Seat No. / Enrolment No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Diploma Engineering – SEMESTER – 1 (NEW) – EXAMINATION – Winter-2024

Subject Code: DI01000021 **Date: 02-01-2025**
Subject Name: Mathematics-I
Time: 10:30 AM TO 01:00 PM **Total Marks: 70**
Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of simple calculators and non-programmable scientific calculators are permitted.**
5. **English version is authentic.**

Q.1 Fill in the blanks/MCQs using appropriate choice from the given options. **14**
(યોગ્ય વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો/ બહુવિકલ્પ પ્રશ્નોનાં જવાબ આપો)

(1) $\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix}$ = _____

(a) 12 (b) 13 (c) 5 (d) 1

(૧) $\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix}$ = _____

(a) 12 (b) 13 (c) 5 (d) 1

(2) If $\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = 0$ then $x =$ _____

(a) 1 (b) 2 (c) 5 (d) -1

(૨) જો $\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} = 0$ તો $x =$ _____

(a) 1 (b) 2 (c) 5 (d) -1

(3) If $f(x) = x^2$ then $f(-1) =$ _____

(a) 1 (b) 0 (c) -1 (d) 2

(3) જો $f(x) = x^2$ તો $f(-1) =$ _____

(a) 1 (b) 0 (c) -1 (d) 2

(4) $\log_{10} 1 =$ _____

(a) 1 (b) 0 (c) 10 (d) -1

(૪) $\log_{10} 1 =$ _____

(a) 1 (b) 0 (c) 10 (d) -1

(5) $\sin \dfrac{\pi}{2} + \cos \dfrac{\pi}{2} =$ _____

(a) 2 (b) 3 (c) 1 (d) -2

(૫) $\sin\dfrac{\pi}{2}+\cos\dfrac{\pi}{2}=$ _____

(a) 2 (b) 3 (c) 1 (d) -2

(6) $\tan^{-1}(1)=$ _____

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) 0 (d) $\dfrac{\pi}{3}$

(૬) $\tan^{-1}(1)=$ _____

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) 0 (d) $\dfrac{\pi}{3}$

(7) $\dfrac{2\pi}{3}$ radian = _____degree

(a) 30 (b) 45 (c) 60 (d) 120

(૭) $\dfrac{2\pi}{3}$ રેડિઅન = _____ડિગ્રી

(a) 30 (b) 45 (c) 60 (d) 120

(8) $\hat{i}\cdot\hat{j}=$ _____

(a) $\hat{i}$ (b) $\hat{j}$ (c) $\hat{k}$ (d) 0

(૮) $\hat{i}\cdot\hat{j}=$ _____

(a) $\hat{i}$ (b) $\hat{j}$ (c) $\hat{k}$ (d) 0

(9) $\left|\hat{i}+\hat{j}+\hat{k}\right|=$ _____

(a) 3 (b) 2 (c) $\sqrt{2}$ (d) $\sqrt{3}$

(૯) $\left|\hat{i}+\hat{j}+\hat{k}\right|=$ _____

(a) 3 (b) 2 (c) $\sqrt{2}$ (d) $\sqrt{3}$

(10) Slope of line $2x+y-3=0$ is _____

(a) -2 (b) 3 (c) -1 (d) 2

(૧૦) રેખા $2x+y-3=0$ નો ઢાળ _____ છે.

(a) -2 (b) 3 (c) -1 (d) 2

(11) Radius of circle $x^2+y^2=81$ is _____

(a) 3 (b) 9 (c) 81 (d) 0

(૧૧) વર્તુળ $x^2+y^2=81$ ની ત્રિજ્યા _____ છે.

(a) 3 (b) 9 (c) 81 (d) 0

(12) $\lim\limits_{n\to\infty}\dfrac{1}{n}=$ _____

(a) ∞ (b) -1 (c) 0 (d) 1

(૧૨) $\lim\limits_{n\to\infty}\dfrac{1}{n}=$ _____

(a) ∞ (b) -1 (c) 0 (d) 1

(13) $\lim\limits_{x\to1}\left(x^2+x+1\right)=$ _____

(a) 3   (b) 1   (c) 2   (d) -3

(૧૩) $\lim\limits_{x \to 1} \left(x^2 + x + 1 \right) = $ _____

(a) 3   (b) 1   (c) 2   (d) -3

(14) $\lim\limits_{\theta \to 0} \dfrac{\tan \theta}{\theta} = $ _____

(a) 3   (b) 1   (c) 2   (d) -3

(૧૪) $\lim\limits_{\theta \to 0} \dfrac{\tan \theta}{\theta} = $ _____

(a) 3   (b) 1   (c) 2   (d) -3

Q.2 (A) Attempt any two (કોઇપણ બે ના જવાબ આપો)   06

(1) Find the value of $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 4 & -2 & 5 \end{vmatrix}$

(૧) $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 4 & -2 & 5 \end{vmatrix}$ ની કિમંત શોધો.

(2) If $f(x) = x^3 + 5$ then find $f(0)$, $f(1)$ and $f(-1)$

(૨) જો $f(x) = x^3 + 5$ તો $f(0)$, $f(1)$ અને $f(-1)$ શોધો.

(3) Prove that $\tan^{-1}\left(\dfrac{1}{2} \right) + \tan^{-1}\left(\dfrac{1}{3} \right) = \dfrac{\pi}{4}$

(3) સાબિત કરો $\tan^{-1}\left(\dfrac{1}{2} \right) + \tan^{-1}\left(\dfrac{1}{3} \right) = \dfrac{\pi}{4}$

(B) Attempt any two (કોઇપણ બે ના જવાબ આપો)   08

(1) If $f(x) = \dfrac{x-1}{x+1}$ then prove that $f(x) f(-x) = 1$

(૧) જો $f(x) = \dfrac{x-1}{x+1}$ તો $f(x) f(-x) = 1$ સાબિત કરો.

(2) If $\log\left(\dfrac{x+y}{2} \right) = \dfrac{1}{2}\left(\log x + \log y \right)$ then prove that $x = y$

(૨) જો $\log\left(\dfrac{x+y}{2} \right) = \dfrac{1}{2}\left(\log x + \log y \right)$ તો $x = y$ સાબિત કરો.

(3) Solve $\log(x+3) + \log(x-3) = \log 27$

(3) ઉકેલો $\log(x+3) + \log(x-3) = \log 27$

Q.3 (A) Attempt any two (કોઇપણ બે ના જવાબ આપો) 06

(1) Prove that $\dfrac{\sin\left(\dfrac{\pi}{2}+\theta\right)}{\cos(\pi-\theta)}+\dfrac{\tan(\pi-\theta)}{\cot\left(\dfrac{3\pi}{2}-\theta\right)}+\dfrac{\cosec\left(\dfrac{\pi}{2}-\theta\right)}{\sec(\pi+\theta)}=-3$

(૧) સાબિત કરો $\dfrac{\sin\left(\dfrac{\pi}{2}+\theta\right)}{\cos(\pi-\theta)}+\dfrac{\tan(\pi-\theta)}{\cot\left(\dfrac{3\pi}{2}-\theta\right)}+\dfrac{\cosec\left(\dfrac{\pi}{2}-\theta\right)}{\sec(\pi+\theta)}=-3$

(2) Prove that $\tan 55° = \dfrac{\cos 10° + \sin 10°}{\cos 10° + \sin 10°}$

(૨) સાબિત કરો $\tan 55° = \dfrac{\cos 10° + \sin 10°}{\cos 10° + \sin 10°}$

(3) If $\bar{a}=2\hat{i}+3\hat{j}+\hat{k}$, $\bar{b}=\hat{i}+\hat{j}+\hat{k}$ and $\bar{c}=3\hat{i}+\hat{j}+\hat{k}$ then find $2\bar{a}+\bar{b}-\bar{c}$

(3) જો $\bar{a}=2\hat{i}+3\hat{j}+\hat{k}$, $\bar{b}=\hat{i}+\hat{j}+\hat{k}$ અને $\bar{c}=3\hat{i}+\hat{j}+\hat{k}$ તો $2\bar{a}+\bar{b}-\bar{c}$ શોધો.

(B) Attempt any two (કોઇપણ બે ના જવાબ આપો) 08

(1) Prove that $\dfrac{\sin(x-y)}{\cos x\cos y}+\dfrac{\sin(y-z)}{\cos y\cos z}+\dfrac{\sin(z-x)}{\cos z\cos x}=0$

(૧) સાબિત કરો $\dfrac{\sin(x-y)}{\cos x\cos y}+\dfrac{\sin(y-z)}{\cos y\cos z}+\dfrac{\sin(z-x)}{\cos z\cos x}=0$

(2) Draw graph of $y=\cos x$; $0\le x\le\pi$

(૨) $y=\cos x$; $0\le x\le\pi$ નું આલેખ દોરો.

(3) Find equation of line passing through (1, 2) and (-3, 1)

(3) (1, 2) અને (-3, 1) માંથી પ્રસાર થતી રેખાનું સમીકરણ મેળવો.

Q.4 (A) Attempt any two (કોઇપણ બે ના જવાબ આપો) 06

(1) Find unit vector perpendicular to $\bar{a}=\hat{i}-3\hat{j}+\hat{k}$ and $\bar{b}=2\hat{i}+\hat{j}+2\hat{k}$

(૧) $\bar{a}=\hat{i}-3\hat{j}+\hat{k}$ અને $\bar{b}=2\hat{i}+\hat{j}+2\hat{k}$ ને લંબ એકમ સદિશ શોધો.

(2) Forces (1, 2, 1) and (2, -1, 3) act on a particle and the particle moves from a point (2, 3, 1) to (4, 6, 2). Find the work done.

(૨) એક કણ પર બળ (1, 2, 1) અને (2, -1, 3) લાગતા તે બિંદુ (2, 3, 1) થી (4, 6, 2) સુધી ખસે છે તો થયેલ કાર્ય શોધો.

(3) Show that lines $2x-3y+5=0$ and $8x-12y-3=0$ are parallel lines.

(3) બતાવો કે રેખાઓ $2x-3y+5=0$ અને $8x-12y-3=0$ સમાંતર રેખાઓ છે.

(B) Attempt any two (કોઇપણ બે ના જવાબ આપો) 08

(1) Show that angle between $\bar{a}=\hat{i}+\hat{j}-\hat{k}$ and $\bar{b}=2\hat{i}-2\hat{j}+\hat{k}$ is $\sin^{-1}\left(\sqrt{\dfrac{26}{27}}\right)$

(૧) બતાવો કે $\bar{a} = \hat{i} + \hat{j} - \hat{k}$ અને $\bar{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ વચ્ચેનો ખૂણો $\sin^{-1}\left(\sqrt{\dfrac{26}{27}}\right)$ છે.

(2) If $\bar{a} = (1, 1, 1)$, $\bar{b} = (2, 0, 1)$ and $\bar{c} = (-2, 1, 0)$ then find $\bar{a} \cdot (\bar{b} \times \bar{c})$

(૨) જો $\bar{a} = (1, 1, 1)$, $\bar{b} = (2, 0, 1)$ અને $\bar{c} = (-2, 1, 0)$ તો $\bar{a} \cdot (\bar{b} \times \bar{c})$ મેળવો.

(3) Evaluate $\lim\limits_{\theta \to 0} \dfrac{\sin 4\theta}{\theta}$

(3) કિંમત મેળવો $\lim\limits_{\theta \to 0} \dfrac{\sin 4\theta}{\theta}$

Q.5 (A) Attempt any two (કોઇપણ બે ના જવાબ આપો)   06

(1) Evaluate $\lim\limits_{x \to 9} \dfrac{x^2 - 81}{x - 9}$

(૧) કિંમત મેળવો $\lim\limits_{x \to 9} \dfrac{x^2 - 81}{x - 9}$

(2) Evaluate $\lim\limits_{x \to \infty}\left(1 + \dfrac{3}{x}\right)^{2x}$

(૨) કિંમત મેળવો $\lim\limits_{x \to \infty}\left(1 + \dfrac{3}{x}\right)^{2x}$

(3) Evaluate $\lim\limits_{x \to 1} \dfrac{x - 1}{x^2 + x - 2}$

(3) કિંમત મેળવો $\lim\limits_{x \to 1} \dfrac{x - 1}{x^2 + x - 2}$

(B) Attempt any two (કોઇપણ બે ના જવાબ આપો)   08

(1) Find the equation of line passing through the point (2, -3) and having slope 4.

(૧) (2, -3) માંથી પ્રસાર થતી અને 4 ઢાળ વાળી રેખાનું સમીકરણ શોધો.

(2) For what value of m, lines $7x + y - 1 = 0$ and $3x - my + 2 = 0$ are perpendicular to each other.

(૨) m ની કઇ કિંમત માટે, રેખાઓ $7x + y - 1 = 0$ અને $3x - my + 2 = 0$ એકબીજા ને લંબ થશે.

(3) Find the centre and radius of the circle $4x^2 + 4y^2 + 8x - 12y - 3 = 0$

(3) વર્તુળ $4x^2 + 4y^2 + 8x - 12y - 3 = 0$નું કેંદ્ર અને ત્રિજ્યા શોધો.

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