

# A SIMPLE BIOMECHANICAL ANALYSIS AND ROTARY MOTOR DESIGN OF A LOWER-LIMB EXOSKELETON FOR SIT-TO-STAND MOVEMENT

Leonardo Paul Milián Ccopa

Grupo de Biomecatrónica  
Universidad Nacional de Ingeniería -  
Lima – Perú  
[milepcco@gmail.com](mailto:milepcco@gmail.com)

Diego Alexis Vásquez Maldonado

Grupo de Biomecatrónica  
Universidad Nacional de Ingeniería -  
Lima – Perú  
[dial1992@gmail.com](mailto:dial1992@gmail.com)

Ebert Choquehuanca Perca

Grupo de Biomecatrónica  
Universidad Nacional de Ingeniería -  
Lima – Perú  
[e05.sek@gmail.com](mailto:e05.sek@gmail.com)

Alex Anchivilca Baltazar

Grupo de Biomecatrónica  
Universidad Nacional de Ingeniería -  
Lima – Perú  
[aanchivilcab@gmail.com](mailto:aanchivilcab@gmail.com)

Reynaldo Jesús Diburga Villanueva

Grupo de Biomecatrónica  
Universidad Nacional de Ingeniería -  
Lima – Perú  
[reynaldo.diburga.2021@gmail.com](mailto:reynaldo.diburga.2021@gmail.com)

Mirna Dolores Ampuero Caro

Grupo de Biomecatrónica  
Universidad Nacional de Ingeniería -  
Lima – Perú  
[Mirna.uni.142@gmail.com](mailto:Mirna.uni.142@gmail.com)

## ABSTRACT

This study presents a kinematic and dynamic analysis to calculate the rotary actuator for developing a lower limb exoskeleton for sit-to-stand movement, which is the first challenge of any patient who is in a lower limb rehabilitation process. First of all, it was necessary to track each articulation position by recording a user's video. With registered data, it was calculated linear and angular velocities and accelerations by applying the Denavit-Hartenberg method. Afterwards, based on before studies about the human body mass proportions' and inertial parameters, the Dynamics of this exoskeleton system was calculated by using the Lagrange – Euler equations method and verified by applying and comparing with the Newton – Euler formulation.

## General Terms

Algorithms, Measurement, Documentation, Design, Human Factors, Standardization and verification.

## Keywords

Exoskeleton, Image processing, Denavit-Hartenberg, Lagrange-Euler, Newton-Euler, Sit-to-stand, S2S.

## 1. INTRODUCTION

The aim of this project is providing a simple method to calculate the rotary-motor actuator for a lower limb exoskeleton to assist users with neurological origin problems, or in rehabilitation after illness, trauma or post-operative. The kinematic and dynamic analysis proposed are not only the simplest, but it is also an excellent point to start and verify related studies.

## 2. PROCEDURE

This project is organized into the following three chapters; biomechanical analysis, kinematic analysis and dynamic analysis.

### 2.1 Biomechanical Analysis

#### 2.1.1 Sit-To-Stand Movement

We can divide the sit-to-stand movement in three phases.

2.1.1.1 *Phase I:* This phase comprises since maximum velocity of CG until its maximum vertical speed.

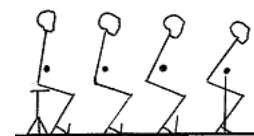


Fig. 1. Initial phase

2.1.1.2 *Phase II:* At this stage the Center of Gravity (CG) accelerates horizontally to acquire the maximum speed in this direction, so some authors as Roebroeckm also call it the acceleration phase



Fig. 2. Take-off phase

2.1.1.3 *Phase III:* This phase comprises since maximum vertical velocity of movement, producing an elevation of the whole body that moves vertically to stabilize the CG within the new base of support. Due the vertical velocity is negative, it's also called deceleration phase.

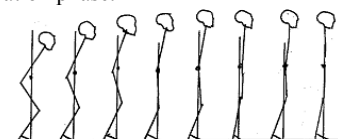


Fig. 3. Stabilization phase

#### 2.1.2 Sit-to-stand muscles' actions

The muscles involved in maintaining the upright attitude are called extensors or antigravity postural muscles, which have red fibbers with high resistant to fatigue, its energy consumption is weak. They have a type of contraction called tonic contraction.

### 2.1.3 Antigravity muscles of the erect posture

The muscles, which are involved into the sit-to-stand movement, are:

- Extensor muscles of the neck.
- Spinal muscles or erector spinal muscles.
- Gluteus maximums muscle.
- Quadriceps muscles.
- Triceps surae muscle (gastrocnemius and soleus).

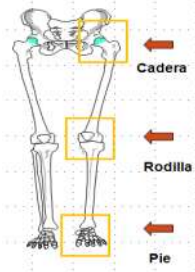


Fig. 4. Implicated Joints

### 2.1.4 Muscular work:

Hip extension and knee movements use the monoarticular-extensor-muscles' concentric work of these ones. (internal and external vast knee and hip gluteus maximums one) During the second phase, the work of the joint muscles, rectus quadriceps and hamstrings starts with a concentric work after taking off.



Fig. 5.

Hamstring's concentric work produces hip extension and helps knee extension (in eccentric) due foot are ground fixed (closed kinetic chain), while the rectus femoris of the quadriceps works almost isometric, moving the moment of inertia from the hip to the knee. At the time of taking off, the activity of both extensor muscles of the knee, monoarticular and two-joint, are very high; it corresponds to the time when the knee has to reach the maximum strength to lift the body weight against the force of the gravity. The force used in this time is equivalent to seven times the body weight.



Fig. 6.

In the ankle, monoarticular muscles and the tibialis anterior muscle are in a isometric contraction throughout the process, presenting activity even before of the takeoff, while the triceps surae, being two-joint, works eccentrically and showing a discrete activity at the start of the manoeuvre, but increasing significantly during takeoff, keeping itself active until the end of the movement



Fig. 7.

## 2.2 Kinematic Analysis:

One multibody system with three degrees of freedom, all rotation and placed on the ankle, knee and hip was considered.

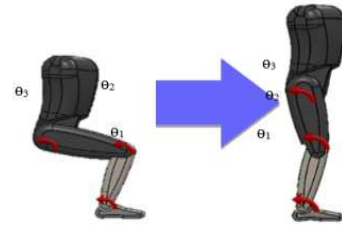


Fig. 8. Rotation axis of the multibody system

Denavit-Hartenberg parameters (D-H) are used for calculation of the transformation matrix

I	$\alpha$	a	d
1	0	L1	0
2	0	L2	0
3	0	L3	0

TABLE 01: Denavit-Hartenberg parameters

$${}^0T_3 = {}^0T_1 {}^1T_2 {}^2T_3 = \begin{bmatrix} \cos(\theta_1 + \theta_2 + \theta_3) & -\sin(\theta_1 + \theta_2 + \theta_3) & 0 & l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ \sin(\theta_1 + \theta_2 + \theta_3) & \cos(\theta_1 + \theta_2 + \theta_3) & 0 & l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In order to get positions, speeds and acceleration peaks patterns, it was made a data recording by motion capture of a healthy person.

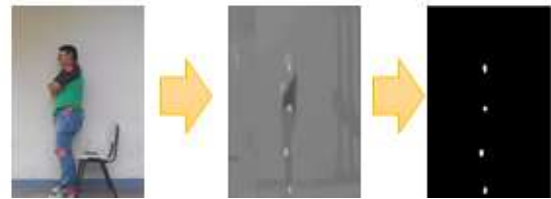


Fig. 9. Motion capture

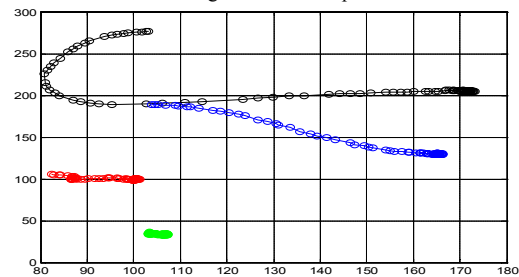


Fig. 10. Representation of registered data

### 2.2.1 Velocities and Acceleration Calculus

Once obtained the register of each joint positions, it is possible to calculate linear and angular velocities and accelerations

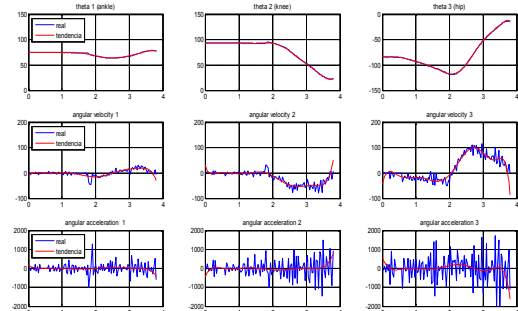


Fig. 11. Representation of theoretical velocities and accelerations

## 2.3 Dynamic analysis:

In order to simplify the analysis, there are some considerations that should be taken before proceeding with the calculations,

### 2.3.1 Initial considerations:

- The human body has been divided in four groups:
  - The first group is formed by the feet, which are considered immobile and serve inertial reference system for the entire Sit-to-stand movement.
  - The second group corresponds to the legs, which rotate on reference of the feet with ankle's axis.
  - The third group is made up of the thighs, which rotate on reference of the legs with knee's axis and finally.
  - The fourth group, which, for reasons of simplifying calculations, consider together the trunk, head and upper limbs. This rotates on reference of the third element (thighs) with knee's axis with hip's axis.
- The relative parameters: weight, center of gravity, location and radius of gyration, which have been used for this work, are based on anthropomorphic models, obtained from samples of body parts, made by Dempster-Winter (1955, amended in 2009) and Zatsiorsky-Seluyanov, (1996, amended in 2002). See Table 02.

Segment	Center of gravity (%)	Relative Weight (%)	Radius of gyration Kxx	Radius of gyration Kyy	Radius of gyration Kzz
Head and neck	60.4	0.0694	0.362	0.312	0.376
Trunk	49.5	0.4346	0.372	0.191	0.347
Arm	43.6	0.0271	0.285	0.158	0.269
Forearm	43	0.0162	0.276	0.121	0.265
Hand	50.6	0.0061	0.628	0.401	0.513
Thigh	43.3	0.1416	0.329	0.149	0.329
Leg	43.3	0.0433	0.251	0.102	0.246
Foot	42.9	0.0137	0.257	0.124	0.245

TABLE 02: Human body parameters

### 2.3.2 Method of Lagrange – EULER formulation

#### 2.3.2.1 Lagrange equation

Let the Lagrange equation:

$$L = \sum E_{c_i} - \sum E_{p_i}$$

Then:

$$\sum E_{c_i} = \frac{1}{2} \sum_i^n [\bar{v}_k^T m_k \bar{v}_k + \bar{w}_k^T D_k \bar{w}_k]$$

$$\sum E_{p_i} = - \sum_i^n [m_k g^T \bar{C}_k]$$

Where:

- $\bar{v}_k$ : Translational speed of the k-th element
- $\bar{w}_k$ : Rotational speed of the k-th element
- $m_k$ : Mass of the k-th element
- $D_k$ : Inertia tensor of the k-th element regarding  $X_0 Y_0 Z_0$  and moved to its center of mass..
- $\bar{C}_k$ : Center of mass of the k-th element
- $g$ : Gravity

#### 2.3.2.2 Kinetic Energy Computational calculation

$$E_c = \frac{1}{2} \dot{q}^T \left\{ \sum_i^n [J_v^{kT} m_k J_v^k + J_w^{kT} [{}^0 R^0]^T \bar{I}_k [{}^0 R^0] J_w^k] \right\} \dot{q}$$

$$\bar{v}_k = J_v^k(q) \dot{q} \quad \bar{w}_k = J_w^k(q) \dot{q}$$

$$E_c = \frac{1}{2} \sum_i^n [\bar{v}_k^T m_k \bar{v}_k + \bar{w}_k^T [{}^0 R^0]^T \bar{I}_k [{}^0 R^0] \bar{w}_k]$$

$$D_k = J_v^{kT} m_k J_v^k + J_w^{kT} [{}^0 R^0]^T \bar{I}_k [{}^0 R^0] J_w^k$$

$$D = \sum_{k=1}^n D_k$$

#### 2.3.2.3 Potential Energy Computational calculation

$$E_p(q) = - \sum_i^n m_k g^T \bar{C}_k(q)$$

$$\bar{C}(q) = \sum_i^n m_k \bar{C}_k(q)$$

$$E_p(q) = -g \bar{C}(q)$$

#### 2.3.2.4 Lagrange Computational calculation

$$E_c(q) = \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

$$E_p(q) = -g \bar{C}(q)$$

The Lagrange equation is:

$$L(q, \dot{q}) = \frac{1}{2} \dot{q}^T D(q) \dot{q} + g^T \bar{C}(q)$$

#### 2.3.2.5 Computational calculation of Torques using Lagrange equations.

$$\frac{d}{dt} \frac{\partial L(q, \dot{q})}{\partial \dot{q}_i} - \frac{\partial L(q, \dot{q})}{\partial q_i} = F_i; \quad 1 \leq i \leq n$$

Where:

$F_i$ : Torque acting on the joint.

$q_i$ : Generalized Coordinate

$\dot{q}_i$ : Derived from the generalized coordinate

$n$ : Degrees of freedom of the robot

#### a) GRAPHICS AND RESULTS

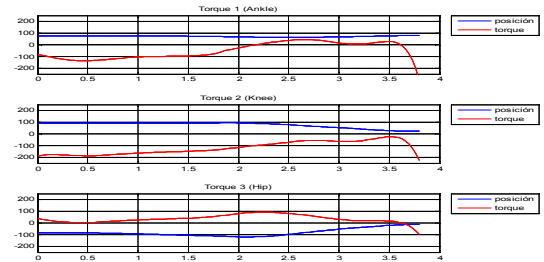


Fig. 12. Torque and position.

### 2.3.3 METHOD OF NEWTON-EULER FORMULATION

1. The  $\theta_1, \theta_2, \theta_3$ , angles were taken on reference of the horizontal axis.
2. Each element is taken as a rigid body.
3. Motion of various part of the body occur in a vector plane so that rotations of the body may be disregarded.
4. Various joint of the body may be expressed as a series of links

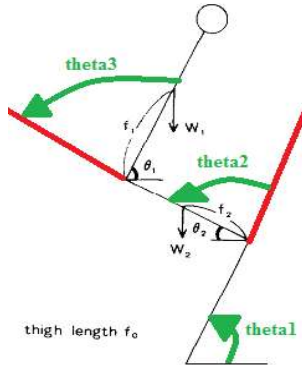


Fig. 13. Newton-Euler dynamic model

5. Each joint has a single axis
6. The center of gravity for each body segment is located along the line extending from one joint to the other.
7. The upper body, including the arms, may be expressed as a single, uniform volume.
8.  $W_1, f_1, W_2, f_2, f_0$  are defined as follows, according to the report by Matsui:  $W_1$ , 56% of body weight;  $f_1$ , 45% of sitting height;  $W_2$ , 10% of body weight;  $f_2$ , 58% of femur length;  $f_0$ , actual measured distance from the outer knee joint to the greater trochanter of the femur.

#### a) CALCULATION OF TORQUES BY JOINT

$$T_{HIP} = w_1 f_1 \cos \theta_1 / 2$$

$$T_{KNEE} = \frac{w_1 [f_0 \cos \theta_2 - f_1 \cos \theta_1]}{2} + w_2 f_2 \cos \theta_2$$

#### b) GRAPHICS AND RESULTS

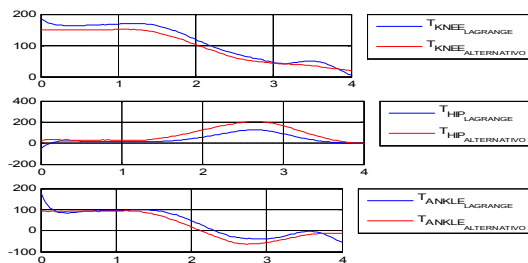


Fig. 14. Comparison between the results of calculation of torque by the Lagrange-Euler equation and the Newton-Euler formulation.

### 3. CONCLUSIONS

Motion capture is an excellent tool for estimating the direct and inverse kinematics of our system; however, this procedure should be standardized, by parameterizing measures and environmental conditions where it is recorded. Once the recording data is made, it should be considered the ankle as a fixed point throughout the sit-to-stand process, allowing this, a better data record.

The Lagrange equation and Denavit-Hartenberg representation let us parameterize the kinematic analysis (position, velocity and acceleration) and dynamic (Forces and Toques) versus time, achieving these, the calculus of maximum torques and forces on each element of our model, by noticing that we can consider that we need 1N-m for each kilogram of user's mass to get manage the sit-to-stand movement. However, it is necessary to apply an additional safety factor when selecting the actuator motor to be used in our exoskeleton.

Newton-Euler formulation and Lagrange-Euler show similar results and graphics, corroborating thus the dynamic and kinematic analysis are correct.

### 4. References

- [1] Rodrigo C. Mirales Marrero, Biomecánica clínica del aparato locomotor.
- [2] Nordin-Frankel, Biomecánica Básica del Sistema Músculo esquelético.
- [3] Roberts – McCollum, Dynamics of the sit-to-stand movement
- [4] Lower Extremity Exoskeletons and Active Orthoses: Challenges and State-of-the-Art Aaron M. Dollar, Member, IEEE, and Hugh Herr, Member, IEEE
- [5] Ahmed. Shabana, Dynamics of Multibody Systems.
- [6] Atlas de anatomia humana - 5ta edicion - Netter.
- [7] Biomechanics and motor control of Human Movement – Winter
- [8] CYBERLEGS: un nuovo sistema robotico per gli amputati di arto inferiore-Nicola Vitiello, Raffaele Molino-Lova
- [9] [www.rmib.somib.org.mx/Htmls/Vol35No2/4/4.html](http://www.rmib.somib.org.mx/Htmls/Vol35No2/4/4.html)
- [10] Dempster's Body Segment Parameter Data for 2-D Studies - *Am J Phys Anthropol* 1959 Vol 17 (289-317)
- [11] <http://www.health.uottawa.ca/biomech/csb/Archives/dempster.pdf>
- [12] Adjustments to Zatsiorsky-Seluyanov's segment inertia parameters - *J Biomech.* 1996
- [13] An Analysis of Sit-to-Stand Movements - Tomomitsu Kotake - 1993”.
- [14] Determination of center of gravity of the human body in various postures. *Taiikugaku tienkyu* (Japan Journal of Physical Education) - Mitsuui H.

### 5. ACKNOWLEDGEMENTS

To our advisors and friends. MSc. David R. Achanccaray Diaz, PhD. Raúl Landeo and Lic. Laura Andrea M. Hidalgo

Who constantly support us with the review and improvement of our Projects.