Cruise Control Design

ENGN3223 Control Systems <Design Project>

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1. Introduction

This report aims to demonstrate the design of cruise control on an arbitrary vehicle in real life. The design started from linearizing the system dynamics ODE using Taylor Series Expansion and ended in simulations via MATLAB Simulink.

Cruise control is a method that can help a vehicle to maintain a target speed automatically without human interference such as stepping on the accelerator pedal. The operating system is complex and sensitive, with the mass of vehicle, cross-sectional area, several natural coefficients, various disturbance forces and more importantly, a control signal involved. To simplify the problem, I directly investigated the non-linear form of the system and compared to the linearized form. The relationship was found and used to model the system in Simulink.

The controller being used was a PI controller. This decision was based on researches on cruise control both non-linearly and linearly. However, the gain parameters were from estimation and trials.

The system response with a PI controller was overall optimistic. With changing target speed, sudden ground grade transition and passengers, the performance was acceptable from the simulation results.

2. System modelling and simulations

2.1 System linearization

From the project description, the dynamic system of a car with engine controller is given by the equation as following.

$$\mathrm{m}\dot{\mathrm{v}} + \frac{1}{2}A\rho c_D v^2 = u + d \tag{1}$$

Here m is the mass of the vehicle in *kg*, v is the velocity in *m/s*, A is the cross-sectional area of the vehicle in m^2, ρ is the density of air in kg/m^3, C_D is a dimensionless drag coefficient. On the righthand side, u is the control input force from the vehicle's engine, and d is the disturbance.

Prior to the controller design, it is noticeable that the system model is in a non-linear form. For modelling simplicity, the linearized form of the system is being used. Yet, linearization introduces assumption. The result is only valid near the equilibrium point.

Here is the procedure of linearization.
Rearrange Eq. (1) and define a function f with variables v and u, I obtain

$$\mathrm{m}\dot{\mathrm{v}} = f(v,u) = -\frac{1}{2}A\rho c_D v^2 + u + d \tag{2}$$

Here the arbitrary equilibrium exists at $f(v_0, u_0) = 0$, where v_0 is a constant initial velocity of the vehicle. Then, easily I have

$$\mathrm{u}_0 = \frac{1}{2}A\rho c_D {v_0}^2 - d \tag{3}$$

To linearize the system, perturbed variables are required.

$$v = v_0 + \delta v \tag{4}$$

$$\dot{v} = \dot{v_0} + \dot{\delta v} = \dot{\delta v} \tag{5}$$

$$\mathrm{u} = u_0 + \delta u \tag{6}$$

Applying Taylor Series Expansion on Eq. (2) with variable substitution, I obtain

$$f(v,u) \approx f(v_0,u_0) + \frac{\partial f}{\partial v}_{(v_0,u_0)} \delta v + \frac{\partial f}{\partial u}_{(v_0,u_0)} \delta u = -A\rho c_D v_0 \delta v + \delta u \tag{7}$$

Substituting Eq. (3) and Eq. (5) into the Taylor Series Expansion, I obtain

$$m\dot{\delta v} = -A\rho c_D v_0 \delta v + u - \frac{1}{2} A\rho c_D {v_0}^2 + d \tag{8}$$

Rearrange the Eq. (8), I have

$$m\dot{\delta v} + A\rho c_D v_0 \delta v = \delta u \tag{9}$$

where $\delta u = u - \frac{1}{2} A\rho c_D {v_0}^2 + d$. Finally, the Eq. (8) is the linearized equation which is being used in further design.

Here it is worthy to briefly setting the constant parameters. The typical constants from nature are $\rho = 1.3\ kg/m^3$ and $C_D = 0.32$ [1]. Since I do not own a car, the typical reference for a vehicle I have selected are $A = 2.4\ m^2$ and $m = 1600\ kg$.

2.2 Disturbance investigation and force calculation

Normally, the disturbance force on a moving vehicle consists of three parts – force due to gravity (F_g), rolling friction (F_r) and aerodynamic drag (F_a) [1]. According to Karl's book, the simplified expression of the three disturbance forces are as following.

$$\mathrm{F}_g = mgsin\theta \tag{10}$$

where θ is the slope of the road and g = 9.8 m/s^2.

$$\mathrm{F}_r = mgC_r sgn(v) \tag{11}$$

where typically $C_r = 0.01$, which is the rolling friction coefficient. And sgn(v) is the sign of v.

$$\mathrm{F}_a = \frac{1}{2} A\rho c_D |v| v \tag{12}$$

Apart from the disturbance, the driving force from engine is given by

$$\mathrm{F} = \mathrm{a}_n u T(\alpha_n v) \tag{13}$$

Notice, the parameter u in Eq. (13) is different to u in section 2.1. Here, u denotes a control signal that used to modify throttle position. Typical α_n values are 40, 25, 16, 12 and 10 for n=1,…,5 respectively.

The torque is given by

$$\mathrm{T}(\omega) = \mathrm{T}_m \left(1 - \beta \left(\frac{\omega}{\omega_m} - 1\right)^2\right) \tag{14}$$

Where the typical parameters $T_m = 190$ Nm, $\omega_m = 420$ rad/s (4000RPM) and $\beta = 0.4$.

Balancing the force on a vehicle, I obtain

$$m\dot{v} = F - F_r - F_a - F_g \tag{15}$$

With the expansion of the driving force and disturbance forces and compared the Eq. (15) with the original system given in Eq. (1), easily I found the disturbance d in Eq. (1) is just the force due to gravity. Also, the control input u in Eq. (1) is $F - F_r$ from Eq. (15). To distinguish the control input, denote the control input from 2.1 to be U instead of u.

$$d = F_g = mgsin\theta \tag{16}$$

$$\mathrm{U} = \mathrm{F} - \mathrm{F}_r \tag{17}$$

Hence, there are two ways to design the controller – using the linearized system in 2.1 or using the original non-linear system originally from Eq. (13). And it is worthy to notice the control signals are different in these systems.

2.3 Simulation results

The simulation results are shown in figure 1 and figure 2 as following. Both plots are generated from Simulink with a feedback controller. The detailed design process will be mentioned in part 3.

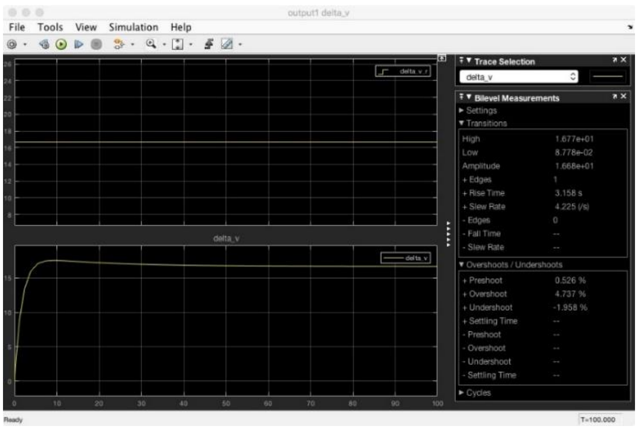

Figure 1: Simulation of delta v

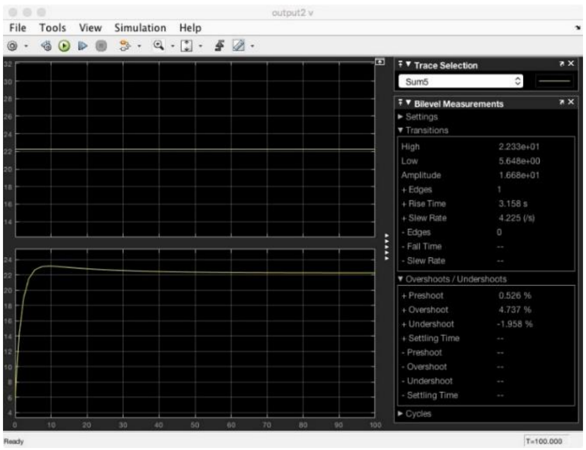

Figure 2: Simulation of v

As shown above, the upper plots are the constant reference speed of δv and v respectively (the plot unit is in m/s instead of km/h). A step signal was used with amplitude $|\delta v| = 60$km/h at time t = 0. The simulation results are plausible for a 1600 kg vehicle directly from the plots. As shown in the right-hand-side bar, the overshoot is slight, the rise time is reasonably short, and the controlled speed becomes stable and reaches the reference at approximately 15s. Hence, at this stage, the design is overall optimistic. Disturbances and more details are included and further discussed in part 3.

3 Controller design and validation

3.1 Linearized model in Simulink

To simplify the design process, I began with testing the linearized system. Here is the linearized system obtained from part 2 using Eq. (8) and Eq. (16).

$$m\dot{\delta v} = -A\rho c_D v_0 \delta v + u - \frac{1}{2} A\rho c_D {v_0}^2 + mgsin\theta \tag{18}$$

Observing the Eq. (18), I found the system input is δv, which is also the signal I intend to control to maintain the speed. As long as δv tends to be stable within a reasonable period, the output speed v becomes stable because it is just an addition of δv and initial speed v_0. Also, it could be inferred that the plot of δv against time and v against time may be only differ in shift along the vertical axis. There is another observation. As v_0 is not changing with time, the last two terms on RHS can be considered as constants when modelling the system in Simulink. But surely v_0 is dependent with δv, so it may influence the system.

The choice of controller was based on researches on non-linear cruise control systems. As mentioned in section 2.1, the non-linear control signal u is highly related to the internal construction of a vehicle, namely engine and throttle. And as it is desirable to maintain a constant velocity when the vehicle is static, a controller with integral function is natural [2]. Also, in Sailan's paper [3], a PID controller was used but with feedforward gain instead of feedback. Considering controller with integral action is included in both designs, it worth to try a PI controller first.

Additionally, this assumption could be valid because the error is given by

$$\mathrm{e(t)} = (\delta v)_r(t) - \delta v(t) \tag{19}$$

which is equal to (v_r – v) as the initial speed v_0 is constant.

Hence, the linearized model built in Simulink is shown in Figure 3.

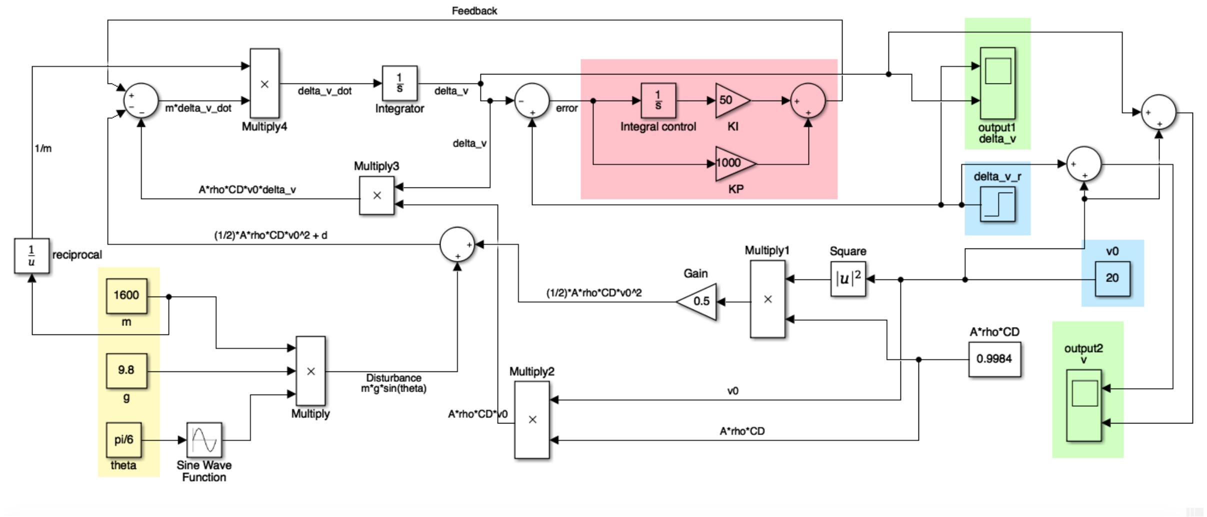

Figure 3: Simulink model with PI control

In the Simulink model, the red block is the PI controller with arbitrary proportional and integral gains. The green boxes are the output scopes for visualising δv and v respectively. The reference speed δv_r and the initial speed v_0 are put in blue areas. The yellow blocks are natural and vehicle's constants.

The model is mainly based on the ODE logic from Eq. (18), and the controller design is inspired by the non-linear Simulink model from FBSwiki [4].

3.2 System dynamics

To further observe the dynamics of the system with the PI controller and disturbances, several situations are tested.

3.2.1 Constant disturbance

From Eq. (16), I know the disturbance is m*g*sin(θ). Let the mass remain the same which is 1600 kg. Then the disturbance mainly comes from the ground grade angle θ. In urban areas in ACT, the maximum grades of roads are varying from 3% to 10% [5]. Assume the vehicle is a common one mostly used in urban areas, I consider a 10% grade is encountered constantly. By changing the parameter θ = 10% in the block diagram from Figure 3, the dynamics plot is obtained in Simulink as following.

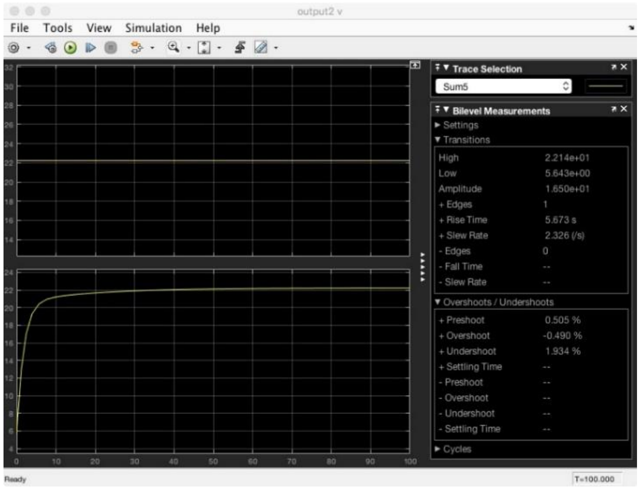

Figure 4: Output velocity with constant disturbance

As shown in Figure 4, the desired speed is precisely achieved with constant disturbance. To notice the reference speed is 80 km/h while unit is in m/s from the graph. Here $|\delta v|_r = 60km/h$.

3.2.2 Changing target speed

In actual cases, there could be several target speeds depending road situations or speed limit. Hence, the dynamics of changing target speed needs to be simulated to ensure performance. In Figure 5, the purple block shows the source of generating reference signal. A repeating sequence stair signal was used by generating δv at 40 km/h, 60 km/h and 90 km/h respectively. As our initial speed v_0 = 20 km/h remains constant, the output speed is just an addition again.

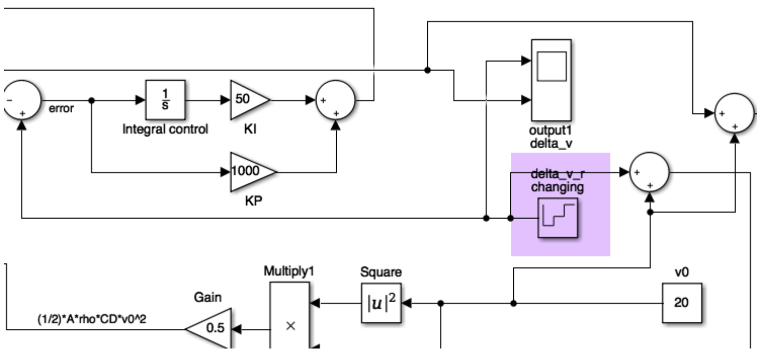

Figure 5: Changing reference speed

By converting all speed units from km/h to m/s, the simulation result is shown in Figure 6.

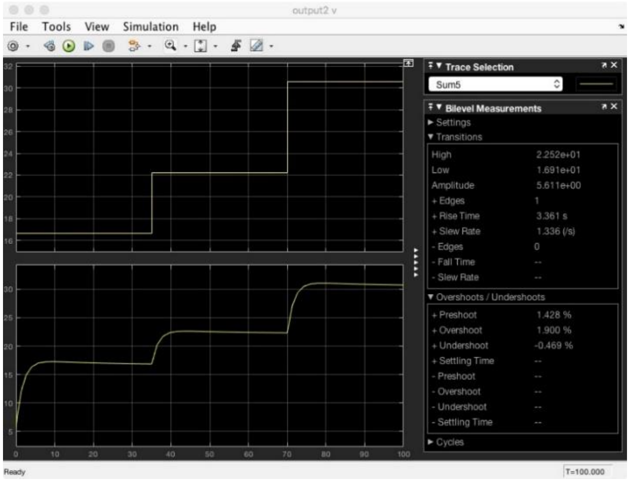

Figure 6: Changing speed result

The time gap between speed changing is set to 35s. As shown in Figure 6, the result is plausible because the vehicle can reach the target speed within a short time period and stay stable every time changing the reference speed.

3.2.3 Sudden grade transition

It is possible a vehicle comes across a hill or any road with steep uphill grade. Hence, A simulation of a sudden transition from horizontal ground to a slope of 35% grade was simulated. The modified model is shown in Figure 7 by changing the constant angle to a step signal in the purple block.

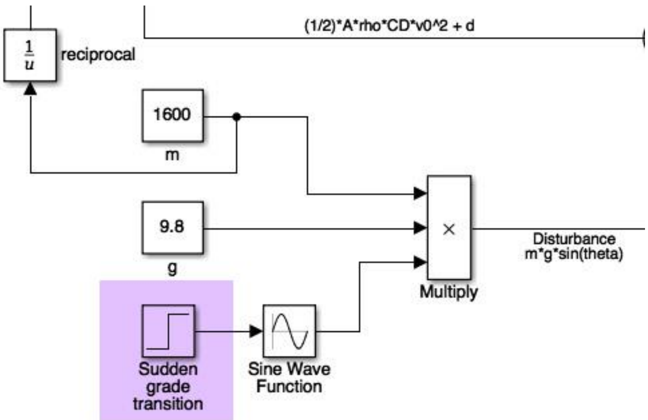

Figure 7: Modified model for sudden transition

With the sudden change of disturbance, the output speed dynamics is shown in Figure 8 as following. The sudden change of slope was set to after 10 seconds from the beginning. Observing the plot, it is found that the target speed was reached before 10s. While right after the sudden change of the slope, the speed dropped significantly and took longer time to reach the reference speed again. This is plausible because the system requires more time and turns to attenuate the larger disturbances through the PI controller. And overall, the dynamics is reasonable at this stage.

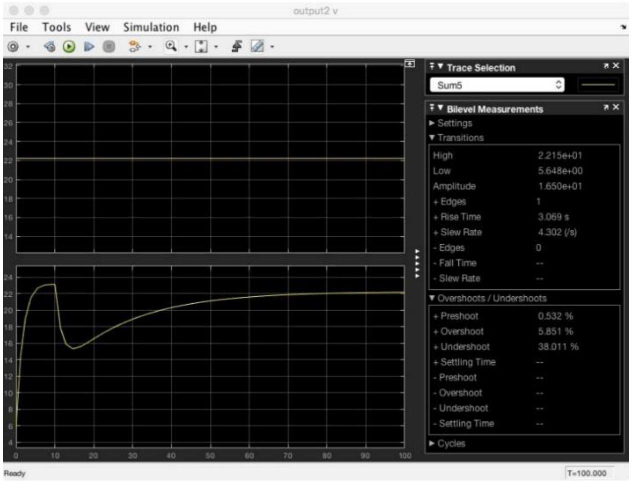

Figure 8: Sudden change response

3.2.4 Uncertainty in mass

Assume the vehicle is a sedan car that can take at most 5 passengers. The average weight of an adult human is approximately 62kg [6]. To observe the influence of taking passengers, the modified model is shown in Figure 9 with other parameters remaining constant.

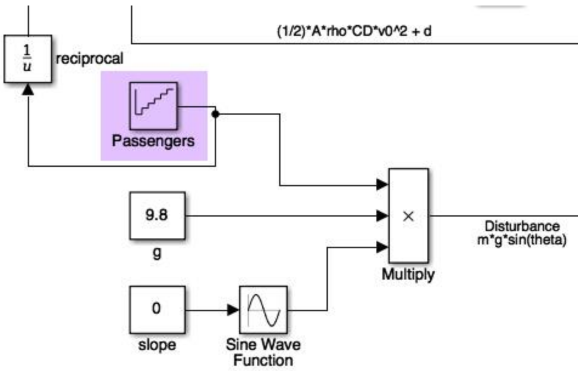

Figure 9: Adding passengers

The repeating sequence generator was set to add one passenger per 20 seconds. It was thought the response could be similar to adding angles each time, yet surprisingly it is not being influenced much. Observing the response in Figure 10, the controlled speed is as stable as without passengers and there is not any fluctuation. This is possible because the mass of vehicle is much larger than a person's weight. Hence, unlike changes in road grade, slightly increasing the overall weight does not affect the total disturbance much.

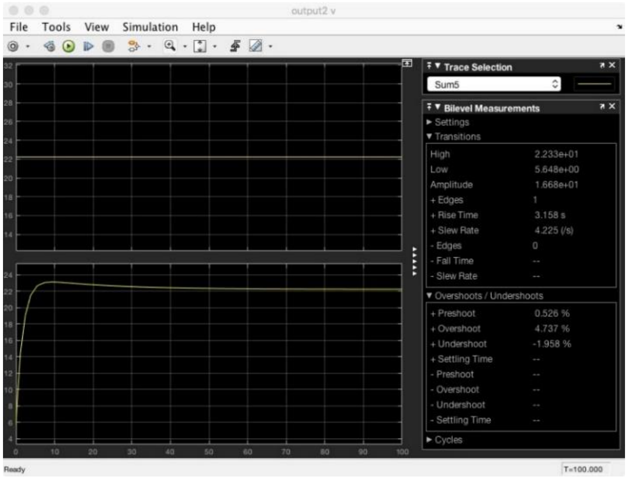

Figure 10: Response of adding passengers

4 Discussion and Conclusions

The overall functionality of the PI controller is relatively optimistic according to the simulations on normal operation and extreme cases. When operating on flat ground with changing targets, the cruise control system can reach the desirable speed within approximately 10 to 15 seconds. When there is a sudden change in the slope of ground, the vehicle can notice and adjust to it through a steady and safe process. With passengers on, the vehicle shows the same performance without passengers, which is optimistic as Ill.

However, there are still some unsolved problems regarding to the design.

Above all, the design is fully based on the linearized form of the true system. Hence, it introduces limitations that the dynamics are only valid near the arbitrary equilibrium. For the situations that far from the equilibrium, the estimation could be inaccurate. Hence, time permitting, a design on the real non-linear system should be implemented to ensure accuracy.

Secondly, the values of proportional gain and integral gain in the PI controller Ire obtained by estimation and enumeration method. Even though optimistic results have been achieved, the process of controller design was not adequately rigorous or persuasive. The gains selected for K_I was 50 and 1000 for K_P respectively for the linear system. Yet, surely there should be a limit on the gains as the vehicle is composed of physical components. Here $K_P = 1000$ is likely to be far too big that beyond some physical limit. It is assumed the limit could be from the throttle or gears according to both the non-linear ODE and common senses. However, there was inadequate research or evidence on it within the limited time.

Finally, as the mass could vary from different types, this design using a constant mass of 1600kg may not adapt to a wide range of vehicles. Also, the initial velocity was set to 20km/h as a constant as Ill, not changing from the target speed. Hence, not all possibilities have been covered in this simulation report. Time permitting, a more comprehensive model could be designed and modified in detail.

References

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