px  
    style B fill:#f9f,stroke:#333,stroke-width:2px  
    style C fill:#f9f,stroke:#333,stroke-width:2px  
{Highlighting}  
{Shaded}
```

#### Equations:

- Mesh 1:  $V_1 = I_1R_1 + I_1R_2 - I_2R_2$
- Mesh 2:  $V_2 = I_2R_2 + I_2R_3 - I_1R_2$

### Mnemonic

“MILK: Mesh Is Loop with KVL”

## Question 3(c) [7 marks]

State and explain Thevenin's theorem.

### Solution

**Thevenin's Theorem:** Any linear network with voltage and current sources can be replaced by an equivalent circuit consisting of a voltage source ( $V_{TH}$ ) in series with a resistance ( $R_{TH}$ ).

### Mermaid Diagram (Code)

{Shaded}

```

{Highlighting} []
graph TD
    subgraph "Original Network"
        A((A)) --- B[Complex Network] --- C((B))
    end
    subgraph "Thevenin Equivalent"
        D((A)) --- E[VTH] --- F(({}+))
        F --- G[RTH]
        G --- H((B))
    end
    {Highlighting}
    {Shaded}

```

#### Steps to Find Thevenin Equivalent:

1. Remove the load from the terminals of interest
2. Calculate the open-circuit voltage (VOC) across these terminals ( $= V_{TH}$ )
3. Calculate the resistance looking back into the circuit with all sources replaced by their internal resistances ( $= R_{TH}$ )
4. The Thevenin equivalent consists of  $V_{TH}$  in series with  $R_{TH}$

#### Example Application:

- Original complex circuit with load  $RL$
- Remove  $RL$  and find  $VOC = V_{TH}$
- Deactivate sources and find  $R_{TH}$
- Reconnect  $RL$  to simplified Thevenin equivalent

#### Mnemonic

“TORV: Thevenin’s Open-circuit Resistance and Voltage”

### Question 3(a) OR [3 marks]

State and explain reciprocity theorem.

#### Solution

**Reciprocity Theorem:** In a linear, bilateral network, if a voltage source in one branch produces a current in another branch, then the same voltage source, if placed in the second branch, will produce the same current in the first branch.

#### Mermaid Diagram (Code)

```

{Shaded}
{Highlighting} []
graph LR
    subgraph "Original Circuit"
        direction LR
        A((A)) --- B[V] --- C((B))
        C --- D[Network]
        D --- E((C))
        E --- F[Ammeter]
        F --- A
    end

    subgraph "Reciprocal Circuit"
        direction LR
        G((A)) --- H[Ammeter]
        H --- I((B))
        I --- J[Network]
        J --- K((C))
        K --- L[V]
        L --- G
    end
    {Highlighting}
    {Shaded}

```

**Mathematically:** If a voltage  $V_1$  in branch 1 produces current  $I_2$  in branch 2, then voltage  $V_1$  in branch 2 will produce current  $I_2$  in branch 1.

**Limitations:** Applies only to networks with:

- Linear elements

- Bilateral elements (no diodes, transistors)
- Single independent source

### Mnemonic

“RESWAP: RECiprocity SWAPs Position with identical results”

## Question 3(b) OR [4 marks]

Explain nodal analysis with required equations.

### Solution

**Nodal Analysis:** A circuit analysis technique that uses node voltages as variables to solve a circuit.

**Steps:**

1. Choose a reference node (ground)
2. Assign voltage variables to remaining nodes
3. Apply KCL at each non-reference node
4. Solve the resulting system of equations

**Example Circuit:**

### Mermaid Diagram (Code)

```
{Shaded}
{Highlighting} []
graph LR
    A((Node 1)) --- G_1 --- B((Ground))
    C((Node 2)) --- G_2 --- B
    A --- G_3 --- C
    A --- I_1 --- B
    C --- I_2 --- B
    style A fill:#f9f,stroke:#333,stroke-width:2px
    style C fill:#f9f,stroke:#333,stroke-width:2px
    style B fill:#f9f,stroke:#333,stroke-width:2px
{Highlighting}
{Shaded}
```

**Equations:**

- Node 1:  $I_1 = V_1G_1 + (V_1 - V_2)G_3$
- Node 2:  $I_2 = V_2G_2 + (V_2 - V_1)G_3$

### Mnemonic

“NKCV: Nodal uses KCL with Voltage variables”

## Question 3(c) OR [7 marks]

State and prove maximum power transfer theorem.

### Solution

**Maximum Power Transfer Theorem:** A load connected to a source will extract maximum power when its resistance equals the internal resistance of the source.

### Mermaid Diagram (Code)

```
{Shaded}
{Highlighting} []
graph LR
    A((+)) --- B[VS] --- C((X))
    D[RS] --- E((Y))
    F[RL] --- G((Z))
```

```

G {-{-}{-} A}
style C fill:#f9f,stroke:#333,stroke-width:2px
style E fill:#f9f,stroke:#333,stroke-width:2px
{Highlighting}
{Shaded}

```

### Proof:

1. Current in the circuit:  $I = VS/(RS + RL)$
2. Power delivered to load:  

$$P = I^2 RL = (VS^2 RL)/(RS + RL)^2$$
3. For maximum power,  $dP/dRL = 0$
4. Solving:  $(VS^2(RS + RL)^2 - VS^2 RL^2(RS + RL))/(RS + RL)^4 = 0$
4. Simplifying:  $(RS + RL)^2 = 2RL(RS + RL)$
4. Further simplifying:  $RS + RL = 2RL$
5. Therefore:  $RS = RL$   
**Maximum Power:**  $P_{max} = VS^2/(4RS)$

### Mnemonic

“MaRLRS: Maximum power when load Resistance equals Source Resistance”

## Question 4(a) [3 marks]

Why series resonance circuit act as voltage amplifier and parallel resonance circuit act as current amplifier?

### Solution

#### Series Resonance as Voltage Amplifier:

- At resonance, series circuit impedance is minimum (just R)
- Voltage across L or C can be much larger than source voltage
- Voltage magnification factor =  $Q = XL/R = 1/R\sqrt{(L/C)}$
- Voltage across L or C =  $Q \times \text{Source voltage}$

#### Parallel Resonance as Current Amplifier:

- At resonance, parallel circuit impedance is maximum
- Current in L or C can be much larger than source current
- Current magnification factor =  $Q = R/XL = R\sqrt{(C/L)}$
- Current through L or C =  $Q \times \text{Source current}$

Table:

Circuit Type	Impedance at Resonance	Amplification
Series	Minimum (R only)	Voltage ( $VL$ or $VC = Q$ )
Parallel	Maximum ( $R^2/r$ )	Current ( $IL$ or $IC = Q$ )

### Mnemonic

“SeVoPa: Series Voltage, Parallel current amplification”

## Question 4(b) [4 marks]

Derive equation of Q of coil.

### Solution

#### Q-factor of a Coil:

#### Mermaid Diagram (Code)

```

{Shaded}
{Highlighting} []
graph LR

```

```

A((A)) {-{-}{-} B[R] {-}{-}{-}{-} C((B)))
C {-{-}{-}} D[L] {-}{-}{-}{-} A}
style C fill:#f9f,stroke:#333,stroke{-width:2px}
{Highlighting}
{Shaded}

```

#### Derivation:

1. Q-factor is defined as:  $Q = \text{Energy stored} / \text{Energy dissipated per cycle}$
2. Energy stored in inductor  $= (1/2)LI^2$
2. Power dissipated in resistor  $= I^2R$
2. Energy dissipated per cycle  $= \text{Power} \times \text{Time period} = I^2R \times (1/f)$
2. Therefore:  $Q = ((1/2)LI^2) / (I^2R \times (1/f))$
2. Simplifying:  $Q = 2 \times (1/2)LI^2 \times f / (I^2R)$
2.  $Q = 2 f \times L / R$

$$R = L / R$$

**Final Equation:**  $Q = L / R = 2 fL / R = XL / R$

#### Mnemonic

“QualityEDR: Quality equals Energy stored Divided by energy lost per Radian”

### Question 4(c) [7 marks]

Derive equation of series resonance frequency for series R-L-C circuit.

#### Solution

##### Series R-L-C Circuit:

##### Mermaid Diagram (Code)

```

{Shaded}
{Highlighting} []
graph LR
    A((Input)) {-{-}{-} B[R] {-}{-}{-}{-} C[L] {-}{-}{-}{-} D[C] {-}{-}{-}{-} E((Output)))
    style A fill:#f9f,stroke:#333,stroke{-width:2px}
    style E fill:#f9f,stroke:#333,stroke{-width:2px}
{Highlighting}
{Shaded}

```

#### Derivation:

1. Impedance of series RLC circuit:  $Z = R + j(XL - XC)$
2. Where:  $XL = L$  and  $XC = 1/C$
3. At resonance,  $XL = XC$  (inductive and capacitive reactances are equal)
4. Therefore:  $L = 1/C$
5. Solving for  $\omega^2 = 1/LC$
5. Resonant frequency:  $\omega_0 = 1/\sqrt(LC)$
5. In terms of frequency  $f$ :  $f_0 = 1/(2\sqrt(LC))$

#### Characteristics at Resonance:

- Impedance is minimum (purely resistive:  $Z = R$ )
- Current is maximum ( $I = V/R$ )
- Power factor is unity (circuit appears resistive)
- Voltages across L and C are equal and opposite

#### Mnemonic

“RES: Reactances Equal at Series resonance”

### Question 4(a) OR [3 marks]

What is coupled circuits? Define self-inductance and mutual inductance.

## Solution

**Coupled Circuits:** Two or more circuits that are magnetically linked such that energy can be transferred between them through their mutual magnetic field.

### Mermaid Diagram (Code)

```
{Shaded}  
{Highlighting} []  
graph TD  
    subgraph "Primary"  
        A((A)) --- B[L1] --- C((B))  
    end  
  
    subgraph "Secondary"  
        D((C)) --- E[L2] --- F((D))  
    end  
  
    G[M] --- B  
    G --- E  
{Highlighting}  
{Shaded}
```

**Self-inductance (L):** The property of a circuit whereby a change in current produces a self-induced EMF in the same circuit.  $L = \Phi/I$  (ratio of magnetic flux to the current producing it)

**Mutual inductance (M):** The property of a circuit whereby a change in current in one circuit induces an EMF in another circuit.  $M = \Phi_{21}/I_1$  (ratio of flux in circuit 2 due to current in circuit 1)

## Mnemonic

“SiMu: Self in Mine, Mutual in Yours”

## Question 4(b) OR [4 marks]

Derive equation for co-efficient of coupling (K).

## Solution

**Coefficient of Coupling (k):**

### Mermaid Diagram (Code)

```
{Shaded}  
{Highlighting} []  
graph LR  
    subgraph "Coupled Coils"  
        A((A)) --- B[L1] --- C((B))  
        D((C)) --- E[L2] --- F((D))  
        G[M] --- B  
        G --- E  
    end  
{Highlighting}  
{Shaded}
```

## Derivation:

1. The mutual inductance (M) between two coils depends on:
  - Self-inductances of the coils ( $L_1$  and  $L_2$ )
  - Physical arrangement (proximity and orientation)
2. Maximum possible mutual inductance:  $M_{max} = \sqrt{L_1 L_2}$
3. Coefficient of coupling is defined as:  $k = M/M_{max}$
4. Therefore:  $k = M/\sqrt{L_1 L_2}$

## Characteristics:

- k ranges from 0 (no coupling) to 1 (perfect coupling)

- $k$  depends on geometry, orientation, and medium
- Typical transformers:  $k = 0.95$  to  $0.99$
- Air-core coils:  $k = 0.01$  to  $0.5$

### Mnemonic

"KMutual: K Measures Mutual linkage proportion"

### Question 4(c) OR [7 marks]

A series RLC circuit has  $R=30\Omega$ ,  $L=0.5H$ , and  $C=5\mu F$ . Calculate (i) series resonance frequency (2) Q Factor (3)BW

### Solution

Given:

- Resistance,  $R = 30\Omega$
- Inductance,  $L = 0.5H$
- Capacitance,  
 $C = 5\mu F = 5 \times 10^{-6}F$

Calculations:

(i) Series Resonance Frequency:

- $f_0 = 1/(2\sqrt{LC})$
- $f_0 = 1/(2\sqrt{(0.5 \times 5 \times 10^{-6})})$
- $f_0 = 1/(2\sqrt{(2.5 \times 10^{-6})})$
- $f_0 = 1/(2 \times 1.58 \times 10^{-3})$
- $f_0 = 1/(9.9 \times 10^{-3})$
- $f_0 = 100.76Hz$
- $f_0 \approx 100Hz$

(ii) Q Factor:

- $Q = (1/R)\sqrt{L/C}$
- $Q = (1/30)\sqrt{(0.5/(5 \times 10^{-6}))}$
- $Q = (1/30)\sqrt{(100,000)}$
- $Q = (1/30) \times 316.23$
- $Q = 10.54$

(iii) Bandwidth (BW):

- $BW = f_0/Q$
- $BW = 100.76/10.54$
- $BW = 9.56 Hz$

Table:

Parameter	Formula	Value
Resonant Frequency ( $f_0$ )	$1/(2\sqrt{LC})$	100 Hz
Quality Factor (Q)	$(1/R)\sqrt{L/C}$	10.54
Bandwidth (BW)	$f_0/Q$	9.56 Hz

### Mnemonic

"RQB: Resonance Quality determines Bandwidth"

### Question 5(a) [3 marks]

Classify various types of attenuators.

### Solution

**Attenuators:** Network of resistors designed to reduce (attenuate) signal level without distortion.

**Types of Attenuators:**

Mermaid Diagram (Code)

```

{Shaded}
{Highlighting} []
graph TD
    A[Attenuators] --> B[Fixed Attenuators]
    A --> C[Variable Attenuators]
    B --> D[T{-}type]
    B --> E[{-}type]
    B --> F[Bridged{-}T]
    B --> G[Lattice]
    C --> H[Step Attenuators]
    C --> I[Continuously Variable]
{Highlighting}
{Shaded}

```

Based on configuration:

- **T-type:** Three resistor T-shaped configuration
- **-type:** Three resistor -shaped configuration
- **Bridged-T:** T-type with a resistor bridging across
- **Lattice:** Balanced configuration with four resistors

Based on symmetry:

- **Symmetrical:** Equal input and output impedance
- **Asymmetrical:** Different input and output impedance

### Mnemonic

“ATP Fixed: Attenuator Types include Pad, Tee, Lattice”

## Question 5(b) [4 marks]

Derive relation between attenuator and neper.

### Solution

#### Relationship between Attenuation and Neper:

- **Attenuation ( ):** Ratio of input voltage (or current) to output voltage (or current), expressed in different units.
- **Neper (Np):** Natural logarithmic unit of ratios, used mainly in transmission line theory.

#### Derivation:

1. For a voltage ratio  $V_1/V_2$  :
  - Attenuation in Nepers =  $\ln(V_1/V_2)$
  - Attenuation in Decibels =  $20\log_{10}(V_1/V_2)$
2. For a power ratio  $P_1/P_2$  :
  - Attenuation in Nepers =  $(1/2)\ln(P_1/P_2)$
  - Attenuation in Decibels =  $10\log_{10}(P_1/P_2)$
3. Relationship between dB and Neper:
  - 1 Neper = 8.686 dB
  - 1 dB = 0.115 Neper

#### Table:

Unit	Voltage Ratio	Power Ratio
Neper (Np)	$\ln(V_1/V_2)$	$(1/2)\ln(P_1/P_2)$
Decibel (dB)	$20\log_{10}(V_1/V_2)$	$10\log_{10}(P_1/P_2)$

### Mnemonic

“NED: Neper Equals Decibel divided by 8.686”

## Question 5(c) [7 marks]

Derive equations of R1 and R2 for symmetrical T attenuator.

## Solution

**Symmetrical T Attenuator:**

Mermaid Diagram (Code)

```

{Shaded}
{Highlighting} []
graph LR
    A((Input)) --- B[R1]
    B --- C((Junction))
    C --- D[R1]
    D --- E((Output))
    C --- F[R2]
    F --- G((Ground))
    style A fill:#f9f,stroke:#333,stroke-width:2px
    style E fill:#f9f,stroke:#333,stroke-width:2px
    style C fill:#f9f,stroke:#333,stroke-width:2px
{Highlighting}
{Shaded}

```

**Derivation:**

1. For a symmetrical T-attenuator with characteristic impedance  $Z_0$  :

- Input and output impedance must both equal  $Z_0$
- Attenuation ratio

$$N = V_1/V_2 = I_2/I_1$$

2. From circuit analysis:

- $Z_0 = R_1 + (R_2(R_1))/(R_2 + R_1)$
- $N = (R_1 + R_2 + R_1)/R_2 = (2R_1 + R_2)/R_2$

3. Solving for  $R_1$  and  $R_2$  :

- $R_1 = Z_0(N - 1)/(N + 1)$
- $R_2 = 2Z_0N/(N^2 - 1)$

4. For attenuation in dB ( ):

- $N = 10^{( /20)}$
- $R_1 = Z_0 \tanh(/2)$
- $R_2 = Z_0 / \sinh()$

**Final Equations:**

- $R_1 = Z_0(N - 1)/(N + 1)$
- $R_2 = 2Z_0N/(N^2 - 1)$

## Mnemonic

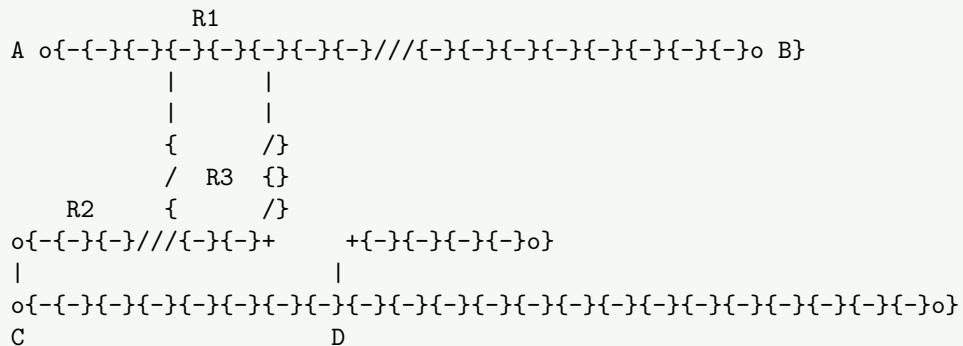
“TSR: T-attenuator Symmetry Requires equal R1 values”

## Question 5(a) OR [3 marks]

Draw circuit diagram of symmetrical Bridge T and symmetrical Lattice attenuator.

## Solution

**Symmetrical Bridge-T Attenuator:**



**Symmetrical Lattice Attenuator:**

R1

```

A o{-{-}{}{-}{}{-}///{-}{-}{-}{-}o B}
  {           /}
  {
  {           /}
  R2 {           / R2}
    {           /}
    {
    {           /}
    {/}
    /{ }
    /
    { }
    /
    { }

R1   /
      { }

C o{-{-}///{-}{-}{-}o D}

```

## Characteristics:

1. **Bridge-T**: Combines features of T and π attenuators, suitable for high-frequency applications
  2. **Lattice**: Balanced configuration with excellent phase and frequency response, commonly used in balanced lines

## Mnemonic

“BL-BA: Bridge Ladder, Balanced Attenuators”

**Question 5(b) OR [4 marks]**

**Write classification of filter based on frequency with their frequency responses showing pass band and stop band.**

## Solution

#### Classification of Filters Based on Frequency:

## Mermaid Diagram (Code)

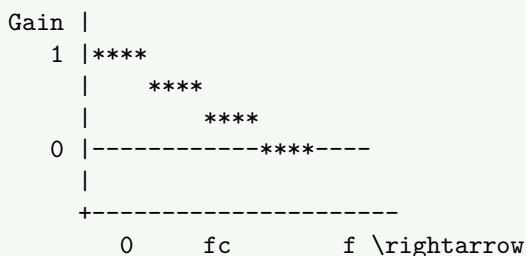
```

{Shaded}
{Highlighting} []
graph TD
    A[Passive Filters] --> B[Low Pass Filter]
    A --> C[High Pass Filter]
    A --> D[Band Pass Filter]
    A --> E[Band Stop Filter]
    A --> F[All Pass Filter]
{Highlighting}
{Shaded}

```

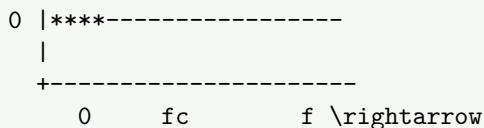
#### Frequency Responses:

1. Low Pass Filter: Passes frequencies below cutoff, attenuates above

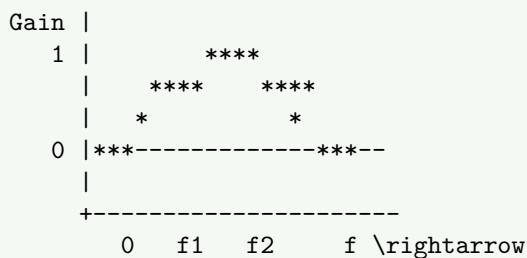


- 2. High Pass Filter:** Passes frequencies above cutoff, attenuates below

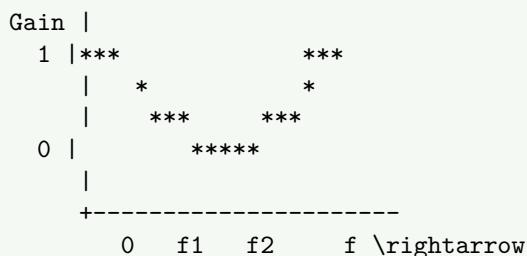




**3. Band Pass Filter:** Passes frequencies within a specific band



**4. Band Stop Filter:** Rejects frequencies within a specific band



## Mnemonic

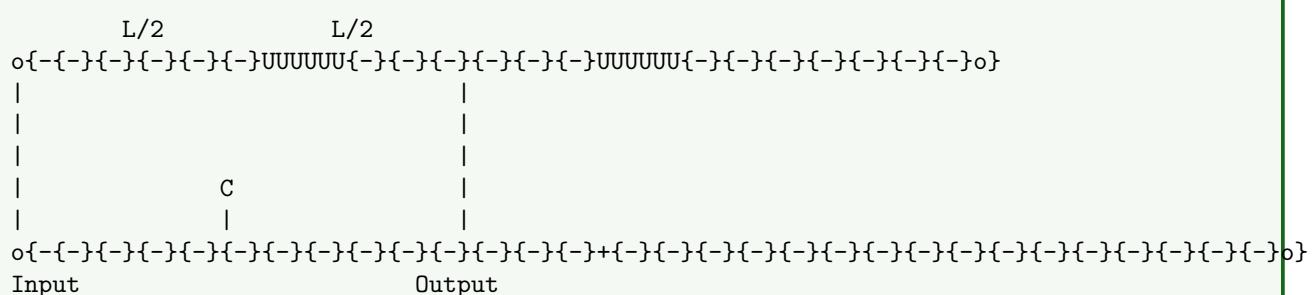
“LHBBA: Low High Band-pass Band-stop All-pass”

**Question 5(c) OR [7 marks]**

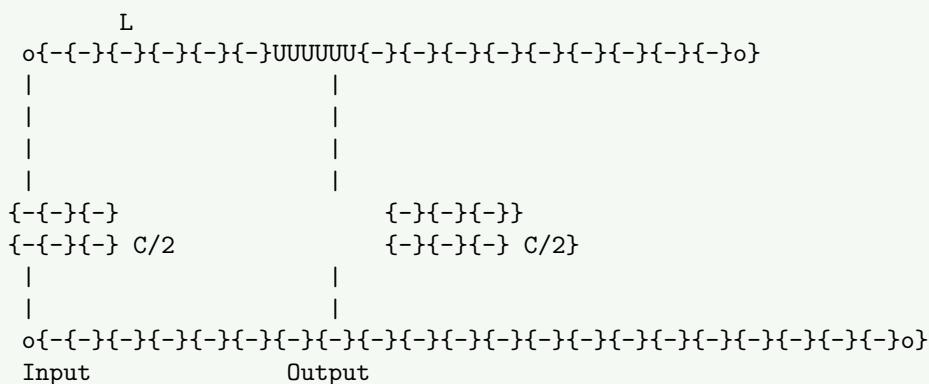
Draw the circuit for T-section and -section constant-K low pass filter and Derive equation of cut-off frequency.

## Solution

### T-section Constant-K Low Pass Filter:



### -section Constant-K Low Pass Filter:



**Derivation of Cutoff Frequency:**

1. For a constant-K filter:
  - $Z_1 \times Z_2 = R_0^2$  (characteristic impedance squared)
  - $Z_1 = jL$  (series impedance)
  - $Z_2 = 1/jC$  (shunt impedance)
2. Therefore:
  - $R_0^2 = Z_1 \times Z_2 = jL \times 1/jC = L/C$
  - $R_0 = \sqrt{L/C}$
3. Pass band condition:
  - $-1 < Z_1/4Z_2 < 0$
  - $-1 < j L/(4 \times 1/jC) < 0$
  - $-1 < -^2 LC/4 < 0$
4. At cutoff frequency:
  - $^2 LC/4 = 1$
  - $c^2 = 4/LC$
  - $c = 2/\sqrt{LC}$
  - $f_c = c/2 = 1/\sqrt{LC}$

**Final Equation:**

- Cutoff frequency  $f_c = 1/\sqrt{LC}$

**Mnemonic**

“KCLP: Konstant-k Cutoff in Low Pass depends on L and C product”