```
In[1]:= (* stability matrix *)
M = {{-Du * k^2, chi * k^2, 0}, {0, -k^2, 0}, {0, 0, -Dw * k^2}}
```

Out[1]= $\{\{-\mathrm{Du}\,k^2, \mathrm{chi}\,k^2, 0\}, \{0, -k^2, 0\}, \{0, 0, -\mathrm{Dw}\,k^2\}\}$

```
In[2]:= (* jacobian matrix *)
J = {{0, 0, 0}, {fu, fv, fw}, {gu, gv, gw}}
```

Out[2]= {{0, 0, 0}, {fu, fv, fw}, {gu, gv, gw}}

```
In[3]:= (* characteristic polynomial *)
P[x_] = -Collect[Det[(M + gamma * J) - x * IdentityMatrix[3]], x]
```

Out[3]= $-\mathrm{chi\ fw\ gamma}^2\,\mathrm{gu}\,k^2 - \mathrm{Du\ fw\ gamma}^2\,\mathrm{gv}\,k^2 + \mathrm{chi\ fu\ gamma}^2\,\mathrm{gw}\,k^2 + \mathrm{Du\ fv\ gamma}^2\,\mathrm{gw}\,k^2 - \mathrm{chi\ Dw\ fu\ gamma}\,k^4 - \mathrm{Du\ Dw\ fv\ gamma}\,k^4 - \mathrm{Du\ gamma\ gw}\,k^4 + \mathrm{Du\ Dw}\,k^6 - (\mathrm{fw\ gamma}^2\,\mathrm{gv} - \mathrm{fv\ gamma}^2\,\mathrm{gw} + \mathrm{chi\ fu\ gamma}\,k^2 + \mathrm{Du\ fv\ gamma}\,k^2 + \mathrm{Dw\ fv\ gamma}\,k^2 + \mathrm{gamma\ gw}\,k^2 + \mathrm{Du\ gamma\ gw}\,k^2 - \mathrm{Du}\,k^4 - \mathrm{Dw}\,k^4 - \mathrm{Du\ Dw}\,k^4)\,x - (\mathrm{fv\ gamma} + \mathrm{gamma\ gw} - k^2 - \mathrm{Du}\,k^2 - \mathrm{Dw}\,k^2)\,x^2 + x^3$

```
In[4]:= (* parameters *)

In[5]:= (* Schnackenberg *)


v0 = a + c + a * e1 + c * e2
w0 = c * (1 + e2) / (v0^2)
```

Out[5]= $a + c + a\,e1 + c\,e2$

Out[6]= $\dfrac{c\,(1 + e2)}{(a + c + a\,e1 + c\,e2)^2}$

```
In[7]:= fv = -1 + 2 v0 w0
```

Out[7]= $-1 + \dfrac{2\,c\,(1 + e2)}{a + c + a\,e1 + c\,e2}$

```
In[8]:= fw = v0^2
```

Out[8]= $(a + c + a\,e1 + c\,e2)^2$

```
In[9]:= fu = a * e1
```

Out[9]= a e1

```
In[10]:= gu = c * e2
```

Out[10]= c e2

```
In[11]:= gv = -2 * v0 * w0
```

Out[11]= $-\dfrac{2\,c\,(1 + e2)}{a + c + a\,e1 + c\,e2}$

```
In[12]:= gw = -v0^2
```

Out[12]= $-(a + c + a\,e1 + c\,e2)^2$

In[13]:=

In[14]:=
```
(* the polynomial coefficients *)
Ak = Collect[-(fv gamma + gamma gw - k^2 - Du k^2 - Dw k^2), {k^2, gamma, chi, Du}]
```

Out[14]=
$$\left(1-\frac{2\,c\,(1+e2)}{a+c+a\,e1+c\,e2}+(a+c+a\,e1+c\,e2)^2\right)\text{gamma}+(1+\text{Du}+\text{Dw})\,k^2$$

In[15]:=
```
Bk =
 Collect[-(fw gamma^2 gv - fv gamma^2 gw + chi fu gamma k^2 + Du fv gamma k^2 + Dw fv gamma k^2 +
     gamma gw k^2 + Du gamma gw k^2 - Du k^4 - Dw k^4 - Du Dw k^4), {k^2, gamma, chi, Du}]
```

Out[15]=
$$\left(2\,c\,(1+e2)\,(a+c+a\,e1+c\,e2)-(a+c+a\,e1+c\,e2)^2\left(-1+\frac{2\,c\,(1+e2)}{a+c+a\,e1+c\,e2}\right)\right)\text{gamma}^2+\left(-a\,\text{chi}\,e1+(a+c+a\,e1+c\,e2)^2-\text{Dw}\left(-1+\frac{2\,c\,(1+e2)}{a+c+a\,e1+c\,e2}\right)+\text{Du}\left(1-\frac{2\,c\,(1+e2)}{a+c+a\,e1+c\,e2}+(a+c+a\,e1+c\,e2)^2\right)\right)\text{gamma}\,k^2+(\text{Dw}+\text{Du}\,(1+\text{Dw}))\,k^4$$

In[16]:=
```
Ck = Collect[
  -chi fw gamma^2 gu k^2 - Du fw gamma^2 gv k^2 + chi fu gamma^2 gw k^2 + Du fv gamma^2 gw k^2 -
   chi Dw fu gamma k^4 - Du Dw fv gamma k^4 - Du gamma gw k^4 + Du Dw k^6, {k^2}]
```

Out[16]=
$$\left(2\,c\,\text{Du}\,(1+e2)\,(a+c+a\,e1+c\,e2)\,\text{gamma}^2-a\,\text{chi}\,e1\,(a+c+a\,e1+c\,e2)^2\,\text{gamma}^2-c\,\text{chi}\,e2\,(a+c+a\,e1+c\,e2)^2\,\text{gamma}^2-\text{Du}\,(a+c+a\,e1+c\,e2)^2\left(-1+\frac{2\,c\,(1+e2)}{a+c+a\,e1+c\,e2}\right)\text{gamma}^2\right)k^2+\left(-a\,\text{chi}\,\text{Dw}\,e1\,\text{gamma}+\text{Du}\,(a+c+a\,e1+c\,e2)^2\,\text{gamma}-\text{Du}\,\text{Dw}\left(-1+\frac{2\,c\,(1+e2)}{a+c+a\,e1+c\,e2}\right)\text{gamma}\right)k^4+\text{Du}\,\text{Dw}\,k^6$$

In[17]:=
```
(* conditions for having PATTERNS *)
(* Ck > 0*)
```

In[18]:=
```
b1 = gamma (-chi Dw fu + Du (-Dw fv - gw))
```

Out[18]=
$$\left(-a\,\text{chi}\,\text{Dw}\,e1+\text{Du}\left((a+c+a\,e1+c\,e2)^2-\text{Dw}\left(-1+\frac{2\,c\,(1+e2)}{a+c+a\,e1+c\,e2}\right)\right)\right)\text{gamma}$$

In[19]:=
```
c1 = gamma^2 (chi (-fw gu + fu gw) + Du (-fw gv + fv gw))
```

Out[19]=
$$\left(\text{chi}\left(-a\,e1\,(a+c+a\,e1+c\,e2)^2-c\,e2\,(a+c+a\,e1+c\,e2)^2\right)+\text{Du}\left(2\,c\,(1+e2)\,(a+c+a\,e1+c\,e2)-(a+c+a\,e1+c\,e2)^2\left(-1+\frac{2\,c\,(1+e2)}{a+c+a\,e1+c\,e2}\right)\right)\right)\text{gamma}^2$$

In[20]:=
```
a1 = Dw
```

Out[20]= Dw

In[21]:=
```
Ckmin = -(b1^2 - 4 * a1 * c1) / 4 * a1
```

Out[21]=

$$\frac{1}{4}\,\text{Dw}\left(-\left(-\text{a chi Dw e1}+\text{Du}\left(\left(\text{a}+\text{c}+\text{a e1}+\text{c e2}\right)^2-\text{Dw}\left(-1+\frac{2\,\text{c}\,\left(1+\text{e2}\right)}{\text{a}+\text{c}+\text{a e1}+\text{c e2}}\right)\right)\right)^2\text{gamma}^2+4\,\text{Dw}\left(\text{chi}\left(-\text{a e1}\left(\text{a}+\text{c}+\text{a e1}+\text{c e2}\right)^2-\text{c e2}\left(\text{a}+\text{c}+\text{a e1}+\text{c e2}\right)^2\right)+\text{Du}\left(2\,\text{c}\,\left(1+\text{e2}\right)\left(\text{a}+\text{c}+\text{a e1}+\text{c e2}\right)-\left(\text{a}+\text{c}+\text{a e1}+\text{c e2}\right)^2\left(-1+\frac{2\,\text{c}\,\left(1+\text{e2}\right)}{\text{a}+\text{c}+\text{a e1}+\text{c e2}}\right)\right)\right)\text{gamma}^2\right)$$

In[22]:=

In[23]:=

In[24]:=

In[25]:=

In[26]:=

In[27]:=

In[28]:=

In[29]:=

In[30]:=

In[31]:=

In[32]:=

In[33]:=

In[34]:=

In[35]:=

In[36]:=
```
fs = 28
(* parameters *)
(* OUT
  a=1
c=0.5*)
(* IN *)
a = 0.2
c = 1.3
Du = 1
gamma = 2200
u0 = 1

f1[e1_, e2_] = c1;
f2[e1_, e2_] = b1;
f3[e1_, e2_] = Ckmin;
nn = 15;
```

Out[36]= 28

Out[37]= 0.2

Out[38]= 1.3

Out[39]= 1

Out[40]= 2200

Out[41]= 1

In[46]:=

In[47]:=
```
(* OUT tab = Table[(Evaluate[f1[nn-e1,e2]]≤0||Evaluate[f2[nn-e1,e2]]≤0)&&
    Evaluate[f3[nn-e1,e2]]≤0,{chi,{0,0.1,0.15,0.5,10}}];*)
(*IN *)
tab = Table[(Evaluate[f1[e1, e2]] ≤ 0 || Evaluate[f2[e1, e2]] ≤ 0) &&
    Evaluate[f3[e1, e2]] ≤ 0, {chi, {0, 0.05, 0.1, 0.4, 10}}];
(* 0,
0.05,
0.1,
0.4,
10*)
```

In[48]:=
```
plotss1 = Table[RegionPlot[Evaluate@tab, {e1, 0, nn}, {e2, 0, nn},
    PlotStyle →
     Directive[RGBColor[0.47000000000000003, 0.44, 0.71], Opacity[0.32]],
    BoundaryStyle → Directive[RGBColor[0.47000000000000003, 0.44, 0.71],
      Thickness[0.006]], FrameStyle → Directive[GrayLevel[0],
      fs, FontFamily → "Helvetica", AbsoluteThickness[0.8]],
    FrameTicks → {{{#, ToString[#]} & /@ Range[0, nn, nn / 5], None},
      {{#, ToString[#]} & /@ Range[0, nn, nn / 5], None}},
    FrameLabel → {{ToExpression["\epsilon_{2}", TeXForm, HoldForm], None},
      {ToExpression["\epsilon_{1}", TeXForm, HoldForm], None}}], {Dw, {1}}];
```

In[49]:=
```
comb1 = Show[plotss1];
```

In[50]:=

In[51]:=

In[52]:=

In[53]:=
```
plotss2 = Table[RegionPlot[Evaluate@tab, {e1, 0, nn}, {e2, 0, nn}, PlotStyle →
     Directive[RGBColor[0.47000000000000003, 0.44, 0.71], Opacity[0.32]],
    BoundaryStyle → Directive[RGBColor[0.47000000000000003, 0.44, 0.71],
      Thickness[0.006]], FrameStyle → Directive[GrayLevel[0],
      fs, FontFamily → "Helvetica", AbsoluteThickness[0.8]],
    FrameTicks → {{{#, ToString[#]} & /@ Range[0, nn, nn / 5], None},
      {{#, ToString[#]} & /@ Range[0, nn, nn / 5], None}},
    FrameLabel → {{ToExpression["\epsilon_{2}", TeXForm, HoldForm], None},
      {ToExpression["\epsilon_{1}", TeXForm, HoldForm], None}}], {Dw, {40}}];


comb2 = Show[plotss2];
```

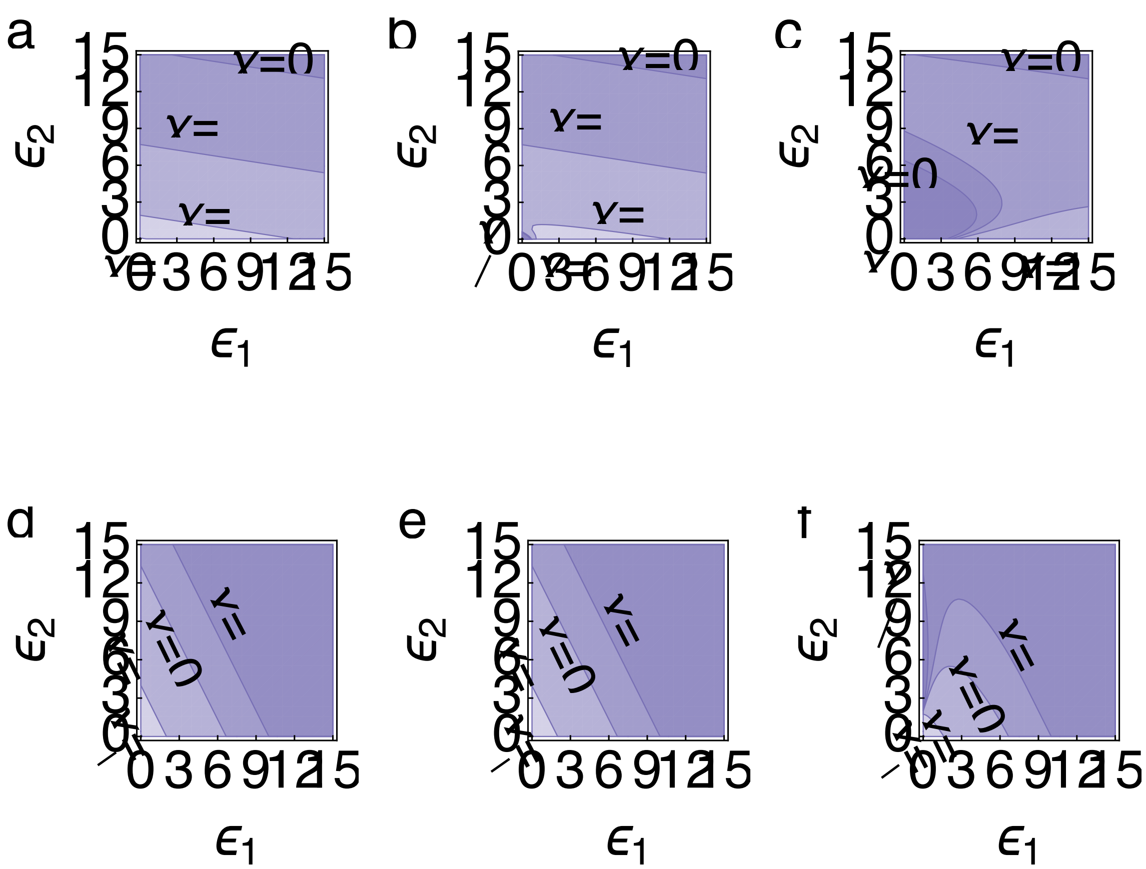
In[55]:=
```
plotss3 = Table[RegionPlot[Evaluate@tab, {e1, nn, 0}, {e2, nn, 0}, PlotStyle →
     Directive[RGBColor[0.47000000000000003, 0.44, 0.71], Opacity[0.32]],
    BoundaryStyle → Directive[RGBColor[0.47000000000000003, 0.44, 0.71],
      Thickness[0.006]], FrameStyle → Directive[GrayLevel[0],
      fs, FontFamily → "Helvetica", AbsoluteThickness[0.8]],
    FrameTicks → {{{#, ToString[#]} & /@ Range[0, nn, nn / 5], None},
      {{#, ToString[#]} & /@ Range[0, nn, nn / 5], None}},
    FrameLabel → {{ToExpression["\epsilon_{2}", TeXForm, HoldForm], None},
      {ToExpression["\epsilon_{1}", TeXForm, HoldForm], None}}], {Dw, {600}}];
```

In[56]:=
```
comb3 = Show[plotss3];

new = {Show[comb1], Show[comb2], Show[comb3]};
Show[GraphicsRow[new, Spacings → Scaled[0.15], ImageSize → 1000]]
```

In[59]:=

Out[60]=



In[61]:=

In[62]:=

In[63]:=

In[64]:=

In[65]:=

In[66]:=

In[67]:=

In[68]:=

In[69]:=

In[70]:=

In[71]:=

In[72]:=

In[73]:=

In[74]:=

In[75]:=

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In[98]:=

In[99]:=

In[100]:=

In[101]:=

In[102]:=

In[103]:=

In[104]:=

In[105]:=

In[106]:=

In[107]:=

In[108]:=