

Q.1 Fill in the blanks [14 marks]

Q1.1 [1 mark]

If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ then $4A = \dots$

Answer: (b) $\begin{bmatrix} 4 & 8 \\ 12 & -4 \end{bmatrix}$

Solution:

$$4A = 4 \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & -4 \end{bmatrix}$$

Q1.2 [1 mark]

Order of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ -3 & 2 & 3 \end{bmatrix}$ is ...

Answer: (a) 2×3

Solution:

Matrix has 2 rows and 3 columns, so order is 2×3 .

Q1.3 [1 mark]

If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then $A^2 = \dots$

Answer: (d) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Solution:

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Q1.4 [1 mark]

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ then adjoint of $A = \dots$

Answer: (c) $\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

Solution:

For matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\text{adj}(A) = \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

Q1.5 [1 mark]

$\frac{d}{dx}(\tan x) = \dots$

Answer: (d) $\sec^2 x$

Solution:

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Q1.6 [1 mark]

$\frac{d}{dx}(\sin 5x) = \dots$

Answer: (b) $5\cos 5x$

Solution:

$$\frac{d}{dx}(\sin 5x) = 5 \cos 5x \text{ (using chain rule)}$$

Q1.7 [1 mark]

If function $y = f(x)$ is maximum at $x = a$ then $f'(a) = \dots$

Answer: (c) 0

Solution:

At maximum point, first derivative equals zero: $f'(a) = 0$

Q1.8 [1 mark]

$$\int \sin x \, dx = \dots + C$$

Answer: (a) $-\cos x$

Solution:

$$\int \sin x \, dx = -\cos x + C$$

Q1.9 [1 mark]

$$\int 1/(x^2+4) \, dx = \dots + C$$

Answer: (d) $(1/2)\tan^{-1}(x/2)$

Solution:

$$\int \frac{1}{x^2+4} dx = \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + C$$

Q1.10 [1 mark]

$$\int_1^2 x^2 \, dx = \dots$$

Answer: (a) $7/3$

Solution:

$$\int_1^2 x^2 dx = \left[\frac{x^3}{3} \right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

Q1.11 [1 mark]

Order of differential equation $(d^3y/dx^3)^4 + dy/dx + 5y = 0$ is ...

Answer: (c) 3

Solution:

Order is the highest derivative present = 3

Q1.12 [1 mark]

Integrating factor of $dy/dx + y/x = 1$ is ...

Answer: (b) \times

Solution:

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Q1.13 [1 mark]

Mean of 39,23,58,47,50,16,61 is ...

Answer: (b) 42

Solution:

$$\text{Mean} = \frac{39+23+58+47+50+16+61}{7} = \frac{294}{7} = 42$$

Q1.14 [1 mark]

Mean of first five natural numbers is ...

Answer: (a) 3

Solution:

$$\text{Mean} = \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

Q.2 Attempt any two [14 marks total]

Q2(A).1 [3 marks]

If $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & 5 \\ 5 & 4 & 3 \\ 3 & 5 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, find $3A+2B-4C$

Solution:

$$3A = \begin{bmatrix} 3 & 9 & 15 \\ -3 & 0 & 6 \\ 12 & 9 & 18 \end{bmatrix}$$

$$2B = \begin{bmatrix} 6 & 8 & 10 \\ 10 & 8 & 6 \\ 6 & 10 & 8 \end{bmatrix}$$

$$4C = \begin{bmatrix} 4 & 8 & 4 \\ 12 & 12 & 12 \\ 16 & 20 & 24 \end{bmatrix}$$

$$3A + 2B - 4C = \begin{bmatrix} 5 & 9 & 21 \\ -5 & -4 & 0 \\ 2 & -1 & 2 \end{bmatrix}$$

Q2(A).2 [3 marks]

If $A = \begin{bmatrix} 7 & 5 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$, show that $(A+B)^T = A^T + B^T$

Solution:

$$A + B = \begin{bmatrix} 8 & 4 \\ 2 & 4 \end{bmatrix}$$

$$(A + B)^T = \begin{bmatrix} 8 & 2 \\ 4 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 7 & -1 \\ 5 & 2 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 8 & 2 \\ 4 & 4 \end{bmatrix}$$

Hence proved: $(A + B)^T = A^T + B^T$

Q2(A).3 [3 marks]

Solve the differential equation $xy \, dy = (x+1)(y+1)dx$

Solution:

Separating variables:

$$\frac{y}{y+1} dy = \frac{x+1}{x} dx$$

$$\left(1 - \frac{1}{y+1}\right) dy = \left(1 + \frac{1}{x}\right) dx$$

Integrating:

$$y - \ln|y+1| = x + \ln|x| + C$$

Final answer: $y - x = \ln|y+1| + \ln|x| + C$

Q2(B).1 [4 marks]

Find the inverse of matrix $\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

Solution:

$$\text{Let } A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$|A| = 3(-3 - (-2)) - 1(2 - (-1)) + 2(4 - (-3)) = 3(-1) - 1(3) + 2(7) = -3 - 3 + 14 = 8$$

Cofactors:

- $C_{11} = -1, C_{12} = -3, C_{13} = 7$
- $C_{21} = 3, C_{22} = 1, C_{23} = -5$
- $C_{31} = 5, C_{32} = 7, C_{33} = -11$

$$\text{adj}(A) = \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

Q2(B).2 [4 marks]

Solve $3x - 2y = 8$, $5x + 4y = 6$ using matrix method

Solution:

$$\begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$$|A| = 3(4) - (-2)(5) = 12 + 10 = 22$$

$$A^{-1} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 44 \\ -22 \end{bmatrix}$$

Answer: $x = 2, y = -1$

Q2(B).3 [4 marks]

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, find $A \cdot \text{adj}(A)$

Solution:

$$|A| = 1(6 - 2) - 2(4 - 1) + 1(4 - 3) = 4 - 6 + 1 = -1$$

For any matrix A: $A \cdot \text{adj}(A) = |A| \cdot I$

$$A \cdot \text{adj}(A) = (-1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Q.3 Attempt any two [14 marks total]

Q3(A).1 [3 marks]

If $y = \log(\sin x / (1 + \cos x))$, find dy/dx

Solution:

$$y = \log(\sin x) - \log(1 + \cos x)$$

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x - \frac{1}{1 + \cos x} \cdot (-\sin x)$$

$$= \frac{\cos x}{\sin x} + \frac{\sin x}{1 + \cos x}$$

$$= \cot x + \frac{\sin x}{1 + \cos x}$$

$$\text{Using identity: } \frac{\sin x}{1 + \cos x} = \tan\left(\frac{x}{2}\right)$$

$$\text{Answer: } \frac{dy}{dx} = \cot x + \tan\left(\frac{x}{2}\right)$$

Q3(A).2 [3 marks]

If $y = \sin(x+y)$, find dy/dx

Solution:

Differentiating both sides:

$$\frac{dy}{dx} = \cos(x+y) \cdot \left(1 + \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \cos(x+y) + \cos(x+y) \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} - \cos(x+y) \cdot \frac{dy}{dx} = \cos(x+y)$$

$$\frac{dy}{dx} [1 - \cos(x+y)] = \cos(x+y)$$

$$\text{Answer: } \frac{dy}{dx} = \frac{\cos(x+y)}{1-\cos(x+y)}$$

Q3(A).3 [3 marks]**Obtain $\int x^2 \log x \, dx$** **Solution:**Using integration by parts: $\int u \, dv = uv - \int v \, du$ Let $u = \log x$, $dv = x^2 \, dx$ Then $du = (1/x) \, dx$, $v = x^3/3$

$$\int x^2 \log x \, dx = \log x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} \, dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

$$\text{Answer: } \frac{x^3}{3} \left(\log x - \frac{1}{3} \right) + C$$

Q3(B).1 [4 marks]**Motion equation $s = 2t^3 - 3t^2 - 12t + 7$. Find s and t when acceleration is zero****Solution:**

$$s = 2t^3 - 3t^2 - 12t + 7$$

$$\text{Velocity: } v = \frac{ds}{dt} = 6t^2 - 6t - 12$$

$$\text{Acceleration: } a = \frac{dv}{dt} = 12t - 6$$

When acceleration = 0:

$$12t - 6 = 0$$

$$t = \frac{1}{2}$$

At $t = 1/2$:

$$s = 2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 - 12\left(\frac{1}{2}\right) + 7 = \frac{1}{4} - \frac{3}{4} - 6 + 7 = \frac{1}{2}$$

$$\text{Answer: } t = 1/2, s = 1/2$$

Q3(B).2 [4 marks]**If $y = 2e^{3x} + 3e^{-2x}$, prove $d^2y/dx^2 - dy/dx - 6y = 0$**

Solution:

$$y = 2e^{3x} + 3e^{-2x}$$

$$\frac{dy}{dx} = 6e^{3x} - 6e^{-2x}$$

$$\frac{d^2y}{dx^2} = 18e^{3x} + 12e^{-2x}$$

$$\text{Now: } \frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y$$

$$= (18e^{3x} + 12e^{-2x}) - (6e^{3x} - 6e^{-2x}) - 6(2e^{3x} + 3e^{-2x})$$

$$= 18e^{3x} + 12e^{-2x} - 6e^{3x} + 6e^{-2x} - 12e^{3x} - 18e^{-2x}$$

$$= (18 - 6 - 12)e^{3x} + (12 + 6 - 18)e^{-2x} = 0$$

Hence proved

Q3(B).3 [4 marks]

Find maximum and minimum values of $f(x) = x^3 - 3x + 11$ **Solution:**

$$f(x) = x^3 - 3x + 11$$

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x - 1)(x + 1)$$

Critical points: $x = 1$, $x = -1$

$$f''(x) = 6x$$

At $x = 1$: $f''(1) = 6 > 0 \rightarrow$ Local minimumAt $x = -1$: $f''(-1) = -6 < 0 \rightarrow$ Local maximum

$$f(1) = 1 - 3 + 11 = 9 \text{ (minimum)}$$

$$f(-1) = -1 + 3 + 11 = 13 \text{ (maximum)}$$

Answer: Maximum = 13 at $x = -1$, Minimum = 9 at $x = 1$

Q.4 Attempt any two [14 marks total]

Q4(A).1 [3 marks]

Obtain $\int \sin 5x \sin 6x \, dx$ **Solution:**

$$\text{Using identity: } \sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin 5x \sin 6x = \frac{1}{2} [\cos(5x - 6x) - \cos(5x + 6x)]$$

$$= \frac{1}{2} [\cos(-x) - \cos(11x)] = \frac{1}{2} [\cos x - \cos(11x)]$$

$$\int \sin 5x \sin 6x \, dx = \frac{1}{2} \int [\cos x - \cos(11x)] \, dx$$

$$= \frac{1}{2} \left[\sin x - \frac{\sin(11x)}{11} \right] + C$$

$$\text{Answer: } \frac{1}{2} \sin x - \frac{\sin(11x)}{22} + C$$

Q4(A).2 [3 marks]**Obtain $\int (1+x)e^x / \cos^2(xe^x) dx$** **Solution:**Let $u = xe^x$, then $du = (1+x)e^x dx$

The integral becomes:

$$\int \frac{du}{\cos^2 u} = \int \sec^2 u du = \tan u + C$$

Substituting back:

$$= \tan(xe^x) + C$$

Answer: $\tan(xe^x) + C$ **Q4(A).3 [3 marks]****Find standard deviation for data: 6,7,10,12,13,4,8,12****Solution:**

Data: 6, 7, 10, 12, 13, 4, 8, 12

 $n = 8$

$$\text{Mean} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$$

x	x-9	(x-9)²
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	-5	25
8	-1	1
12	3	9

$$\Sigma(x-9)^2 = 74$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{74}{8}} = \sqrt{9.25} = 3.04$$

Answer: $\sigma = 3.04$ **Q4(B).1 [4 marks]****Obtain $\int (2x+1)/[(x+1)(x-3)] dx$**

Solution:

Using partial fractions:

$$\frac{2x+1}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$2x + 1 = A(x - 3) + B(x + 1)$$

$$\text{When } x = -1: 2(-1) + 1 = A(-4) \Rightarrow -1 = -4A \Rightarrow A = \frac{1}{4}$$

$$\text{When } x = 3: 2(3) + 1 = B(4) \Rightarrow 7 = 4B \Rightarrow B = \frac{7}{4}$$

$$\int \frac{2x+1}{(x+1)(x-3)} dx = \frac{1}{4} \int \frac{1}{x+1} dx + \frac{7}{4} \int \frac{1}{x-3} dx$$

$$= \frac{1}{4} \ln |x+1| + \frac{7}{4} \ln |x-3| + C$$

$$\text{Answer: } \frac{1}{4} \ln |x+1| + \frac{7}{4} \ln |x-3| + C$$

Q4(B).2 [4 marks]Obtain $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ **Solution:**

$$\text{Let } I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$$

$$\text{Using property: } \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cot(\pi/2-x)}}{\sqrt{\cot(\pi/2-x)} + \sqrt{\tan(\pi/2-x)}} dx$$

$$\text{Since } \cot(\pi/2-x) = \tan x \text{ and } \tan(\pi/2-x) = \cot x:$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

Adding both expressions:

$$2I = \int_0^{\pi/2} \frac{\sqrt{\cot x} + \sqrt{\tan x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$$

$$\text{Answer: } I = \frac{\pi}{4}$$

Q4(B).3 [4 marks]

Find mean deviation for grouped data

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Solution:

$$N = \sum f_i = 3+5+9+5+4+3+1 = 30$$

$$\text{Mean} = \frac{\sum f_i x_i}{N} = \frac{3(4)+5(8)+9(11)+5(17)+4(20)+3(24)+1(32)}{30}$$

$$= \frac{12+40+99+85+80+72+32}{30} = \frac{420}{30} = 14$$

$ x_i $	$ f_i $	$ x_i-14 $	$ f_i x_i-14 $
4	3	10	30
8	5	6	30
11	9	3	27
17	5	3	15
20	4	6	24
24	3	10	30
32	1	18	18

$$\Sigma f_i |x_i - 14| = 174$$

$$\text{Mean deviation} = \frac{\Sigma f_i |x_i - \bar{x}|}{N} = \frac{174}{30} = 5.8$$

Answer: Mean deviation = 5.8

Q.5 Attempt any two [14 marks total]

Q5(A).1 [3 marks]

Find mean deviation for grouped data

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Freq	3	7	12	15	8	3	2

Solution:

Class	Mid-value	f_i	$f_i x_i$
30-40	35	3	105
40-50	45	7	315
50-60	55	12	660
60-70	65	15	975
70-80	75	8	600
80-90	85	3	255
90-100	95	2	190

$$N = 50, \Sigma f_i x_i = 3100$$

$$\text{Mean} = 3100/50 = 62$$

Class	x_i	f_i	$ x_i-62 $	$f_i x_i-62 $
30-40	35	3	27	81
40-50	45	7	17	119

50-60	55	12	7	84
60-70	65	15	3	45
70-80	75	8	13	104
80-90	85	3	23	69
90-100	95	2	33	66

Mean deviation = $568/50 = 11.36$

Answer: Mean deviation = 11.36

Q5(A).2 [3 marks]

Find standard deviation for given data

Class	60	61	62	63	64	65	66	67	68
Freq	2	1	12	29	25	12	10	4	5

Solution:

$N = 100$, Mean = $(2 \times 60 + 1 \times 61 + \dots + 5 \times 68)/100 = 6380/100 = 63.8$

x_i	f_i	$(x_i - 63.8)$	$(x_i - 63.8)^2$	$f_i(x_i - 63.8)^2$
60	2	-3.8	14.44	28.88
61	1	-2.8	7.84	7.84
62	12	-1.8	3.24	38.88
63	29	-0.8	0.64	18.56
64	25	0.2	0.04	1.00
65	12	1.2	1.44	17.28
66	10	2.2	4.84	48.40
67	4	3.2	10.24	40.96
68	5	4.2	17.64	88.20

$$\sum f_i(x_i - \bar{x})^2 = 290$$

Standard deviation = $\sqrt{(290/100)} = \sqrt{2.9} = 1.70$

Answer: $\sigma = 1.70$

Q5(A).3 [3 marks]

Find mean for grouped data

Class	0-20	20-40	40-60	60-80	80-100	100-120
Freq	26	31	35	42	82	71

Solution:

Class	Mid-value	f_i	$f_i x_i$
0-20	10	26	260
20-40	30	31	930
40-60	50	35	1750
60-80	70	42	2940
80-100	90	82	7380
100-120	110	71	7810

$$N = 287, \sum f_i x_i = 21070$$

$$\text{Mean} = \frac{\sum f_i x_i}{N} = \frac{21070}{287} = 73.42$$

Answer: Mean = 73.42

Q5(B).1 [4 marks]

Solve differential equation $(x + y + 1)^2 dy/dx = 1$

Solution:

Let $z = x + y + 1$, then $dz/dx = 1 + dy/dx$

So $dy/dx = dz/dx - 1$

Substituting: $z^2(dz/dx - 1) = 1$

$$z^2 dz/dx - z^2 = 1$$

$$z^2 dz/dx = 1 + z^2$$

$$\frac{z^2}{1+z^2} dz = dx$$

Integrating:

$$\int \frac{z^2}{1+z^2} dz = \int dx$$

$$\int \left(1 - \frac{1}{1+z^2}\right) dz = x + C$$

$$z - \tan^{-1} z = x + C$$

Substituting back $z = x + y + 1$:

$$(x + y + 1) - \tan^{-1}(x + y + 1) = x + C$$

Answer: $y + 1 = \tan^{-1}(x + y + 1) + C$

Q5(B).2 [4 marks]

Solve $dy/dx + y/x = e^x$, $y(0) = 2$ **Solution:**

This is a linear differential equation of the form $dy/dx + P(x)y = Q(x)$

Here $P(x) = 1/x$, $Q(x) = e^x$

Integrating factor: $I.F. = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = |x| = x$ (for $x > 0$)

Multiplying the equation by x :

$$x \frac{dy}{dx} + y = x e^x$$

$$\frac{d}{dx}(xy) = x e^x$$

Integrating both sides:

$$xy = \int x e^x dx$$

Using integration by parts for $\int x e^x dx$:

Let $u = x$, $dv = e^x dx$

Then $du = dx$, $v = e^x$

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x = e^x(x - 1)$$

$$\text{So: } xy = e^x(x - 1) + C$$

$$y = \frac{e^x(x-1)+C}{x}$$

Using initial condition $y(0) = 2$:

As $x \rightarrow 0$, we need to use L'Hôpital's rule or series expansion.

From the original equation at $x = 0$: $dy/dx = e^x - y/x$

This suggests we need to be more careful with the initial condition.

Alternative approach: Since the equation has a singularity at $x = 0$, we solve in the neighborhood where $x \neq 0$.

Answer: $y = \frac{e^x(x-1)+C}{x}$ where C is determined by boundary conditions.

Q5(B).3 [4 marks]**Solve $y \, dy/dx = \sqrt{1 + x^2 + y^2 + x^2 y^2}$** **Solution:**

$$y \frac{dy}{dx} = \sqrt{1 + x^2 + y^2 + x^2 y^2}$$

$$y \frac{dy}{dx} = \sqrt{(1 + x^2)(1 + y^2)}$$

$$\frac{y dy}{\sqrt{1+y^2}} = \sqrt{1+x^2} dx$$

Integrating both sides:

$$\int \frac{y dy}{\sqrt{1+y^2}} = \int \sqrt{1+x^2} dx$$

For the left side, let $u = 1 + y^2$, then $du = 2y \, dy$:

$$\int \frac{y dy}{\sqrt{1+y^2}} = \frac{1}{2} \int \frac{du}{\sqrt{u}} = \sqrt{u} = \sqrt{1+y^2}$$

For the right side:

$$\int \sqrt{1+x^2} dx = \frac{x\sqrt{1+x^2}}{2} + \frac{1}{2} \ln |x + \sqrt{1+x^2}| + C$$

Therefore:

$$\sqrt{1+y^2} = \frac{x\sqrt{1+x^2}}{2} + \frac{1}{2} \ln |x + \sqrt{1+x^2}| + C$$

$$\text{Answer: } \sqrt{1+y^2} = \frac{x\sqrt{1+x^2}}{2} + \frac{1}{2} \ln |x + \sqrt{1+x^2}| + C$$

Formula Cheat Sheet

Matrix Operations

- $(A + B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$
- $A \cdot \text{adj}(A) = |A| \cdot I$
- For 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$: $\text{adj} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Differentiation Formulas

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$
- $\frac{d}{dx}(e^x) = e^x$
- Chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Integration Formulas

- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int \frac{1}{x} dx = \ln |x| + C$
- $\int e^x dx = e^x + C$
- $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Differential Equations

- **Linear DE:** $\frac{dy}{dx} + P(x)y = Q(x)$
- **Integrating Factor:** $I.F. = e^{\int P(x)dx}$
- **Variable Separable:** $\frac{dy}{dx} = f(x)g(y) \Rightarrow \frac{dy}{g(y)} = f(x)dx$

Statistics

- **Mean:** $\bar{x} = \frac{\sum x_i}{n}$ (ungrouped), $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ (grouped)
- **Mean Deviation:** $M.D. = \frac{\sum |x_i - \bar{x}|}{n}$
- **Standard Deviation:** $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

Problem-Solving Strategies

Matrix Problems

1. **Always check dimensions** before operations
2. **For inverse:** Calculate determinant first, then adjoint
3. **For system of equations:** Use $X = A^{-1}B$ where $AX = B$

Differentiation Problems

1. **Identify the type:** Chain rule, product rule, quotient rule
2. **For implicit differentiation:** Differentiate both sides, collect dy/dx terms
3. **For parametric:** Use $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Integration Problems

1. **Try substitution** if you see function and its derivative
2. **Use integration by parts** for products (LIATE rule)
3. **For definite integrals:** Check for symmetry properties

Differential Equations

1. **Identify type:** Separable, linear, exact
2. **For linear DE:** Find integrating factor first
3. **Always verify** your solution by substitution

Statistics Problems

1. **Find mean first** for deviation calculations
2. **Use grouped data formulas** when data is in classes
3. **Create frequency table** to organize calculations

Common Mistakes to Avoid

1. **Matrix multiplication:** Order matters ($AB \neq BA$ generally)
2. **Chain rule:** Don't forget to multiply by derivative of inner function
3. **Integration by parts:** Choose u and dv carefully using LIATE
4. **Differential equations:** Don't forget the constant of integration

5. **Statistics:** Use correct formula for grouped vs ungrouped data

Exam Tips

1. **Read questions carefully** - especially for OR questions
2. **Show all steps** - partial marks are awarded
3. **Check units and signs** in your final answers
4. **Verify solutions** when possible by substitution
5. **Manage time wisely** - attempt questions you're confident about first
6. **Use standard formulas** - memorize the formula sheet content
7. **For fill-in-blanks:** Eliminate obviously wrong options first