Sample Mathematics Solutions

Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options

Q1.1 [1 mark]

If $A_{2\times3}$ and $B_{3\times4}$ are two matrices then find order of $AB =$ ___

a. 4×2 b. 2×4 c. 3×3 d. AB is not possible

Answer: a. 2×4

Solution:
For matrix multiplication AB to be possible, the number of columns in matrix A must equal the number of rows in matrix B.

Given:

- Matrix A has order 2×3 (2 rows, 3 columns)
- Matrix B has order 3×4 (3 rows, 4 columns)

Since the number of columns in A (3) equals the number of rows in B (3), multiplication is possible.

The order of the resultant matrix AB will be:
$AB_{(2\times3)} \times B_{(3\times4)} = (AB)_{(2\times4)}$

Therefore, the order of AB is 2×4.

Q1.2 [1 mark]

If $A = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ then find $AB =$ ___

a. Not possible b. 9 c. $\begin{bmatrix} 1 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 6 & 2 \end{bmatrix}$

Answer: b. 9

Solution:
Given:
$A = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$ (order: 1×3)
$B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ (order: 3×1)

Since A is 1×3 and B is 3×1, multiplication is possible and the result will be 1×1 (a scalar).

$$AB = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$AB = (1)(1) + (3)(2) + (2)(1)$
$AB = 1 + 6 + 2 = 9$

Q1.3 [1 mark]

If $A \cdot I_2 = A$ then $I_2 =$ ____

a. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Answer: c. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution:
The condition $A \cdot I_2 = A$ means that I_2 is the identity matrix of order 2×2.

The identity matrix has the property that when any matrix is multiplied by it, the original matrix remains unchanged.

The 2×2 identity matrix is:
$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Verification: For any 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$:

$$A \cdot I_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$$

Calculus Problems

Q1.4 [1 mark]

If $\frac{d}{dx}(\sin^2 x + \cos^2 x) =$ ____

a. 1 b. 0 c. -1 d. x

Answer: b. 0

Solution:
We know that $\sin^2 x + \cos^2 x = 1$ (fundamental trigonometric identity).

Since the derivative of a constant is zero:
$\frac{d}{dx}(\sin^2 x + \cos^2 x) = \frac{d}{dx}(1) = 0$

Alternative approach (step by step):
$\frac{d}{dx}(\sin^2 x + \cos^2 x)$
$= \frac{d}{dx}(\sin^2 x) + \frac{d}{dx}(\cos^2 x)$

$= 2\sin x \cos x + 2\cos x(-\sin x)$
$= 2\sin x \cos x - 2\sin x \cos x = 0$

Integration Example

Evaluate: $\int x^5 dx$

Solution:
Using the power rule for integration: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ where $n \neq -1$

$\int x^5 dx = \frac{x^{5+1}}{5+1} + C = \frac{x^6}{6} + C$

Complex Numbers

Sample Problem

Simplify: $(3 + 4i)(4 - 5i)$

Solution:
$(3 + 4i)(4 - 5i)$
$= 3(4) + 3(-5i) + 4i(4) + 4i(-5i)$
$= 12 - 15i + 16i - 20i^2$

Since $i^2 = -1$:
$= 12 - 15i + 16i - 20(-1)$
$= 12 - 15i + 16i + 20$
$= 32 + i$

Differential Equations

Sample Problem

Solve: $x\frac{dy}{dx} + y = 0$

Solution:
This is a separable differential equation.

Rearranging: $x\frac{dy}{dx} = -y$

Separating variables: $\frac{dy}{y} = -\frac{dx}{x}$

Integrating both sides:
$\int \frac{dy}{y} = \int -\frac{dx}{x}$
$\ln|y| = -\ln|x| + C_1$
$\ln|y| = \ln|x^{-1}| + C_1$
$|y| = e^{\ln|x^{-1}|+C_1} = e^{C_1} \cdot |x^{-1}|$

Let $C = \pm e^{C_1}$, then:
$y = \frac{C}{x}$

This is the general solution.